Bistable behavior of a two-mode Bose–Einstein condensate in an optical cavity

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Abstract

We consider a two-component Bose–Einstein condensate in a one-dimensional optical cavity. Specifically, the condensate atoms are taken to be in two degenerate modes due to their internal hyperfine spin degrees of freedom and they are coupled to the cavity field and an external transverse laser field in a Raman scheme. A parallel laser also excites the cavity mode. When the pump laser is far detuned from its resonance atomic transition frequency, an effective nonlinear optical model of the cavity–condensate system is developed under the discrete mode approximation (DMA), while matter–field coupling has been considered beyond the rotating wave approximation. By analytical and numerical solutions of the nonlinear dynamical equations, we examine the mean cavity field and population difference (magnetization) of the condensate modes. The stationary solutions of both the mean cavity field and normalized magnetization demonstrate bistable behavior under certain conditions for the laser pump intensity and matter–field coupling strength.

(Some figures may appear in colour only in the online journal)

1. Introduction

Recently, due to experimental advances in coupling a dilute gas of bosons to a single mode of an optical cavity, many theoretical and experimental works have been performed in order to explore and explain the physics of such complex systems. Bose–Einstein condensate (BEC) itself is a very rich platform which allows us to examine different properties of quantum systems such as quantum turbulence [1–3], quantum chaos [4] and entanglement [5]. A Bose–Einstein condensate can also be used to study nonequilibrium dynamics and decoherence in finite quantum systems [6] as well as for the calculation of fluctuation indices for atomic systems [7]. Moreover, if confined in an optical lattice [8], Bose–Einstein condensates provide the possibility to observe solid-state physics processes, such as localization [9, 10], in these atomic systems. Further uses of BECs are exemplified in transport problems [11], interaction driven instabilities [12],

the stability of boson–fermion gaseous mixtures [13], and Raman pump–probe experiments [14].

In a more complex setup, where a dilute condensate of bosons is confined inside a high-finesse optical cavity subject to external laser fields, if the cavity mode and laser fields are detuned far from the atomic transition frequency of the condensate atoms, the system is in the dispersive regime. Under the dispersive regime conditions, atom–field interaction provides an optical lattice for the condensate atoms which affects their mechanical motion. On the other hand, the atoms cause a position-dependent phase-shift of the cavity mode. As a result, the condensate–cavity system is highly nonlinear, with nonlocal nonlinearities which give rise to a series of interesting physical phenomena such as self-organization of condensate atoms, cavity enhanced superradiant scattering, Dicke quantum phase transition [15, 16] and optical bistability. The nonlinearity caused by atom–field interaction can be more effective [17] than
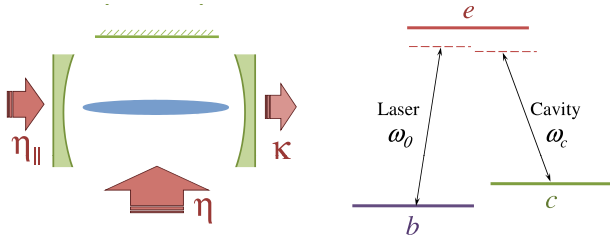
Figure 1. Left: schematic drawing of a BEC in a one-dimensional optical cavity subject to parallel and transverse laser fields. The cavity has a decay rate of κ. Right: two internal modes (b and c) of the BEC atoms are coupled by cavity field and laser field via the atomic excited state e in a Raman scattering manner. Both the laser field and cavity field are detuned from the atomic transition frequency.

the nonlinearity from atom–atom interaction, which itself can play a crucial role in the scattering dynamics of condensates [18].

The optical bistability, which is the focus of this paper, has been studied in spinor BECs [19, 20] and a two-component BEC with two modes coupled by a classical field [21]. In a system consisting of a single-mode BEC in an optical cavity, a transverse laser pump has also been used to control the bistability of cavity photons induced by a parallel pump [17]. In this work we consider a two-mode BEC in a one-dimensional optical cavity, where two laser fields are applied to the system, one parallel to the cavity axis and the other one perpendicular to it. Specifically we assume that the transverse pump is scattered to the cavity mode by condensate atoms in the Raman scheme [22, 23]. Under this condition, we examine the effect of transverse pump strength on the bistability of both the mean cavity photon number and normalized population difference (magnetization) of the two modes. The stationary state of the cavity field and condensate wavefunction are obtained under the discrete mode approximation (DMA), which has been shown to be reliable for similar systems [17, 24].

