International Journal of Production Research
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tprs20

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Published online: 09 Sep 2010.

To cite this article: Nitesh Khilwani, M.K. Tiwari & Ihsan Sabuncuoglu (2011) Hybrid Petri-nets for modelling and performance evaluation of supply chains, International Journal of Production Research, 49:15, 4627-4656, DOI: 10.1080/00207543.2010.497173

To link to this article: http://dx.doi.org/10.1080/00207543.2010.497173

International Journal of Production Research
Vol. 49, No. 15, 1 August 2011, 4627–4656

Hybrid Petri-nets for modelling and performance evaluation of supply chains

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(*Received 6 December 2009; final version received 30 April 2010*)

Modelling and analysis of complex and co-ordinated supply chains is a crucial task due to its inherent complexity and uncertainty. Therefore, the current research direction is to devise an efficient modelling technique that maps the dynamics of a real life supply chain and assists industrial practitioners in evaluating and comparing their network with other competing networks. Here an effective modelling technique, the hybrid Petri-net, is proposed to efficiently handle the dynamic behaviour of the supply chain. This modelling methodology embeds two enticing features, i.e. cost and batch sizes, in deterministic and stochastic Petri-net for the modelling and performance evaluation of supply chain networks. The model is subsequently used for risk management to investigate the issues of supply chain vulnerability and risk that has become a major research subject in recent years. In the test bed, a simple productive supply chain and an industrial supply chain are modelled with fundamental inventory replenishment policy. Subsequently, its performance is evaluated along with the identification and assessment of risk factors using analytical and simulation techniques respectively. Thus, this paper presents a complete package for industrial practitioners to model, evaluate performance and manage risky events in a supply chain.

Keywords: Petri-nets; supply chain networks; risk management; simulation technique; hybrid Petri-net

1. Introduction

Supply chains are often visualised as a link among its facilities including suppliers, manufacturing plants, logistics, final assembly plants, packaging centres and trans-shipment points (Chopra and Meindl 2001, Levi *et al.* 2004). Traditionally, each facility performs its activities independently thereby optimising their own functional objectives and belittling the importance of others (Munson and Rosenblatt 2001). In order to overcome these shortcomings, a process oriented approach is implied that co-ordinates the process across all the departments and all functions involved in value delivery process (Chopra and Meindl 2001). The series of activities in the supply chain facilities are often triggered and synchronised for satisfying customer orders for products. In general, supply chains are triggered by two strategies; namely, fixed order quantity and fixed time

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ISSN 0020–7543 print/ISSN 1366–588X online

DOI: 10.1080/00207543.2010.497173
http://www.informaworld.com

period technique. These techniques define two different inventory control policies, i.e. ways of handling source and delivery function. The goal of this study is to present a modelling technique for supply chain networks that facilitate strategic and tactical decision making.

This paper uses a powerful tool, i.e. Petri-net for modelling and analysing real life supply chain often viewed as a discrete event dynamical system. It can be envisaged from the literature that Petri-nets have been the most frequent tool for modelling the dynamical systems (Murata 1989, Viswanadham and Narahari 1992, Desrochers and Al-Jaar 1995, Proth and Xie 1997). A few PN models of a supply chain are available in the literature (Viswanadham and Raghavan 2000, Dong and Chen 2001, Chen *et al.* 2003, Singh *et al.* 2003), but they ignore certain important features of a supply chain, such as cost flow in network, batch features of inventory replenishment, distribution, etc. Chen *et al.* (2002, 2005) introduced a batch deterministic and stochastic Petri-net (BDSPN) as a tool for modelling and performance evaluation of a supply chain. These BDSPN models, however, overlooked an important feature of a supply chain, i.e. financial flow, which helps in determining the factors creating disturbance in material and information flow of a supply chain.

It is to be noted that all the activities occurring in a supply chain incur certain costs, such as, materials, transportation, inventory, etc., that directly affect the profit of an enterprise. In a static supply chain, due to fixed resources and production schedule, the financial flow is a constant quantity. Thus in literature, most of the papers have modelled the supply chain without considering the flow of monetary terms (Viswanadham and Raghavan 2000, Chen *et al.* 2002, 2005). This paper analyses the supply chain as a dynamic system that often comes across certain changes due to variation in material and information flow. Uncertainty plays an important role in a dynamic supply chain system. For handling uncertainty in an effective and efficient way, this paper introduces a new PN model entitled hybrid Petri-net (HPN) for modelling a real life supply chain. HPNs are introduced to represent material, information and financial flow of a supply chain in an integrated way. The proposed HPN modelling methodology embeds the enticing features of cost places and cost tokens in BDSPNs and efficiently maps the financial flow of a supply chain. The HPN proposed in this paper is a viable mathematical and graphical tool, named after its inherent features comprising cost flow and time flow, batch featured and discrete places, and deterministic and stochastic Petri-nets.

The model introduced in this paper differs from HPNs presented in the literature (Giua *et al.* 2001, Tsinarakis *et al.* 2006). Giua *et al.* (2001) introduced HPN as a tool to model and study hybrid systems. These HPN tools consist of discrete and continuous transitions that are capable of modelling material flow. Subsequently, Tsinarakis *et al.* (2006) introduced hybrid timed Petri-nets (HTPNs) for modelling complex production systems. Unlike the cited works, this paper introduces batch deterministic and stochastic cost Petri-nets for modelling supply chains consisting of either of production systems, viz. productive system, assembly and disassembly system. For brevity only, it is referred to as hybrid Petri-net. The behaviour of a discrete event dynamic system is nicely modelled using the proposed HPN modelling methodology. This HPN is later used to evaluate the performance of the supply chain in order to determine various performance measures such as inventory level, stock out rates, etc. Performance analysis of a supply chain is carried out for the determination of their inherent parameters of, for example, inventory level, backorders, inventory cost, supply chain cost, etc. (Viswanadham and Raghavan 2000, Chan 2003, Li *et al.* 2004). The undertaken supply chain system is characterised by the complex interaction of the timing of various discrete events such as arrival of customers, manufacturing of products, etc., that changes only at discrete events in time (Viswanadham and Raghavan 2000, Desrochers *et al*. 2005). The performance evaluation methods are developed by emphasising the marking process of the model and by extending the method of BDSPNs (Chen *et al*. 2005). In this paper an analytical method, based on steady state distribution, is used for evaluating the performance of dynamic system. In addition, a simulation technique is also provided for the evaluation of complex and large supply chains that arise in a state space explosion problem in their analytical evaluation. The marking process of the model is further used to identify the risky events in supply chains. The risky events responsible for vulnerable supply chain are envisaged to analysis and assess the culprit within the network.

1.1 *Objectives and organisation*

The objectives of the present paper are:

- To model a dynamic supply chain network with fixed time and fixed order inventory replenishment policy using an HPN model which can vividly capture the basic features of supply chain cost, e.g. cost, batch, etc.
- To evaluate the performance of an HPN model for measuring all the basic building blocks of supply chain.
- To identify the risk factors present in the network and finally assess the probability of occurrence and severity of consequences.

The rest of this paper is organised as follows: Section 2 provides a brief description of a supply chain and the inventory replenishment mechanism. Section 3 illustrates the building blocks of the proposed HPN along with the basic details about the proposed methodology and its structural properties. In Section 4, a simple productive supply chain is modelled, evaluated and subsequently its risk events are identified and assessed using analytical method. In Section 5, a real life industrial supply chain is modelled using the introduced HPN approach. The performance evaluation and risk assessment of supply chains using simulation technique is also provided in this section. After describing the overall methodology in upper section, the special features of proposed methodology is provided in Section 6. Finally, the paper is concluded in Section 7.

2. Supply chain description

The supply chain is basically an extensive continuous process in which each process entails a certain cost for converting raw material into a finished product and adds value to the product. The prime objective in a supply chain is to minimise the cost and design an efficient and cost effective network that incurs minimum system-wide cost, including material cost, transportation cost, inventory cost and manufacturing cost. In a supply chain the minimisation of system-wide cost not only emphasises facilities present in a network, but rather encompasses the entire supply chain configuration that too defines the interconnections pattern among its facilities (Angerhofer and Angelides 2000, Baiman *et al*. 2001, Levi *et al*. 2004). However, all supply chains do not have the same configuration. Depending on the production system a supply chain is generally categorised into three fundamental modules, viz. productive, assembly and disassembly system, as shown in Figure 1 (Viswanadham and Narahari 1992, Tsinarakis *et al*. 2006). In this figure

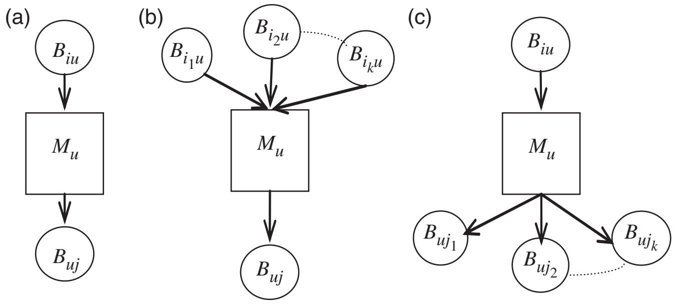

Figure 1. Fundamental modules of supply chain (a) productive, (b) assembly and (c) disassembly.

the three shapes, viz. rectangle, circle and arrow, represent the production facility, buffer and transport facility respectively.

The modules presented in Figure 1 are basically the fundamental sub-system with a set of input and output connecting arcs defining interaction with other modules (Demongodin 2001, Tsinarakis *et al.* 2006). In a productive system (Figure 1(a)) one facility of the network serves as an input for the other facility. Here, the facility either stores the product (buffer) or adds value to the product (production facility). It implies that production facility M_u receives parts from one of the upstream buffers B_{iu} and after processing it, transfers the value-added product to the downstream buffer B_{uj}. Further, in an assembly module (Figure 1(b)), the facility M_u receives parts from the two or more buffer B_{i_lu} (where l is the number of upstream buffer) and transports the value added parts to the downstream buffer B_{uj}. Similarly, a disassembly module (Figure 1(c)) resembles a cone in which the production facility M_u receives the parts from B_{iu} and transfers the processed parts to the downstream buffer B_{uj_m} (where m is the number of the downstream buffer). In the above-mentioned modules the transaction between buffer and production facility takes place through a transport medium.

In the above-mentioned configurations, the prime concern emphasises the transportation facility and inventory system that monitors the level of buffer and determines its optimum level. The former is concerned with the transaction of material between buffer and transportation facilities. However, inventory management is also of major importance and the co-ordination of the individual inventory control mechanism has a significant impact on customer service level and supply chain system-wide cost. In general, two types of inventory system are used, viz. fixed order quantity model (Q-model), fixed time period model (P-model) (Demongodin 2001). The former is an event triggered technique in which order size Q is placed when the inventory level (IL) drops below the re-order point (R). The size of order and re-order point is determined with an aim to minimise the total inventory cost given by (Anderson *et al.* 1997, Demongodin 2001):

$$TC = \frac{D * C_O}{Q} + \frac{Q * C_H}{2} \tag{1}$$

where, TC, C_H, C_O represents the total cost, holding cost and ordering cost respectively; D represents the demand for certain planning horizon and Q represents the order size. The optimal order size that offers minimum cost is given by:

$$Q_{\text{opt}} = \sqrt{\frac{2.D.C_O}{C_H}} \tag{2}$$

The Q_{opt} is placed when the IL drops below a reorder point given by $RP = \overline{d}.L$, where $\overline{d}$ represents the demand in the smallest unit of the planning horizon and L is the lead time required in replenishing the inventory.

