false alarm is declared when the underlying machine is fault-free (top) and the fraction of times the rule fails to declare a fault given that the underlying machine is faulty (bottom). The results shown are aggregated over 10^3 runs; in each run, the machine is supplied with appropriate random inputs for 5000 time steps.

V. Conclusion

In this note, we analyzed a probabilistic scheme that can detect a fault in the state-transition mechanism of a deterministic FSM. The novelty of this scheme is that the detector does not need to know the inputs that are applied to the FSM or the order in which states appear. The detecting mechanism only requires the input probability distribution and measurements of the (empirical) frequencies with which different states are occupied. Fault detection is achieved by analyzing how the obtained state occupancy measurements deviate from the stationary distribution probabilities that are expected from a fault-free machine. In applications where the input distribution is not precisely known, our decision mechanism would also need to incorporate the uncertainty introduced in our knowledge of the (fault-free and faulty) stationary distributions; an interesting question is then to characterize how the insufficient knowledge of the input distribution affects the requirements on the length of the observation window. Other interesting future research directions include the study of the applicability of these techniques as the size of the system increases, the development of tighter bounds on the required length for the observation window, and the adjustment of these techniques so that they can perform fault diagnosis (detection and identification).

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Remarks on Strong Stabilization and Stable $\mathcal{H}^\infty$ Controller Design

Suat Gümüşsoy and Hitay Özbay

***Abstract*—A state space based design method is given to find strongly stabilizing controllers for multiple-input–multiple-output plants (MIMO). A sufficient condition is derived for the existence of suboptimal stable $\mathcal{H}^\infty$ controller in terms of linear matrix inequalities (LMIs) and the controller order is twice that of the plant. A new parameterization of strongly stabilizing controllers is determined using linear fractional transformations (LFTs).**

***Index Terms*—Linear matrix inequality (LMI), stable $\mathcal{H}^\infty$ controller design, strong stabilization.**

I. Introduction

Strong stabilization problem is known as the design of a stable feedback controller which stabilizes the given plant. For practical reasons, a stable controller is desired [1], [2]. In this note, we derive a simple and effective design method to find stable $\mathcal{H}^\infty$ controllers for multiple-input–multiple-output (MIMO) systems.

A stable controller can be designed if and only if the plant satisfies the parity interlacing property (PIP) [3], i.e., the plant has even number of poles between any pair of its zeros on the extended positive real axis. There are several design procedures for strongly stabilizing controllers [4]–[16].

The result in this note is the generalization of the work in [11] using linear matrix inequalities (LMIs). The procedure is quite simple, efficient and easy to solve by using the LMI Toolbox of MATLAB [17]. In the next section, it is shown that if a certain LMI has a feasible solution, then it is possible to obtain a stable $\mathcal{H}^\infty$ controller whose order is twice the order of the plant. Moreover, a parameterization of strongly stabilizing controllers can be given in terms of linear fractional transformations (LFTs).

The note is organized as follows. The main results are given in Section II. Stable $\mathcal{H}^\infty$ controller design procedure is proposed in Section III. Numerical examples and comparison with other methods can

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S. Gümüşsoy was with the Department of Electrical and Computer Engineering, The Ohio State University, Columbus, OH 43210 USA. He is now with MIKES, Inc., Ankara TR-06750, Turkey (e-mail: suat.gumussoy@mikes.com.tr).

H. Özbay is with the Department of Electrical and Electronics Engineering, Bilkent University, Ankara TR-06800, Turkey, on leave from the Department of Electrical and Computer Engineering, The Ohio State University, Columbus, OH 43210 USA (-mail: hitay@bilkent.edu.tr; ozbay.1@osu.edu).

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be found in Section IV and concluding remarks are made in the last section.

