

tion. The solutions are represented in convergent series form, and numerical computations are performed to show the charge-density distribution through the slit.

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### A Numerically Efficient Technique for the Analysis of Slots in Multilayer Media

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**Abstract**—A numerically efficient technique for the analysis of slot geometries in multilayer media is presented using closed-form Green's functions in spatial domain in conjunction with the method of moments (MoM). The slot is represented by an equivalent magnetic-current distribution, which is then used to determine the total power crossing through the slot and the input impedance. In order to calculate power and current distribution, spatial-domain closed-form Green's functions are expanded as power series of the radial distance  $\rho$ , which makes the analytical evaluation of the spatial-domain integrals possible, saving a considerable amount of computation time.

**Index Terms**—Green's function, moment methods, multilayers.

#### I. INTRODUCTION

Slot geometries have a broad spectrum of applications either as transmission lines or radiating elements, and have been examined extensively in the literature [1]–[4]. The most commonly used numerical technique for analyzing the slot geometries is the method of moments (MoM), which can be applied in either the spatial or spectral domains. Although the MoM is preferred over the differential equation methods because it is relatively efficient in terms of the computation time, it is still time consuming because of the slow convergence and the oscillatory nature of the integrals involved. One approach to overcome these difficulties is to employ the closed-form Green's functions in the spatial domain, which can speed up the computation of the MoM matrix elements by several orders of magnitude as compared to the numerical evaluation of the Sommerfeld integral [5]–[8].

In this paper, the Galerkin's MoM analysis of the slot geometries in multilayer media has been developed by employing the closed-form Green's functions for the vector and scalar potentials of a

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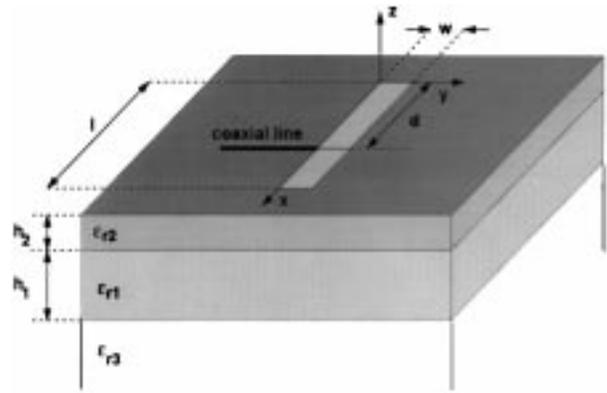


Fig. 1. A slot structure on a multilayer medium. The region above the slot is free space.

horizontal magnetic dipole (HMD) in the spatial domain [9]. The formulation is presented for narrow slot geometries excited with coaxial-line feed; however, it can be applied to slot geometries of any kind of excitation without any major modification. The equivalent magnetic-current distribution of the slot is computed and used for the computation of power crossing the slot and the input impedance. Numerical calculation of power crossing the slot and the equivalent magnetic slot current is computationally a very demanding procedure because the numerical evaluation of the integrals involved is very time consuming in either the spatial or spectral domains. Here, the spatial-domain Green's functions are approximated as a power series of radial distance  $\rho$ , and integrals involving the Green's functions are carried out analytically, saving a considerable amount of computational time both in current and power calculations [10].

#### II. FORMULATION

An example of a narrow slot placed in a multilayer medium is shown in Fig. 1. It is assumed that the layers extend to infinity in the transverse direction and the slot is excited with a coaxial line of current  $I_{in}$  amperes at the feeding point. It is also assumed that there is no conducting or dielectric losses. Therefore, the only loss mechanism is the radiation.

