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Possible Worlds: A Neo-Fregean Alternative

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Abstract I outline a neo-Fregean strategy in the debate on the existence of possible worlds. The criterion of identity and the criterion of application are formulated. Special attention is paid to the fact that speakers do not possess proper names for worlds. A broadly Quinean solution is proposed in response to this difficulty.

Keywords Possible worlds · Realism · Context principle · Possibility semantics · Wright · Dummett · Stalnaker

I

1. The debate over the nature of possible worlds has been driven largely by David Lewis' 'modal realism'. On Lewis' view, possible worlds are spatiotemporally and causally isolated universes. The majority agreed that Lewis' view was untenable. Some good arguments have been produced against it. And because modal realism was perceived as a leading version of any realism about possible worlds, some theorists were led to declare that possible worlds 'don't really exist'.

In general, any viable account of possible worlds must satisfy two demands. One is the clarification of the ontological status of these entities. The other lies in explaining the notion of truth in a world. These demands are related. It is commonly thought that commitment to possible worlds—or at least a *prima facie* commitment—springs from the endorsement of the following principles governing modal locutions:

Leibniz' Principles. $\ulcorner$Necessarily, $S\urcorner$ is true iff for every possible circumstance w, S is true in w. $\ulcorner$Possibly, $S\urcorner$ is true iff there is a possible circumstance w such that S

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is true in w. $\ulcorner$Contingently, $S\urcorner$ is true iff $\ulcorner$Possibly, $S\urcorner$ is true and $\ulcorner$Possibly, $\neg S\urcorner$ is true.

The principles mention novel entities—possible worlds—and a novel notion of truth in a world. The latter difficult issue I shall here put aside and focus exclusively on ontology.

Before enquiring about the nature of possible worlds, it is proper to ask: what is the reason for believing in their existence in the first place? A straightforward answer comes in the form of the *Master Argument* which relies on Leibniz' Principles:

1. There are contingent truths. (Assumption)
2. If there is a contingent truth, then there is at least one true statement quantifying over more than one possible world. (From Leibniz' Principles)
3. Therefore, some statements quantifying over more than one possible world are true. (From #1 and #2)
4. Therefore, there are merely possible worlds. (From #3)

Anti-realists reject the Master Argument, but for different reasons. Yablo's figuralist and Forbes' modalist reject the very idea that statements quantifying over possible worlds are truth-apt, let alone true. Rosen's fictionalist believes they are truth-apt, but uniformly false. The bi-conditionals of Leibniz' Principles must, therefore, be reinterpreted in some suitable way if we are to assign them usual classical truth-conditions. Since it is not guaranteed that any such reinterpretation is available, the Principles will be regarded by them as fundamentally misleading (such, e.g., is the view of a figuralist).

Allied to the anti-realists is someone like a Carnapian ontologist. According to him, the argument, if literally taken, is valid. But its conclusion is still misleading. The notion of truth employed in the premisses is 'internal'. 'Externally', it derives from pragmatic notions of usefulness and simplicity. As such, it is insufficient for establishing any philosophically significant conclusion about the existence of possible worlds.[1] Finally, a Spinozist can reject premiss #1, but, as far as I know, this option found little sympathy in the contemporary debate.

These views I want to put aside at the outset.[2] Much interesting discussion can be produced by arguing with the anti-realist, but my goal here is different. It lies primarily in articulating a particular form of realism and in stressing its advantages. I want to suggest that a sensible answer to our ontological question can be extracted from Frege's context principle. I suggest that we can imitate the success of the neo-logicist programme in arithmetic and formulate an analogous proposal for the metaphysics of possible worlds. The ontological view that will emerge will be non-reductionist and rather orthodox. Its guiding idea is that the theoretical role of possible worlds must dictate our conception of them. Possible worlds must be precisely those entities suitable for providing truth conditions for modal discourse. The most we can say about the nature of a possible world is that it is an entity which could have been the actual world. The eventual account would, first, allow us the

[1] This may be Stalnaker's ultimate view. See Stalnaker (1996).

[2] For some doubts about the transition from #3 to #4 of this argument see Girle (2003: 160–163).

use of possible worlds and, on the other hand, would avoid the metaphysical controversy generated by the reduction of the notion of a possible world to other notions. A note of caution: our account, though it is inspired by Frege's work in the *Grundlagen*, might not be acceptable to Frege himself, so far as Frege would deny the existence of intensional objects. However, Frege's criterion of ontological commitment, as developed by modern neo-Fregeans, is logically independent of his views on intensional semantics.

2. Our story begins with Frege's context principle. There are two ways to interpret it. On the surface it is formulated by Frege as a claim about meaning (sense). It states that parts of a sentence acquire their meaning through the meaning of the sentence as a whole.[3] The sense of an expression is identified with its contribution to the truth-conditions of any sentence where it occurs.

Here we are interested in the neo-Fregean interpretation which takes the context principle to be a claim about reference. As such, it amounts to the view that so far as the sense is established for every sentence in which a singular term occurs and some sentences of appropriate syntactic structure are true, nothing else is required for assigning the reference to that term.[4] Therefore, if one regards *X* as an object, one has to show that we refer to *X* with a singular term, and that the name for *X* appears in true statements. The test of objecthood based on syntactic considerations was labelled by Crispin Wright the 'syntactic priority thesis':

> [T]he category of objects is to be explained as comprising everything which might be referred to by a singular term, where it is understood that possession of reference is imposed on a singular term by its occurrence in true statements of an appropriate type. (Wright 1983: 53)

The power of this approach is clear at once: we gain a uniform criterion for identifying the class of objects and map our discussion into the domain of the philosophy of language. This is an elegant twist in the nominalist-platonist controversy, especially in the light of torturous attempts to define the issue of this controversy. But is there any substantive argument in its favour? Wright admits that there is not. There is, none the less, a sort of a 'negative' argument. If we endorse an opposite view, we seem to believe in expressions functioning as singular terms in a wide range of statements, but having no reference.[5] Adopting the neo-logicist view in a broad sense, then, as a programme, means adopting the context principle as a thesis about reference. The criterion of objecthood is interpreted syntactically. In so far as an entity *X* is represented in a language by a singular term, *X* should qualify as an object.

With the syntactic priority thesis in place, the next step should consist in a procedure for introducing novel concepts into the language. The contextual definition embodied in Hume's Principle achieves that for numbers. But for every concept whose introduction into language is done by a contextual definition we will need a separate bi-conditional equivalence parallel to Hume's Principle. When we

[3] See Frege (1980: §60) and Dummett (1981: 369).

