

Optimum Power Randomization for the Minimization of Outage Probability

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Abstract—The optimum power randomization problem is studied to minimize outage probability in flat block-fading Gaussian channels under an average transmit power constraint and in the presence of channel distribution information at the transmitter. When the probability density function of the channel power gain is continuously differentiable with a finite second moment, it is shown that the outage probability curve is a nonincreasing function of the normalized transmit power with at least one inflection point and the total number of inflection points is odd. Based on this result, it is proved that the optimum power transmission strategy involves randomization between at most two power levels. In the case of a single inflection point, the optimum strategy simplifies to on-off signaling for weak transmitters. Through analytical and numerical discussions, it is shown that the proposed framework can be adapted to a wide variety of scenarios including log-normal shadowing, diversity combining over Rayleigh fading channels, Nakagami- m fading, spectrum sharing, and jamming applications. We also show that power randomization does not necessarily improve the outage performance when the finite second moment assumption is violated by the power distribution of the fading.

Index Terms—Power randomization, outage probability, fading, jamming, wireless communications.

I. INTRODUCTION

UNDER fading, the signal power at a wireless receiver varies randomly over distance or time due to shadowing and/or multipath propagation [1], [2]. Depending on the properties of the channel and the delay-constraint of the application, various performance metrics have been proposed in the literature to assess system performance under fading. When the coherence time is much smaller than the codeword duration such that the fading process is fast enough to reveal its statistics during the transmission of a single codeword, ergodic capacity is an appropriate performance criterion [3], [4]. In this case, a few simultaneous errors due to a deep fade within a codeword transmission period can be corrected using error correction and interleaving techniques. On the other hand, for several practical situations in wireless communications, such as wireless local area networks (LANs) and mobile users moving at walking speed, the channel coherence time is comparable to the coding block length. In this case, a block-fading

(BF) channel model is assumed where the channel fading coefficient is constant over the entire duration of a codeword but changes randomly from codeword to codeword [4]–[7]. Due to slow variations of the channel, a deep fade lasts for the entire duration of a codeword transmission. Although burst error-correcting codes that can achieve the ergodic capacity for the slow fading case still exist, they usually require longer codewords that take longer time to transmit. This may not be an adequate choice in the case of delay-sensitive applications such as voice and video for which long delays due to channel variation cannot be tolerated [3], [8]. Since the messages must be transmitted and decoded successfully within a certain time to satisfy the delay constraint, the frequency with which the instantaneous channel parameters cannot support the transmitted data rate arises as a natural performance criterion to assess the quality of communications. To this end, the information outage probability has been defined as the probability that the instantaneous mutual information of the channel is less than the considered code rate [3], [9].

In delay-sensitive applications, it is desirable to maintain a minimum mutual information rate over all fading conditions through optimal transmit power control [8]. This is possible only when the instantaneous channel gains, called the channel state information (CSI), are available both at the receiver and transmitter. This goal may still not be achievable under a finite average transmit power constraint since much power is needed to maintain a constant rate during severe fading (e.g., the Rayleigh fading channel [10]). However, by suspending data transmission under severe fading conditions, a higher instantaneous mutual information rate can be supported continuously during non-outage without violating the average transmit power constraint. In this particular case, the outage probability is described as the probability of suspending transmission to prevent data loss, and the transmit power during non-outage is adjusted such that the instantaneous mutual information is exactly equal to the required rate of transmission for reliable communications [3], [8], [9].

Over the last two decades, the design of transmit power control mechanisms that aim to minimize the information outage probability for a given rate has been studied extensively under various constraints and frameworks assuming that perfect channel state information is available at the receiver (CSIR) while either perfect or partial channel state information is available at the transmitter (CSIT) [3], [9], [11]–[38]. In the following, we briefly mention some of these. A single-user M -block block-fading additive white Gaussian noise (BF-AWGN) channel is considered in [9], with an extension to transmit and receive antenna diversity in [12]. Optimal power control for outage minimization is addressed for flat-

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fading broadcast channels under different spectrum sharing techniques in [13], and for fading multiple-access channels (MACs) in [16]. The optimal power allocation problem for minimizing outage probability has also been studied extensively in more recent research areas such as cooperative communications, secure communications and cognitive radio (CR). The case for the BF relay channel is analyzed in [17] for different relaying protocols, and under the low signal-to-noise ratio (SNR) and low outage probability regimes in [21]. The same problem is also addressed for fading broadcast channels with confidential messages (BCC) in [23] by utilizing different interpretations for outage. In [20] and [29], the authors derive the optimum power allocation policy that minimizes the outage probability of the cognitive user in a spectrum-sharing CR network assuming various fading distributions and average/peak transmit/interference power constraints. In [35], outage minimization and optimal power control are investigated in state estimation of linear dynamical systems using multiple sensors. It is assumed that the messages are propagated to a fusion center over wireless fading channels and an outage is described as the event that the state estimation error exceeds a pre-determined threshold.

In the absence of CSIT, the transmitter cannot employ any form of power control and always transmits at a constant rate. Since the channel experiences block-fading, the channel gain stays constant during a codeword transmission [4], [6], [7], [9]. When the instantaneous mutual information falls below the designated rate due to a deep-fade, the channel code designed for this rate cannot successfully recover the transmitted codeword. In this particular case without CSIT, the outage probability can be interpreted as a close approximation to the decoding error probability implying its operational significance [9], [16], [39].

In general, both the accuracy and the availability of the CSIT are limited due to channel measurement errors (e.g. low SNR) and limitations of the feedback channel. The outage minimization problem and the corresponding optimal power control strategies have been studied in great detail for the cases of perfect CSIT as mentioned above. Most results that take into account the uncertainty of CSIT are limited to the ergodic capacity due to its concavity and analytical tractability [40]–[43]. In an attempt to alleviate this problem, simplifying assumptions are employed to establish convexity and/or analytically tractable special cases for which optimal power control strategies can be derived to minimize the outage probability [6], [7], [44]–[47]. In [44], independent and identically distributed Rayleigh fading is considered for a multiple-input-single output (MISO) communications system. In [45], the CSIT is restricted to the feedback of channel fading mean and covariance, and the optimum transmit covariance matrix is designed based on the assumption that the channel coefficients are jointly Gaussian. In [46], a similar problem is discussed for a MIMO system. Assuming that the channel distribution is known to the transmitter as a sampled data set with equally likely channel instances, a convex optimization problem is formulated via relaxation. In [7], it is assumed that the channel or its distribution is not known at the transmitter but the fading distribution belongs to a class of distributions that are within a certain distance from a nominal distribution in relative entropy

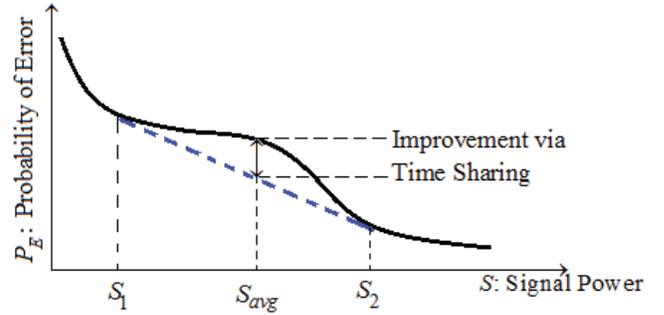


Fig. 1. Illustrative example demonstrating the benefits via time sharing between two power levels under an average power constraint.

sense. It is shown that the input distribution optimized for the nominal outage probability under the power constraint is also optimal over the class.