The tangential component of the magnetic field on the slot can be expressed in terms of an equivalent magnetic-current density  $\vec{J}^m$  using the mixed-potential integral equation (MPIE) formulation [11] as follows:

$$H_x = -j\omega G_{xx}^F * J_x^m + \frac{1}{j\omega} \frac{\partial}{\partial x} (G_x^{qm} * \nabla \cdot \vec{J}^m) \quad (1)$$

where  $J_x^m$  is the longitudinal component of the current density  $\vec{J}^m$ , and  $G_{xx}^F$  and  $G_x^{qm}$  are the spatial-domain Green's functions for the vector and scalar magnetic potentials for an HMD, respectively. To solve for the equivalent magnetic current density  $J_x^m$  using the MoM, the current density is expressed as a linear combination of suitable subdomain basis functions in the following form:

$$J_x^m = \sum_{n=1}^N I_{xn} B_{xn}(x, y) \quad (2)$$

where  $B_{xn}$ 's are the basis functions which are chosen in this paper to be rooftops. Since a narrow slot is assumed, the current variation in  $y$ -direction is considered to be constant. Enforcing the boundary

conditions for the tangential fields, the following equation is obtained:

$$\begin{aligned} & \frac{-1}{jw} \langle T_{xm}, J_s \delta(x-d) \rangle \\ &= \langle T_{xm}, (G_{xx}^F|_{z<0} + G_{xx}^F|_{z>0}) * J_x^m \rangle \\ &+ \frac{1}{w^2} \left\langle T_{xm}, \frac{\partial}{\partial x} \left[ (G_x^{qm}|_{z<0} + G_x^{qm}|_{z>0}) * \frac{\partial J_x^m}{\partial x} \right] \right\rangle \quad (3) \end{aligned}$$

where  $T_{xm}$  denotes the testing functions expressed by subdomain basis functions,  $\langle \cdot \rangle$  designates the inner product, and  $*$  designates the convolution integral. Note that the Green's functions appearing in (3) are the spatial-domain closed-form Green's functions which can be obtained from the closed-form spectral-domain Green's functions [6], [12]. The spatial-domain closed-form Green's functions are expressed in the following form:

$$G_{xx,x}^{F,qm} \cong \sum_{m=1}^N a_m \frac{e^{-jk_i r_m}}{r_m} \quad (4)$$

where  $r_m = \sqrt{\rho^2 - b_m^2}$ , and  $k_i^2 = k_{z_i}^2 + k_\rho^2$ . Here,  $a_m$ 's and  $b_m$ 's are complex constants, in general. Consequently, for a slot geometry given in Fig. 1, the spatial-domain Green's functions encountered in (3) can be written as follows:

$$G_{xx}^F = \begin{cases} 2 \frac{\epsilon_0}{4\pi} \frac{e^{-jk_0 r}}{r}, & z > 0 \\ \sum_{m=1}^M a_m \frac{e^{-jk_r r_m}}{r_m}, & z < 0 \end{cases} \quad (5a)$$

$$G_x^{qm} = \begin{cases} 2 \frac{1}{4\pi\mu_0} \frac{e^{-jk_0 r}}{r}, & z > 0 \\ \sum_{n=1}^N a_n \frac{e^{-jk_r r_n}}{r_n}, & z < 0. \end{cases} \quad (6a)$$

$$(6b)$$

After having obtained the closed-form Green's functions, the remaining integrals need to be evaluated. In this paper, each exponential term in the above Green's functions is expanded as a power series of  $\rho$  which makes the analytical evaluation of the inner products in (3) possible, saving considerable amount of computational time [10]. Also note that since the spatial-domain Green's functions have a surface integrable singularity at the origin, analytical evaluation of these integrals does not need the extraction of singularity.

#### A. Calculation of the Total Power and Input Impedance

Once the equivalent magnetic-current density on the slot is obtained, then the power crossing through the slot can be calculated by using the following integral:

$$-P_c = \iint_{\text{slot}} \vec{E} \times \vec{H}^* \cdot d\vec{s} \quad (7)$$

where

$$(\vec{E} \times \vec{H}^*)_z = E_x H_y^* - E_y H_x^* \quad (8)$$

$$E_y = J_x^m \quad (9)$$

$$E_x = 0, \quad (10)$$

Here,  $H_x$  and  $J_x^m$  are given by (1) and (2), respectively, and  $(\cdot)^*$  denotes the complex conjugate. It should be noted that since the geometry is physically separated into two half-spaces by replacing the slot with a perfect electric conductor and equivalent densities, the power should be computed for each region separately, and then should be combined to get the total power. Hence,

$$P_c^{\text{total}} = P_c|_{z>0} + P_c|_{z<0}. \quad (11)$$

Note that  $P_c|_{z>0}$  and  $P_c|_{z<0}$  are evaluated on the slot surface at  $z = 0^+$  and  $z = 0^-$ , but with different sets of Green's functions, as given in (5) and (6). By substituting (1), (9), and (10) into (7),