[4] See Dummett (1981: 380).

[5] See Wright (1983: 52).

turn to possible worlds, Leibniz' Principles appear to be an obvious candidate. The concept POSSIBLE WORLD is seen as being governed by the bi-conditional equivalences. And no one seems to dispute their truth, at least given that modal statements are to be assigned truth values at all. The dividing issue is the cognitive status of these equivalences, namely, which part is supposed to explain which part, and which part is epistemically prior. On the well-known modalist account advocated by Graeme Forbes, the explanatory role is performed by modal statements. The meaning of the statements apparently quantifying over possible worlds is derived from the meaning of corresponding modal statements.[6] On the Lewisean account, the explanatory route is reversed: the meaning of the statements apparently quantifying over possible worlds provides the meaning of corresponding modal statements. Both accounts are reductionist. On the neo-logicist view, the meanings of both parts of bi-conditionals are autonomous. Neither is to be reduced to the other.

The purpose of using Leibniz' Principles is in imitating the success of the neo-logicist approach in arithmetic. According to Hume's Principle, the number of Fs is equal to the number of Gs if and only if there are just as many Fs as there are Gs. In its standard symbolic version it becomes:

$$Nx\colon Fx = Nx\colon Gx \leftrightarrow Fx \approx Gx. \tag{N=}$$

One immediately notes a dissimilarity between Hume's Principle and Leibniz' Principles. The former sets up a criterion of identity for numbers. The way to establish whether the number of Fs equals the number of Gs is to look for a bijection between Fs and Gs. But Leibniz' Principles, as they stand, do not give us a recipe for verifying whether $w = u$. They do not even contain the equality sign in its formulation. A revision is required.

3. Leibniz' Principles tell us two things about possible worlds. First, they are quantified over. This information offers us preliminary evidence for singular representation of possible worlds in language. Secondly, statements are true or false on them. This bit suggests that worlds are precisely those entities in which statements are true or false. We express this idea as follows. Let σ range over statements and **w** and **u** be proper names for worlds. Let the first-order predicate $G(\sigma, w)$ be the relativisation of the statement σ to the world w, so that, e.g., for $\sigma =$ 'Snow is white', we get: $G(\sigma, w) =$ 'Snow is white in w'). Then the desired condition would be:

$$\mathbf{w} = \mathbf{u} \leftrightarrow \forall\sigma(G(\sigma, \mathbf{w}) \leftrightarrow G(\sigma, \mathbf{u})). \tag{W=)}$$

But we still have not got a contextual definition which would be parallel to (N=). If we wanted to introduce the terms for worlds into a language, (W=) would clearly be inadequate, since these terms occur at the right-hand side of the bi-conditional. The difficulty is not surprising: the predicate '① true in a world ②' is suggested by model-theoretic treatment of modal semantics. There we begin with the set of worlds and by selecting members from that set lay down rules of interpretation for

[6] See Forbes (1985: 80ff). I believe that pursuing the modalist strategy further will result in a version of fictionalism. The details cannot be elaborated here.

atomic sentences and connectives. Once the concept of a world, or in any case of a set of worlds, is grasped, we can of course use (W=) to determine the identity between any two worlds.

A straightforward transcription of (N=) would not do, but the remedy may still be located within Leibniz' Principles. It does establish a link between world terms and modal notions. It tells that when, and only when a modal statement is true, we should quantify over something to which it is relativised. Why cannot we take the Principle itself as constitutive of the use of world terms? We are not yet granted the use of names for that 'something' ('*w*' is a variable there), and generally we should not be. For suppose we are given the following principle:

Tom is in pain iff there is x which caused Tom's pain.

The principle declares commitment to the causes of pain, but affirming that pain has causes is insufficient for being able to identify and name these causes. Nevertheless we can claim that we introduce quantification over pain causes into our discourse, even though there is as yet no means for distinguishing between different pain causes. And before we are able to achieve the latter, we have not mastered the concept of a pain cause in full.

Similarly with worlds. We can claim that Leibniz' Principle, on its own, introduces relativisation of statements to certain entities, thus inserting terms for these entities into our discourse, whilst (W=) furnishes us with the criterion of identity for these entities. The legality of such procedure will turn on the behaviour of first-order quantifiers. If we are permitted the use of existential instantiation, then, for example, the possibility clause will yield names for worlds. Once we have obtained names, we are within our rights to use (W=). According to this idea, (W=), in contrast to (N=), is a supplementary clause to Leibniz' Principles. That the latter should be assigned a central role in our account becomes evident from the difficulties discussed next.

4. Suppose we have established a two-stage procedure leading from Leibniz' Principles to (W=). We have introduced world terms (henceforth PW-terms) into our discourse. If we remain faithful to the neo-logicist proposal, we have to show that PW-terms are singular terms. PW-terms can be shown to satisfy Dummett's syntactic criteria for singular terms (see "Appendix A"). But on its own, this step only means that we have separated one syntactic category from all the rest. The neo-Fregean claim, to repeat, is that objects are denoted by singular terms, and only by singular terms. Then, if the account is not to be an arbitrary one, we must explain what exactly in the *semantic behaviour* of singular terms makes them so intimately related to particulars.

The response in general must be that the very use of proper names is tailored to regarding their semantic values as objectual. One unique feature is reflected in the use of ordinary proper names for middle-sized objects. The reason why the term t refers to a middle-sized X is that we explicitly introduce t into the language by pointing at X. This is a basic feature of linguistic practice. No doubt it has its parasites. There are cases of mythological or fictional objects represented by proper names; they are to be discarded as bogus names.

We seem to have singular terms for natural numbers. The unique identification of the natural number n is delivered via (N=) by the mere counting of Fs and Gs and the resulting bijective correspondence between their respective collections. What about worlds? The question is whether one can uniquely identify a world w in the plurality of worlds. It is best stated with a specific example. Suppose I say:

$$\text{Socrates could have been a pirate.} \tag{1}$$

I can be understood as saying that there is a possible world designated with a singular term '**w**', where Socrates is a pirate. One worry with (1) results from an apparent incompleteness of the list of statements by which the speaker identifies the world **w**. Saying that **w** is such that 'Socrates is a pirate' is true on it the speaker may be understood as tacitly presupposing the truth in **w** of the statements such as: 'Socrates is human', 'There are ships', 'There is water', 'There are people other than Socrates', and so forth. All those statements are semantically (and pragmatically) presupposed by the statement 'Socrates is a pirate'. Referring to the world **w** is secured by listing statements true in it. It will always be possible to refine the list further by producing a statement inferentially dependent upon the statements in the original list. No real trouble occurs in this instance. Even though worlds are identified by the statements true in them, the speaker is not obliged to produce a full list of those statements. He is only required to satisfy the discrimination constraint and to distinguish two worlds by means of such a list.