Recently, power randomization techniques have been applied successfully to decrease the average probability of error in M -ary communications systems [48]–[50], to improve the average detection probability in a Neyman-Pearson framework [51]–[54] and to reduce the Bayesian cost of a given estimator [55] under average power constraints. Fig. 1 depicts how power randomization helps improve the error probability under an average power constraint via a simple illustration. Suppose that the average power constraint is denoted with S_{avg} . It is seen that the average probability of error can be reduced by randomizing between power levels S_1 and S_2 with respect to the constant power transmission with S_{avg} . More precisely, power randomization exploits the nonconvexity of the plot of error probability with respect to the transmitted signal power. Although the area of optimizing the transmit power over a fading channel is well-studied even with imperfect CSIT and the assumption of an average power constraint is widely employed in the analysis of outage performance, the benefits of power randomization are not addressed to the best of our knowledge.

In this paper, we propose the idea of optimum power randomization to minimize the outage probability of an average power constrained communications system that operates over a flat BF-AWGN channel. We assume that the channel distribution information (CDI) is perfectly known at the transmitter but the instantaneous CSI is not available. In order to focus on the power randomization technique without the technicalities associated with diversity, it is assumed that a single antenna is used at the receiver. The proposed approach exploits the nonconvexity of the outage probability with respect to the transmit power to improve the outage performance over the fixed power transmission scheme, which is the only alternative in the absence of CSI for this model. In Section II, it is shown that when the probability density function (PDF) of the channel power gain is continuously differentiable with a finite second moment, the outage probability curve is a nonincreasing function of the normalized transmit power with at least one inflection point and the total number of inflection points is odd. Based on this result, optimum power randomization strategies are proposed to minimize the outage probability under an average transmit power constraint. In Section III, we apply the proposed power transmission strategy to a variety of fading scenarios including log-normal shadowing, diversity combining over Rayleigh fading channels, Nakagami- m

fading, spectrum sharing, and jamming applications. We also present a CR system in Section III.D, for which we show that the power randomization does not necessarily improve the outage performance when the finite second moment condition is violated by the power distribution of the fading. Some concluding remarks are provided in Section IV.

Notation: Throughout this paper, we use $p_h(\cdot)$ to denote the PDF of the continuous channel power gain h . $\Pr(\cdot)$ denotes the probability of the event inside the parentheses. $P_{\text{out}}(\beta)$ denotes the outage probability as a function of the normalized transmit power β . The prime symbol ' and the double prime symbol '' denote the first and second derivative of a function, respectively, e.g., $P'_{\text{out}}(\beta) = \frac{d(P_{\text{out}}(\beta))}{d\beta}$ and $P''_{\text{out}}(\beta) = \frac{d^2(P_{\text{out}}(\beta))}{d\beta^2}$. $\hat{\beta}$ and β_t denote an inflection point and a tangent point of the outage probability curve, respectively. $\bar{\beta}$ and β_p denote the average and peak power constraints, respectively.

II. CONVEXITY PROPERTIES OF OUTAGE PROBABILITY AND OPTIMUM POWER RANDOMIZATION

Consider a communications system operating over a flat BF-AWGN channel. Due to the Gaussian channel assumption, information outage probability can equivalently be described as the probability that the instantaneous received SNR falls below a minimum target SNR value required for proper operation [1], [2]. We express the received SNR as $\gamma \triangleq \rho h/N$, where ρ denotes the transmit power, h is the channel power gain between the transmitter and the receiver, and N represents the effective noise power at the receiver. The channel power gain h is described with the PDF $p_h(\cdot)$. We also assume that p_h belongs to the class of continuously differentiable PDFs and h has a finite second moment. Mathematically stated, $p_h \in \mathcal{P} \triangleq \{p(x) \in \mathcal{C}^1 : p(x) \geq 0, \int p(x)dx = 1, \int x^2 p(x)dx < \infty\}$, where \mathcal{C}^1 is the class of continuously differentiable functions on $(0, \infty)$. Almost all fading distributions employed in practice such as Rayleigh, Hoyt, Rice, Nakagami- m , and log-normal have power distributions that belong to this class [56].

Suppose that a target minimum SNR level γ_0 is imposed to ensure acceptable communication performance. If the received SNR value at the detector is below this value, outage is declared. In this paper, we consider an average power constrained communications system in which the transmitter, having perfect knowledge of the channel distribution, can randomize/time-share its transmit power in order to decrease the outage probability. To this end, it is also assumed that the transmitter is informed of the noise power N at the receiver via a feedback mechanism. Due to sufficiently long coherence time of block-fading, the receiver can learn the channel gain although it is not essential for our purposes [4], [7].

For a fixed noise power N and a target SNR γ_0 , let $\beta \triangleq \rho/(N\gamma_0)$ represent the normalized transmit power. The outage probability as a function of β is given by

$$\begin{aligned} P_{\text{out}}(\beta) &= \Pr(\gamma < \gamma_0) = \Pr\left(h < \frac{N\gamma_0}{\rho}\right) \\ &= \Pr(h < \beta^{-1}) = \int_0^{\beta^{-1}} p_h(x)dx. \end{aligned} \quad (1)$$

Similar to [7], we assume that a particular channel realization stays fixed for the whole duration of codeword transmission and changes from codeword to codeword due to block-fading.¹ At the beginning of each codeword transmission, a normalized transmit power value is selected randomly from a given finite set according to the probability distribution $p_\beta(x) = \alpha_1 \delta(x - \beta_1) + \alpha_2 \delta(x - \beta_2) + \dots + \alpha_k \delta(x - \beta_k)$, where $\delta(x)$ denotes the Kronecker delta function which is equal to one if $x = 0$ and to zero otherwise. More precisely, the probability that any given codeword is transmitted using normalized power β_i is equal to α_i . The actual transmit power is obtained from the relation $\rho = \beta N \gamma_0$ based on the randomly selected value β . Assuming that the transmitted signal, channel fading and the receiver noise are independent of each other, the optimal transmit power randomization problem can be stated as

$$\begin{aligned} \min_{k, \{\alpha_i, \beta_i\}_{i=1}^k} & \sum_{i=1}^k \alpha_i P_{\text{out}}(\beta_i) \\ \text{subject to} & \sum_{i=1}^k \alpha_i \beta_i \leq \bar{\beta} \\ & \sum_{i=1}^k \alpha_i = 1 \text{ and } \alpha_i \geq 0 \quad \forall i \in \{1, 2, \dots\} \end{aligned} \quad (2)$$

where α_i denotes the probability that a codeword is transmitted with normalized power β_i , $\bar{\beta}$ denotes the average normalized transmit power limit, and k is the cardinality of the set of β_i 's. The objective function in (2), $\sum_{i=1}^k \alpha_i P_{\text{out}}(\beta_i)$, is the average probability of outage over all possible power allocations.² Therefore, the aim is to find the optimal power randomization scheme (i.e., $p_\beta(x)$) that minimizes the average probability of outage under an average transmit power constraint.