Notation: The notation is fairly standard. A state–space realization of a transfer function, $G(s) = C(sI - A)^{-1}B + D$, is shown by $G(s) = [\begin{array}{c|c} A & B \\ \hline C & D \end{array}]$ and the linear fractional transformation of G by K is denoted by $\mathcal{F}_l(G, K)$ which is equivalent to $G_{11} + G_{12}K(I - G_{22}K)^{-1}G_{21}$ where G is partitioned as $G = [\begin{array}{cc} G_{11} & G_{12} \\ G_{21} & G_{22} \end{array}]$. As a shorthand notation for LMI expressions, we will define $\Gamma(A, B) := B^T A^T + AB$ where A, B are matrices with compatible dimensions.

II. Strong Stabilization of MIMO Systems

Consider the standard feedback system with generalized plant G, which has state–space realization

$$G(s) = \left[\begin{array}{c|cc} A & B_1 & B \\ \hline C_1 & D_{11} & D_{12} \\ C & D_{21} & 0 \end{array}\right] \tag{II.1}$$

where $A \in \mathcal{R}^{n\times n}$, $D_{12} \in \mathcal{R}^{p_1\times m_2}$, $D_{21} \in \mathcal{R}^{p_2\times m_1}$ and other matrices are compatible with each other. We suppose the plant satisfies the standard assumptions

A.1) (A, B) is stabilizable and (C, A) is detectable;
A.2) $[\begin{array}{cc} A - \lambda I & B \\ C_1 & D_{12} \end{array}]$ has full-column rank for all $\mathrm{Re}\{\lambda\} \geq 0$;
A.3) $[\begin{array}{cc} A - \lambda I & B_1 \\ C & D_{21} \end{array}]$ has full-row rank for all $\mathrm{Re}\{\lambda\} \geq 0$;
A.4) has no eigenvalues on the imaginary axis.

Let the controller has state space realization, $K_G(s) = [\begin{array}{c|c} A_K & BA_K \\ \hline C_K & 0 \end{array}]$ where $A_K \in \mathcal{R}^{n\times n}$, $B_K \in \mathcal{R}^{n\times p_2}$, and $C_K \in \mathcal{R}^{m_2\times n}$. Define the matrix $X \in \mathcal{R}^{n\times n}$, $X = X^T > 0$ as the stabilizing solution of

$$A^T X + XA - XBB^T X = 0 \tag{II.2}$$

(i.e., $A - BB^T X$ is stable) and the "A-matrix" of the closed-loop system as $A_{CL} = \begin{bmatrix} A & BC_K \\ B_K C & A_K \end{bmatrix}$. Note that since (A, B) is stabilizable, X is unique and $A_X := (A - BB^T X)$ is stable. Also, the closed-loop stability is equivalent to whether A_{CL} is stable or not.

Lemma 2.1: Assume that the plant (II.1) satisfies the assumptions A.1)–A.4). There exists a stable stabilizing controller, $K_G \in \mathcal{RH}^\infty$ if there exists $X_K \in \mathcal{R}^{n\times n}$, $X_K = X_K^T > 0$, and $Z \in \mathcal{R}^{n\times p_2}$ for some $\gamma_K > 0$ satisfying the LMIs

$$\Gamma(X_K, A) + \Gamma(Z, C) < 0 \tag{II.3}$$

$$\begin{bmatrix} \Gamma(X_K, A_X) + \Gamma(Z, C) & -Z & -XB \\ -Z^T & -\gamma_K I & 0 \\ -B^T X & 0 & -\gamma_K I \end{bmatrix} < 0 \tag{II.4}$$

where X is the stabilizing solution of (II.2) and A_X is as defined previously. Moreover, under the previous condition, a stable controller can be given as $K_G(s) = \left[\begin{array}{c|c} A_X + X_K^{-1}ZC & -X_K^{-1}Z \\ \hline -B^T X & 0 \end{array}\right]$ and this controller satisfies $\|K_G\|_\infty < \gamma_K$.