This paper is organized as follows. In section 2 first the model for our system is introduced, then the Hamiltonian of the system and equations of motion of cavity field and condensate are derived. Using the DMA, we solve the equations of motion for the steady state of the system in section 3, where we show how mutual bistability of the mean cavity photon number and magnetization take place under certain conditions for laser field intensities and cavity–atom coupling strength. Finally, we summarize our work in section 4.

2. Two-mode BEC in one-dimensional cavity

We consider a condensate of N atoms, each with two internal degrees of freedom, shown as states b and c, and an excited state e, in a one-dimensional cavity along the x axis. The cavity has a single mode with frequency ω_c and is subjected to a laser field with frequency ω_0, which is detuned far from the atomic transition, in directions parallel and perpendicular to its axis. If the transverse laser field interacts with matter

through Raman scattering (figure 1), then the full Hamiltonian of the system will have the following form

$$H = \sum_{j=b,c} \int \mathrm{d}x\, \psi_j^\dagger \left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V_j(x) + \hbar\omega_{bc}\delta_{j,c} \right) \psi_j$$
$$+ \sum_{i,j=b,c} \int \mathrm{d}x\, \frac{u_{ij}}{2} \psi_j^\dagger \psi_i^\dagger \psi_i \psi_j$$
$$+ \hbar\omega_c a^\dagger a - \mathrm{i}\hbar\eta_\parallel (a\,\mathrm{e}^{\mathrm{i}\omega_0 t} - a^\dagger\,\mathrm{e}^{-\mathrm{i}\omega_0 t}) + H_{\mathrm{Raman}}, \quad (1)$$

where ω_{bc} is the frequency of transition between modes b and c and the $\{u_{ij}\}$ are the interaction strengths of atoms in modes i and j. The parallel laser field intensity is shown by $\eta_\parallel$ and a and $a^\dagger$ are the annihilation and creation operators of the cavity mode. The interaction of atoms with the transverse pump is shown by the Raman scattering Hamiltonian (H_{Raman}) and has the following form

$$H_{\mathrm{Raman}} = -\mathrm{i}\hbar \int \mathrm{d}x\, \psi_e^\dagger h_0 (\mathrm{e}^{-\mathrm{i}\omega_0 t} + \mathrm{e}^{\mathrm{i}\omega_0 t}) \psi_b + \mathrm{H.c.}$$
$$- \mathrm{i}\hbar \int \mathrm{d}x\, \psi_e^\dagger g_0 \cos(kx)(a + a^\dagger)\psi_c + \mathrm{H.c.}, \quad (2)$$

where h_0 and g_0 are the atom–pump and atom–cavity coupling strengths, respectively, and the Hamiltonian is written without the rotating wave approximation. After adiabatically eliminating the excited state ψ_e and introducing $U_0 = g_0^2/\Delta_0$ and $\eta = h_0 g_0/\Delta_0$, with $\Delta_0 = \omega_0 - \omega_{be}$ being the pump detuning from the atomic transition, one obtains

$$H_{\mathrm{Raman}} = \hbar\eta \int \mathrm{d}x\, (a + a^\dagger)(\mathrm{e}^{-\mathrm{i}\omega_0 t} + \mathrm{e}^{\mathrm{i}\omega_0 t})$$
$$\times \cos(kx)(\psi_c^\dagger \psi_b + \psi_b^\dagger \psi_c)$$
$$+ \frac{\hbar h_0^2}{\Delta_0} \int \mathrm{d}x\, (\mathrm{e}^{-\mathrm{i}\omega_0 t} + \mathrm{e}^{\mathrm{i}\omega_0 t})^2 \psi_b^\dagger \psi_b$$
$$+ \hbar U_0 \int \mathrm{d}x\, \cos^2(kx)(a + a^\dagger)^2 \psi_c^\dagger \psi_c. \quad (3)$$