As an initial observation, if $P_{\text{out}}(\beta)$ is nonincreasing and convex, the power randomization does not provide any improvements over the constant power transmission strategy at the average power limit as can be noted from Jensen's inequality [57]:

$$\sum_{i=1}^k \alpha_i P_{\text{out}}(\beta_i) \geq P_{\text{out}}\left(\sum_{i=1}^k \alpha_i \beta_i\right) \geq P_{\text{out}}(\bar{\beta}). \quad (3)$$

However, in general, it is possible to reduce the average probability of outage via power randomization. To discover such scenarios, the problem in (2) is investigated for generic forms of function $P_{\text{out}}(\cdot)$.

Although the optimization problem in (2) is quite challenging to solve in its current form, the following arguments can be used to simplify it significantly for practical scenarios. Assume that the normalized transmit power is finite and takes values from a closed interval in the form of $[0, \beta_p]$. Consider the set of all possible $(\beta_i, P_{\text{out}}(\beta_i))$ pairs, and denote this set as U . The average probability of outage and the average

¹This model can be generalized to the case where multiple codeword transmissions see a fixed channel realization [4].

²Alternatively, (2) can be interpreted as time-sharing among different power levels. If we consider a time interval $[0, T]$ that is split into k subintervals each of which spans multiple codeword transmissions, then one can view α_i as the fractional length of the i th subinterval and β_i as its normalized transmitted signal power.

normalized transmit power expressions in (2) are the convex combinations of $P_{\text{out}}(\beta_i)$ and β_i terms, respectively. Therefore, the set of all possible $(\sum_{i=1}^k \alpha_i \beta_i, \sum_{i=1}^k \alpha_i P_{\text{out}}(\beta_i))$ pairs is recognized as the convex hull of set U . From Caratheodory's theorem in convex analysis [58], it follows that any $(\sum_{i=1}^k \alpha_i \beta_i, \sum_{i=1}^k \alpha_i P_{\text{out}}(\beta_i))$ pair at the boundary of the convex hull of set U can be obtained as a convex combination of at most two elements in U ; that is, $k \leq 2$. Since a minimum value of $\sum_{i=1}^k \alpha_i P_{\text{out}}(\beta_i)$ must lie at the boundary of the convex hull, an optimal solution to (2) can be obtained via the following simpler problem:

$$\begin{aligned} & \min_{\alpha_1, \beta_1, \beta_2} \alpha_1 P_{\text{out}}(\beta_1) + (1 - \alpha_1) P_{\text{out}}(\beta_2) \\ & \text{subject to } \alpha_1 \beta_1 + (1 - \alpha_1) \beta_2 \leq \bar{\beta}, \quad \alpha_1 \in [0, 1]. \end{aligned} \quad (4)$$

Compared to (2), the optimization problem in (4) is significantly simpler since it is only over three variables. Also, it is noted that $\alpha_1 = 1$ (equivalently, $k = 1$ in (2)) corresponds to the trivial case of no power randomization.

In the following, the convexity properties of the outage probability in (1) are investigated in order to determine whether improvements in outage performance are possible via power randomization.

Proposition 1: For any PDF of the channel power gain that belongs to set \mathcal{P} , $P_{\text{out}}(\beta)$ is a nonincreasing function of the normalized transmit power β with at least one inflection point. Furthermore, the total number of inflection points is odd.

Proof: The proof can be established in a similar manner to that of [59, Theorem 2]. Differentiating $P_{\text{out}}(\beta)$ given in (1) with respect to β ,

$$P'_{\text{out}}(\beta) = -\beta^{-2} p_h(\beta^{-1}) \leq 0, \quad \forall \beta > 0. \quad (5)$$

It is observed that P_{out} is nonincreasing in normalized transmit power β . Differentiating once more, we have

$$P''_{\text{out}}(\beta) = \beta^{-3} (2p_h(\beta^{-1}) + \beta^{-1} p'_h(\beta^{-1})). \quad (6)$$

If we let $z \triangleq \beta^{-1}$ and $g(z) \triangleq 2p_h(z) + z^2 p'_h(z)$, then for any $t > 0$ we have

$$\int_0^t z g(z) dz = \int_0^t (2z p_h(z) + z^2 p'_h(z)) dz = t^2 p_h(t). \quad (7)$$

The fact that the fading channel power gain h has a finite second moment implies that $\lim_{t \rightarrow \infty} t^2 p_h(t) = 0$. Since the function $z g(z)$ integrates to 0 over $(0, \infty)$, $g(z)$ must change sign. Recalling that p_h is continuously differentiable ($p_h \in \mathcal{C}^1$), $g(z) = 0$ must have at least one positive root. Consequently, from (6), $P_{\text{out}}(\beta)$ has at least one inflection point.

To analyze the behavior of $P_{\text{out}}(\beta)$ at the high transmit power region (large β), we take a sufficiently small value for $t > 0$ in (7). Since $t^2 p_h(t) \geq 0$ and t is very small, we can conclude that $g(z) \geq 0$ for small z by continuity. Hence, $P''_{\text{out}}(\beta) \geq 0$ and $P_{\text{out}}(\beta)$ is convex in the high transmit power region. For the low transmit power case (small β), we take a sufficiently large yet arbitrary value for $t > 0$. Employing the finite second moment argument once more, we have

$$\int_t^\infty z g(z) dz = -t^2 p_h(t) \leq 0 \quad (8)$$

which implies that $g(z) \leq 0$ for large z . Hence, $P''_{\text{out}}(\beta) \leq 0$ and $P_{\text{out}}(\beta)$ is concave in the low transmit power region. Finally, since P_{out} is concave for small β and P_{out} is convex for large β , there must be an odd number of inflection points, $P''_{\text{out}}(\beta) = 0$, in between by the continuity of the second derivative. \square

Proposition 1 implies that it is possible to improve outage performance via power randomization under the fixed average transmit power, unless the average transmit power limit is large, in which case the best strategy is to always transmit at the fixed average power limit. This conclusion can also be made based on the formulation in (4) since the optimum outage probability is expressed as a convex combination of (at most) two outage probabilities corresponding to different power levels. Therefore, due to the presence of the concave regions in P_{out} (as implied by Proposition 1), it is possible to achieve a lower outage probability via power randomization (convex combination) than the minimum outage probability that is obtained without power randomization (i.e., transmitting always at the fixed average power limit). Also, since P_{out} is nonincreasing and convex for high transmit powers (as stated in the proof of Proposition 1), no improvements can be achieved via power randomization if the average transmit power limit is sufficiently large.

In the following, we investigate the optimum power randomization strategy in more detail for the case of a single inflection point. As shown in Section III, this assumption is valid for a wide range of outage scenarios including log-normal shadowing, Nakagami- m fading, and diversity combining over Rayleigh fading channels. Before stating the optimal strategy in this case, we derive the following lemma in a similar manner to that in [59, Lemma 2].

Lemma 1: Let $\hat{\beta}$ be the only inflection point obtained from the solution of $P''_{\text{out}}(\beta) = 0$ given in (6). There exists a unique point β_t with $\beta_t \geq \hat{\beta}$ such that the tangent to $P_{\text{out}}(\beta)$ at β_t passes through the point $(0, 1)$ and this tangent lies below $P_{\text{out}}(\beta)$ for all $\beta > 0$.