The trouble with (1) may lie elsewhere. There is a possible world $\mathbf{w}'$ where Socrates is a pirate and Plato is a philosopher, and there is a possible world $\mathbf{w}''$ where Socrates is a pirate and Plato is a businessman. Since no philosopher is a businessman (of course!), it follows from our criterion (W=) that $\mathbf{w}' \neq \mathbf{w}''$. The question is to which of these two the speaker refers in his assertion of (1). That is, should we say that $\mathbf{w} = \mathbf{w}'$ or that $\mathbf{w} = \mathbf{w}''$? In general:

The problem of indefinite assertion. Let the world $\mathbf{w}_1$ be identified by the set Γ_1 of statements true in it. Let $\Gamma_1 = \{A_1, A_2, \ldots, A_n\}$ and consider $\Gamma_2 = \{A_1, A_2, \ldots, A_n, B\}$ and $\Gamma_3 = \{A_1, A_2, \ldots, A_n, C\}$, such that:

1. Neither B, nor C are semantically presupposed by $A_1, A_2, \ldots, A_n$;
2. B and C are not simultaneously true.

Then there are two worlds w_2 and w_3 identified by Γ_2 and Γ_3, such that $w_2 \neq w_3$. Therefore, there is no meaningful answer as to whether $w_1 = w_2$ or $w_1 = w_3$.

Our problem here relates to the statements that are not semantically linked to any of the statements on the original list L. There will be no way of deciding which of the worlds—the one with the businessman Plato, or the one with the philosopher Plato—the singular term **w** denotes. The speaker's mastery of that term will then be impaired.

The situation we face here is different from the case of ordinary proper names. The competent use of the name 'Clint Eastwood' similarly does not require knowledge of every property of Clint Eastwood. It is demonstrative identification which provides a swift route for singling out the bearer in the multitude of objects. A competent user must either himself be able to single out Eastwood by ostension, or at least receive the name from the earlier users with that ability.[7] Unlike the users of ordinary proper names, one has no non-descriptive resources of referring to worlds. There is no reason to assume that, indeed, any speaker can competently use the names for possible worlds.

Plainly our problem is not the mystery of epistemic access to possible worlds. If it were such, a neo-Fregean could easily brush it aside: one major significance of the context principle is in dispensing with epistemic concerns. We are not asking *how* the speaker came into possession of PW-terms. Our worry is that he does not possess them in the first place. The real significance of the problem of indefinite assertion is, therefore, the direct threat of modal ignorance. The inability to identify **w** uniquely signals our ignorance of the truth-conditions of both sides of (W=). And perhaps not only that. Perhaps the speaker's failure in referring to **w** is a result of the assertoric failure on the left-hand side of Leibniz' Principles. There are many ways in which Socrates could be a pirate, but we fail to identify uniquely any one of them. That is, in asserting the statement 'Socrates could be a pirate' we fail simultaneously to take into account the truth value of the statements 'Plato is a businessman', 'Aristotle is a doctor', 'Cicero is an engineer', and so forth. All of these are inferentially linked to the truth of 'Socrates is a pirate'. The speaker who pretends to assert the statement 'Socrates could be a pirate' *alone* does not really assert anything. To be able to assert the statement, he must follow up and see how Socrates' piracy impinges on the mosaic of the whole world, and how a change in the mosaic of the whole world could lead to Socrates' piracy in the first place. That would constitute an impossible feat for human speakers.

5. I anticipate several reactions, of varying strength, to the problem of indefinite assertion just posed. One would be to offer an intransigent response and to insist on the absolute priority of syntactic criteria. So far as those are satisfied, it is legitimate to go along with (W=). The knowledge of truth-conditions of modal statements is secured by logical reflection. Neo-Fregeans may sometimes be interpreted as offering precisely such a response for (N=).[8] Applied in the modal case, the conservative logicist response says that our knowledge of the truth-conditions of the left-hand side of (W=) guarantees our knowledge of the right-hand side. And there is no mystery in knowing the truth-conditions of the left-hand side, since any such knowledge is analytic.[9] However, to give this response is in effect to maintain the explanatory priority of the modal discourse over the possible-worlds discourse. The role of the possible-worlds discourse will then be reduced to burdening us with additional ontic commitments without providing explanatory benefits.

Another reaction may draw its inspiration from the well-known remarks by Kripke.[10] The problem of indefinite assertion results from a misguided attempt, encapsulated in the condition (W=), to describe possible worlds qualitatively. We are not sure to which world we refer in (1) because of the uncertainty over which properties attribute to it. Instead we must simply *stipulate* that the world to which

[7] The full story is spelled out in the works of Kripke, Dummett, and Evans.

[8] See Wright (1983: 139).

[9] Compare e.g. Wright (1990: 154) and Hale and Wright (2001a: 12).

[10] See Kripke (1980: 19, 44).

we refer is the world where Socrates is a pirate. Clearly the response will have a point if we were to doubt whether Socrates is really a *pirate* in the world **w**, or whether it is really *Socrates* who is a pirate there. Yet our concern is not about epistemic warrant or transworld identity. Stipulations may work. But to distinguish a particular world in the pool of other worlds and to ensure our mastery of their names will again, at least on occasions, require infinitely many stipulations.

The third reaction would echo some other remarks of Kripke's.[11] The real culprit is the condition of maximality. We must settle for a notion of a 'possible circumstance', or 'possible situation'. Possible circumstances will still be identified by the list of statements true on them, but not every statement will be evaluated on them. Therefore, these incomplete entities will play the same theoretical role as possible worlds, but referring to them will not require knowing their properties in minute detail.

Here it is convenient to revert back to the very beginning of this paper and ask a simple question: why do Leibniz' Principles mention *worlds*, rather than circumstances? We should be given some minimal intuitive justification of the Principles.[12] One answer here is that the commitment to possible worlds is rooted in ordinary discourse. Let me examine this idea in some detail.

