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How supply chain coordination affects the environment: a carbon footprint perspective

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Abstract Environmental responsibility has become an important part of doing business. Government regulations and customers' increased awareness of environmental issues are pushing supply chain entities to reduce the negative influence of their operations on the environment. In today's world, companies must assume joint responsibility with their suppliers for the environmental impact of their actions. In this paper, we study coordination between a buyer and a vendor under the existence of two emission regulation policies: cap-and-trade and tax. We investigate the impact of decentralized and centralized replenishment decisions on total carbon emissions. The buyer in this system faces a deterministic and constant demand rate for a single product in the infinite horizon. The vendor produces at a finite rate and makes deliveries to the buyer on a lot-for-lot basis. Both the buyer and the vendor aim to minimize their average annual costs resulting from replenishment set-ups and inventory holding. We provide decentralized and centralized models for the buyer and the vendor to determine their ordering/production lot sizes under each policy. We compare the solutions due to independent and joint decision-making both analytically and numerically. Finally, we arrive at coordination mechanisms for this system to increase its profitability. However, we show that even though such coordination mechanisms help the buyer and the vendor decrease their costs without violating emission regulations, the cost minimizing solution may result in increased carbon emission under certain circumstances.

Keywords Environmental regulations · Buyer–vendor coordination · Supply chains

1 Introduction and literature

Since the Industrial Revolution, the levels of greenhouse gases in the atmosphere have increased due to human activities. The World Meteorological Organization (WMO) (2013a)

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reports that the atmospheric concentrations of the greenhouse gases exhibited an upward and accelerating trend and reached a record high in 2012. Greenhouse gases slow or prevent the loss of heat to space, which increases the temperature of Earth's surface, leading to global warming. Greenhouse gases are emitted as a result of the activities of energy industries, transportation, residential and commercial activities, manufacturing, construction, industrial processes, and agriculture. Carbon dioxide (CO_2) is the main greenhouse gas emitted as a result of the human activities; it is responsible for 85 % of the increase in global warming. The effect of CO_2 is followed by methane (CH_4,) and then nitrous oxide (N_2O) (WMO 2013b). To decrease greenhouse gases (particularly CO_2) emissions, policy makers and international organizations have proposed agreements and regulations. In this paper, we study the independent and coordinated inventory replenishment decisions of a buyer and a vendor under two different emission regulation policies (i.e., cap-and-trade and tax), and investigate the impact of coordinated decisions on the environment.

Under a cap-and-trade mechanism, the government sets a fixed value for the maximum amount of carbon that can be emitted in each period (i.e., the cap) and firms are free to buy or sell allowances in trading markets. Emission trading systems (ETSs) are currently implemented in the EU (EU ETS), Australia, New Zealand (NZ ETS), Northeastern United States, and Tokyo (Tokyo ETS), as well as in other countries (see the International Emissions Trading Association's web site International Emissions Trading Assosication 2013). The carbon tax mechanism puts a price on each tonne of greenhouse gas (e.g., CO_2) emitted. According to the Center for Climate and Energy Solutions (2013), Finland, the Netherlands, Norway, Sweden, the UK and Australia are among countries that have implemented a carbon tax.

Issues related to environmental policies, such as regulation design and the effect of a domestic environmental policy on international trade or social welfare, and others, have been widely investigated in environmental economics since the late 1960s (Cropper and Oates 1992). In contrast, the literature in operations management that considers environmental concerns is fairly new, and focuses on tactical or operational planning decisions. Some of these studies do not particularly assume the existence of environmental regulations; rather, they optimize an objective function that incorporates terms dependent on environmental performance metrics (e.g., Bonney and Jaber 2011; Bouchery et al. 2011; Chan et al. 2013; Saadany et al. 2011), or investigate the impact of supply chain members' greening efforts on their profitability in different settings with environmentally conscious consumers (e.g., Liu et al. 2012; Swami and Shah 2013). Another group of papers studies problems such as single-item inventory replenishment, product mix, or green investment decisions, while considering a specific environmental regulation policy (e.g., Benjaafar et al. 2013; Chen et al. 2013; Dong et al. 2014; Du et al. 2011; Hoen et al. 2014; Hua et al. 2011; Jaber et al. 2013; Krass et al. 2013; Letmathe and Balakrishnan 2005; Song and Leng 2012; Toptal et al. 2014; Zhang and Xu 2013). Of these papers, Benjaafar et al. (2013), Dong et al. (2014), Du et al. (2011), Krass et al. (2013) and Jaber et al. (2013) model supply chain problems in multi-echelon settings, as our study does.