Proof: With a single inflection point and a finite limit, $P_{\text{out}}(\beta)$ is concave for $\beta < \hat{\beta}$, and convex for $\beta > \hat{\beta}$. As a result, the tangent at $\beta = \hat{\beta}$ lies above $P_{\text{out}}(\beta)$ for all $\beta < \hat{\beta}$. The y -axis intercept of the tangent to $P_{\text{out}}(\beta)$ at an arbitrary point $\beta \geq 0$ is given by $f(\beta) = P_{\text{out}}(\beta) - \beta P'_{\text{out}}(\beta)$. Since both $P_{\text{out}}(\beta)$ and $P'_{\text{out}}(\beta)$ are continuously differentiable functions, so is $f(\beta)$, and its derivative is $f'(\beta) = -\beta P_{\text{out}}''(\beta)$. Therefore, $f'(\beta)$ is negative for $\beta > \hat{\beta}$. Furthermore, it can be seen that $f(\hat{\beta}) \geq P_{\text{out}}(0) = 1$ and $\lim_{\beta \rightarrow \infty} f(\beta) = 0$. As a result, $f(\beta)$ is a monotonically decreasing function for $\beta > \hat{\beta}$, with an initial value that is greater than or equal to 1, and the limit at the infinity is equal to 0. This implies that there exists a unique β_t satisfying $f(\beta_t) = 1$.

The proof about the tangent lying below $P_{\text{out}}(\beta)$ for all $\beta > 0$ is as follows. Since $P_{\text{out}}(\beta)$ is convex over $(\hat{\beta}, \infty)$, the tangent at β_t lies below $P_{\text{out}}(\beta)$ for $\beta > \hat{\beta}$. On the other hand, the line segment connecting the point $(0, 1)$ to the point $(\hat{\beta}, P_{\text{out}}(\hat{\beta}))$ lies below $P_{\text{out}}(\beta)$ and its slope can be expressed as $(P_{\text{out}}(\hat{\beta}) - 1)/\hat{\beta}$. Similarly, the line segment connecting the point $(0, 1)$ to the point $(\beta_t, P_{\text{out}}(\beta_t))$ has a

slope of $(P_{\text{out}}(\beta_t) - 1)/\beta_t$. In the interval $\hat{\beta} \leq \beta \leq \beta_t$,

$$\frac{d}{d\beta} \frac{P_{\text{out}}(\beta) - 1}{\beta} = \frac{1 - f(\beta)}{\beta^2} \leq 0. \quad (9)$$

Therefore, line segments originating from the point $(0, 1)$ and passing through the point $(\beta, P_{\text{out}}(\beta))$ have decreasing slopes as β is increased in the interval $[\hat{\beta}, \beta_t]$. This, in turn, suggests that the tangent line lies below the first line segment, and consequently below $P_{\text{out}}(\beta)$ in the interval $[0, \hat{\beta}]$ as well. \square

Next, we state the optimum power transmission strategy for the case of a single inflection point under average power constraint $\bar{\beta}$ and peak power constraint β_p ($\beta_p \geq \bar{\beta}$).

Proposition 2: For $\beta_t \leq \bar{\beta}$ where β_t is as defined in Lemma 1, the best strategy is to exclusively transmit at the average power $\bar{\beta}$, i.e., power randomization does not help. When $\bar{\beta} < \beta_t < \beta_p$ is satisfied, the optimal strategy is to randomize between powers 0 and β_t with the probability of on-power $\bar{\beta}/\beta_t$. For $\beta_t \geq \beta_p$, the optimal solution randomizes between powers 0 and β_p with the probability of on-power $\bar{\beta}/\beta_p$.

Proof: The proposed strategy achieves the following outage probability:

$$P_{\text{out}}^{\text{opt}}(\bar{\beta}) = \begin{cases} P_{\text{out}}(\bar{\beta}), & \beta_t \leq \bar{\beta} \\ 1 - \frac{\bar{\beta}(1 - P_{\text{out}}(\beta_t))}{\beta_t}, & \bar{\beta} < \beta_t < \beta_p \\ 1 - \frac{\bar{\beta}(1 - P_{\text{out}}(\beta_p))}{\beta_p}, & \beta_t \geq \beta_p \end{cases} \quad (10)$$

The proof can be established in a straightforward manner by noticing that $P_{\text{out}}^{\text{opt}}(\beta)$ is the largest convex function that lower-bounds $P_{\text{out}}(\beta)$ for $\beta \in [0, \beta_p]$, and thus the outage probability cannot be further decreased by power randomization [58], [59]. \square

From Proposition 2, it is seen that a weak transmitter can benefit from on-off power randomization to reduce its outage probability. Furthermore, the optimal power randomization strategy is solely determined by the value of tangential point β_t and its relation to average and peak power constraints. Depending on the specific form of the fading distribution, a closed form solution for β_t may not be available. In this case, β_t can be solved numerically under the constraint $\beta_t \geq \hat{\beta}$ via the equation $\beta_t P'_{\text{out}}(\beta_t) = P_{\text{out}}(\beta_t) - 1$, or equivalently solving for x from

$$x p_h(x) = \int_x^{\infty} p_h(\tau) d\tau, \quad (11)$$

and substituting $\beta_t = x^{-1}$.

We also present a numerical algorithm that is guaranteed to converge globally to the true value of β_t with desired accuracy. The proposed method relies on a bisection search algorithm that facilitates rapid convergence and the solution of a convex optimization problem at each iteration [54]. This is given below.

Algorithm

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 $\lambda_{\min} = P'_{\text{out}}(\hat{\beta}), \lambda_{\max} = 0$ 
 $\beta_{\min} = \hat{\beta}, \beta_{\max} = \infty$ 
do

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 $\lambda = (\lambda_{\max} + \lambda_{\min})/2$ 
 $\beta_X = \underset{\beta \in (\beta_{\min}, \beta_{\max})}{\text{argmin}} P_{\text{out}}(\beta) - \lambda\beta$ 
if  $P_{\text{out}}(\beta_X) - P'_{\text{out}}(\beta_X)\beta_X > 1$ ,
  then  $\lambda_{\min} = \lambda, \beta_{\min} = \beta_X$ 
  else  $\lambda_{\max} = \lambda, \beta_{\max} = \beta_X$ 
while  $|P_{\text{out}}(\beta_X) - P'_{\text{out}}(\beta_X)\beta_X - 1| > \epsilon$ 

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At each iteration, either λ_{\min} increases towards $P'_{\text{out}}(\beta_t)$ or λ_{\max} decreases towards $P'_{\text{out}}(\beta_t)$, and $\lambda_{\max} \geq P'_{\text{out}}(\beta_t) \geq \lambda_{\min}$ is assured. Thus, λ converges to $P'_{\text{out}}(\beta_t)$. At convergence, we have $\beta_X = \underset{\beta \in (\beta_{\min}, \beta_{\max})}{\text{argmin}} P_{\text{out}}(\beta) - P'_{\text{out}}(\beta_t)\beta = \beta_t$. In practice, a sufficiently small value is selected for ϵ to control the accuracy of the solution at convergence.

III. VARIOUS APPLICATIONS AND NUMERICAL EXAMPLES

In this section, we apply the results from the previous section to improve the performance of some commonly employed systems in the wireless communications literature, which include log-normal shadowing, diversity combining over Rayleigh fading channels, Nakagami-m fading, cognitive radio, and jamming applications.