Tiger Woods, we can imagine people saying, is a golfer, but he could have been a farmer. So, it is not necessary, but contingent, that TW is a golfer. Similarly, it is only contingent that he is not a farmer. Now you say:

$$\text{It is contingent that TW is a golfer.} \tag{2}$$

What you have said, we believe, is true. We predicate truth of a statement modified by the locution of contingency. But, per above, contingency is paraphrasable. The statement (2) can be interpreted as:

$$\text{TW is a golfer and TW might not have been a golfer.} \tag{3}$$

The second true conjunct again contains a modifier. We take it to be the modifier of possibility. (3) is paraphrased as:

It is possible that TW is not a golfer.

The necessity modifier is introduced analogously. Once we have an agreement on how to parse modal locutions, the task is to state their truth conditions. Why do we believe that Tiger Woods might have been a farmer? Isn't it because we believe there be an alternative circumstance in which he is a farmer? Hence an idea:

> Possibly, TW is a farmer iff there is a possible circumstance *w* such that TW is a farmer in *w*.

Given that necessity and contingency are paraphrasable in terms of possibility, parallel clauses are obtained for them, too. In particular:

[11] See Kripke (1980: 18).

[12] Unless, that is, we are allowed to treat them as postulates useful for all kinds of theoretical purposes. This is the main strategy adopted in Lewis (1986).

Contingently, TW is a golfer iff there is a possible circumstance w such that TW is a golfer in w and there is a possible circumstance $u \neq w$ such that TW is not a golfer in u.

We can generalise and talk about arbitrary statements by conducting semantic ascent. Thus:[13]

$\ulcorner$Possibly, $S\urcorner$ is true iff there is a possible circumstance w such that S is true in w.

Contingency and necessity are interpreted along similar lines:

$\ulcorner$Contingently, $S\urcorner$ is true iff $\ulcorner$Possibly, $S\urcorner$ is true and $\ulcorner$Possibly, $\neg S\urcorner$ is true.
$\ulcorner$Necessarily, $S\urcorner$ is true iff for every possible circumstance w, S is true in w.

We have obtained bi-conditionals mirroring Leibniz' Principles. Several comments are in order. First, the right-hand side of the bi-conditionals is supposed to have explanatory priority over the left-hand side. What we are trying to achieve is an explication of modal notions, albeit a non-reductive one. Second, there is an asymmetry between the left- and right-hand sides in that on the left we find truth *simpliciter* and on the right there is a novel notion of truth in a circumstance. This is not accidental. The intent of Leibniz' Principles is to take a fragment of modal discourse equipped with a generic truth predicate and to show the transition from such a discourse to the discourse about possible circumstances.

While the asymmetry can rightly be viewed with suspicion, the remedy should be easy. Let us introduce a triple $M = \langle W, @, V \rangle$, where W is a set of possible circumstances, @ is a designated member of W, and V is a valuation function assigning truth values to ordered pairs of circumstances and propositions, with the following conditions:

$$V(w, s) = \begin{cases} \text{True only if } s \text{ is true in } w \, (w \in W, w \neq @). \\ \text{True only if } s \text{ is true in } M \, (w \in W, w = @). \end{cases}$$

In other words, truth in M is truth in the element @ of M. With this model-theoretic framework in place, our next step is in introducing the notion of intended model. The domain of such model contains the objects which the speakers refer to and over which they quantify. Other models can be merely isomorphic to the intended model. If we give a model-theoretic treatment to the modal discourse, then, intuitively, the set W is supposed to contain possible circumstances and the world @ is the actual world. But instead, W may well contain times or algebraic objects, provided those satisfy formal conditions we impose on our semantics. In the intended model the latter are ruled out.

The final step is to make truth *simpliciter* equivalent to truth in the intended model:

For all s, s is true iff s is true in the intended model M_I.

[13] Other versions may include:
'Possibly'$^\frown S$ is true iff there is a possible circumstance w such that S is true in w.
'Possibly, …' is true iff there is a possible circumstance w such that '…' is true in w.

In such roundabout way we can restore the symmetry of truth predicates embedded in our bi-conditionals (and by analogy, in Leibniz' Principles too). Of course the outstanding problem remains the problem of representation. Speakers of the language may be trusted with understanding the generic truth predicate. It is not at all clear how we are to interpret the novel notion of truth in a possible circumstance. Indeed, it should depend on how we interpret the notion of a possible circumstance in the first place.

What to do about the representation problem, as admitted earlier, is a difficult issue. The burning question before us now is: can we get from possible circumstances to possible worlds? In investigating the contingency of Tiger Woods being a golfer we were led to examine the circumstance where he becomes a banker. A fairly limited circumstance is identified by the truth of the statements 'TW is a golfer' or 'TW is a banker'. But what if go further and look into a larger chunk of the alternative history? Then we enquire about the state of Tiger's finances, the identity of his wife, or the kind of car he drives. The more details we add, the more refined circumstance we consider. Eventually we should arrive at a maximally refined, or complete, circumstance identified by the truth value of every statement.

The procedure is not guaranteed to succeed. First, it is not clear whether it should yield worlds complete with respect to every statement of every language, or whether worlds will be complete with regard to the statements of one language only. Secondly, there are doubts about the very idea of a totally complete circumstance.[14]

At all events, there is no sufficient evidence to conclude that everyday discourse forces a commitment to possible worlds. At most it yields a commitment to circumstances. Such entities, under the name of 'possibilities', have been described by Lloyd Humberstone (to prevent confusion I shall label them 'H-possibilities').[15] For a language $\mathfrak{L}$ we define a triple $\langle \mathcal{W}, \geqslant, V \rangle$, where $\mathcal{W}$ designates a (non-empty) set of H-possibilities, V assigns truth values to the letters of $\mathfrak{L}$. The relation $\geqslant$ is the relation of refinability. Take a particular $P \in \mathcal{W}$. Since not all the sentences of $\mathfrak{L}$ are assigned truth values at P, there should be $Q \in \mathcal{W}$, where the sentences with truth values at P form a subset of the sentences assigned truth values at Q. The relation $P \geqslant Q$ is formalised by two conditions:

1. Persistence: For every sentence σ, if $V(\sigma, Q) = k$, then $V(\sigma, P) = k$.
2. Refinability: The set of sentences assigned a truth value on Q is a subset of the set of sentences assigned a truth value on P.

The criterion of identity for H-possibilities will mirror the one for worlds:

$$\mathbf{P} = \mathbf{Q} \leftrightarrow \forall \sigma (G(\sigma, \mathbf{P}) \leftrightarrow G(\sigma, \mathbf{Q})). \tag{P=}$$

Using the refinability relation, we can also readily interpret worlds as special cases of H-possibilities:

$$\forall w (Ww \leftrightarrow \forall u (u \geqslant w \supset u = w)). \tag{W^m}$$

[14] As shown by the Kaplan-Peacocke's paradox. See Lewis (1986) and Davies (1981).