In contrast to the above-mentioned Q-model, in time triggered P-model, an order is placed at the end of a predetermined time period. In this system, the ordered quantity varies from period to period depending on the usage rate. The order quantity can be given by:

$$Q = \overline{d}(T + L) + z \cdot \sigma_{T+L} - IL \tag{3}$$

where $\overline{d}$ represents the forecast demand for the smallest unit of the planning horizon (usually days), σ_{T+L} signifies the standard deviation of demand over review and lead time and z is the number of standard deviation for specified service probability (Demongodin 2001).

The previous models determine the buffer size and placement order quantity irrespective of the uncertainty present in the network (Lee *et al.* 2000, Qi *et al.* 2004). However, in a real supply chain there are various factors that disturb the flow of material and information in a supply chain. Although, in the majority of cases safety stocks are maintained to provide some level of protection from such uncertainties, a real supply chain frequently deals with random events, such as uncertainty in judgment and lack of evidence associated with the customer demand, supply deliveries and market supply, etc. The uncertainty present in the network is the key factor behind the risky events present in a supply chain that often ascend due to failure in operation and service management along with variegating customer demand and its delivery time (Hallikas *et al.* 2004, Rossi 2005). These risky factors consequently diminish the customers' satisfaction and enhance the cost of a supply chain. In order to minimise the above consequences of risky factors, generally the firms follow a typical risk management process (Hallikas *et al.* 2004).

A risk management process is characterised by a sequential structure divided into three main phases given by W2H, i.e. risk identification (*what* is risk?), risk assessment (*how* much is risk?) and risk control (*how* to control) (Hallikas *et al.* 2004). The former one is the prime step in risk management practice concerned with the recognition of uncertainties present in a supply chain network. The risk factors identified in the above step are assessed based on their frequency and severity of their outcomes during the risk assessment. Finally, the asperity of risky events is abated through the appropriate risk control process. The strategies often used for implementing the risk control process include risk elimination and/or risk modification (Hallikas *et al.* 2004). In a supply chain network the basic reason behind the risk management is to minimise the cost of a supply chain and finally reach the zenith of profitability.

3. Fundamentals of hybrid Petri-nets

Petri-nets are the powerful graphical and mathematical modelling tool used to describe and analyse complex systems exhibiting concurrency, synchronisation and conflicts (Murata 1989, Desrochers and Al-Jaar 1995, Proth and Xie 1997). Several modifications have been performed in typical Petri-net formalism to model timed and probabilistic behaviour of the system for performance and reliability evaluation (Marsan *et al.* 1995, Wang 1998, Giua *et al.* 2001). A class of stochastic Petri-nets (SPNs) have gained wide acceptance in the literature as a performance analysis tool for the modelling of dynamic systems aided with time delays (Viswanadham and Narahari 1992, Dicesare *et al.* 1993, Wang 1998). The class of SPNs comprises of typical stochastic Petri-nets, generalised stochastic Petri-nets, deterministic and stochastic Petri-nets, etc. (Dicesare *et al.* 1993). However, for brevity only the preliminary definitions and properties of interest are provided in this section. In the following discussion IN is the set of non-negative integers.

Definition 1:

(a) A stochastic Petri-net, defined as a tuple $SPN = (P, T, I, O, M_0, F)$ is the extension of traditional PN, where $P = (p_1, p_{2,\ldots,}p_g)$ and $T = (t_1, t_{2,\ldots,}t_h)$ is a set of finite places and transition respectively such that $P \cup T \neq \phi$ and $P \cap T = \phi$ (i.e. non-empty and disjoint sets); $I : (P \times T) \rightarrow IN$ $(O : (T \times P) \rightarrow IN)$ is the input (output) function that specifies input (output) places of transitions and weights associated with it. The marking of PN is mapping $M : P \rightarrow IN$ and M_0 represents the initial marking. $M(p)$ indicates the number of tokens in p under the marking M. The preset (postset) of a place p is represented by $\bullet p = \{t \in T | O(p, t) > 0\}$ $(p\bullet = \{t \in T | I(p, t) > 0\})$. Similarly $\bullet t = \{p \in P | I(p, t) > 0\}$ $(t\bullet = \{p \in P | O(p, t) > 0\})$ denotes the preset (postset) of transition t. $F : (R(M_0) \times T) \rightarrow \Re$, assigns a random variable with rate $F(M, t)$ to each $t \in T$ in each $M \in R(M_0)$.

(b) A generalised stochastic Petri-net (GSPN) is defined as $GSPN = (SPN, V, S)$ that extends SPN by dividing transitions into two sets viz. immediate transition (T_I) with 0 firing time and exponential transition (T_E) with firing time subjected to exponential distribution. Here, $V : (P \times T_i) : IN$ is the inhibitor arc associated with T_I; $F : (R(M_0) \times T) \rightarrow \Re$ is a firing function that associates an exponential random variable with rate $F(M, t)$ to each $t \in T_E$ in each $M \in R(M_0)$.

(c) A deterministic and stochastic Petri-net (DSPN) is defined as a tuple $DSPN = (P, T, I, O, M_0, F, V, S, \psi)$ that belongs to an extended class of GSPN where $\psi : T \rightarrow \Re$ is the firing priority function for the transitions that assign a priority level to the transitions. Transitions are classified into a finite set, i.e. $T = T_I \cup T_D \cup T_E$, where T_I and T_E are of similar significance, but T_D are introduced to represent the deterministic transitions with constant firing time. Further, $F : (R(M_0) \times (T_E \times T_D)) \rightarrow \Re$ represents the firing function that associates:

- Exponential random variable to each $t \in T_E$ in each $M \in R(M_0)$.
- Constant firing time to each $t \in T_D$ in each $M \in R(M_0)$.
- Zero firing time to each $t \in T_I$ in each $M \in R(M_0)$.

3.1 *Batch deterministic and stochastic Petri-nets (BDSPNs)*

BDSPN is an extended form of DSPN introduced to enhance the flexibility and generality in modelling the dynamical system such as supply chain (Chen *et al.* 2002, 2003). Although various approaches are present in the literature for modelling the dynamical supply chain systems, they ignore the batch feature of a supply chain. A general supply chain involves various operations (such as transportation, buffer replenishment, etc.) that are performed in batch way, i.e. parts are moved as an inseparable entity. Therefore for the effective modelling of a real life supply chain network, the concept of batch (viz. batch tokens, batch firing etc.) has been aided with the concept of DSPN for modelling the batch operations in the framework of Petri-nets (Chen *et al.* 2002, 2003). The following definition is provided in this regard:

Definition 4: A BDSPN can be defined as a tuple $BDSPN = (DSPN, \mu_0)$, which extends DSPN by incorporating the batch features for the modelling of a supply chain. Here, $P = P_D \cup P_B$, is the finite set of places classified into discrete places and batch places. $\mu_0 : P \to IN \cup 2^{IN}$ is the initial μ marking of the net, in which 2^{IN} includes all the subsets of IN present in a batch place.

The μ marking is introduced in BDSPN, for the representation of states of $P = P_D \cup P_B$ simultaneously. In general, μ marking for places can be represented as

$$\mu(p) = \begin{cases} m|m \in IN & \text{if}\, p \in P_d \\ \left\{ \bigcup\limits_{b \in P_b} b|b \in IN \right\} & \text{if}\, p \in P_b \end{cases} \tag{4}$$

However, in the underlying batch network, the M marking is given as:

$$M(p) = \begin{cases} \mu(p) & \text{if}\, p \in P_d \\ \sum\limits_{\forall b \in P_b} b & \text{if}\, p \in P_b \end{cases} \tag{5}$$

From Equations (4) and (5), it is evident that M marking can be evaluated by μ marking. Therefore, the batch Petri-nets are represented as $BDSPN = (P, T, I, O, V, F, \psi, \mu_0)$.

3.2 *Hybrid Petri-nets*

This paper presents a new type of modelling technique that extends BDSPN by incorporating the feature of cost flow in it. This incorporation is carried out for simultaneously analysing the variation in cost with time in each part of a supply chain. This new kind of Petri-net, referred to as hybrid Petri-net, is a sequel of BDSPN with modified set of place $P = P_D \cup P_B \cup P_C$, where, P_B and P_D represents the batch and discrete places and P_C is the set of cost places.

Definition 5: Cost places $P_c \subset P$, are used to store the information about the cost spent in unit time in a supply chain. The cost stored in P_c includes raw material cost (RM), inventory holding cost (IH), ordering cost (OC) and backordering cost (BC) and transportation cost (TC), given by:

$$C(t) = RM(t) + IH(t) + OC(t) + BC(t) + TC(t) \tag{6}$$

In the above equation $C(t)$ depicts the cost used up in $[t-1, t]$.

The cost places do not correspond to any physical element of the network, but record total cost incurred in performing an operation in a certain part of a supply chain. The cost places are the 'virtual places' that help in analysing the disruption occurred in a supply chain, providing the information regarding the fluctuation of cost in a supply chain. The cost places are also used to evaluate the total cost spent during the planning horizon given by

$$C = \sum_{t=0}^{T} C(t).$$

The graphical representation of various elements of HPN are as follows (Chen *et al.* 2003, Tsinarakis *et al.* 2006): places viz. P_D, P_B and P_C are represented by circle, circle embedded in square and oval. Similarly, the three transitions viz. T_I, T_E and T_D are represented by thin vertical lines, empty rectangle and filled rectangle respectively. The places and transitions present in a network are connected by arcs. Generally μ marking is used for simultaneously determining the state of the system that comprises of discrete, batch and cost places. In an HPN, the tokens are used for representing the flow of resources. Here discrete tokens represented by black dots (•), batch tokens $b \in IN$ and money token $c \in +\Re$ are used to define the state of places, i.e. P_D, P_B and P_C respectively.

The dynamic behaviour of the system represented by μ marking is defined in terms of the so called 'token game'. The prime considerations of this game are transition enabling and firing conditions. The former two parameters highly depend on the presence of batch input place, i.e. $p \in \bullet t \cap P_b$. Therefore, the transition firing is classified into two groups, viz.

(1) *Transition with no batch input place:* In such cases, the transition is said to be enabled under marking μ (denoted by $\mu[t >)$ *iff* $\forall p \in \bullet t \cap P_D \rightarrow \mu(p) \geq\ I(p,t) \wedge \forall p \in \circ t \cap P_D \rightarrow \mu(p) \leq I(p,t)\ \wedge \forall t_k \in \Gamma(\mu) \rightarrow \psi_t \geq \psi_{t_k \backslash t}$; where $\Gamma(\mu)$ is the set of k transition enabled under marking μ.