Proof: By using similarity transformation, one can show that A_{CL} is stable if and only if A_X and $A_Z := A + X_K^{-1}ZC$ is stable. Since X is a stabilizing solution, A_X is stable. If we rewrite the LMI (II.3) as

$$\left(A + X_K^{-1}ZC\right)^T X_K + X_K \left(A + X_K^{-1}ZC\right) < 0$$

it can be seen that A_Z is stable since $X_K > 0$. The second LMI (II.4) comes from KYP lemma and guarantees that $\|K_G\|_\infty < \gamma_K$. ■

Remark 1: If the design only requires the stability of closed loop system, it is enough to satisfy the LMI (II.3), $(1, 1)$ block of (II.4), i.e.,

$$A_X^T X_K + X_K A_X + C^T Z^T + ZC < 0 \tag{II.5}$$

and the controller has same structure as before.

Remark 2: Lemma (2.1) is generalization of [11, Th. 2.1]. If the algebraic riccati equation (ARE) (7) in [11] has a stabilizing solution, $Y = Y^T \geq 0$, then there exists a stable controller in the form, $[\begin{array}{c|c} A_X - \gamma_K^2 YC^TC & \gamma_K^2 YC^T \\ \hline -B^T X & 0 \end{array}]$. This structure is the special case of the LMIs (II.3) and (II.4) when $X_K = (\gamma_K Y)^{-1}$ and $Z = -\gamma_K C^T$. Note that our formulation does not assume special structure on Z. Also in [11], the stability of A_Z is guaranteed by the same riccati equation, we satisfy the stability condition of A_Z with another LMI (II.3) which is less restrictive. Therefore, the Lemma (2.1) is less conservative as will be demonstrated in examples.

Corollary 2.1: Assume that the sufficient condition (II.3) and (II.5) holds. Then, all controllers in the set

$$\mathrm{K}_{G,ss} := \left\{K = \mathcal{F}_l\left(K^0_{G,ss}, Q\right) : Q \in \mathcal{RH}^\infty, \|Q\|_\infty < \gamma_Q\right\}$$

are strongly stabilizing where

$$K^0_{G,ss}(s) = \left[\begin{array}{c|cc} A_X + X_K^{-1}ZC & -X_K^{-1}Z & B \\ \hline -B^T X & 0 & I \\ -C & I & 0 \end{array}\right] \tag{II.6}$$

and $\gamma_Q = (\|C(sI - (A_X + X_K^{-1}ZC))^{-1}B\|_\infty)^{-1}$.

Proof: The result is direct consequence of parameterization of all stabilizing controllers [19]. ■

III. Stable $\mathcal{H}^\infty$ Controller Design for MIMO Systems

The standard $\mathcal{H}^\infty$ problem is to find a stabilizing controller K such that $\|\mathcal{F}_l(P, K)\|_\infty \leq \gamma$ where $\gamma > 0$ is the closed loop performance level and P is the generalized plant. It is well known that if two ARE's have unique positive semidefinite solutions and the spectral radius condition is satisfied, then standard $\mathcal{H}^\infty$ problem is solvable. All suboptimal $\mathcal{H}^\infty$ controllers can be parameterized as $K = \mathcal{F}_l(M_\infty, Q)$ where the central controller is in the form

$$M_\infty(s) = \left[\begin{array}{c|cc} A_c & B_{c1} & B_{c2} \\ \hline C_{c1} & D_{c11} & D_{c12} \\ C_{c2} & D_{c21} & 0 \end{array}\right]$$

and Q is free parameter satisfying $Q \in \mathcal{RH}^\infty$ and $\|Q\|_\infty \leq \gamma$. For derivation and calculation of M_∞, see [18] and [19].

If we consider M_∞ as plant and $\gamma = \gamma_K$, by using Lemma (2.1), we can find a strictly proper stable K_{M_∞} stabilizing M_∞ and resulting stable $\mathcal{H}^\infty$ controller, $C_\gamma = \mathcal{F}_l(M_\infty, K_{M_\infty})$ where $\|K_{M_\infty}\|_\infty < \gamma_K$. If sufficient conditions (II.3) and (II.4) are satisfied, then K_{M_∞} can be written as

$$K_{M_\infty}(s) = \left[\begin{array}{c|c} A_c - B_{c2}B_{c2}^T X_c + X_{Kc}^{-1}Z_c C_{c2} & -X_{Kc}^{-1}Z_c \\ \hline -B_{c2}^T X_c & 0 \end{array}\right]$$

and by similarity transformation, we can obtain the state space realization of C_γ as

$$C_\gamma(s) = \left[\begin{array}{c|c} A_{C_\gamma} & B_{C_\gamma} \\ \hline C_{C_\gamma} & D_{c11} \end{array}\right]$$