Substituting (3) into (1), moving to a rotating frame defined by the unitary operator $U = \mathrm{e}^{-\mathrm{i}\omega_0 t a^\dagger a}$, and ignoring two-photon processes result in

$$H = \sum_{j=b,c} \int \mathrm{d}x\, \psi_j^\dagger \mathcal{H} \psi_j + \sum_{i,j=b,c} \int \mathrm{d}x\, \frac{u_{ij}}{2} \psi_j^\dagger \psi_i^\dagger \psi_i \psi_j, \quad (4)$$

for the Hamiltonian of the system, where

$$\mathcal{H} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2}$$
$$+ (\hbar U_0 \cos^2(kx)(aa^\dagger + a^\dagger a) + V_c(x) + \hbar\omega_{bc})\sigma^+\sigma^-$$
$$+ \left(\frac{2\hbar h_0^2}{\Delta_0} + V_b(x) \right) \sigma^-\sigma^+ + \hbar\eta(a + a^\dagger)$$
$$\times \cos(kx)(\sigma^- + \sigma^+) - \hbar\delta_c a^\dagger a - \mathrm{i}\hbar\eta_\parallel(a - a^\dagger). \quad (5)$$

Here $\sigma^+ = |\psi_c \times \psi_b|$ and $\sigma^- = |\psi_b \times \psi_c|$ are the ascending and descending operators in the two-mode manifold and $\delta_c = \omega_0 - \omega_c$ is the pump–cavity detuning. From equations (4) and (5) one can derive the Heisenberg equations of motion for

condensate modes and cavity field as follows

$$
\dot{\psi}_b = -\frac{\mathrm{i}}{\hbar}\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V_b(x) + \frac{2\hbar h_0^2}{\Delta_0}\right.
$$
$$
\left. + u_{bb}\psi_b^\dagger\psi_b + u_{bc}\psi_c^\dagger\psi_c\right)\psi_b
$$
$$
- \mathrm{i}\eta\cos(kx)(a + a^\dagger)\psi_c, \tag{6}
$$

$$
\dot{\psi}_c = -\frac{\mathrm{i}}{\hbar}\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V_c(x) + \hbar\omega_{bc}\right.
$$
$$
+ \hbar U_0\cos^2(kx)(aa^\dagger + a^\dagger a)
$$
$$
\left. + u_{cc}\psi_c^\dagger\psi_c + u_{bc}\psi_b^\dagger\psi_b\right)\psi_c
$$
$$
- \mathrm{i}\eta\cos(kx)(a + a^\dagger)\psi_b, \tag{7}
$$

and

$$
\dot{a} = \mathrm{i}\left(\mathrm{i}\kappa + \delta_c - 2U_0\int \mathrm{d}x\,\psi_c^\dagger\cos^2(kx)\psi_c\right)a
$$
$$
- \mathrm{i}\eta\int \mathrm{d}x\,\cos(kx)(\psi_c^\dagger\psi_b + \psi_b^\dagger\psi_c) + \eta_\parallel, \tag{8}
$$

where we have introduced a decay rate κ for the cavity in (8). In order to obtain the equations of motion for the expectation values of field operators, we further simplify our system. Since we work with a condensate of a large number of atoms, we can safely treat the condensate as a coherent state and disentangle it from photons. We also neglect higher order correlations within the photon subsystem and use $\langle a\rangle\langle a^\dagger\rangle$ and $\langle a^\dagger\rangle\langle a\rangle$ instead of $\langle aa^\dagger\rangle$ and $\langle a^\dagger a\rangle$. At the end, as far as we only deal with the intensities, we treat the photon as a coherent state as well. As a result, the following equations of motion are obtained

$$
\dot{\psi}_b = -\frac{\mathrm{i}}{\hbar}\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V_b(x) + \frac{2\hbar h_0^2}{\Delta_0}\right.
$$
$$
\left. + u_{bb}|\psi_b|^2 + u_{bc}|\psi_c|^2\right)\psi_b - 2\mathrm{i}\eta\cos(kx)\alpha_r\psi_c, \tag{9}
$$