[15] See Humberstone (1981).

This formula serves to express the informal idea that worlds are 'complete' or 'maximal' H-possibilities.

So the theory of H-possibilities has the advantage of dispelling the worry about indefinite assertion. What is more, it was shown that both propositional and first-order semantics of H-possibilities are equivalent to possible worlds semantics satisfying the axioms of **S5**.[16] That is, H-possibilities will have the same theoretical utility as possible worlds. On the other hand, bear in mind that the same kind of nominalistic worries about the nature of worlds may be raised with regard to H-possibilities. These, too, appear to be abstract objects. Thus a nominalist sympathiser may look at them with suspicion. The criterion of identity provides a rejoinder to these skeptics.

6. Our proposal so far may be interpreted as favouring H-possibilities, and I think that would be congruent with both the spirit and the letter of the original neo-Fregean strategy. However, it is not the only option we could pursue. In the first place, as we have noted earlier, there is a continuing tension between Leibniz' Principles and (W=). The Principles do not require proper names. All that is needed for, e.g., the possibility clause is that there is some possible world where a given statement is true. There is no further need to single out a unique world and name it.

Secondly, if unique identification of possible worlds is at the heart of the problem, a simpler escape may be available. We may consider dropping the discrimination constraint altogether. To take the clause for possibility, we must be able to use individual variables bound by the existential quantifier over the domain of possible worlds. The clause itself does not actually introduce proper names for possible worlds. On the other hand, we must be able to apply the existential instantiation rule. We may talk about a possible world where Socrates is a businessman and a possible world where Socrates is a pirate. What we want to say further is that those possible worlds mentioned in the previous sentence are distinct. If we are permitted the full use of the tools of the first-order logic, there must be a procedure at our disposal for eliminating existential quantifiers, that is, for passing from $\exists xWx$ to Wa. The existential instantiation rule should be supplemented with various constraints to block invalid inferences (such as $\forall x\exists yFxy \vdash \exists y\forall xFxy$), but in the process we would still obtain a statement containing a singular term. Then we could eventually get the desired $a \neq b$. And in that case the use of the singular terms 'a' and 'b' must be justified. In some logical textbooks the difficulty is alleviated by using individual parameter, rather than singular terms, for those contexts of arbitrary choice. Formal convenience notwithstanding, no philosophical issue is settled by notational variation. The same problem may be stated for individual parameters as well.

The root of our troubles may be seen in the additional demand to go beyond the syntactic level and seek for a piece of knowledge about possible worlds *qua* referents of singular terms. If now we wish to absolve ourselves from the epistemic obligation entirely, we could settle for a minimal notion of singular terms. This is a familiar strategy expressed in Quine's dictum 'to be is to be the value of a variable'. Quine characterises 'names' as those expressions which are designed to replace

[16] See Humberstone (1981) and Forbes (1985).

bound variables in the quantificational rules of inference.[17] What are these rules? Quine is interested above all in existential generalisation and universal instantiation, for the following reason. Suppose I assert that there is such a thing as possible world. Then the truth of a statement '$\cdots$possible world$\cdots$' entitles me to infer the statement '$\exists x(\cdots x \cdots)$'. If the inference is valid, then I should also be able to infer from the negation of the premiss the negation of the conclusion. Applying the rule of contraposition and replacing the quantifiers, we infer from '$\forall x(\cdots x \cdots)$' the conclusion '$\cdots$possible world$\cdots$'. Eventually, however, no principled distinction between rules of quantification is drawn, so that universal generalisation and existential instantiation can safely be added. Quine concludes:

> Here, then, are five ways of saying the same thing "There is such a thing as appendicitis"; "the word 'appendicitis' designates"; "the word 'appendicitis' is a name"; "The word 'appendicitis' is a substituent for a variable"; "The disease appendicitis is a value of a variable". The universe of entities is the range of values of variables. To be is to be the value of a variable. (Quine 1939: 708)

It is easy to verify that Quine's notion of a name satisfies syntactic criteria for singular terms. The extra-syntactic discrimination constraint is dropped—as are any other constraints which could plausibly be imposed on ordinary proper names and which go beyond the syntactic characterisation of singular terms. There is, as we have seen, only one sense of existence; so that, furthermore, it would be misleading to segregate names into merely syntactic and truly referential ones.

Quine has several reasons for insisting on the central role of bound variables. One is the theory of descriptions. Definite descriptions are masquerading as proper names, but they are not to be regarded as such when properly analysed. Secondly, there are entities, such as real numbers, which are more numerous than the totality of names constructible in language. Thirdly, it is possible to doubt the ontological commitment through names by denying that the terms in question are *really* names. Though names cannot serve as a stable criterion of ontological commitment, we must still insist that the ontological commitment to entities is rooted in our discourse about those entities.[18] The only stable criterion is delivered by existential quantification.

Quine's worries, then, are analogous to our own worries. Possible worlds may be unnameable in principle, because of the problem of indefinite assertion. And the terms for them, on the surface looking like names, might well be bogus names. We may be tempted to base our commitment to possible worlds by justifying our practice of employing singular terms for them. But there is no such need: we can base our commitment on quantified variables. The names for worlds used in (W=) are understood syntactically. The ontological commitment to worlds does not result from their very use. It is rather the fact of our quantifying over worlds according to Leibniz' Principles which established that commitment.

[17] See Quine (1939: 707).

[18] See Quine (1951: 205).

The dispensability of names has a fruitful consequence. We should not tie the extent of our ontological commitments to the act of naming as such. We are then able to express commitment to the objects which are either unnameable in principle (reals, possible worlds), or to those which are nameless as a matter of fact, such as unnamed dogs or ships. Yet we cannot radicalise the proposal and treat variables on their own as referential. Variables are syncategorematic terms, on Quine's view, and they name nothing. Nor quantifiers on their own have any referential role. It is the *alliance* of variables and quantifiers that produces ontological commitment. In this way we are able finally to circumvent the problem of indefinite assertion.[19]

II

7. We may have alleviated some serious worries about the criterion of identity, but this criterion, on its own, is not sufficient for maintaining any meaningful realist position. Whether we use (W=) or (W=), one can very well agree that possible worlds or H-possibilities should be distinguished according to the criterion of identity, and still argue that this has no impact on ontology. (In the remainder of the paper I will focus the discussion on worlds. The case for H-possibilities can be made by analogy, but I will not labour this point.)