A. Log-normal Shadowing

Empirically, the Gaussian (normal) distribution has been found to accurately model the medium-scale variations of the received power, when represented in dB scale, due to changes in the reflecting surfaces and scattering objects in the signal path [1]. More explicitly, the channel power gain h can be modeled by a log-normal random variable where $\log h$ is Gaussian distributed with mean μ and variance σ^2 . In practice, log-normal shadowing is usually identified in terms of its dB-spread via the relation $\sigma = 0.1 \log(10) \sigma_{dB}$. By defining $\gamma = \rho h/N \triangleq \rho e^{\mu + \sigma z}/N$, the outage probability is given by

$$P_{\text{out}}(\beta) = Q\left(\frac{\log \beta}{\sigma}\right) \quad (12)$$

where $\beta \triangleq (\rho e^{\mu})/(N\gamma_0)$ represents the normalized transmit power, and $Q(x) = (\sqrt{2\pi})^{-1} \int_x^{\infty} e^{-t^2/2} dt$ denotes the tail probability of the standard normal distribution. Then, the first and second derivatives of the outage probability can be derived as

$$P'_{\text{out}}(\beta) = -(\sqrt{2\pi}\sigma\beta)^{-1} e^{-(\log \beta)^2/(2\sigma^2)} \quad \text{and} \quad (13)$$

$$P''_{\text{out}}(\beta) = (\sqrt{2\pi}\sigma)^{-1} \beta^{-2} e^{-(\log \beta)^2/(2\sigma^2)} \left(1 + \frac{\log \beta}{\sigma^2}\right). \quad (14)$$

From (13) and (14), it is deduced that P_{out} is a monotonically decreasing function of β with a single inflection point at $\hat{\beta} = e^{-\sigma^2}$. As a result of Proposition 2, the outage probability can be reduced via on-off power randomization for small values of the average power constraint.

In Fig. 2, we investigate the effects of shadow fading standard deviation on the outage performance under the optimum power randomization strategy. The solid lines correspond to the outage probability under fixed power transmission, whereas the dashed lines depict the outage probability under

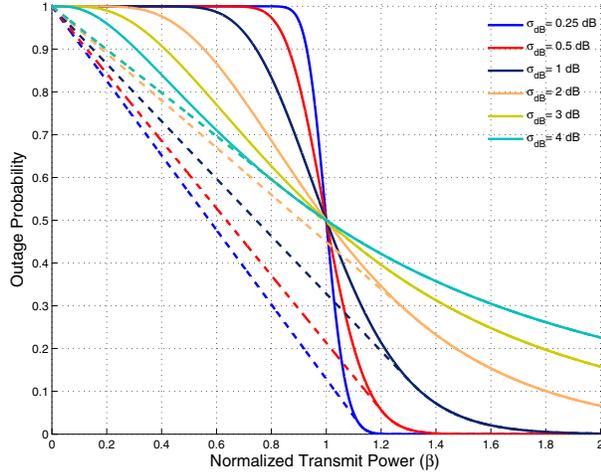


Fig. 2. Outage probability versus normalized transmit power for fixed power transmission (solid lines) and optimum power randomization (dashed lines) for various values of shadow fading standard deviation.

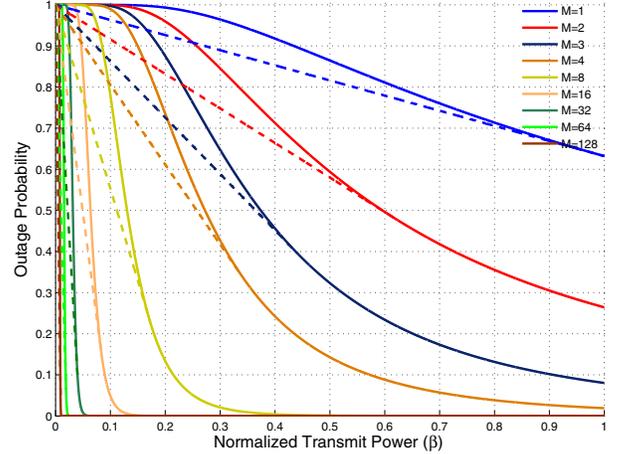
TABLE I
PARAMETER VALUES FOR OPTIMAL POWER RANDOMIZATION UNDER LOG-NORMAL SHADOWING

σ^2	$\bar{\beta}$	β_t	$P_{\text{out}}(\beta_t)$	$P'_{\text{out}}(\beta_t)$
0.25	0.9967	1.1207	0.0239	-0.8710
0.5	0.9868	1.2038	0.0536	-0.7862
1	0.9484	1.3079	0.1218	-0.6714
2	0.8089	1.3165	0.2752	-0.5505
3	0.6205	1.1274	0.4311	-0.5046
4	0.4281	0.8413	0.5744	-0.5059

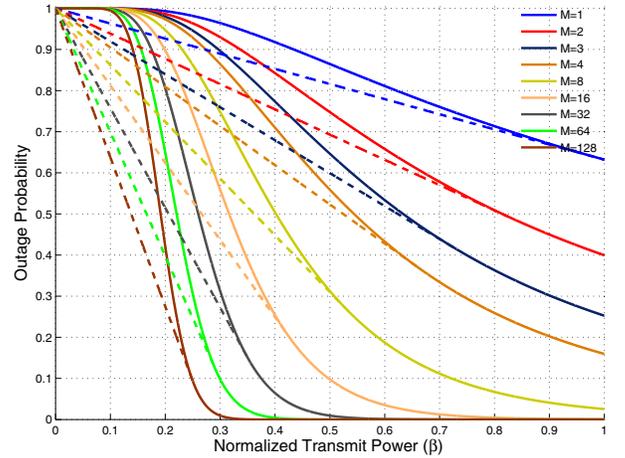
optimum power randomization as stated in Proposition 2. For small σ_{dB} values, the performance improvement due to power randomization becomes much more evident. For $\beta = 1$ and $\sigma_{dB} = 1$ dB, it is possible to decrease the outage probability from 0.5 down to 0.3286. However, if $\sigma_{dB} = 0.5$ dB, the outage probability can be decreased even further down to 0.2138. Table I summarizes the optimal power randomization parameters employed to achieve these performance figures under log-normal shadowing. $\bar{\beta}$ represents the unique inflection point of the outage curve, β_t denotes the normalized transmit power at the tangent point, $P_{\text{out}}(\beta_t)$ is the corresponding outage probability, and $P'_{\text{out}}(\beta_t)$ is the slope of the curve at the tangent point. Using this table, it can be determined that if the average transmit power limit is greater than the tangent value ($\bar{\beta} > \beta_t$), transmission should be continuous at the average power value. On the other hand, if the average transmit power limit is less than the tangent value ($\bar{\beta} < \beta_t$), the optimum solution employs transmit power β_t with probability $\bar{\beta}/\beta_t$ or aborts transmission otherwise. In other words, the optimal on-off transmitter employs the following PDF for the normalized power parameter: $p_{\beta}(x) = (\bar{\beta}/\beta_t)\delta(x - \beta_t) + (1 - \bar{\beta}/\beta_t)\delta(x)$.

B. Diversity Combining over Rayleigh Fading Channels

In this part, we assume that measurements are acquired from M receive antennas associated with independent and identically distributed (i.i.d.) Rayleigh fading paths. Suppose also that the effective noise powers in all branches of the combiner



(a)



(b)

Fig. 3. Outage probability versus normalized transmit power for fixed power transmission (solid lines) and optimum power randomization (dashed lines) for various values of the antenna number M under (a) MRC, (b) SC at the receiver.

are equal. Then, the SNR at the combiner input from branch i is given by $\gamma_i \triangleq \rho h_i/N$ where ρ denotes the transmit power, h_i is the channel power gain between the transmitter and the receive antenna i , and N represents the noise power. Under Rayleigh fading, the channel power gain is exponentially distributed; that is: $p_{h_i}(x) = \lambda^{-1}e^{-x/\lambda}$ for $x \geq 0$, where λ denotes the average channel power gain due to Rayleigh fading [1]. Next, we examine the outage performance of two widely employed combining techniques.