The aforementioned condition reveals that only the transitions with highest priorities are enabled and may be fired at any time. Further, when $((t \cap T_I \in \Gamma(\mu)) \wedge (t \cap T_E \vee T_D \in \Gamma(\mu)))$ then $\mu[t > \mu' : t \cap T_I \in \Gamma(\mu) \wedge \psi_t \geq \psi_{t' \in T_I \backslash t}$. However, when $((t \cap T_I \notin \Gamma(\mu)) \wedge (t \cap T_E \vee T_D \in \Gamma(\mu)))$ then race policies and pre-selection are used for the selection of firing transitions, which is to be fired next. In case of pre-selection the predefined policies are used for the selection of firing transition. And in case of race policies, the probability for the selection of firing transition depends on firing time of transition. Among the three different race policies present in literature, viz. re-sampling, enabling memory and age memory, the race policy with enabling memory is utilised in this paper, for the selection of firing transition.

The firing of the transition, selected from the above conditions, results in new marking μ given by:

$$\forall p \in \bullet t \quad \mu'(p) = \mu(p) - I(p,t) \tag{7}$$

$$\forall p \in t \bullet \cap P_d \quad \mu'(p) = \mu(p) + O(p,t) \tag{8}$$

$$\forall p \in t \bullet \cap P_b \quad \mu'(p) = \mu(p) + \{O(p,t)\} \tag{9}$$

$$\forall p \notin t \bullet \wedge \bullet t \quad \mu'(p) = \mu(p) \tag{10}$$

(2) *Transition with batch input places*: In such a network, a transition is defined to be enabled *iff* $\forall p \in \bullet t \cap P_b \rightarrow q = (b/I(p,t))$, where $q \in IN$ is the batch firing index. And $\forall p \in \bullet t \cap P_d \rightarrow (\mu(p) \geq q \times I(p,t)) \wedge (\forall p \in \circ t \rightarrow \mu(p) < I(p,t))$.

The selection of transition on the basis of transition priorities is similar to that of the previous case. The firing of a selected transition results in new μ marking evaluated by:

$$\forall p \in \bullet t \cap P_d \quad \mu'(p) = \mu(p) - q \times I(p,t) \tag{11}$$

$$\forall p \in \bullet t \cap P_b \quad \mu'(p) = \mu(p) - q \times \{I(p,t)\} \tag{12}$$

$$\forall p \in t \bullet \cap P_d \quad \mu'(p) = \mu(p) + q \times O(p,t) \tag{13}$$

$$\forall p \in t \bullet \cap P_b \quad \mu'(p) = \mu(p) + q \times \{O(p,t)\} \tag{14}$$

In this case, the firing of a transition depends on a few assumptions delineated as follows:

(1) If there exists more than 1 batch input place, then batch token with equal sizes are selected for the evaluation of q.
(2) If there exists more than one batch tokens in an input place, then selection for the batch token for the firing of a transition is based on the FIFO (First In-First Out) rule.

Example 1: An illustrative example is provided in Figure 2 to explain the above-mentioned two cases. The subnet provided in this figure consists of three input and two output places with initial marking $\mu_0 = (1, 6, \{4, 4, 2\}, \{2\}, 2)$. Transition t is enabled with firing index $q = 2$ that results in marking $\mu_1 = (1, 2, \{4, 2\}, \{2, 6\}, 6)$. Then t is enabled with firing index $q = 1$ that results in marking $\mu_2 = (1, 0, \{4\}, \{2, 3, 6\}, 8)$.

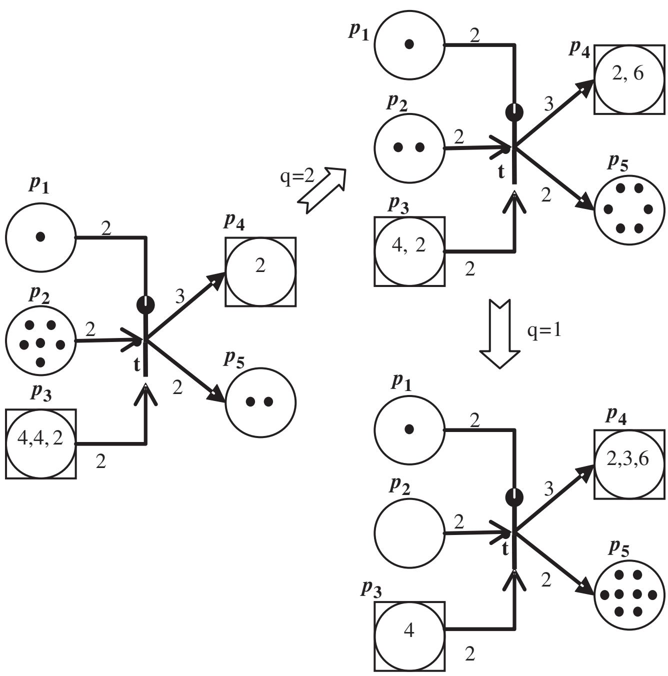

Figure 2. Transition enabling and firing of HPNs, Example 1.

3.3 *Structural analysis of HPNs*

This section lucidly delineates the structural property of an HPN model based on state space equation and reachability analysis which provides a basis for its structural analysis. Structural properties mainly deal with the qualitative and logical properties of the net, thus time is ignored while analysing these properties. For a clear explanation in this regard the following properties are provided:

Property 1: A marking μ' is said to be reachable from other marking μ *iff* $\exists$ a firing sequence of transitions $\sigma = \{t_1, t_2, \ldots, t_{g'} | \, g' \le g \wedge \mu[t_1 > \mu_1[t_2 > \mu_3[t_3 > \cdots [t_{g'} > \mu'\}$ also shown as $\mu[\sigma > \mu'$.

Property 2: Boundness and liveness are other important properties of HPN that correspond to overflow freeness and deadlock freeness respectively. An HPN is said to bound *iff* $\exists$ a fixed number

$$\beta \in IN | \forall \mu \in R(\mu_0) \wedge \forall p \in P : (\neg p \in P_d \rightarrow \mu(p) \le \beta) \wedge \left(\neg p \in P_b \rightarrow \sum_{\forall b \in \mu(p)} b \le \beta \right).$$

Similarly, HPN is said to be live *iff* $\exists \mu : t$ is enabled $\forall t \in T \wedge M \in R(M_0)$.

Further the reachability analysis plays an important role for analysing the structural properties of HPN. Before getting into the details of this analysis the following definitions are provided in this regard (Viswanadham and Narahari 1992, Dicesare *et al*. 1993, Chen *et al*. 2005):

Definition 6: The transitive closure of the reachability relation comprising of all marking reachable from μ_0 by firing a sequence of transition is called reachability set $R(\mu_0)$.

Definition 7: For a marked HPN, with initial marking μ_0, the 'reachability graph' is a directed graph (V_μ, E_μ), where $V_\mu = R(\mu_0)$ is the set of vertices and E_μ is the set of directed arcs given by $\mu_1, \mu_2 \in E_\mu$, *iff* $\mu_1, \mu_2 \in R(\mu_0)$ and $\neg\exists \, \sigma = \{t_1, t_2, \ldots, t_n\}$: $\mu_1[\sigma > \mu_2$, where $n \in IN$.

This μ reachability graph is used for evaluating the performance of HPN. The procedure followed for the evaluation is exemplified in the next section, with a lucid example.

4. Modelling, performance evaluation and risk management of a supply chain using proposed HPNs

This section lucidly delineates the modelling, performance evaluation and risk management technique for a generic supply chain comprising of a productive system. As discussed earlier, a productive system is a network structure of several facilities, in which each facility serves as an input for the succeeding facility (Tsinarakis *et al*. 2006). As an example, a single item-4 stage supply chain is used for exemplifying the proposed approach. The supply chain taken into consideration comprises of 1-supplier, 1-manufacturer, 1-retailer and 1-customer. Before modelling the network it is necessary to collect the information that helps in modelling the supply chain network.

4.1 *Prerequisite*

Each element of the supply chain is characterised by various elements that affect the efficiency and effectiveness of the network. Generally, these elements are based on the inventory systems and logistic network adopted in a supply chain. The logistic system is based on the availability/unavailability of transportation resources and the inventory system, which is decided on the basis of model (viz. Q-model or P-model) adopted for the flow of material and information in the network (Anderson *et al.* 1997). For instance, a supply chain network adopts a fixed order quantity model for the inventory replenishment. In such a network, the parameters concerned with supplier and manufacturers are transportation resources, its economic order quantity, re-order point, demands of the manufacturer, etc. Similarly, the elements characterising the retailers are the daily demand of the customers, number of items bought by customers, inventory level and delivery lead of the transportation resources of manufacturers, etc. In a similar manner, the elements characterising the levels of supply chain with a fixed time period model can be defined. The Petri-net model designed on the basis of the abovementioned factors is detailed in the upcoming section.

4.2 *Modelling*

The relevance of hybrid Petri-net for supply chain modelling is provided in this section. As discussed earlier, a supply chain is a dynamic system and thus Petri-nets have been proven to be a powerful tool for its modelling. In this modelling technique, the prime feature of a supply chain, e.g. inventory replenishment, manufacturing and distribution processes are modelled using discrete places. Further, the batch features (e.g. orders, deliveries, inventory level, etc.) of discrete processes are described using the batch places. The financial flow occurring in the supply chain is described using the cost places.

Using the introduced HPNs a compact model of the productive supply chain is provided in Figures 3 and 4. These two figures show the HPN model of the supply chain with fixed order quantity and fixed time period inventory system respectively. The two models differ from each other on the basis of their replenishment technique adopted for the inventory system. In the model shown in Figure 3, replenishment is performed by placing optimal order (Q_{opt}), when the position of inventory drops below reorder point (RP). However, in the model provided in Figure 4, inventory is replenished at the end of a predetermined time period (e.g. T_R, T_S, etc.) only. A detailed description of the HPN model is given in an upcoming section. In the following discussion, the places and transitions are named after their types, e.g. p_d for discrete place, p_b for batch places, etc. Further, for ease of the reader, arrows are used for material and information flow, whereas dashed arcs are used for financial flow.