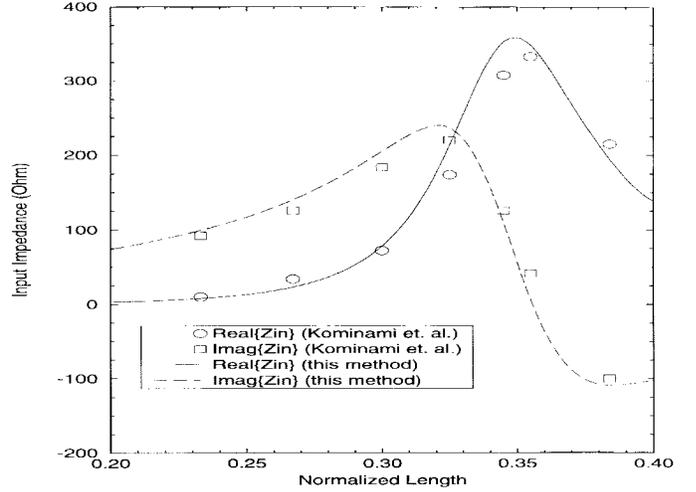


Fig. 2. Input impedance versus normalized length ( $l/\lambda_0$ ) for the slot given in Fig. 1 ( $w/l = 0.02$  cm,  $h_1 = h_2 = 0.0$  cm,  $\epsilon_{r3} = 2.55$ , and  $d = 5.0$  cm).

and taking the complex conjugate of both sides, one can obtain the following expression:

$$P_c^*|_{z \leq 0} = \iint_{\text{slot}} J_x^{m*} \left\{ -jw G_{xx}^F * J_x^m + \frac{1}{jw} \frac{\partial}{\partial x} \left[ G_x^{qm} * \frac{\partial}{\partial x} J_x^m \right] \right\} dx dy \quad (12)$$

and expanding the convolution integrals, the following expression is obtained:

$$\begin{aligned} P_c^*|_{z \leq 0} = & -jw \iint_{\text{slot}} dx dy J_x^{m*} \iint dx' dy' G_{xx}^F(x-x', y-y') J_x^m \\ & + \frac{1}{jw} \iint_{\text{slot}} dx dy J_x^{m*} \frac{\partial}{\partial x} \iint dx' dy' \\ & \cdot G_x^{qm}(x-x', y-y') \frac{\partial}{\partial x'} J_x^m. \end{aligned} \quad (13)$$

Note that (13) contains quadruple integrals in spatial domain, therefore, numerical methods to evaluate (13) would be very inefficient. On the other hand, it is also possible to carry the convolution in (12) to the spectral domain, thus eliminating one of the double integrals. In that case, although the spectral-domain Green's functions are in closed form, they are still oscillatory functions and the limits of the integrals extend to infinity yielding computationally expensive numerical integrals. To overcome these difficulties, (13) can be written in matrix form as follows:

$$P_c^*|_{z \leq 0} = [I_{x1} \ I_{x2}, \dots, I_{xn}]^* [A_{nn}'] \begin{bmatrix} I_{x1} \\ I_{x2} \\ \vdots \\ I_{xn'} \end{bmatrix} \quad (14)$$

where

$$\begin{aligned} A_{nn}' = & -jw \iint_{\text{slot}} dx dy B_{xn}(x, y) \iint dx' dy' \\ & \cdot G_{xx}^F(x-x', y-y') B_{xn'}(x', y') \\ & - \frac{1}{jw} \iint_{\text{slot}} dx dy \frac{\partial}{\partial x} B_{xn}(x, y) \iint dx' dy' \\ & \cdot G_x^{qm}(x-x', y-y') \frac{\partial}{\partial x'} B_{xn'}(x', y'). \end{aligned} \quad (15)$$