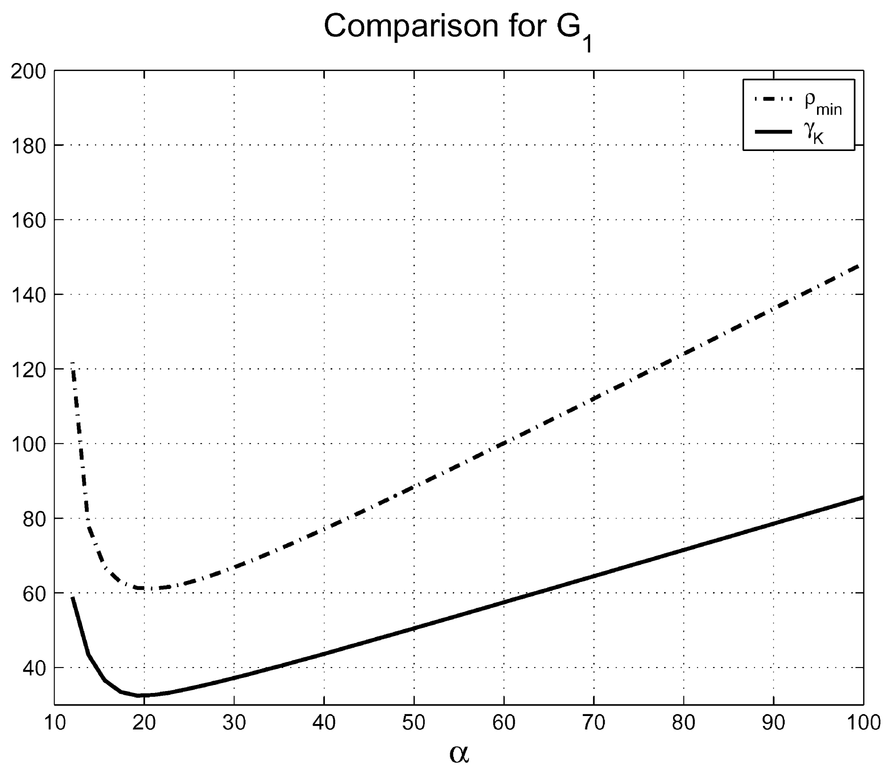

Fig. 1. Comparison for plant $\boldsymbol{G}_1$.

where X_c is the stabilizing solution of

$$A_c^T X_c + X_c A_c - X_c B_{c2} B_{c2}^T X_c = 0$$

as in (II.2) and X_{Kc}, Z_c are the solution of (II.3) and (II.4), respectively, and

$$\begin{aligned} A_{C_\gamma} &= \begin{bmatrix} A_c - B_{c2}B_{c2}^T X_c & -B_{c2}B_{c2}^T X_c \\ 0 & A_c + X_{Kc}^{-1} Z_c C_{c2} \end{bmatrix} \\ B_{C_\gamma} &= \begin{bmatrix} B_{c1} \\ -B_{c1} - X_{Kc}^{-1} Z_c D_{c21} \end{bmatrix} \\ C_{C_\gamma} &= [\, C_{c1} - D_{c12} B_{c2}^T X_c \quad -D_{c12} B_{c2}^T X_c \,]. \end{aligned}$$

Note that C_γ is stable stabilizing controller such that $\|\mathcal{F}_l(P, C_\gamma)\|_\infty < \gamma$.