In the Q-model, the places p_{D_1} and p_{B_1} represent on-hand inventory of supplier and outstanding orders – the orders that have been dispatched but not reached to the inventory – equivalent to EOQ of p_{D_1}. These places are linked with t_{E_1} that fire when inventory level drops below the re-order point. The firing of t_{E_1} creates tokens in p_{D_1} as per the information received p_{B_1}. Note that once this condition is satisfied the transition keeps on firing indefinitely. To avoid this loophole, p_{D_5} is introduced that represents the sum of on-hand inventory and outstanding orders, i.e.

$$\mu(p_{D_5}) = \sum_{b_{p\in\mu(p_{B_2})}} b_{p_{B_2}} + \mu(p_{D_1}).$$

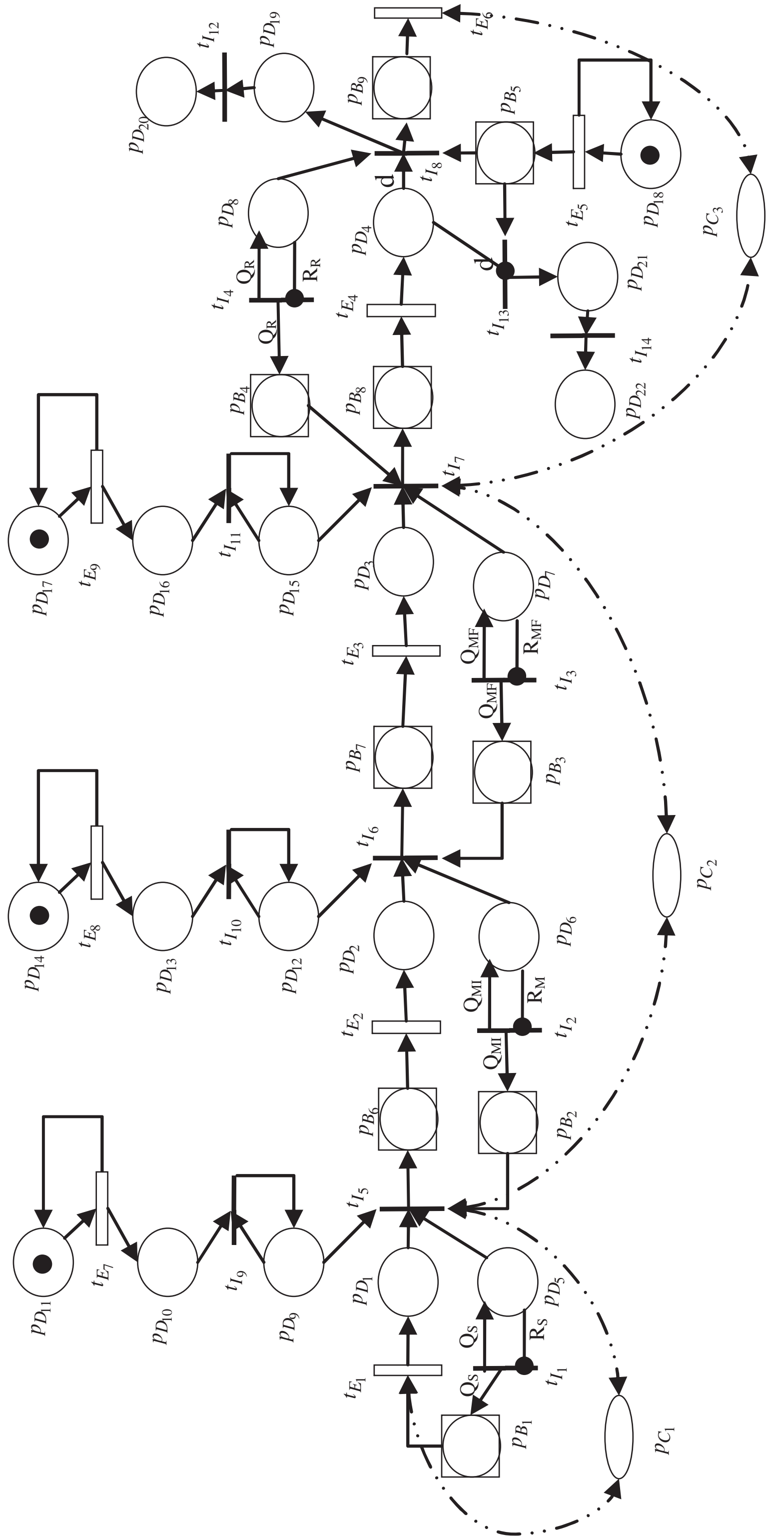

Figure 3. Q-model of a simple productive supply chain.

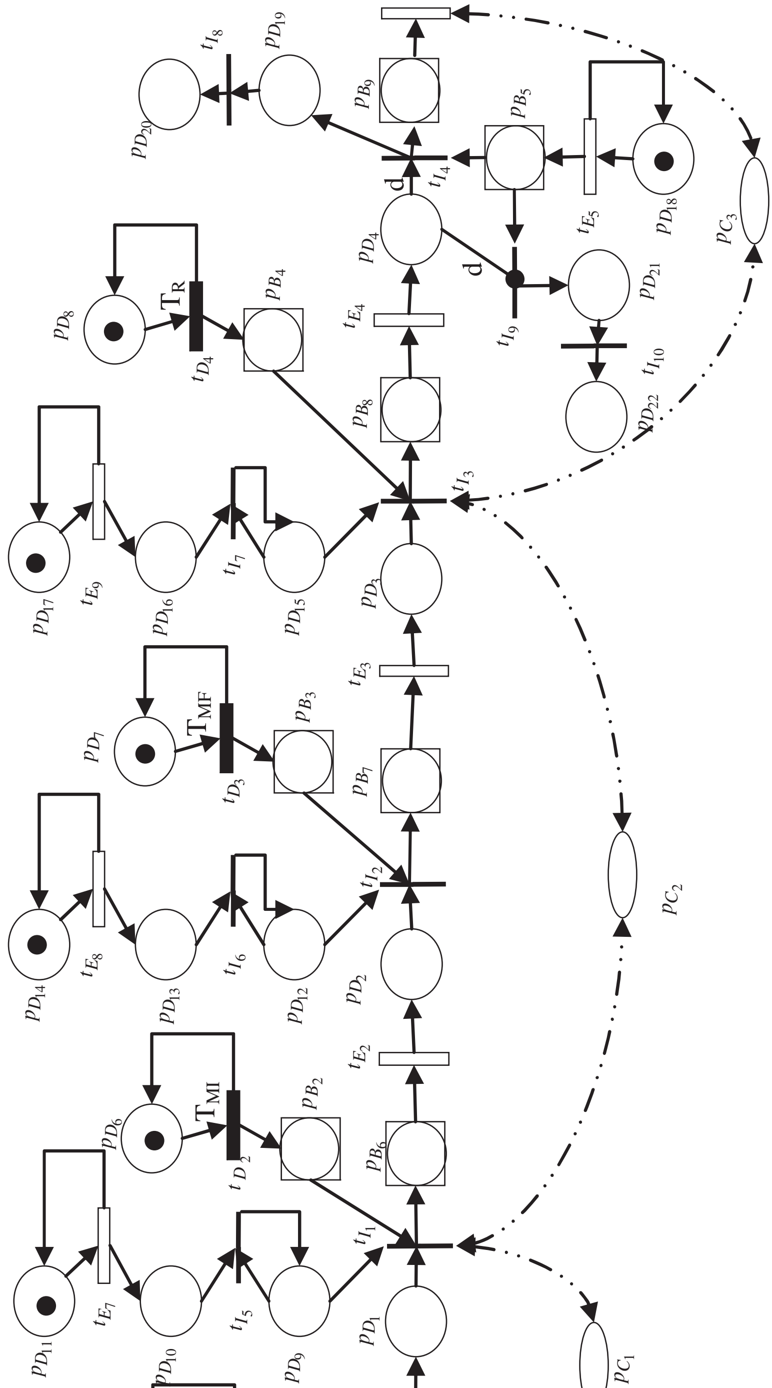

Figure 4. P-model of a simple productive supply chain.

Similarly, $p_{D_{2-4}}$, $p_{B_{2-4}}$ and $t_{E_{2-4}}$ are used to represent the on-hand inventory, outstanding orders and inventory replenishment for manufacturers (viz. raw material and end product) inventory and retailers respectively. Here $p_{D_{6-8}}$ are of similar significance, i.e. to represent the sum of on-hand and outstanding orders of respective inventory. It is also evident from Figure 3 that $p_{D_{1-8}}$ are linked with $t_{I_{1-4}}$ that fires as per the information received from the $p_{B_{2-5}}$. It implies that respective inventories offer products to their succeeding department on their requirement to fill the inventory from reorder point to EOQ level. The transaction between two successive inventories, performed through transportation or manufacturing system is referred to by $p_{B_{6-9}}$ and $t_{E_{2-4}}$, where the former depicts batch size of products to be transferred and the latter is used to portray the transportation facility.

In the supply chain model portrayed in Figure 3, the final destination for products is 'customer' that fulfils their demand from the retailer's inventory. Customers' arrival is represented by place $p_{D_{18}}$ and transition t_{E_5} that fires after a duration drawn from the probability of customer inter-arrival time and batch place p_{B_5} reveals demand of the customers. From the model, it is evident that every time a customer arrives, his demand (d) is fulfilled if $\mu(p_{D_4}) \geq d$. However, the satisfaction of customer's demand vanishes tokens from p_{D_4} and creates 1 token in $p_{D_{19}}$ and then in $p_{D_{20}}$ to record the number of satisfied customers. However, if the $\mu(p_{D_4}) < d$, i.e. quantity of items at the retailer's inventory is not sufficient to fulfil customer demand, then $p_{D_{21}}$, and $t_{I_{13}}$ and $t_{I_{14}}$ fires to create a token in $p_{D_{22}}$ where the record of unsatisfied customers is kept.

Therefore, when the inventory level of the retailer drops below its re-order point, i.e. $\mu(p_{D_4}) \wedge \mu(p_{D_8}) < RP_R$ then t_{I_7} fires and creates tokens in p_{D_4}, equivalent to its optimal order quantity. The firing of t_{I_7} depends on the presence of a single token in $p_{D_{17}}$ (i.e. on the availability of transportation resources at the manufacturer's end) and adequate products in the manufacturer's finished goods inventory, i.e. $\mu(p_{D_3}) \geq EOQ_R$. Its firing creates EOQ_R tokens in p_{D_4} and p_{D_8} by simultaneously eliminating EOQ_R token from p_{D_3} and p_{D_7}, and 1 token from $p_{D_{15}}$ via p_{B_8} after a time delay of transport delivery lead time. Similarly, when the manufacturer's inventory level drops below its re-order point, i.e. $\mu(p_{D_3}) \wedge \mu(p_{D_7}) < RP_{MF}$ (re-order point of finished goods inventory) then t_{I_6} fires by creating tokens in p_{D_3} and p_{D_7}, and eliminating it from p_{D_2}, p_{D_6} and $p_{D_{14}}$. Note that the firing of t_{I_6} depends on the presence of production resources (i.e. $\mu_{D_{12}} = 1$) and adequate inventory level of the manufacturer's raw material inventory (i.e. $\mu(p_{D_2}) \geq EOQ_{MI}$; MI is the manufacturer raw material inventory). In a similar pattern, tokens are created in $p_{D_{1-2}}$ and $p_{D_{5-6}}$. The set of places and transition represented by $\{p_{D_9}, t_{I_9}, p_{D_{10}}, t_{E_7}, p_{D_{11}}\}$, $\{p_{D_{12}}, t_{I_{10}}, p_{D_{13}}, t_{E_8}, p_{D_{14}}\}$ and $\{p_{D_{15}}, t_{I_{11}}, p_{D_{16}}, t_{E_9}, p_{D_{17}}\}$ represent the availability of the supplier's transportation resources, the manufacturer's production resources and the manufacturer's transportation resources respectively.

The remainder of this section deals with the description of a P-model supply chain network for making a clear distinction between the two inventory replenishment models (i.e. P-model and Q-model) that guides the flow of material, information and money in a supply chain. In a supply chain with a fixed time period inventory replenishment system (as shown in Figure 4) all places and transitions are of similar significance as that of a Q-model supply chain, except $p_{D_{5-8}}$ and $t_{I_{1-4}}$ of Figure 3. In a fixed time period supply chain system $t_{I_{1-4}}$ fires after a predetermined duration of time (i.e. inventory are replenished after a fixed time period) defined by the firing delay of $t_{D_{1-4}}$. Here a single token is created in $p_{D_{5-8}}$ after a deterministic time that enables $t_{D_{1-4}}$ and later t_{E_1} and $t_{I_{1-3}}$, respectively by collecting the information from $p_{B_{1-4}}$ regarding the tokens to be removed from $p_{D_{1-3}}$. For instance, t_{D_1} fires after T_s unit of time and enables t_{I_1} for firing, thus removing tokens from p_{D_1} and creating it in p_{D_2} after an exponential time delay drawn from the distributor delivery lead time.