The idea is that many different kinds of things may play a certain theoretical role, e.g., serving to provide truth-conditions for modal statements. To the extent that they play this role we call them 'possible worlds'. But the fact that they are adapted to play that role cannot be used to extend our original ontological commitment. That is, the concept POSSIBLE WORLD is a 'structural' or 'functional' concept indicating the role of a certain previously identified thing in a given theoretical construction. Or in Robert Stalnaker's words:

> The concept of possible worlds that I am defending is not a metaphysical conception… The concept is a formal or functional notion, like the notion of an *individual* presupposed by the semantics for extensional quantification theory. An individual is not a particular kind of thing; it is a particular role that things of any kind may occupy the role of subject of predication.
> Similarly, a possible world is not a particular kind of thing or place. The theory leaves the *nature* of possible worlds as open as extensional semantics leaves the nature of individuals. (Stalnaker 1984: 57)

It seems uncontroversial that individuals in this sense are not part of any ontological commitment. One does not say that there are chairs and tables and molecules—*and* individuals. For if a question is asked 'What are these individuals?', the answer is inevitable: 'Chairs and tables and molecules are individuals'. Even a metaphysical solipsist would presumably agree that, at the very least, he himself should be treated as an individual. Such a concession would be easy, because it does not at all impact on ontological commitments.

[19] For an account that assigns referential role to variables see Fine (1985).

Secondly, chairs are individuals in the sense that the variables for chairs occupy name-positions in one's logical syntax. On Quine's influential account, only those variables which occupy name-positions are quantifiable.[20] Accordingly, the statement 'There are individuals' should be interpreted as claiming that there are entities the variables for which occupy name-position. The issue of *what* these entities are is still not settled. And then, if one person believes that *X*'s and possible worlds exist, and another person believes that just *X*'s exist, they do not disagree about what exists. The first person's claim 'Possible worlds exist' must be interpreted along the lines of the claim '*X*'s exist and they have a certain theoretical role.' The structuralist view is congenial to 'magical ersatzism'. From (W=) one can presumably conclude that possible worlds are 'abstract' objects, since it is difficult to see how statements can be true or false on any 'concrete' objects. But nothing more specific about their nature can legitimately be inferred from (W=). In fact there is no one single 'nature' that all possible worlds can share.

The neo-Fregean response to the structuralist challenge is based on the idea that the concept POSSIBLE WORLD is a *sortal concept*. It can informally be illustrated as follows. Suppose I wish to know whether my laptop is Julius Caesar. I might start by comparing them. Caesar walks (in a tenseless way), talks, eats. My laptop does not eat, does not talk, and does not walk. I conclude, by the indiscernibility of identicals, that my laptop is not Julius Caesar. But if I am careful, I should see at once that it is not the end of the story. The interesting question remains whether *any* laptop is Julius Caesar. How can I be sure that there is no laptop in some Chinese province which is in fact Julius Caesar? Or conversely, how can I be sure that no Chinese farmer is in fact a laptop? The query seems odd, but it is unclear exactly why. It is not odd to ask whether my laptop is your laptop stolen yesterday. Such a query is a daily police routine. And it is not odd to ask whether Augustus is, or could be, Julius Caesar. That is a daily historical and philosophical routine. But there is something amiss in asking whether my laptop is Julius Caesar. The queer element in it seems to be the doubt whether any laptop *could be* a man.

Here is the clue, then: so far as the the object before me is a laptop, it is not, and could not be, Julius Caesar. The same goes for any other laptop. We freeze this insight into the following terminology:

> F is a sortal concept only if for any x falling under F, necessarily there is a property G such that necessarily, x is F just in case x is G.

To make the condition informative we must also specify that G is not a formal property such as being identical to oneself. That is, there must be x which could be not-G. It is, of course, a difficult problem which G would suit a given sortal F. The perennial question what makes humans human is precisely the question which G is suitable for the sortal concept HUMAN. The condition above simply states that, whatever G there is for HUMAN, the analogous property G' for, say, the sortal PLANET will be different.

[20] See Quine (1969: 95).

The oddity in my earlier query was due to our regarding COMPUTER and PERSON as sortals, whereas the query presented the property G for PERSON as blending into the property G' for COMPUTER.[21]

Now, if possible worlds are introduced into the language solely through the criterion (W=), we apparently possess no means for determining whether, for example, configurations of dice or chess pieces are, or could be, possible worlds. (W=) fails to introduce the concept G needed for justifying POSSIBLE WORLD as a sortal. Therefore, the failure to answer the worry opens the door to structuralism. To maintain the neo-Fregean position we will have to go beyond (W=).

8. We have been labouring the familiar 'Julius Caesar problem' originally posed by Frege for natural numbers. If natural numbers are introduced into the language through (N=), there is no means for determining whether a person is, or may be, a number. Just like (W=), the condition (N=) fails to introduce the concept G needed for justifying NUMBER as a sortal. Crispin Wright has argued that the same principle (N=) can be employed for giving the solution of the Julius Caesar problem.[22] I think Wright's solution can work just as well with (W=). Let me paraphrase it as follows. To say, for example, that chess configurations are possible worlds is to say that chess configurations fall under the concept POSSIBLE WORLD. And the ability to subsume a given object under the concept F is conditioned on our possession of the criterion of application for that concept. But what should this possession amount to? It amounts to nothing but the mastery of applying the intra-sortal identity condition. That is, if the speaker is presented with a and b allegedly falling under F, the identity $a = b$ must be spelled out by the criterion of identity for Fs. The criterion of application thus takes the form:

> F is a sortal concept under which possible worlds are subsumed only if there is x and there is y putatively falling under F such that for every sentence σ the truth-conditions of the identity x = y are adequately explained by the correlation between the truth-values of σ on x and on y. (W↓)

The significance of the criterion (W↓) is in blocking any further reductionist attempts in explicating the nature of possible worlds. Suppose we identify a world w by a set of statements true on it, and suppose we identify a world u by the same set of statements. According to (W↓), $w = u$. But, one might press, how can we be sure that in absolutely no respect the worlds u and w are distinct? The answer is that to allow this possibility is simply to believe that worlds are entities determined (or individuated) in some way other than through (W↓). Such supposition could make perfect sense within Lewis' modal realism, but not within the neo-Fregean approach.

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[21] Clearly LAPTOP is not a stand-alone sortal concept. It is rather an impure sortal concept in Wiggins' terminology. See Hale and Wright (2001b: 387) and references therein. I also ignore the view on which people are sophisticated computers. The concept COMPUTER in the text must be understood as FACTORY-BUILT COMPUTER, not as TURING MACHINE.