IV. Numerical Examples and Comparisons

A. Strong Stabilization

The numerical example is chosen from [11]. In order to see the performance of our method, we calculated the minimum γ_K satisfying the sufficient conditions in Lemma (2.1) for the following plants:

$$G_1(s) = \begin{bmatrix} \frac{(s+5)(s-1)(s-5)}{(s+2+j)(s+2-j)(s-\alpha)(s-20)} \\ \frac{(s+1)(s-1)(s-5)}{(s+2+j)(s+2-j)(s-\alpha)(s-20)} \end{bmatrix}$$

$$G_2(s) = \begin{bmatrix} \frac{(s+1)(s-2-j\alpha)(s-2+j\alpha)}{(s+2+j)(s+2-j)(s-1)(s-5)} \\ \frac{(s+5)(s-2-j\alpha)(s-2+j\alpha)}{(s+2+j)(s+2-j)(s-1)(s-5)} \end{bmatrix}.$$

For various α values, the minimum γ_K is found. Figs. 1 and 2 illustrates the conservatism of [11] mentioned in Remark 2 (where $\rho_{\min}$ is the minimum value of the free parameter γ_K corresponding to the method of [11]).

B. Stable $\mathcal{H}^\infty$ Controllers

We applied our method to stable $\mathcal{H}^\infty$ controller design. As a common benchmark example, the following system is taken from [15]:

$$P = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & 0 \end{array}\right] \tag{IV.7}$$

where

$$\begin{aligned} A &= \begin{bmatrix} -2 & 1.7321 \\ 1.7321 & 0 \end{bmatrix} \\ [B_1 \,|\, B_2] &= \left[\begin{array}{cc|c} 0.1 & -0.1 & 1 \\ -0.5 & 0.5 & 0 \end{array}\right] \\ \left[\begin{array}{c} C_1 \\ \hline C_2 \end{array}\right] &= \left[\begin{array}{cc} 0.2 & -1 \\ 0 & 0 \\ \hline 10 & 11.5470 \end{array}\right] \\ \left[\begin{array}{cc} D_{11} & D_{12} \\ \hline D_{21} & 0 \end{array}\right] &= \left[\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & 1 \\ \hline 0.7071 & 0.7071 & 0 \end{array}\right]. \end{aligned}$$

$$\begin{aligned} z_1 &= \frac{0.03s^7 + 0.008s^6 + 0.19s^5 + 0.037s^4 + 0.36s^3 + 0.05s^2 + 0.18s + 0.015}{s^8 + 0.161s^7 + 6s^6 + 0.582s^5 + 9.984s^4 + 0.407s^3 + 3.9822s^2}(w_1 + u) \\ z_2 &= \beta u \\ y &= w_2 + \frac{0.0064s^5 + 0.0024s^4 + 0.071s^3 + s^2 + 0.1045s + 1}{s^8 + 0.161s^7 + 6s^6 + 0.582s^5 + 9.984s^4 + 0.407s^3 + 3.9822s^2}(w_1 + u) \end{aligned} \tag{IV.8}$$