4.3 *Performance evaluation*

After designing the HPN model of a supply chain network, the next step is to evaluate the performance of the modelled system. The performance of an HPN model is evaluated by studying its inherent qualitative and quantitative properties. These properties can easily be verified by analysing the marking process of an HPN, characterised by probabilistic selection of immediate, exponential and deterministic transitions to be fired next. The qualitative properties such as reachability, liveness, boundness, etc, can be studied by analysing the marking process of HPN similar to those of traditional Petri-net approaches (Murata 1989, Desrochers and Al-Jaar 1995, Proth and Xie 1997, Viswanadham and Raghavan 2000, Chen *et al.* 2005). However, in order to deal with quantitative approaches, analysis is performed by dealing with the stochastic marking process, which is a continuous time Markov chain without any deterministic transition (Viswanadham and Raghavan 2000, Chen *et al.* 2005). The basis for analysing such a class of HPN without any deterministic transition is its steady state μ marking, usually expressed in terms of performance indices.

The performance of HPN is evaluated by constructing the μ-reachability graph $G = (\mu, t, q)$; where, μ marking reachable from initial marking represents nodes of the graph, t represent the label of transition and q, i.e. the discrete firing index is marked with transition batch firing (Viswanadham and Narahari 1992, Chen *et al.* 2005). The graph is constructed with two types of marking, viz. 'vanishing' and 'tangible' which are associated with the firing of immediate and timed transition respectively. Since immediate transitions offer zero firing time, they are therefore eliminated from G for generating a reduced μ reachability graph G' (Viswanadham and Narahari 1992, Chen *et al.* 2005). It is then used to construct a transition rate matrix

$$\Xi = [\varsigma_{ij}] : \sum_{\forall j} \varsigma_{ij} = 0 \wedge \varsigma_{ii} = -\sum_{\forall j \backslash i} \varsigma_{ij},$$

where ς_{ij} is the transition rate from μ_i to μ_j. This matrix is further used to determine the steady state distribution vector $\chi = \{\chi_1, \chi_2, \ldots, \chi_k\}$ with the help of a linear equation system given by $\chi.\Xi = 0$ such that $\sum_{j=1}^{k} \chi_j. = 1$. This vector is further used for evaluating the performance of the model. Some of the common performance indices measured using the steady state vector χ are discussed as follows:

(a) The mean number of tokens in a place is given by (Chen *et al.* 2005):

$$\forall p \in P_d \quad \overline{\mu}(p) = \sum_{j:\mu_j \in R(\mu_0)} \mu_i(p)\chi_i \tag{15}$$

$$\forall p \in P_b \quad \overline{\mu}(p) = \sum_{j:\mu_j \in R(\mu_0)} |\mu_i(p)|\chi_i \tag{16}$$

$$\forall p \in P_c \quad \overline{\mu}(p) = \sum_{t=0}^{T} C_t / T \tag{17}$$

where $|\mu_i(p)|$ represents the cardinality of set $\mu_i(p)$; t represents the time slot decided by decision makers by dividing the planning horizon T for storing the information in cost places regarding the cost used up in their respective departments.

(b) Probability that place p have k number of tokens (Viswanadham and Narahari 1992):

$$prob(p, k) = \sum_{j:\mu_j \in R(\mu_0)} \chi_j \tag{18}$$

The details regarding the measurement of other performance measures can be obtained in Viswanadham and Narahari (1992), Dicesare *et al.* (1993), and Chen *et al.* (2005). In order to clarify the procedure of performance evaluation, a section of a supply chain is considered for performance evaluation.

Example 2: Consider an inventory system of the supplier having a fixed order inventory replenishment policy with an initial inventory level of 8, EOQ level of 5 and re-order point 3. It is assumed that all the timed transitions in its HPN model fire with exponential time distribution. The reduced μ-reachability graph of concern HPN model is shown in Figure 5, where λ_1 and λ_2 represents the exponential time distribution of transition t_{E_1} and t_{E_2}respectively. Note that the transition t_{E_2} and t_{I_5} are synthesised to form a closed inventory system. The transition matrix of this system is given by:

$$\Xi = \begin{bmatrix} -\lambda_1 & \lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda_1 & \lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\lambda_1 & \lambda_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\lambda_1 & \lambda_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\lambda_1 & \lambda_1 & 0 & 0 & 0 \\ \lambda_2 & 0 & 0 & 0 & 0 & -(\lambda_1+\lambda_2) & \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 & 0 & 0 & -(\lambda_1+\lambda_2) & \lambda_1 & 0 \\ 0 & 0 & \lambda_2 & 0 & 0 & 0 & 0 & -(\lambda_1+\lambda_2) & \lambda_1 \\ 0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & 0 & -(\lambda_1+\lambda_2) \end{bmatrix}$$

This transition rate matrix is further used to evaluate the steady state probability vector $\chi = \{\chi_0, \chi_2, \ldots, \chi_8\}$. This probability vector is further used to evaluate the performance of the system, using Equations (15)–(18). For instance, average inventory level, i.e. mean number of tokens in p_{D_1} given by $\overline{\mu}(p_{D_1})$, probability of stock-outs, i.e. $prob(p_{D_1}, 0)$, etc.

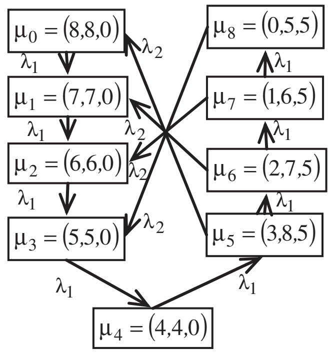

Figure 5. Reduced-μ reachability graph for Example 2.

4.4 *Risk assessment*

The aforementioned procedure for evaluating the performance of an HPN model provides a measure for several indices that reduce the efficiency of a supply chain such as stock outs, etc. In order to deal with these shortcomings this section presents an effective methodology for identifying the risky events that are responsible for reducing the efficacy of supply chain (Hallikas *et al.* 2004, Rossi 2005). While executing this process, initially risky events are identified by analysing the sequence of markings.

For example, starting with the initial sequence of marking of an HPN model of a productive supply chain, the sequential firing of transitions given by $\{t_{E_5}, t_{I_8}, t_{E_6}, t_{I_{12}}\}$, leads to a marking when $t_{I_{13}}$ fires, which shows a risky event namely forecasting inaccuracy of retailers. Similarly, other risk factors, such as forecast inaccuracy of retailer, supplier, etc, and manufacturing lead time higher than standard value, etc., can also be identified with the help of firing sequence of transitions.

Once the risky events present in a supply chain are identified, the next step is the assessment of these events, responsible for a vulnerable supply chain. These risk factors are generally assessed on the basis of two parameters namely: probability of occurrence and their severity of consequences. Here the former is evaluated by computing the probability of occurrence of risky events (RE) given by Viswanadham and Narahari (1992) and Chen *et al.* (2005):

$$Prob(RE) = \sum_{j \in S} \chi_j \qquad (19)$$

where $\{S = (j \in (1, 2, \ldots, s) : C \textit{ is satisfied in marking } \mu_j)\}$. Similarly, the consequence of risky events is assessed by computing the extra cost used up in a supply chain due to their occurrence (Hallikas *et al.* 2004). The extra cost is computed by summating the cost stored in cost places for a particular span of time (Definition 5). It implies that severity of risky events is compared on the basis of average cost dissipated in a particular part of the supply chain or total supply chain, with respect to cost used up in ideal conditions. Subsequently, the risky events are appraised on a five class scale; namely, probability assessment scale and consequence assessment scale, designed for the aforementioned two parameters. Among the two assessment scales, the former one is planned for categorising the risky events based on their occurrence named as rare, improbable, moderate, probable and frequent events. Similarly, the asperity of outcomes of risky events is ranked based on their – very low, low, medium, high and very high – impact on supply chain network (Hallikas *et al.* 2004, Rossi 2005). After categorising the risky events based on the former two assessment scales, the net influence of these events is estimated to provide an aid to the decision makers for reducing the cost of a supply chain. The net assessment of risky factors provides an overall view of a vulnerable supply chain and indicates to the experts the most important risks that require major attention for the enhancement of profitability of the network. However, in the case of a dynamic supply chain, the assessment of risky events using the statistical technique is quite an abstruse task. Therefore, the next section deals with a simulation technique for the assessment of risky factors present in a supply chain network (Dicesare *et al.* 1993, Tsinarakis *et al.* 2006). Prior to it, the next section portrays the modelling and performance evaluation of a real life industrial supply chain network.

5. Case study and simulation results

The applicability of the proposed methodology is demonstrated on a real life industrial supply chain, shown in Figure 6. As apparent from Figure 6, a supply chain system is composed of two suppliers (S1 and S2), two manufacturers (M1 and M2), one retailer (R) and several customers. Here the prime sources of raw materials are supplier 1 and 2, providing solid and hollow cylinders, respectively. Solid cylinders are transferred to M1 for making screws and bolts, which are further assembled with a hollow cylinder by M2. The end product of M2 is then transferred to the retailer which is in direct contact with the customers. The above-mentioned supply chain is mainly characterised by three types of flows: material flow, information flow and financial flow. The material flow takes place through transport facilities included in the network, but it directly or indirectly affects the information and financial flow of the system. It is obvious that material flow depends on the inventory levels maintained in the supply chain. Thus the present case study deals with two types of inventory models namely Q-model and P-model. In both these models, the upstream inventories are used to fulfil the demand of downstream customers as per the information received from them.

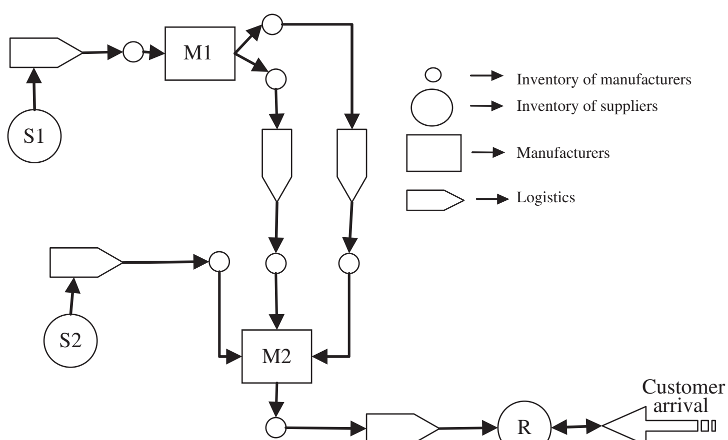

Figure 6. Configuration of industrial supply chain.