1) *Maximal-Ratio Combining (MRC)*: In this case, the signals in all the branches are combined coherently to maximize the output SNR. The resulting combiner SNR is given by $\gamma_{\Sigma} = \sum_{i=1}^M \gamma_i = (\rho/N) \sum_{i=1}^M h_i \triangleq \rho \lambda h_{eq}/N$, where the distribution of h_{eq} is Erlang with shape parameter M and scale parameter 1 [1]:

$$p_{h_{eq}}(x) = \frac{x^{M-1}e^{-x}}{(M-1)!}, \quad x \geq 0 \quad (15)$$

Let $\beta \triangleq (\rho\lambda)/(N\gamma_0)$ denote the normalized transmit power. The corresponding outage probability can be calculated from:

$$\begin{aligned} P_{\text{out}}(\beta) &= \Pr(\gamma_{\Sigma} < \gamma_0) = \Pr\left(h_{\text{eq}} < \frac{N\gamma_0}{\rho\lambda}\right) \\ &= 1 - e^{-1/\beta} \sum_{m=1}^M \frac{\beta^{1-m}}{(m-1)!}. \end{aligned} \quad (16)$$

The first and second derivatives of the outage probability can be obtained by directly differentiating (16), or equivalently from (5) and (6), which give

$$P'_{\text{out}}(\beta) = -\frac{e^{-1/\beta}}{(M-1)!\beta^{M+1}}, \quad \text{and} \quad (17)$$

$$P''_{\text{out}}(\beta) = \frac{e^{-1/\beta}}{(M-1)!\beta^{(M+3)}} ((M+1)\beta - 1). \quad (18)$$

From the equations above, it is observed that $P_{\text{out}}(\beta)$ is a monotonically decreasing function for all $\beta > 0$ with a single inflection point at $\hat{\beta} = 1/(M+1)$. Since $P_{\text{out}}(\beta)$ is concave for $\beta < 1/(M+1)$, it is possible to improve the outage performance via power randomization for weak transmitters or under strict average transmit power constraints.

In Fig. 3(a), the outage probability is plotted versus the normalized transmit power for various values of the number of antennas M under MRC at the receiver. In accordance with Proposition 2, power randomization results in superior outage performance over the fixed power transmission scheme for small values of the average transmit power constraint. Also noted from the figure is that as the number of antennas employed at the receiver increases, the improvements due to power randomization becomes more pronounced. For example, when $M = 8$ and $\beta = 0.03$, the probability of outage drops from 0.6146 to 0.3253 under the optimum power randomization strategy. If $M = 32$ and $\beta = 0.02$ are selected, the improvement in outage probability is even higher, from 0.9973 down to 0.5502.

2) *Selection Combining (SC)*: In selection combining, the combiner outputs the signal on the branch with the highest SNR. The combiner output has an SNR equal to the maximum SNR of all branches which can be expressed as $\gamma_{\Sigma} = \max_{i \in \{1, 2, \dots, M\}} \gamma_i = (\rho/N) \max_{i \in \{1, 2, \dots, M\}} h_i \triangleq \rho\lambda h_{\text{eq}}/N$, where the distribution of h_{eq} is given by

$$p_{h_{\text{eq}}}(x) = M(1 - e^{-x})^{M-1} e^{-x}, \quad x \geq 0. \quad (19)$$

Similar to the previous case, let $\beta \triangleq (\rho\lambda)/(N\gamma_0)$. Then, the outage probability is expressed as a function of the normalized transmit power as follows:

$$P_{\text{out}}(\beta) = (1 - e^{-1/\beta})^M. \quad (20)$$

The first and second derivatives of the outage probability are

$$P'_{\text{out}}(\beta) = -M\beta^{-2}e^{-1/\beta}(1 - e^{-1/\beta})^{M-1}, \quad \text{and} \quad (21)$$

$$P''_{\text{out}}(\beta) = -M\beta^{-4}e^{-1/\beta}(1 - e^{-1/\beta})^{M-2}g(\beta), \quad (22)$$

where $g(\beta) \triangleq 1 - 2\beta + (2\beta - M)e^{-1/\beta}$. Again, $P_{\text{out}}(\beta)$ is a monotonically decreasing function for $\beta > 0$. However, it is difficult to find an analytical expression for the inflection point. Below, we show that there exists a unique point satisfying $P''_{\text{out}}(\beta) = 0$ (and equivalently $g(\beta) = 0$). Notice that

$\lim_{\beta \rightarrow 0} g(\beta) = 1$, $\lim_{\beta \rightarrow \infty} g(\beta) = -(M+1)$, and the derivative of $g(\beta)$ with respect to β is

$$\begin{aligned} g'(\beta) &= -2 + 2 \underbrace{\left(1 + \frac{1}{\beta}\right) e^{-1/\beta}}_{<1 \quad \forall \beta > 0} - \frac{M}{\beta^2} e^{-1/\beta} \\ &< -\frac{M}{\beta^2} e^{-1/\beta} < 0 \quad \forall \beta > 0, \end{aligned} \quad (23)$$

which altogether indicate that the zero of $g(\beta)$ occurs at a single point for $\beta \in (0, \infty)$. Therefore, the outage probability can be decreased via transmit power randomization in the case of SC diversity technique as well. Fig. 3(b) demonstrates this fact for various numbers of antenna.

C. Nakagami- m Fading

Nakagami- m distribution provides an excellent fit to a wide variety of empirical measurements. The fading parameter m represents the ratio of the power in the line-of-sight (LOS) component to the power in the multipath components. Let the average channel gain be denoted by λ . In the absence of any diversity combining techniques, the average SNR at the receiver is given by $\gamma = \rho h/N \triangleq \rho\lambda h_{\text{eq}}/N$. The power distribution under Nakagami- m fading corresponding to a unit-mean channel gain h_{eq} is expressed as [1]:

$$p_{h_{\text{eq}}}(x) = \frac{m^m x^{m-1}}{\Gamma(m)} e^{-mx}, \quad x \geq 0 \text{ and } m > 0, \quad (24)$$

where $\Gamma(m) = \int_0^{\infty} t^{m-1} e^{-t} dt$ denotes the Gamma function. By defining $\beta \triangleq (\rho\lambda)/(N\gamma_0)$, the first and second derivatives of the outage probability can be computed as

$$P'_{\text{out}}(\beta) = -\frac{m^m}{\Gamma(m)} \beta^{-(m+1)} e^{-m/\beta}, \quad \text{and} \quad (25)$$

$$P''_{\text{out}}(\beta) = \frac{m^m}{\Gamma(m)} \beta^{-(m+3)} e^{-m/\beta} ((m+1)\beta - m), \quad (26)$$

which confirm that P_{out} is a monotonically decreasing function of β with a single inflection point at $\hat{\beta} = m/(m+1)$. Hence, power randomization can help reduce the outage probability for weak transmitters as depicted in Fig. 4. Since the Rayleigh distribution is a special case of the Nakagami distribution with $m = 1$, this result agrees with that of Section III-B.