[22] See Hale and Wright (2001b: 368) and Wright (1983: 116–117).

Similarly, the criterion (W↓) helps to rule out chess configurations as possible worlds. The identity of chess configurations must be decided by reference to the positions particular chess pieces occupy in them. In the same way, temporal instants cannot count as worlds, since they leave statements of mere possibility undetermined (and also because the identity between instants must be established either by reference to their position in temporal sequence, or else by reference to the identity of simultaneous states of the universe). Since we already possess the condition (W^m), we can also distinguish between worlds and incomplete H-possibilities.

I believe that the condition (W↓) delivers a sufficiently convincing response to the structuralist challenge. There is, however, one controversial element in this account. We have been talking about possible worlds. Should the actual world receive the same treatment? Many theorists wish to draw no metaphysical contrast between the actual and merely possible worlds. The actual world is 'everything that is the case', and a merely possible world is everything that might have been the case. There are, of course, physical individuals, all of them being actual, yet the actual world itself is an abstract entity. Each world has a domain of individuals associated with it, and among the domain of some merely possible world *w* some physical individuals can be found, too.[23]

For those theorists, then, *nothing more is required* for the criterion of application than the conditions (W↓) and (W^m). Others may disagree. Others may think that the actual world is the spatiotemporal experience we inhabit. Merely possible worlds are all abstract entities. One may, therefore, adopt one half of Lewis' realism, assimilating the actual world to the spatiotemporal universe, but resisting the further move of turning merely possible worlds into spatiotemporal universes as well. On this account, necessarily existing entities, such as numbers, are actual without being spatiotemporal. It is the contingently actual entities which are all spatiotemporal.

Now, the actual world is actual contingently. That is the whole purpose of taking modal talk seriously. So, it is contingent that the actual world is the spatiotemporal universe. Hence we obtain the desired link between the actual world and the spatiotemporal universe. The term 'universe' here cannot be interpreted as 'one of isolated universes'. There is only one universe in the sense intended here: if there are chunks of causally isolated matter floating in absolute space and time, or alternatively, if there are disconnected spacetimes, they all still comprise one single universe, perhaps indeed better expressed in the vernacular as 'the way things are'. On the other hand, to be a possible world is to be a possibly actual world. So to be a possible world is to be something non-concrete that could have been the actual world. Thus, to be a possible world is to be something that could have been a spatiotemporal universe.

Our claim is that possible worlds are exactly those entities which could have been spatiotemporal universes:

$$\forall x(Wx \leftrightarrow \blacklozenge\mathrm{Universe}(x)), \qquad (\mathrm{W}^u)$$

where $\blacklozenge$ is a generic possibility operator to be explained in a moment. Such a condition, if we recall the discussion of sortals above, alone suffices to fix the sortal

[23] See Stalnaker (1998: 99) with a nod to Wittgenstein's *Tractatus*.

concept POSSIBLE WORLD. One can no longer state that instants, or chess configurations, or goldfish are possible worlds. None of these could be universes, at least so far as they themselves are subsumed under a sortal concept. Its second role is in establishing the link between the actual and possible worlds. But we still need our condition (W↓). It is to be used against the intransigent opponent insisting on the question 'What are possible worlds?' Such an opponent would not be satisfied with the answer explaining what they *could be*. The condition (W↓) will settle the dispute with the intransigent opponent.

Therefore, for those theorists who assimilate the actual world to the spatiotemporal universe the criterion of application consists of the following components:

1. The condition (W↓): F is a sortal concept under which possible worlds are subsumed only if there is x and there is y putatively falling under F such that for every sentence σ the truth-conditions of the identity $x = y$ are adequately explained by the correlation between the truth-values of σ on x and on y.
2. The condition (W^m): $\forall w(W(w) \leftrightarrow \forall u(u \geqslant w \supset u = w))$.
3. The condition (W^u): $\forall x(Wx \leftrightarrow \blacklozenge \mathrm{Universe}(x))$.

9. The last bit to be patched up is the condition (W^u). It employs a non-standard notion of possibility expressed by the operator $\blacklozenge$. There is a reason why we cannot interpret it as the standard possibility operator. We theorise about the modal properties of worlds. And once we fix the actual world, we can no longer state that other worlds are possibly actual. Kripke's semantics of quantified modal logic cannot therefore capture the modal talk about the worlds. We need additional logical tools for handling the operator $\blacklozenge$. They can be located in the so-called 'two-dimensionalist' approach.

In Kripke's semantics, necessity is understood as truth in all possible worlds. When we start making modal claims about actuality, or the actual world, the situation changes. We have to insert the actuality operator into our object-language. The semantics of such an operator $\mathcal{A}$ will be given by the clause:

$$\mathcal{M}[w] \models \mathcal{A}\alpha \text{ iff } \mathcal{M}[@] \models \alpha,$$

where $\mathcal{M}$ is a modal model and @ is the actual world in $\mathcal{M}$. Then we can provide a competing notion of necessity interpreted as truth in whichever world is considered actual.[24] To make this intuitive notion precise, call the model $\mathcal{M}$ a *variant* of the model $\mathcal{M}'(\mathcal{M} \approx \mathcal{M}')$ just in case they differ at most over which world in them is rendered actual. Then we introduce a novel 'fixedly' operator $\mathcal{F}$ interpreted by the clause:

$$\mathcal{M}[w] \models \mathcal{F}\alpha \text{ iff for any } \mathcal{M}' \approx \mathcal{M},\ \mathcal{M}'[w] \models \alpha.$$

The definition of $\approx$ ensures that w is contained both in $\mathcal{M}$ and $\mathcal{M}'$. Applying these clauses consecutively, we obtain the clause for $\mathcal{FA}$:

[24] See Crossley and Humberstone (1977: 19) and Davies and Humberstone (1980: 2–3).

$$\mathcal{M}[w] \models \mathcal{FA}\alpha \text{ iff for any } \mathcal{M}' \approx \mathcal{M},\ \mathcal{M}'[@] \models \alpha.$$

It turns out that $\mathcal{FA}$ satisfies the intuitive requirements we have for a necessity operator. In particular, the **S4** and **S5** axioms

$$\mathcal{FA}\alpha \supset \mathcal{FAFA}\alpha$$
$$\neg\mathcal{FA}\alpha \supset \mathcal{FA}\neg\mathcal{FA}\alpha$$

are both provable in the axiomatic system of **S5** $\mathcal{AF}$ that may be given for $\mathcal{FA}$ (see "Appendix" for details).