The HPN model of the industrial supply chain taken into consideration is shown in Figures 7 and 8. The interpretation of places and transitions used in the model are provided in Tables 1 and 2. The former Figure 7 shows the HPN model of the supply chain with a fixed order quantity inventory replenishment policy and Figure 8 exhibits the model having a fixed time period replenishment policy. The places and transitions used in the HPN model of the industrial supply chain are of similar significance as that of a simple productive supply chain, detailed in Section 4. The overall HPN models of a supply chain consist of 54 transitions and 83 places including five cost places. In the model, different cost places are provided for supplier and manufacturers present in the network. These cost places stores the information about the cost used in their respective department per unit of time slot.

Once the HPN model is established, the second step is the dissection of the model for its performance evaluation. The procedure for the analytical evaluation of a supply chain system, using reachability graphs and state transition equations, is provided in Section 4.3. However, in the present case, the construction of its reachability matrix and state transition equation is a complicated task (Chen *et al.* 2005, Desrochers *et al.* 2005). Therefore, the performance of the model is evaluated via simulation for assessing its inherent properties. Some of the possible goals of simulation are to evaluate the inventory levels, total cost incurred in a supply chain and their individual departments and

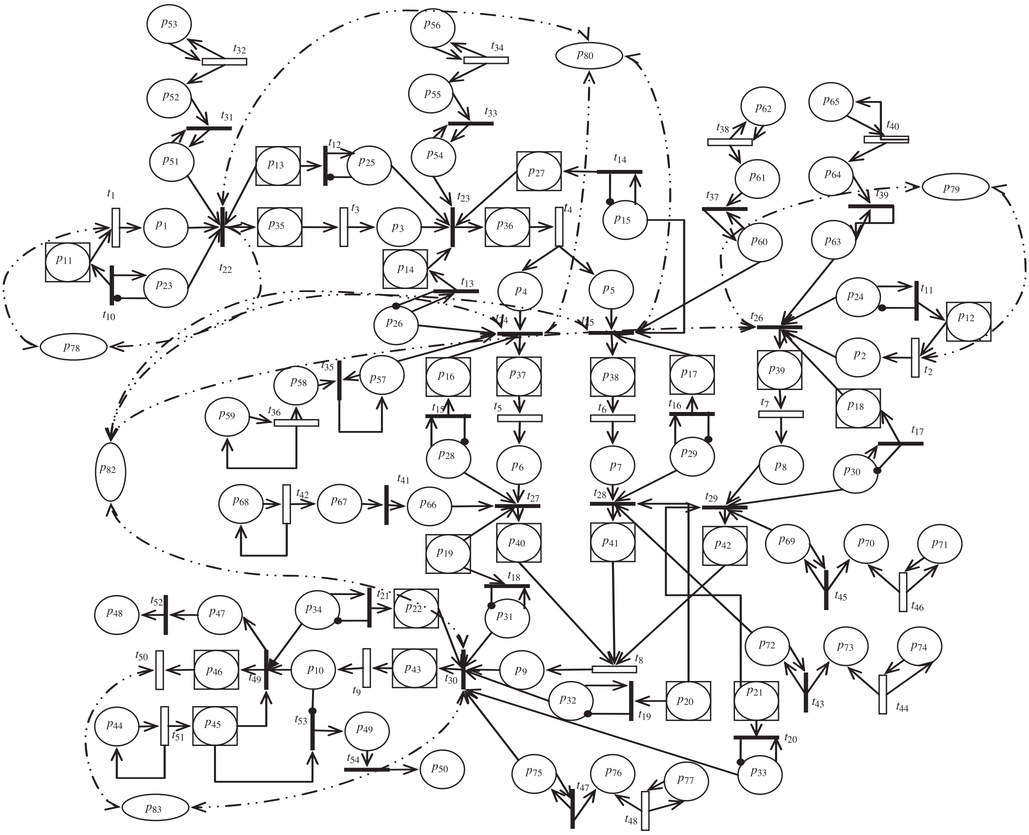

Figure 7. HPN model of the industrial supply chain with Q-model for inventory replenishment.

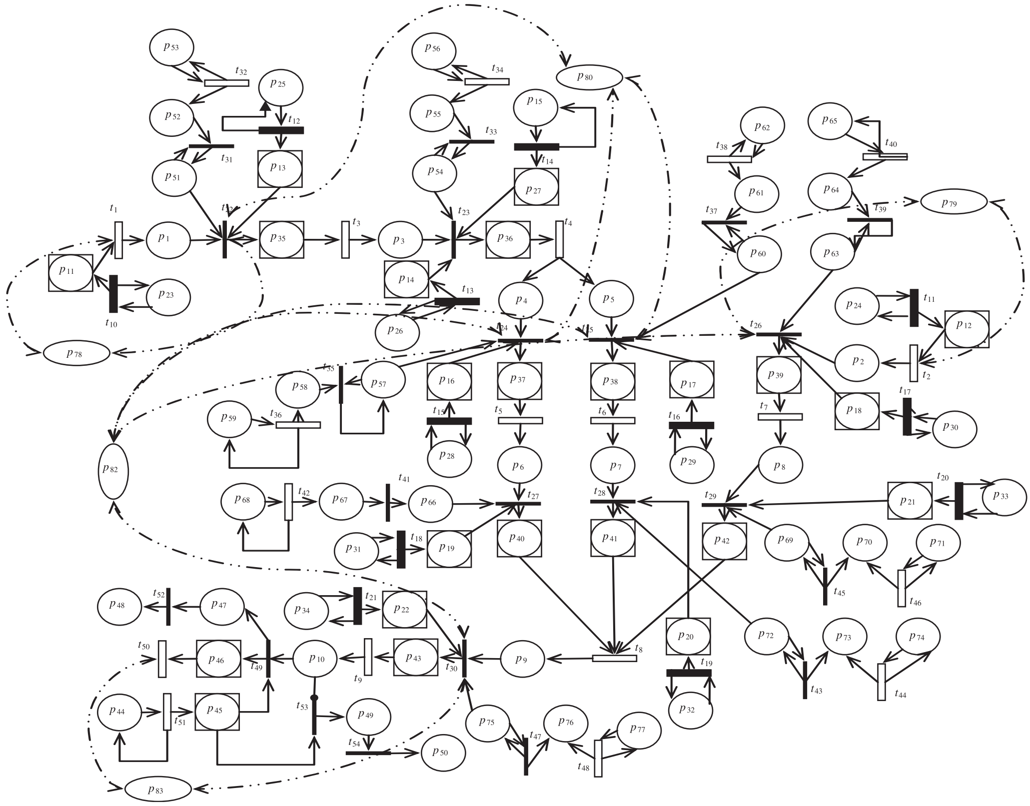

Figure 8. HPN model of the industrial supply chain with P-model for inventory replenishment.

measuring the performance of two inventory models (Desrochers *et al.* 2005). The simulation of an HPN model is performed by analysing the marking process of the model and firability of the transitions. In this paper, simulation is performed by implementing it on a C++ program, and the output is considered as the average of multiple replications.

In order to perform the simulation on an HPN model of a supply chain, it is necessary to define the parameters utilised in the simulation process. Due to the dynamic nature of the system, the simulation is performed 20 times, each with a planning horizon of 250 days. Other details used to accomplish the simulation are provided in Table 3. The data tabulated in Table 3 can easily be correlated with the structural elements of an HPN model, e.g. the lead time is considered as the firing delay of transitions defined for the transportation system and manufacturing system. Similarly, the forecasting horizon and other costs provided in Table 3 are used to calculate the EOQ and re-order point of the inventory level, particularly in the case of a Q-model supply chain. However, in the case of the P-model, the forecasting horizon is considered as the review period for the inventory replenishment. In both the cases, the customers are assumed to arrive after a time lag of exp(0.2) days with a demand of *discrete* $(5-10)$ products. The demand is used to determine the EOQ level of inventories for a planning horizon, whereas the time lag in customer arrival is considered as the time slot for storing the data in the cost places. In a large planning horizon, storing the data for each unit of time (especially seconds) would be

Table 1. Interpretation of places in Figures 7 and 8.

Places	Description	Model
I, F	Notations for initial and final inventory of products	Both
ISC	{S1, S2, M1I, M1F1, M1F2, M2I1, M2I2, M2I3, M2F,R}	Both
(A/U)T	Availability/unavailability of transportation resources	Both
(A/U)P	Availability/unavailability of production resources	Both
$p_{1\text{–}10}$	Available inventory of **ISC**	Both
$p_{11\text{–}18}$	Outstanding orders of {S1, S2, M1I, M1F1, M1F2, M2I1, M2I2, M2I3}	Q
$p_{19\text{–}21}$	Outstanding orders for M2F (individual place for raw products to be assembled)	Q
P_{22}	Outstanding orders for R	Q
$p_{23\text{–}30}$	Inventory level of {S1, S2, M1I, M1F1, M1F2, M2I1, M2I2, M2I3} i.e. $\mu(p_{23-30}) = \left(\mu(p_{1-8}) + \sum_{\forall b \in \mu(p_{11-18})} b\right)$	Q
$p_{31\text{–}33}$	Inventory level of M2F i.e. $\mu(p_{31-33}) = \left(\mu(p_9) + \sum_{\forall b \in \mu(p_{19-21})} b\right)$	Q
p_{34}	Inventory level of R, i.e. $\mu(p_{34}) = \left(\mu(p_{10}) + \sum_{\forall b \in \mu(p_{22})} b\right)$	Q
$p_{11-13,16-18,22}$	Order quantity to replenish S1, S2, M1I, M2I1, M2I2, M2I3, R	P
$p_{14-15,19-21}$	Quantity of products being disassembled/assembled	P
p_{23-34}	Represents the status of t_{10-21}	P
p_{35-43}	Orders transported from {S1-M1I, M1I-M1F, M1F1-M2I1,M1F2-M2I2, S2-M2I3, M2I1-M2F, M2I2-M2F, M2I3-M2F, M2F-R}	Both
p_{44}	Arrival of customer	Both
p_{45}	Outstanding orders of customers	Both
p_{46}	Order delivered to customers	Both
p_{47-48}	Number of satisfied customers	Both
p_{49-50}	Number of unsatisfied customers	Both
p_{51-53}	**(A/U)T** from S1-M1I	Both
p_{54-56}	**(A/U)P** M1	Both
p_{57-59}, p_{60-62}, p_{63-65}	**(A/U)T** from M1F1- M2I1, M1F2-M2I2, S2-M2I3	Both
p_{66-68}	**(A/U)P** for M2I1, M2I2, M2I3 p_{69-71}, p_{72-74}	Both
p_{75-77}	**(A/U)T** from M2F-S2	Both
p_{78-83}	Cost places for {S1, S2, M1, M2, R}	
	For elucidation	

Types	Places (maximum no. of tokens)
Inventory	p_{1-10} (EOQ), p_{23-34} (EOQ + outstanding orders)
Information	{p_{11-22}, p_{35-43}}Batch size of order
Status	{p_{44}, p_{47-50},p_{51-77}} (1)
Cost	p_{78-83} (Stores cost per unit time)

Table 2. Interpretation of transition in Figures 7 and 8.