D. Spectrum Sharing in Fading Environments (Cognitive Radio)

In this part, we consider a communications scenario in which a secondary user operates simultaneously within a licensee's spectrum under a constraint on the average interference power at the primary receiver. Let h_s and h_p represent independent channel power gains from the secondary and primary transmitters to the secondary receiver, respectively. The secondary transmitter needs to know the power of the primary transmitter ρ_p , which is assumed to be fixed. Additionally, the secondary transmitter does not have the perfect knowledge of h_s and h_p instantaneously, but just their joint statistical distribution (i.e., CDI). This information can be supplied by the licensee via a feedback mechanism or by a management body which mediates the two parties [60]. For simplicity, we

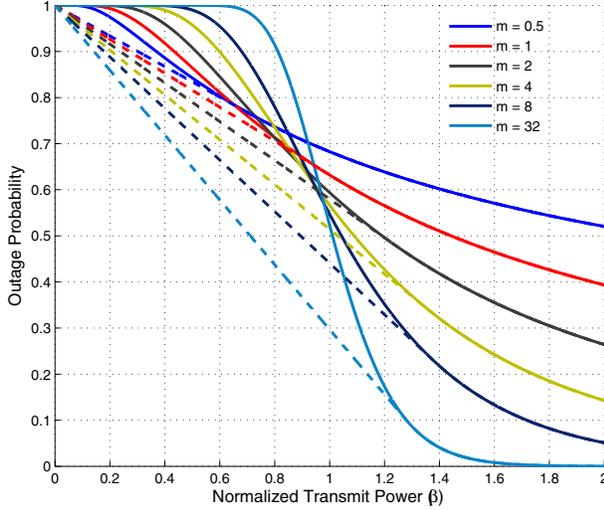


Fig. 4. Outage probability versus normalized transmit power for fixed power transmission (solid lines) and optimum power randomization (dashed lines) under Nakagami- m fading for various values of m .

assume that the noise at the secondary receiver is dominated by the interference from the primary user, hence can be neglected. The SNR at the secondary receiver can then be expressed as $\gamma \triangleq (\rho_s h_s)/(\rho_p h_p)$, where ρ_s is the power of the secondary transmitter for which the optimal power transmission strategy is sought. In this case, outage probability at the secondary receiver is given by

$$\begin{aligned} P_{\text{out}}(\beta) &= \Pr(\gamma < \gamma_0) = \Pr\left(\frac{h_s}{h_p} < \frac{\rho_p \gamma_0}{\rho_s}\right) \\ &= \Pr\left(\frac{h_s}{h_p} < \beta_s^{-1}\right) = \Pr\left(\frac{h_p}{h_s} > \beta_s\right), \end{aligned} \quad (27)$$

where $\beta_s \triangleq \rho_s/(\rho_p \gamma_0)$ is the normalized power of the secondary transmitter. Since the transmitted signal power is independent of the instantaneous value of the fading distribution, the average interference power constraint at the primary receiver can be equivalently described as an average power constraint at the secondary transmitter after proper scaling with the expected value of the channel power gain between secondary transmitter and primary receiver.

1) *Log-normal Shadowing*: Let h_s and h_p be independent log-normal random variables such that $\log h_s$ and $\log h_p$ are zero-mean Gaussian random variables with variances σ_s^2 and σ_p^2 , respectively. Then, $h_{eq} \triangleq \log(h_s/h_p)$ is also Gaussian distributed with zero-mean and variance $\sigma^2 \triangleq \sigma_s^2 + \sigma_p^2$. The rest of the analysis is exactly the same as in Section III-A, which indicates that power randomization can be employed to improve the outage performance under stringent transmit power constraints in this framework as well.

2) *Nakagami Fading*: With channel fading following the Nakagami distribution, suppose that h_s and h_p are independently distributed as shown in (24) with fading parameters m_s and m_p , respectively.³ In this case, $h_{eq} \triangleq h_p/h_s$ is known to

³Means of h_s and h_p can be captured into β_s if they are not equal to one as discussed earlier in Sections III-A, III-B and III-C.

have the beta-prime distribution [60, Appendix I]:

$$p_{h_{eq}}(x) = \left(\frac{m_s}{m_p}\right)^{m_s} \frac{x^{m_p-1}}{\mathcal{B}(m_s, m_p) \left(x + \frac{m_s}{m_p}\right)^{m_s+m_p}}, \quad (28)$$

where $\mathcal{B}(v, \varphi) = \Gamma(v)\Gamma(\varphi)/\Gamma(v + \varphi)$ is the Beta function. From the nonpositivity of the first derivative $P'_{\text{out}}(\beta_s) = -p_{h_{eq}}(\beta_s) \leq 0$, it is observed that the outage probability decreases monotonically with increasing β_s . The second derivative is given by

$$\begin{aligned} P''_{\text{out}}(\beta_s) &= \left(\frac{m_s}{m_p}\right)^{m_s} \frac{x^{m_p-2}}{\mathcal{B}(m_s, m_p) \left(x + \frac{m_s}{m_p}\right)^{m_s+m_p+1}} \\ &\quad \cdot \left((m_s + 1)x - \frac{m_s}{m_p}(m_p - 1) \right). \end{aligned} \quad (29)$$

From (29), it is noted that $m_p \leq 1$ (severe fading over the channel from the primary transmitter to the secondary receiver) provides a necessary and sufficient condition for non-improvability of the outage probability via secondary transmit power randomization for all $\beta_s \geq 0$. For $m_p > 1$, there exists a single inflection point at $\hat{\beta}_s = \frac{m_s(m_p-1)}{m_p(m_s+1)}$, which suggests that power randomization can help reduce the outage probability of the secondary user when the average transmit power should be limited. As an example, consider identical and independent Rayleigh fading on both channels, i.e., $m_s = m_p = 1$. In this case, h_{eq} has a log-logistic distribution [60]:

$$p_{h_{eq}}(x) = \frac{1}{(1+x)^2}, \quad x \geq 0. \quad (30)$$

Correspondingly, the outage probability and its second derivative are given by

$$P_{\text{out}}(\beta) = \frac{1}{1+\beta}, \quad P''_{\text{out}}(\beta) = \frac{2}{(1+\beta)^3} > 0 \quad \forall \beta > 0. \quad (31)$$

As expected from the condition $m_p \leq 1$, the power randomization does not help reduce the outage probability in this scenario due to convexity. From another point of view, log-logistic distribution does not have a finite second moment: $(\lim_{x \rightarrow \infty} x^2/(1+x)^2 = 1)$, which justifies why Propositions 1 and 2 are not applicable.

E. Jammer's Perspective

In this part, we investigate the convexity properties of the outage probability in the presence of an average power constrained Gaussian jammer. Assuming that the jammer has only the knowledge of the fading distribution (contrary to the cases in which the jammer has access to perfect CSI [61]–[63]), the optimum jammer power allocation strategy is studied in order to maximize the outage probability of the victim system under different fading scenarios.