$$
\dot{\psi}_c = -\frac{\mathrm{i}}{\hbar}\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V_c(x) + \hbar\omega_{bc}\right.
$$
$$
\left. + 2\hbar U_0\cos^2(kx)|\alpha|^2 + u_{cc}|\psi_c|^2 + u_{bc}|\psi_b|^2\right)\psi_c
$$
$$
- 2\mathrm{i}\eta\cos(kx)\alpha_r\psi_b, \tag{10}
$$

$$
\dot{\alpha} = \mathrm{i}\left(\mathrm{i}\kappa + \delta_c - 2U_0\int \mathrm{d}x\,|\psi_c|^2\cos^2(kx)\right)\alpha
$$
$$
- \mathrm{i}\eta\int \mathrm{d}x\,\cos(kx)(\psi_c^*\psi_b + \psi_b^*\psi_c) + \eta_\parallel, \tag{11}
$$

where α_r is the real part of the cavity field α.

3. Bistability of photon number and magnetization

To solve the equations of motion derived in section 2 for the steady state of the system, we employ the discrete mode

approximation. The ground state of the condensate, without any laser field, is a homogeneous macroscopic state with zero momentum, which we refer to by ϕ_0. Above the superradiance threshold one can assume that the condensate is fragmented to a symmetric superposition of the states with momentum $\pm\hbar k$ due to the transverse laser field. On the other hand, since the cavity mode is excited by the parallel pump, absorption and emission of the cavity photons can excite the condensate to a superposition of states with momentum $\pm 2\hbar k$. Therefore if we consider a first-order perturbation on the homogeneous wavefunction ϕ_0, the following functions can be used as the basis for the DMA:

$$
\phi_0 = \sqrt{1/L} \qquad \phi_1 = \sqrt{2/L}\cos(kx)
$$
$$
\phi_2 = \sqrt{2/L}\cos(2kx) \tag{12}
$$

with L being the length of the cavity (condensate). Now the wavefunctions of the two modes of the condensate can be expanded in this basis as follows

$$
\psi_b(x, t) = \sum_{i=0}^{2}\phi_i b_i \qquad \psi_c(x, t) = \sum_{i=0}^{2}\phi_i c_i. \tag{13}
$$

If we substitute these wavefunctions into the equations of motion of the two modes, by ignoring external potentials and atom–atom interaction, the equation of motion for the condensate wavefunction can be written in a compact form:

$$
\mathrm{i}\hbar\frac{\mathrm{d}}{\mathrm{d}t}X = H(\alpha)X = [H_0 + H_1 + H_2 + 2|\alpha|^2 H_3 + 2\alpha_r H_4]X, \tag{14}
$$

where $X = (b_0, b_1, b_2, c_0, c_1, c_2)^{\mathrm{T}}$ and

$$
H_0 = \hbar\omega_r\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{pmatrix} \tag{15}
$$

$$
H_1 = \frac{2\hbar h_0^2}{\Delta_0}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{16}
$$