Transition	Description	Model
$t_{1-3,5-7,9}$	Inventory replenishment of {S1, S2, M1I, M2I1, M2I2, M2I3, R}	Both
$t_{4/5}$	Assembly/disassembly operation	Both
$t_{10-12,15-17,21}$	Placement of orders for {S1, S2, M1I, M2I1, M2I2, M2I3, R}	Both
$t_{13-14/18-20}$	Placement of orders for disassembly/assembly	Both
$t_{22,24-26,30}$	Interface with Transport {S1-M1I, M1F1-M2I1,M1F2-M2I2, S2-M2I3, M2F-R}	Both
$t_{23/27-29}$	Start of disassembly/assembly operation	Both
t_{30-48}	Used for **(A/U)R** and **(A/U)P**	Both
t_{49}	Preparation for delivery of orders	Both
t_{50}	Order delivered to customers	Both
t_{51}	Arrival of customer	Both
t_{52}	Used for satisfied customers	Both
t_{53-54}	Used for unsatisfied customers	Both

Table 3. Data set for the performance evaluation of industrial supply chain network.

	Inventory	Forecasting horizon (accuracy)	Holding cost per unit	Setup cost	Lead time (SD)	Inventory cost
Retailer		15 (0.9)	5.0	50	1.0 (0.1)	0.333
Manufacturer 2 (assembler)	EP	35 (0.9)	3.5	100	5.0 (0.1)	0.100
	RP 1	50 (0.85)	3.0	140	2.5 (0.3)	0.060
	RP 2	50 (0.85)	3.0	140	3.0 (0.3)	0.060
	RP 3	60 (0.85)	3.0	140	3.0 (0.3)	0.060
Manufacturer 1 (disassembly)	EP 1	60 (0.85)	2.5	160	4.0 (0.3)	0.042
	EP 2	60 (0.85)	2.5	160	4.0 (0.1)	0.042
	RP	80 (0.85)	2.0	200	5.0 (0.05)	0.025
Supplier 2		130 (0.8)	1.5	400	8.0 (0.0)	0.012
Supplier 1		130 (0.8)	1.3	400	8.0 (0.0)	0.010

Note: SD, standard deviation.

an abstruse task, therefore the customer arrival time is considered as the time slot for recording the cost used up in different sections of a supply chain.

The simulation process is started with an assumption that inventories are filled up to their EOQ level. The EOQ levels of inventories are determined using Equation (2) and the data provided in Table 3. Due to the dynamic nature of the system, it is often that the upstream level is unable to satisfy the requirements of the lower level, thus orders are backordered. In this case study the backorder cost is assumed to be 10 times the holding cost.

Using the aforementioned considerations, the simulation is performed on Q- and P-models and the mean value over the 20 replications is provided in Tables 4 and 5. The simulation is performed on an ideal supply chain system with 100% accuracy in

Table 4. Performance evaluation of industrial supply chain with Q-model inventory replenishment policy.

Inventory	Average inventory	Standard deviation	BO level	Standard deviation	Average cost per unit day	Standard deviation
Retailer	54.93	0.89	0.25	0.03	324.23	0.42
M2E	228.45	3.00	1.36	0.19	6187.57	28.11
M2I1	249.20	12.88	31.28	1.64		
M2I2	251.09	10.48	31.28	1.64		
M2I3	251.09	10.48	31.28	1.64		
M1F1	429.07	37.50	20.93	2.57	4849.68	15.79
MIF2	429.07	37.50	20.93	2.57		
M1I	566.52	13.00	38.90	4.18		
Supplier 1	1164.03	37.60	26.37	1.59	2155.58	13.99
Supplier 2	1011.00	17.69	3.32	1.13	1646.86	5.83
Total					15,163.92	164.12

Table 5. Performance evaluation of industrial supply chain with P-model inventory replenishment policy.

Inventory	Average inventory	Standard deviation	BO level	Standard deviation	Average cost per unit day	Standard deviation
Retailer	61.98	1.11	0.31	0.18	389.25	0.33
M2E	256.54	15.46	1.84	0.52	6721.22	22.15
M2I1	298.55	26.22	24.22	2.01		
M2I2	310.12	28.83	29.32	2.01		
M2I3	310.12	28.83	29.21	2.01		
M1F1	479.25	40.07	32.23	5.57	5002.25	13.87
MIF2	479.25	40.07	32.23	5.57		
M1I	617.25	29.18	40.02	7.21		
Supplier 1	1313.22	54.52	119.12	8.95	2242.25	12.12
Supplier 2	1151.12	28.22	121.21	5.41	1846.58	4.51
Total					16201.55	149.21

forecasting horizon and 0% deviation in lead time excluding the discrepancy of customer arrival. Tables 4 and 5 show the mean of the basic performance indexes, viz. inventory level, backorder level and cost per day, etc., along with their standard deviations. It is evident from the two tables that the average values of basic performance indices for P-model are higher than Q-model. This additional cost and inventory level is mainly due to the extra stock maintained in case of time triggered inventory replenishment technique to take care of the stock-outs during the review period. Although, the results provided in Tables 4 and 5, show the superiority of Q-model, but in real life both the models offer approximately equal cost. It is basically

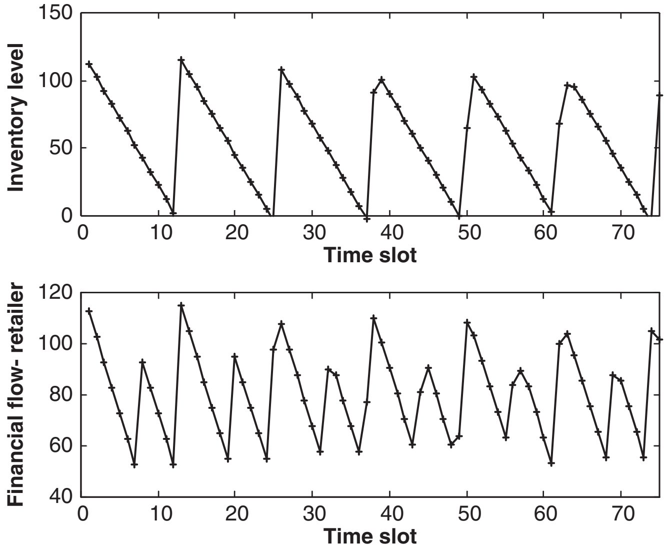

Figure 9. Discrepancy in inventory level and cost incurred in the retailer section.

due to a lesser amount of cost incurred in the transportation and manufacturing sections due to their use after a fixed time horizon.

The cost provided in Tables 4 and 5, shows the average cost incurred in five sections of the supply chain. It is obvious that it is evaluated by summating the cost stored in cost places. The variation in the cost stored in cost places are also shown in Figures 9 and 10. The figures clearly show that the variation in cost of different departments is relied on customers demand. The sudden variations in cost, shown by circles in Figure 10, used up in other departments are basically due to stock-outs accounted at their inventories level, transportation facilities, etc. The prime reasons behind the variations are the risky events present in the supply chain that are analysed and assessed in an upcoming section (Hallikas *et al*. 2004).

Risk management process primarily deals with the risk identification and its assessment. The former is the fundamental phase determined by analysing the marking process of the HPN modules. Starting from the initial marking of HPN, the transitions are fired as per their firability sequence and risky events present in the network. While analysing the sequence of fired transitions, the risky events obtained in the network are tabulated in Table 6. These factors depict the risk factors present in a supply chain network characterised by fixed order and fixed time period inventory replenishment policy.

The risk factors identified in the network are the prime reason behind the vulnerability of supply chains that creates numerous stock outs as well as an increase in cost of the supply chain (Hallikas *et al*. 2004, Rossi 2005). Therefore, in the subsequent step these factors are assessed on a probability assessment scale and consequence assessment scale. The similar simulation technique is used for assessing the risky events by evaluating the number of stock-outs cost of supply chain. During the experimentation procedure, simulation is conducted on the system by concerning with individual risky events separately and finally comparing the results with the ideal one. The comparison of results

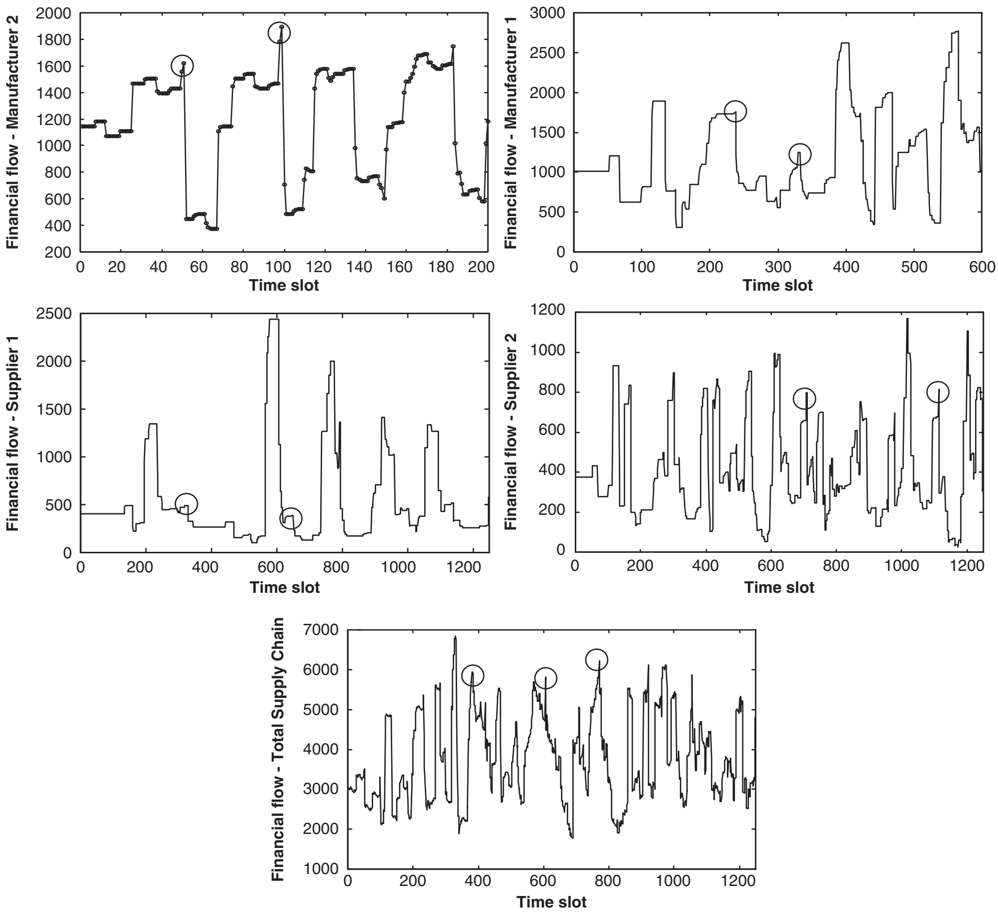

Figure 10. Variation in cost used up in different parts of a supply chain and total supply chain.

for the two supply chain networks, viz. Q-model and P-model are provided in Figures 11 and 12 respectively. The former two figures portray the average number of stock-outs (averaged over forecasting horizon) obtained at retailer's inventory w.r.t. the increasing level of risky events. Whereas, comparison of results in terms of average cost (averaged over per day) incurred in a supply chain are shown in Figure 13.