Armed with this second (or 'deep') notion of necessity, we can assert that the actual world is deeply contingently a spatiotemporal universe, whereas merely possible worlds could have been spatiotemporal universes. Every world is a universe only in the model where it itself is designated as the actual world, and it is an abstract object in all the rest. Similarly, if we take the model in which *our* world is designated as actual, our world comes out concrete, and all other worlds abstract, at every world in this model. This is the traditional, 'superficial' kind of necessity. And therefore, the possibility expressed by the operator $\blacklozenge$ is two-dimensional. The operator $\blacklozenge$ must be read as an abbreviation of $\neg\mathcal{FA}\neg$.

III

10. The neo-Fregean treatment of possible worlds offers a realist response to the question of their existence. It upholds the conclusion of the Master Argument. The advantage it holds over reductionist realist accounts, such Lewis' modal realism or linguistic ersatzism, is in avoiding entirely the torturous debate on the 'nature' of possible worlds. Their notion is not interpreted in terms of some other, more basic notions. Equally, in contrast to magical ersatzism, the neo-Fregean account does not merely deny all substantive answers. It exploits the connection between the notion of possible worlds and the modal discourse which generated it. The idea here is to insist that any metaphysics of possible worlds going beyond what can be extrapolated from the modal discourse is unwarranted.

On the other hand, as with any similar realist account of abstract objects, the neo-Fregean realism holds advantage over anti-realist theories. Modal talk is truth-apt, and so is *prima facie* possible-worlds talk. Some modal statements are true, and so are some possible-world statements. We save time and effort on reinterpreting, reconfiguring and adjusting these linguistic data to our metaphysical beliefs and doubts.

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Appendix A

A general strategy for defining the category of singular terms was suggested by Dummett and later refined by Wright and Hale. The essence of their proposal consists in demarcating singular terms from other expressions in accordance with their inferential role. As the first step we segregate singular terms from substantival terms, such as ‘nothing’, ‘someone’, or ‘everything’, which can grammatically be put in name-positions. The tactic is to contrast directly the inferential behaviour of singular and substantival terms by applying the familiar rules of passage. For instance, if one says, ‘Jim is perfect’, we can infer that there is somebody who is perfect. But if one says, ‘Nobody is perfect’, we cannot infer that there is somebody who is perfect. This still leaves with the problem of higher generality where predicates can masquerade as singular terms. For example, if we say, ‘Jim is good at tennis’, you can still infer that there is something at which Jim is good. To deal with predicates we can use the Aristotelian intuition that qualities have their opposites, but substances do not. For example, the opposite of the predicate ‘① is white’ would be ‘① is black’, but there will not be any opposite for ‘Socrates’. The proposal can therefore be put as follows:[25] Syntactic criteria of singular termhood. An expression t functions as a singular term in a sentential context $A(t)$ just in case:

1. The following conditions are satisfied:
   (a) The inference is valid from $A(t)$ to ‘Something is such that A(it)’.
   (b) For some sentence $B(t)$ the inference is valid from $\{A(t), B(t)\}$ to ‘Something is such that A(it) and B(it)’.
   (c) For some sentence $B(t)$, the inference is valid from ‘It is true of t that A(it) or B(it)’ to the disjunction ‘$A(t)$ or $B(t)$’.
2. There are no terms ‘opposite’ to t:

$$\neg\Sigma\alpha\Pi\beta((\alpha,\beta)\leftrightarrow\neg(t,\beta)),$$

where the class β contains any expression which can be fitted into the sentential construct $A(t)$ save those that fail the conditions of the first part.

The formalism of the second part demands some explanation. Suppose t is an expression that could be part of a sentence. Let $\mathfrak{S}()$ be a sentential function. We then use $\Sigma\alpha$ and $\Pi\beta$ as substitutional quantifiers, aimed at replacing t and $\mathfrak{S}()$ respectively in the complete expression $\mathfrak{S}(t)$, where α and β comprise the classes of the grammatically legitimate substitutions of t and $\mathfrak{S}()$ respectively. The pair (α, β) designates the sentential construction containing one expression from α and one expression from β. The condition demarcates between singular terms and predicates: for every predicate it is possible to find an opposite predicate applied to the same quasi-singular term (*i.e.* the term certified by the three conditions of the first part of our definition). For genuine singular terms no such opposite term is to be found.

Let us see whether PW-terms qualify syntactically as singular terms. The first part of the test should not present difficulties. Consider, for example, condition (1b). Suppose that the following premisses hold:

1. Socrates is wise in w.
2. Socrates is fat in w.

[25] I omit various qualifications made by Hale in response to criticisms. For the latest version see Hale (1994: 68).

There is no problem to infer:

3. Something is such that Socrates is fat and Socrates is wise in it.

Some doubts may persist about the second part of the test. Since worlds are the entities in which statements are true or false, could there be a world u in which every statement true in w is false? The answer is in the negative. We are guaranteed to have necessarily true statements, with the notion of necessity fixed appropriately, that will be true in every possible world.

Appendix B

For the axioms and rules of inference see Davies and Humberstone (1980).

Lemma 1 $\mathbf{S5}\mathcal{AF} \vdash \mathcal{FA}\alpha \supset \mathcal{FAFA}\alpha$ (*analogue of* $\Box\alpha \supset \Box\Box\alpha$).

Proof

1.$\mathcal{FA}\alpha$	hyp
2.$\Box\mathcal{FA}\alpha$	1, Nec
3.$\mathcal{AFA}\alpha$	2, A3, MP
4.$\mathcal{FAFA}\alpha$	3, Fix
5.$\mathcal{FA}\alpha \supset \mathcal{FAFA}\alpha$	$1, 4, \supset \mathbf{I}$

Lemma 2 $\mathbf{S5}\mathcal{AF} \vdash \neg\mathcal{FA}\alpha \supset \mathcal{FA}\neg\mathcal{FA}\alpha$ (*analogue of* $\Diamond\alpha \supset \Box\Diamond\alpha$).

Proof

1.$\neg\mathcal{FA}\alpha$	hyp
2.$\Box\neg\mathcal{FA}\alpha$	1, Nec
3.$\mathcal{A}\neg\mathcal{FA}\alpha$	2, A3, MP
4.$\mathcal{FA}\neg\mathcal{FA}\alpha$	3, Fix
5.$\neg\mathcal{FA}\alpha \supset \mathcal{FA}\neg\mathcal{FA}\alpha$	$1, 4, \supset \mathbf{I}$

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