1) *Fading over only Jammer-Receiver Channel*: This scenario considers the case when the received power due to jamming varies while the received power due to signal transmission is fixed. The random fluctuations in the received jamming power may result from the inaccuracy of the jammer to resolve the parameters of the victim receiver such as the center frequency or the operating band. It may also be the

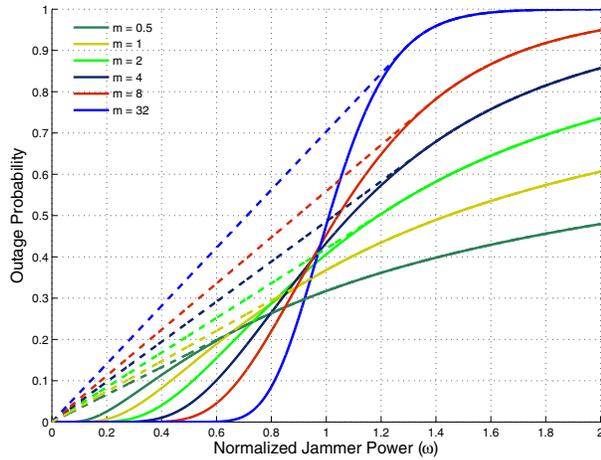


Fig. 5. Outage probability versus normalized jammer transmit power for fixed power jamming (solid lines) and optimum jammer power randomization (dashed lines) for various values of fading parameter m .

case that the jammer is moving with respect to the receiver while the transmitter and the receiver stay at fixed locations, which allows us to assume that the received jammer power changes much faster than the received signal power. Under such circumstances, we express the SNR at the receiver as $\gamma \triangleq \rho/(\Omega h)$, where ρ denotes the fixed received signal power, Ω is the jammer transmit power, and h is the channel power gain between the jammer and the receiver. Given a target SNR γ_0 , let $\omega \triangleq (\Omega\gamma_0)/\rho$ represent the normalized jammer power. Then, the outage probability as a function of ω is given by

$$\begin{aligned} P_{\text{out}}(\omega) &= \Pr(\gamma < \gamma_0) = \Pr\left(h > \frac{\rho}{\Omega\gamma_0}\right) \\ &= \Pr(h > \omega^{-1}) = \int_{\omega^{-1}}^{\infty} p_h(x) dx. \end{aligned} \quad (32)$$

Comparing (32) with (1), it is observed that the outage probability in the latter case equals one minus the outage probability of the former assuming the same values for ω and β . This implies a sign reversal for all the first and second derivative expressions obtained so far. Therefore, similar conclusions can be deduced in a straightforward manner. As an example, Fig. 5 illustrates the performance degradation in the outage probability due to jammer power randomization under Nakagami fading for various values of the parameter m . In practice, it is desired that the outage probability should be less than 1% [1]. From Fig. 5, it is observed that jammer power randomization strategy is very effective in degrading the outage performance over these regions. For example, when $m = 4$ and $\omega = 0.4$, the outage probability under constant power jamming is 0.0103, whereas the outage probability can be increased up to 0.1942 via the optimum jammer power randomization. Also noted from the figure is that the jammer power randomization strategy is more effective for higher values of m which indicates less severe fading conditions.

2) *Fading over only Transmitter-Receiver Channel*: Similar to the previous case, it is possible to construct scenarios in which the received signal power varies much faster than the received jammer power (e.g., the jammer and the receiver are

at fixed locations whereas the transmitter is moving). In other words, we can assume that the channel between the transmitter and the receiver is subject to fading while the channel between the jammer and the receiver introduces a fixed power gain. In such cases, the SNR at the receiver can be specified as $\gamma \triangleq (\rho h)/(\Omega)$, where ρ denotes the transmitted signal power, h is the channel power gain between the transmitter and the receiver, and Ω is the received jammer power. For a target SNR γ_0 , the normalized jammer power is defined as $\omega \triangleq (\Omega\gamma_0)/\rho$ and the corresponding outage probability can be obtained from

$$\begin{aligned} P_{\text{out}}(\omega) &= \Pr(\gamma < \gamma_0) = \Pr\left(h < \frac{\Omega\gamma_0}{\rho}\right) \\ &= \Pr(h < \omega) = \int_0^{\omega} p_h(x) dx. \end{aligned} \quad (33)$$

Differentiating with respect to ω , the first and second derivatives are given as

$$P'_{\text{out}}(\omega) = p_h(\omega), \quad \text{and} \quad P''_{\text{out}}(\omega) = p'_h(\omega), \quad (34)$$

which indicate that the outage probability is nondecreasing in the normalized jammer power and the inflection points are the stationary points of the PDF $p_h(\omega)$ assuming continuous differentiability. For Nakagami- m fading with a unit-mean channel power gain,

$$p'_h(\omega) = \frac{m^m}{\Gamma(m)} x^{m-2} e^{-mx} ((m-1) - m\omega) \quad (35)$$

implies that the outage probability is concave for $m < 1$. Therefore, jammer power randomization would not help degrade the outage performance under severe fading. When $m > 1$, the outage probability has a single inflection point at $\hat{\omega} = (m-1)/m$ suggesting that weak jammers can degrade the outage performance via power randomization in comparison to the constant power jamming strategy. Similarly for log-normal shadowing with parameter $\mu = 0$, $\Pr(h < \omega) = \Pr(h > \omega^{-1})$ and the single inflection point occurs at $\hat{\omega} = e^{-\sigma^2}$, which points out that benefits from power randomization are limited to a very small interval for high values of σ .

3) *Fading over Both Channels*: This case can be treated in an analogous way to that in Section III-D by noting that the SNR at the receiver is expressed as $\gamma \triangleq (\rho h_t)/(\Omega h_j)$, where h_t and h_j represent the channel power gains from the transmitter and the jammer to the receiver, respectively. Hence, same results are valid.

IV. CONCLUDING REMARKS

In this work, we have analyzed the convexity/concavity properties of the outage probability curve for flat BF-AWGN channels in terms of the normalized transmitted signal power. It has been shown that when the PDF of the channel power gain is continuously differentiable with a finite second moment, the outage probability is nonincreasing with at least one inflection point and the total number of inflection points is odd. For the case of a single inflection point, we have shown that the outage probability can be reduced via the optimum on-off type transmit power randomization in low transmit power regime. Examples from commonly adopted shadowing, fading, and diversity combining models point out significant performance improvements over the common practice which

is restricted to constant power transmission in the absence of CSI. Similar studies show that an average power constrained jammer can degrade the outage performance of the victim communications system considerably. For cognitive radio and jammer applications under Nakagami- m fading, sufficient and necessary conditions are also provided for the nonimprovability of the outage performance in terms of the fading parameter m , and its relation to the finite second moment assumption is discussed. A future work is to investigate the effects of covariance matrix randomization to minimize the probability of outage for a given target data rate vector over a fading MIMO channel, the distribution of which is known to the transmitter.

It should be noted that the proposed power randomization strategy is optimal when the channel distribution is perfectly known at the transmitter but additional information about the instantaneous state of the channel is not available. On the other hand, if instantaneous CSI is available, the transmitter can adapt its power accordingly. This type of power adaptation that utilizes CSI will perform superior to the proposed approach which relies solely on CDI. Nevertheless, results of this paper can be extended to variable power transmission strategies that utilize CSI. Channel fading varies in a continuous manner, while power adaptation needs to be performed at discrete time instants. When the channel power is adapted according to the current state of the channel fading, the transmitter employs fixed power transmission for a certain period of time (e.g., until the next update of the sensed channel statistics). When the channel state information is not updated frequently, power randomization can be employed to help improve performance by partially compensating for the variations in the channel fading between consecutive updates.

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