$$
H_2 = \hbar\omega_{bc}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{17}
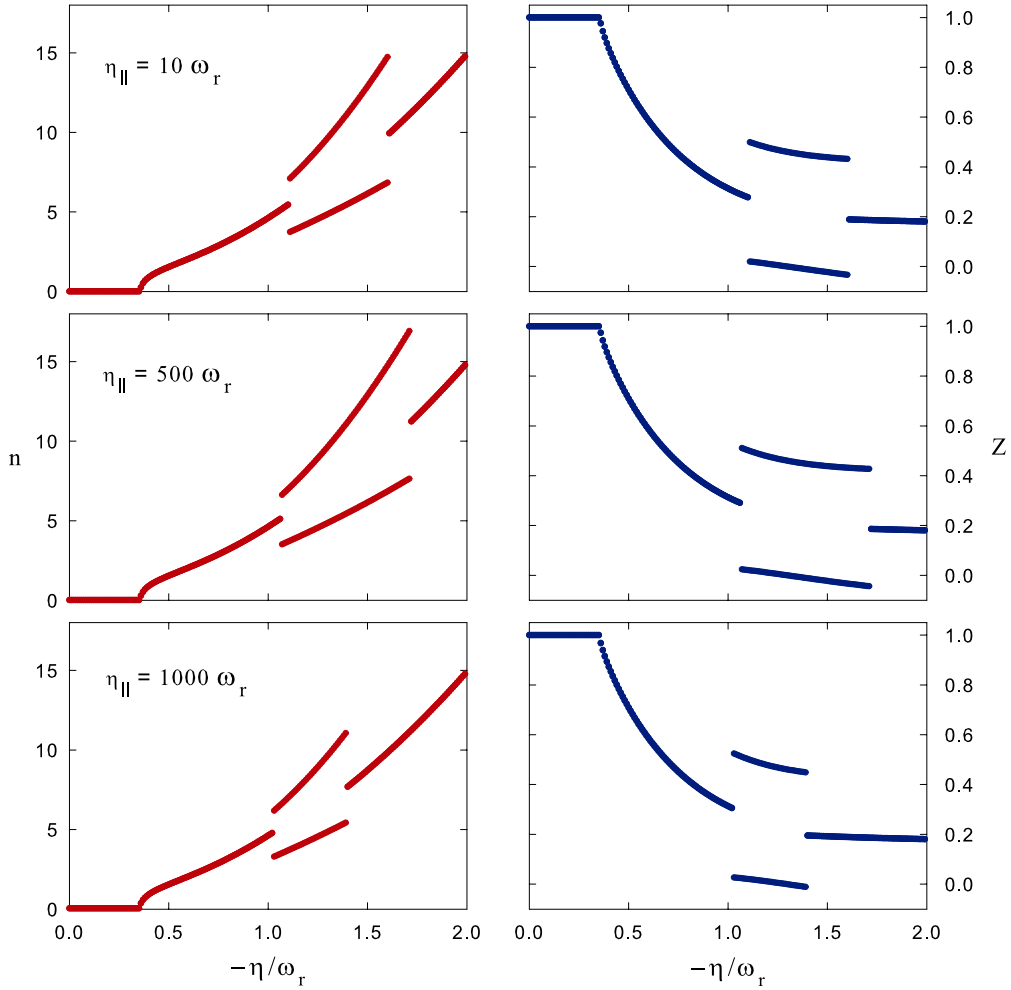$$

Figure 2. Mean cavity field n (left) and normalized magnetization Z (right) as functions of η for different values of parallel pump strength $\eta_\parallel$. Note that η is proportional to transverse pump strength and atom–cavity coupling strength. In all plots, in units of ω_r, $U_0 = -0.5$, $\Delta_0 = -4 \times 10^6$, $\kappa = 400$, $\delta_c = 4800$, $\omega_{bc} = 1$, and $N = 4.8 \times 10^4$.

$$H_3 = \frac{\hbar U_0}{4} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & \sqrt{2} \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & \sqrt{2} & 0 & 2 \end{pmatrix} \tag{18}$$

$$H_4 = \frac{\hbar \eta}{2} \begin{pmatrix} 0 & 0 & 0 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & \sqrt{2} & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & \sqrt{2} & 0 & 0 & 0 & 0 \\ \sqrt{2} & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{19}$$

Here $\omega_r = \hbar k^2 / 2m$ is the recoil frequency.

To examine the equilibrium properties of the system, we set $\dot\alpha = 0$ in (11), which results in

$$\alpha = \frac{\eta \int dx \cos(kx)(\psi_c^* \psi_b + \psi_b^* \psi_c) + i\eta_\parallel}{i\kappa + \delta_c - 2U_0 \int dx \cos^2(kx)\psi_c^* \psi_c}. \tag{20}$$

On the other hand, one can easily obtain the following alternative expressions for the integrals in the above equation

$$\int dx \cos^2(kx)\psi_c^* \psi_c = \frac{1}{\hbar U_0} X^\dagger H_3 X \tag{21}$$

$$\int dx \cos(kx)(\psi_b^* \psi_c + \psi_c^* \psi_b) = \frac{1}{\hbar \eta} X^\dagger H_4 X. \tag{22}$$