The above-mentioned results provide an aid to decision makers to identify the most relevant risky events present in a supply chain. While dealing with Q-model, it is observed that the events concerned with the delivery and manufacturing lead time are the prime risk factors behind the vulnerability of a supply chain. Whereas in case of P-model, the risk factors that increases the number of stock-outs and cost of a supply chain are related to forecasting made for inventory replenishments. In the present case study it is observed that the risky event 1 and 2, i.e. forecasting inaccuracy in evaluation of EOQ for retailer and transportation time from M2-R, are the critical factors that require the immediate attention of decision makers for P- and Q-model respectively. Thus it can be concluded that the order triggered model requires greater attention towards the accuracy of lead time, whereas the network characterised by time triggered inventory replenishment policy requires a careful eye on the accuracy of order size ordered for inventory replenishment.

Table 6. Risky events identified in Q-model and P-model of the industrial supply chain.

S.no.	Parts of supply chain	Risky events	Events numbers
1	Retailer	Forecast Inaccuracy in evaluation of *EOQ*	1
2	Manufacturer 2 (assembly)	Transportation time (M2-R) and production time (M2I-M2F) higher than standard value Forecast inaccuracy at the initial* and end product inventory	2–5
3	Manufacturer 1 (disassembly)	Transportation time (M2-R) and production time (M2I-M2F) higher than standard value, forecast inaccuracy at the initial and end product inventory*	6–9
4	Supplier 2	Transportation time (S2-M2) higher than standard value Forecast inaccuracy at the S2 inventory	10–11
5	Supplier 1	Transportation time (S1-M1) higher than standard value Forecast inaccuracy at the S1 inventory	12–13

Note: *Input/output inventory more than 1 are treated as single inventory.

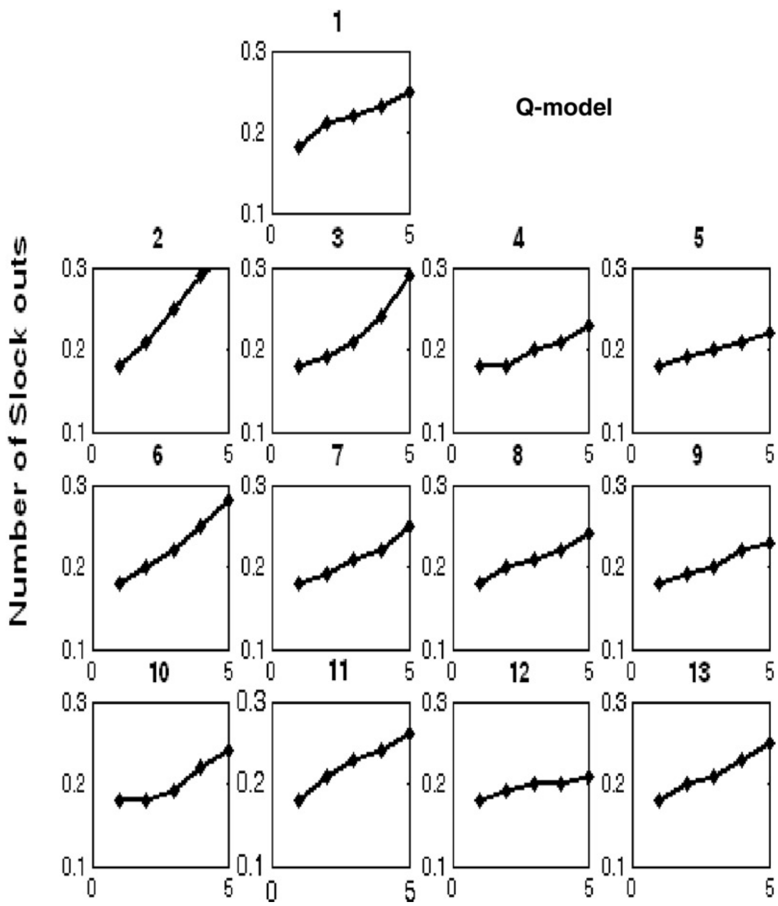

Figure 11. Effect of risky events prevailing in Q-model supply chain, on number of stock-outs.

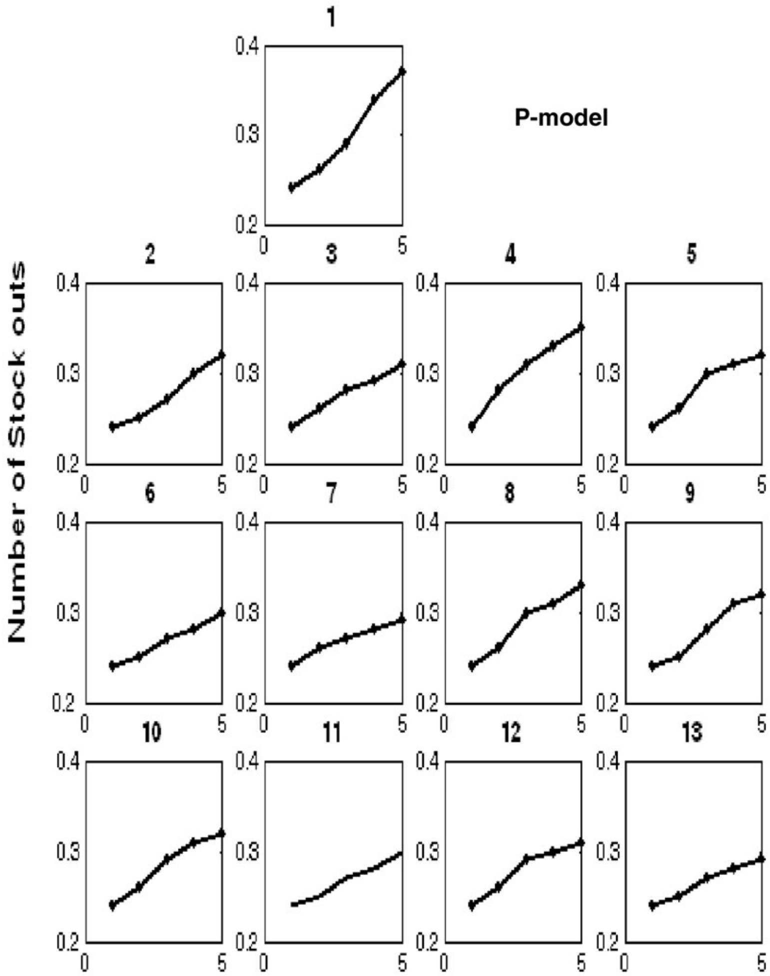

Figure 12. Effect of risky events prevailing in P-model supply chain, on number of stock-outs.

6. Deepening of the approach

The methodology proposed in this paper is suggested for modelling and studying the supply chain networks with stochastic demands. The HPN models are capable of modelling the complex supply chains comprising of all the basic production system. These models are analytically and efficiently evaluated for analysing their structural properties. However, the performance evaluation of such complex models using reachability analysis is not trivial, since its complexity explodes with the rise in number of reachable markings. Further, no papers concerned with this regard are present in literature that deals with the reachability graphs for such complex problems. Therefore, a simulation technique with multiple replications is used for evaluating the performance of a supply chain.

Similar simulation technique is used for identification and assessment of risk factors present in supply chain networks. The former two steps of risk management process reveal the gist of perilous factors and provide a focused approach for decision makers to resolve

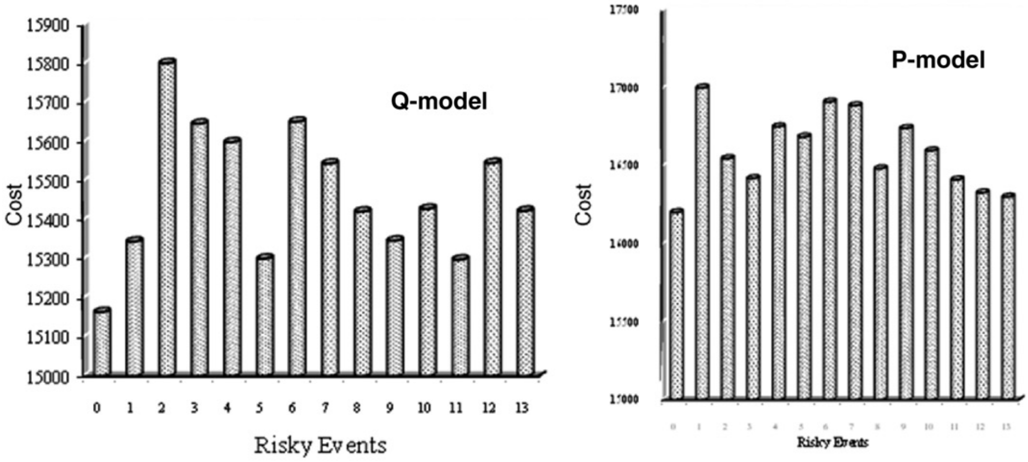

Figure 13. Comparison of cost used up in Q-model and P-model supply chains due to various risky events.

these factors. Mainly the assessment of risk helps the enterprises to decide 'where to focus on' and 'how to resolve them' for curtailing the vulnerability of a supply chain.

Albeit the proposed approach is effective and efficient for portraying the supply chain network along with the assessment of their risk factors. But it is not appropriate and sufficient for the deeper analysis of risk. Various inherent risk factors are also needed to be considered that can create a complete package for the firms to lucidly visualise the interruption, quality failures and delivery fluctuations in their networks. However, the factors excluded in this paper can only be portrayed in a real life environment. Thus the present approach is ample for firms to minimise the harmful consequences of risk factors bulging in their network.

One more drawback of the proposed model is the inherent complexity concerned with the analysis of an HPN model. This negative aspect is mainly due to the large amount of memory swallowed by the cost of places for storing data of cost spent in supply chain per unit of time slot. These memory usages have been reduced by increasing the span of the time slot and making it dynamic by deciding the time slots as per the arrival of customers. But the present authors decided not to use further reduction techniques, since it would have resulted in loss of significant details.

7. Conclusion

In this paper HPNs are introduced as an enticing tool for modelling, performance evaluation and risk analysis and assessment of a supply chain. The new hybrid Petri-net model is developed by incorporating the features of cost places and cost tokens in batch deterministic and stochastic Petri-nets. Application of the proposed methodology on real life supply chain problems demonstrates that the proposed approach is proficient in modelling the complex supply chain networks, including all basic production systems (viz. productive, assembly and disassembly) and all the basic inventory replenishment policy (viz. Q-model and P-model). The implementation of this methodology shows that the methods for evaluating the performance of HPNs are capable of the identification and assessment of risky factors present in the network. The assessment of risky events on probability and consequence assessment scale provides a helping aid for experts to focus on the prime aspects responsible for the vulnerability of a supply chain. Therefore, the overall methodology presented in the paper can be gauged as a complete package for industrial experts to model, evaluate the performance and identify and assess the risk factors prevalent in their network.

In an extension to this paper, the authors plan to develop a new modelling methodology as a sequel to hybrid Petri-net. The authors are working to develop hybrid Petri-nets with changeable structures (Jiang *et al*. 2000) and that are capable of modelling the changes performed in the network during the risk management process. Further, the paper can be extended to develop the new Petri-net models for the representation of supply chain models with demand disruptions, supply chain co-ordination, etc.

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