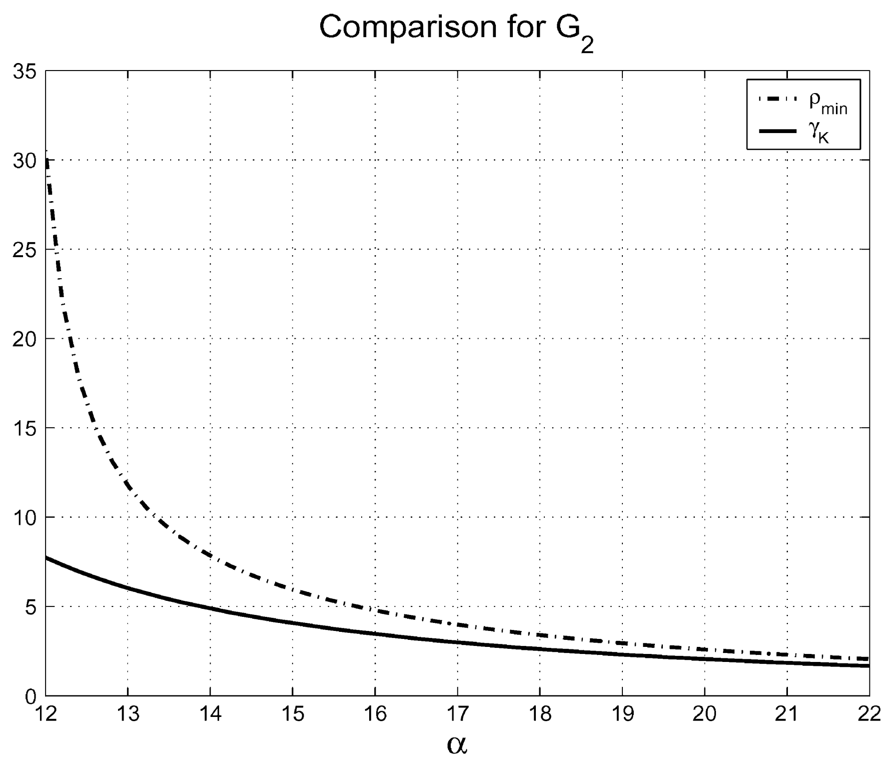

Fig. 2. Comparison for plant $\boldsymbol{G}_2$.

The optimal γ value for standard $\mathcal{H}^\infty$ problem is $\gamma_{\rm opt} = 1.2929$. Using the synthesis in [15], a stable $\mathcal{H}^\infty$ controller is found at $\gamma_{\min} = 1.369\,94$. When our method applied, we reached stable $\mathcal{H}^\infty$ controller for $\gamma_{K,\min} = 1.369\,57$. Although it seems slight improvement, our method is much more simpler with help of LMI problem formulation. Apart from standard problem solution (finding M_∞), the algorithm in [15] finds the stable $\mathcal{H}^\infty$ controller by solving an additional $\mathcal{H}^\infty$ problem.

Another common benchmark example (see [12] and its references) is to find a stable $\mathcal{H}^\infty$ controller for the generalized plant described by (IV.8), as shown at the bottom of the previous page.

In [10], it is noted that for this problem, the sufficient condition in [7] is not satisfied for even large values of γ and the method is not applicable. As we can see from Table I, the performance of our method is better than the method in [10] except the last case. For all cases, the result of [12] is superior from all other methods. However, the controller order in [12] is 24 which is greater than our controller order, 16. To address this problem, in [12] a controller order reduction is performed, that results in lower order (e.g., tenth-order for the case $\beta = 0.1$) stable controllers without significant loss of performance. Furthermore, the method in [12] involves solution of an additional $\mathcal{H}^\infty$ problem which is complicated compared to our simple LMI formulation. If the algorithm in [12] fails, selection of a new parameter Q is suggested which is an *ad-hoc* procedure. Although the performance of the controller suggested in the present note is slightly worse, it is numerically stable and easily formulated.

The following example is taken from [13]. Design a stable $\mathcal{H}^\infty$ controller for the plant

$$P(s) = \frac{s^2 + 0.1s + 0.1}{(s - 0.1)(s - 1)(s^2 + 2s + 3)}.$$

For the mixed sensitivity minimization problem the weights are taken to be as in [13]. A comparison of the methods [10], [13], and [14] and our method can be seen in Table II. There is a compromise between the methods. The performance of the method in [13] is worse than our method, but the order of our controller has twice order of the controller in [13]. Although the method in [14] gives better results than our method, the order of the controller in [14] is considerably higher than our controller order. However, this example clearly shows that our method is superior than [10].

TABLE I
STABLE $\mathcal{H}^\infty$ CONTROLLER DESIGN FOR (IV.8)

		Gumussoy-Ozbay(GO)	[10]	[12]
β	γ_{opt}	γ_{GO}	γ_{ZO}	γ_{CZ}
0.1	0.232	0.241	0.245	0.237
0.01	0.142	0.176	0.178	0.151
0.001	0.122	0.170	0.170	0.132

TABLE II
STABLE $\mathcal{H}^\infty$ CONTROLLER DESIGN FOR EXAMPLE IN [13]

	Gumussoy-Ozbay	[10]	[13]	[14], Thm 7
γ_{min}	32.557	37.551	43.167	21.787
Order	$2n$	$2n$	n	$3n$