Therefore the cavity field α and averaged photon number $n = |\alpha|^2$ will read as

$$\alpha = \frac{\frac{1}{\hbar} X^\dagger H_4 X + i\eta_\parallel}{i\kappa + \delta_c - \frac{2}{\hbar} X^\dagger H_3 X} \tag{23}$$

$$n = \frac{(\frac{1}{\hbar} X^\dagger H_4 X)^2 + \eta_\parallel^2}{\kappa^2 + (\delta_c - \frac{2}{\hbar} X^\dagger H_3 X)^2}. \tag{24}$$

Here one should notice that if there is no transverse laser field ($h_0 = 0$) then $\eta = 0$ and $H_4 = 0$. As a result, there will be no transition between the two modes and if atoms are initially in the ground state b, they will stay there. Therefore expectation values of both H_4 and H_3 in (24) or, equivalently, the integrals in (20) would be zero and no bistability in the
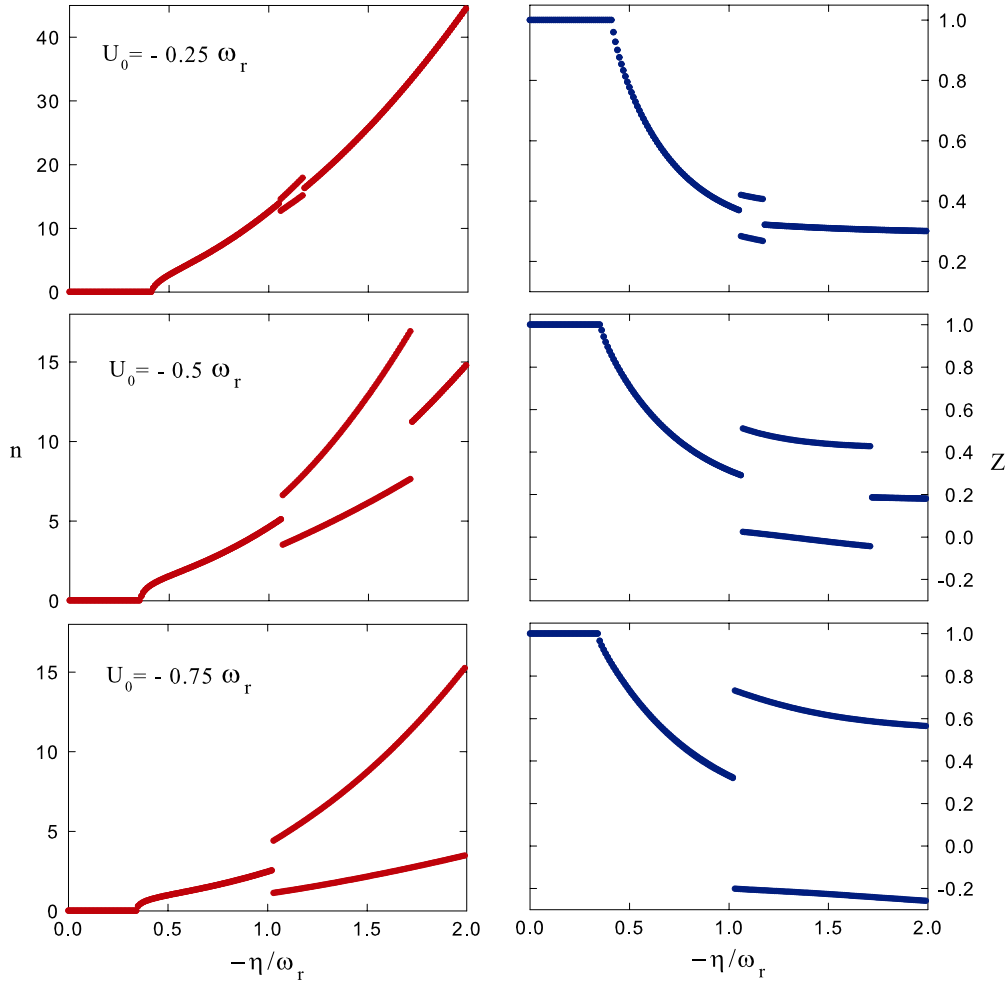
Figure 3. Mean cavity field n (left) and normalized magnetization Z (right) as functions of η for different values of atom–cavity coupling strength U_0. In all plots, in units of ω_r, $\eta_\parallel = 500$, $\Delta_0 = -4 \times 10^6$, $\kappa = 400$, $\delta_c = 4800$, $\omega_{bc} = 1$, and $N = 4.8 \times 10^4$.