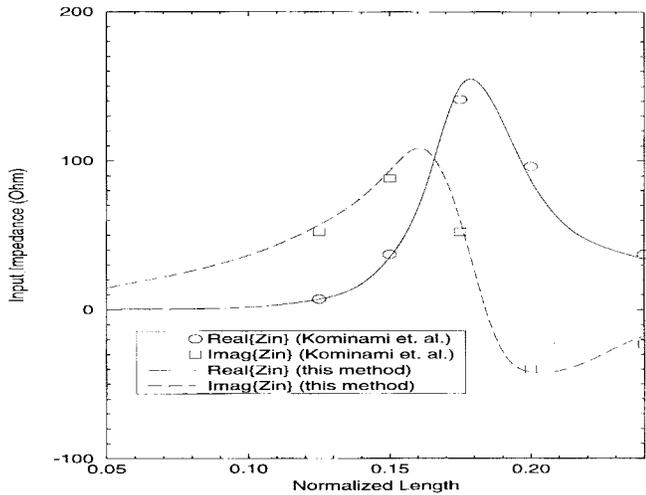


Fig. 3. Input impedance versus normalized length ( $l/\lambda_0$ ) for the slot given in Fig. 1 ( $w/l = 0.02$  cm,  $h_1 = h_2 = 0.0$  cm,  $\epsilon_{r3} = 12.8$ , and  $d = 5.0$  cm).

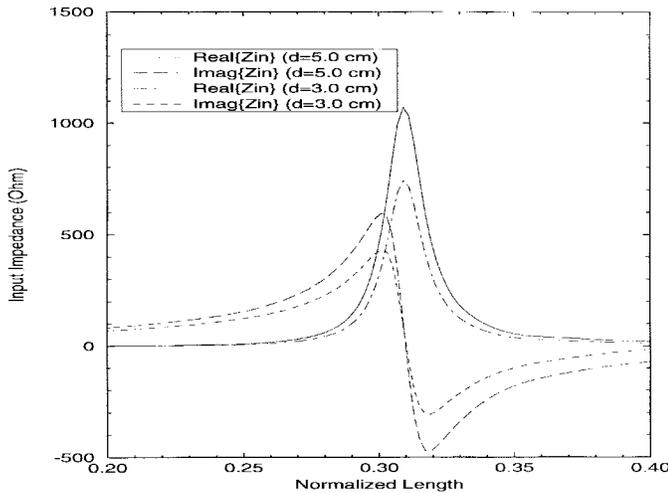


Fig. 4. Input impedance versus normalized length ( $l/\lambda_0$ ) for the slot given in Fig. 1 ( $w/l = 0.02$  cm,  $h_1 = 0.2$  cm,  $h_2 = 0.05$  cm,  $\epsilon_{r1} = 2.55$ ,  $\epsilon_{r2} = 12.8$ ,  $\epsilon_{r3} = 1.0$ ).

Now, one can notice that the matrix entry which appears in (15) is similar to the MoM impedance matrix entries. Hence, the Green's functions in (15) can also be approximated by power series of  $\rho$ , making analytical integration possible. Finally, the input impedance seen from the feed point is obtained by

$$Z_{in} = \frac{P_c^{total}}{I_{in}^2}. \quad (16)$$

### III. RESULTS AND CONCLUSIONS

The first investigated geometry consists of an infinitely large ground plane with a narrow rectangular center-fed slot, which sepa-

rates the geometry into two infinite half-spaces. This can be achieved by setting  $h_2 = h_1 = 0$  in the geometry shown in Fig. 1. The input impedance of the slot is calculated and is compared with the results presented in [2] and, as can be seen in Figs. 2 and 3, there is a good agreement between the results. The difference between the results near resonance could be due to the fact that Kominami *et al.* uses a different set of basis functions which forces the edge singularity, and they obtained the results for a lossy dielectric, whereas in our case, the dielectric is assumed to be lossless. As a second example, a multilayer geometry is selected by setting  $h_1 = 0.2$  cm and  $h_2 = 0.05$  cm, and both center-fed and offset-fed configurations are analyzed. The input impedances of the slot for both cases are given in Fig. 4. It should be noted that offset-feeding has not changed the resonance frequency of the structure.

In conclusion, it can be stated that the use of closed-form spatial-domain Green's functions increases the computational efficiency in the analysis of slot geometries in multilayer media. Although the formulation is presented for narrow slot geometries with coaxial feeding, it can be applied to general slot geometries placed in a multilayer geometry, such as slot-coupled microstrip patch antennas.

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