As a remark, the method also gives very good results for single-input–single-output (SISO) systems. The following SISO example is taken from [11]:

$$P(s) = \frac{(s+5)(s-1)(s-5)}{(s+2+j)(s+2-j)(s-20)(s-30)}$$
$$W_1(s) = \frac{1}{s+1}$$
$$W_2(s) = 0.2$$

the optimal $\mathcal{H}^\infty$ problem is defined as

$$\gamma_{\rm opt} = \inf_{K \text{ stabilizing } P} \left\| \begin{bmatrix} W_1(1+PK)^{-1} \\ W_2K(1+PK)^{-1} \end{bmatrix} \right\|_\infty$$

and the optimal performance for the given data is $\gamma_{\rm opt} = 34.24$. A stable $\mathcal{H}^\infty$ controller can be found for $\gamma = 42.51$ using the method of [11], whereas our method, which can be seen as a generalization of [11], gives a stable controller with $\gamma = 35.29$. In numerical simulations, we observed that when γ approaches to the minimum value satisfying sufficient conditions, the solutions of algebraic riccati equations of [11] become numerically ill-posed. However, the LMI-based solution proposed here does not have such a problem. Same example is considered in [20] and stable $\mathcal{H}^\infty$ controller found for $\gamma = 34.44$. The method in [20] is a two-stage algorithm with combination of genetic algorithm and quasi-Newton algorithm and gives slightly better performance than our method. The method finds stable $\mathcal{H}^\infty$ controllers with a selection of low-order controller for free parameter Q. Since the example considered in the note is for SISO case, it may be difficult to achieve good performance with low-order controller for MIMO case. Due to nonlinear optimization problem structure, the solution of the method may converge to local minima and in general, genetic algorithms give solution for longer time.

V. Concluding Remarks

In this note, sufficient conditions for strong stabilization of MIMO systems are obtained and applied to stable $\mathcal{H}^\infty$ controller design. Our conditions are based on linear matrix inequalities which can be easily solved by the LMI Toolbox of MATLAB. The method is very efficient from numerical point of view as demonstrated with examples. The benchmark examples show that the proposed method is a significant improvement over the existing techniques available in the literature. The exceptions to this claim are the methods of [12], [14], and [20]. In [12], the controller design is based on *ad-hoc* search method, and both [13] and [14] result in higher order controllers than the one designed by our method. In [20], selection of low-order controller for Q gives good results for SISO structure of Q. However, in MIMO structure, Q may not result in good performance.

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The Well-Posedness and Stability of a Beam Equation With Conjugate Variables Assigned at the Same Boundary Point

Bao-Zhu Guo and Jun-Min Wang

Abstract—A Euler–Bernoulli beam equation subject to a special boundary feedback is considered. The well-posedness problem of the system proposed by G. Chen is studied. This problem is in sharp contrast to the general principle in applied mathematics that the conjugate variables cannot be assigned simultaneously at the same boundary point. We use the Riesz basis approach in our investigation. It is shown that the system is well-posed in the usual energy state space and that the state trajectories approach the zero eigenspace of the system as time goes to infinity. The relaxation of the applied mathematics principle gives more freedom in the design of boundary control for suppression of vibrations of flexible structures.

Index Terms—Boundary control, Euler–Bernoulli beam, C_0-semigroup, Riesz basis, stability.

I. Introduction

It has been known (see, e.g., [1] and [6]) that the following Euler–Bernoulli beam subject to the boundary shear force feedback

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B.-Z. Guo is with the Academy of Mathematics and System Sciences, Academia Sinica, Beijing 100080, P.R. China, and also with the School of Computational and Applied Mathematics, University of the Witwatersrand, Wits 2050, Johannesburg, South Africa (e-mail: bzguo@iss.ac.cn).

J.-M. Wang is with the Department of Mathematics, Beijing Institute of Technology, Beijing 100081, P. R. China, and also with the School of Computational and Applied Mathematics, University of the Witwatersrand, Wits 2050, Johannesburg, South Africa (e-mail: wangjc@graduate.hku.hk).

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