system is expected. The importance of the transverse laser pump could be predicted from the crucial role it plays in the Raman scattering. On the other hand to better understand the contribution of a parallel laser field in bistability, we note that due to the dependence of X on α, the expectation values in the numerator and denominator of (24) are functions of n. To lowest order, we can assume that these terms depend on n linearly. Therefore (24) is a cubic equation of n, which in the case of $\eta_\parallel = 0$ has always a zero root. Therefore, to avoid a zero root in (24), we will consider cases where $\eta_\parallel \neq 0$.

To obtain the mean cavity field n for the steady state of the system we need to find the wavefunction X_s in the steady state. Since $H(\alpha)X_s = E_0X_s$ is nonlinear due to the dependence of X on α, we first solve it for E_0 and X_s with a guess for the value of α. Then, by substituting the resulting X_s into (23), a new α is obtained. If this new α is equal to the guessed value then the steady state is reached, otherwise we repeat the procedure, using this new α as the guess, until the steady state is attained.

In addition to the average photon number, in a system consisting of two-mode condensate there is another quantity which reveals the nonlinear effect of matter–field interaction; this is the normalized population difference (magnetization Z) of two modes. In our system magnetization is defined as

$Z = \int dx \, (|\psi_b|^2 - |\psi_c|^2)/N$, where $N = \int dx \, (|\psi_b|^2 + |\psi_c|^2)$ is the total number of atoms, which is fixed.

Figure 2 shows the mean cavity field n and normalized magnetization Z as functions of η for different values of parallel pump strength $\eta_\parallel$. We have considered a condensate of $N = 48\,000$ atoms in a cavity with decay rate $\kappa = 400\,\omega_r$. The laser field is detuned from the cavity mode by $\delta_c = 4800\,\omega_r$ and from the atomic transition by $\Delta_0 = -4 \times 10^6\,\omega_r$. In all plots, the atom–cavity coupling is assumed to be $U_0 = -0.5\,\omega_r$. As one can observe from these plots, the effect of a parallel pump on widening the area of bistability is not monotonic. While increasing $\eta_\parallel$ from 10 to 500 results in a wider interval of η in which bistability happens, further increasing it to 1000 has the opposite effect. Moreover, $\eta_\parallel$ has a larger impact on the width of the area in which bistability takes place than on the values of n and Z in the bistable region. The interesting point about this system is the fact that nonlinear effects of matter–field coupling result in bistable behavior of both matter and field, such that bistability occurs for magnetization of condensate atoms exactly at the same region of η where n has shown bistable behavior. Moreover, it can be seen in figure 2 that, in some points, two states not only

with different values of magnetization but also with different signs of magnetization are stable.

In the next step, to better understand the role of atom–cavity coupling on bistable behavior of the system, we keep the value of parallel pump strength $\eta_\parallel$ constant and increase the strength of atom–cavity coupling. Figure 3 shows the average photon number n and normalized magnetization Z as functions of η for the same values of parameters used in figure 2 but this time with $\eta_\parallel = 500\,\omega_r$ and for three different values of U_0. Clearly, atom–cavity coupling has a significant effect on the width of the area in which bistability occurs. More importantly, U_0 can change the distance between the two stable branches. It is worth mentioning that η itself is proportional to g_0 and, therefore, for a fixed value of η, larger U_0 means smaller value for h_0. As a result, increasing U_0 makes it possible to achieve bistability with smaller values and a wider range of atom–laser coupling h_0.

4. Conclusion

In this work we developed an effective Hamiltonian and equations of motion for a cavity–condensate system consisting of a two-mode BEC in a one-dimensional cavity, while parallel and transverse laser fields are applied to the system. Under the DMA, simultaneous and mutual bistability (multistability) of the cavity field and population difference (magnetization) of the two modes has been observed for different values of transverse and parallel pump strengths. The system shows bistable behavior for quite a wide range of parameters. Moreover it has been shown that, with strong enough cavity–matter coupling strength, bistability occurs between two states with different signs of magnetization.

Acknowledgments

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