Benjaafar et al. (2013) propose an integrated model to solve the joint lot-sizing decisions of multiple firms subject to emission caps. Krass et al. (2013) consider a two-echelon system in which the upper echelon is the policy maker who maximizes social welfare and the lower echelon is a firm that maximizes its profits. In this setting, the authors analyze a Stackelberg game under three different environmental polices: tax-only, tax-and-subsidy, and a joint policy that also includes rebates given to consumers who buy products manufactured with green technologies. The policy maker, as the Stackelberg leader, decides the parameters of the different policies and the firm chooses the emission-reducing technology and the selling price. Jaber et al. (2013) investigate the impact of coordination on some environmental measures in a manufacturer-retailer setting. In this setting, manufacturer is the only party who is subject to an environmental policy. Manufacturer's emissions due to his/her production rate are penalized with a per-unit emission cost and a fixed penalty if the total amount of emissions exceeds a limit. This combination policy allows for the modeling of a tax policy and a variant of the cap-and-trade policy. Specifically, as in cap-and-trade policy, a cost is incurred when an upper bound on the emissions is exceeded. However, unlike the typical cap-and-trade policy, it does not allow for the possibility of gains when the emissions are under the upper bound. In analyzing the impact of coordination on the environment, the authors look into how the solution to the integrated problem changes the sum of emissions and penalty costs in comparison to independently-made decisions of the parties. They observe over a set of examples that total system costs reduce with no change in the sum of emissions and penalty costs.

As opposed to Benjaafar et al. (2013), Krass et al. (2013) and Jaber et al. (2013), the studies of Dong et al. (2014) and Du et al. (2011) consider stochastic demand environments. Du et al. (2011) analyze a two-echelon system in which the upper echelon, as the permit supplier, decides the permit selling price, and the lower echelon, as the manufacturer, decides his/her production quantity. In this system, if the manufacturer needs more carbon allowance, he/she purchases it from the permit supplier, but does not have the option to sell if he/she has excess carbon allowance. In the manufacturer-retailer setting considered by Dong et al. (2014), the retailer decides the order quantity in response to the manufacturer's decision regarding the sustainability investment. The manufacturer in this setting is subject to a cap-and-trade policy, and both the selling and purchasing prices of the unit carbon allowance are the same. The authors also examine some of the traditional contracting mechanisms and show that revenue-sharing contracts can be used for coordinating this supply chain system.

In this paper, we consider a buyer–vendor system with deterministic and steady demand rate in the infinite horizon. Our paper exhibits relative similarities to each of the reviewed papers that model the existence of an environmental regulation policy in a multi-echelon setting. However, different from the majority of the papers in this area (i.e., Benjaafar et al. 2013; Krass et al. 2013; Jaber et al. 2013; Du et al. 2011), we focus on coordination within the context of inventory replenishment decisions, and we consider a cap-and-trade and a tax policy. We propose coordination mechanisms to align each firm's objective with the supply chain's objective. Dong et al. (2014) is the only paper with a similar focus under a cap-and-trade policy, but unlike those authors, we assume that both the buyer and the vendor are subject to the policy, and our modeling allows for cases in which the purchasing price of a unit carbon allowance is greater than its selling price.

As the world economy becomes increasingly conscious of the environmental concerns, it is more likely that we will evidence complex settings where several parties in the supply chain may be subject to emission policies. In fact, Center for Climate and Energy Solutions reports California cap-and-trade program and European Union (EU) Emission Trading Scheme as examples of multi-sector cap-and-trade programs (see Center for Climate and Energy Solutions 2014). Electricity, heat and steam production, oil, iron and steel, cement, glass, pulp and paper are industries in EU's Emission Trading System, and electricity, ground transportation, heating fuels are industries in California's cap-and-trade program. This obviously indicates a need for models that analyze multiple parties in the supply chain being subject to emission policies.

Another distinguishing characteristic of our models is the difference between the purchasing and selling prices of unit carbon allowance, which leads to a piecewise objective function in both the decentralized and centralized models. Through a careful analysis of the structural properties of the objective functions, we propose finite-time exact solution procedures for these optimization problems. Our consideration of the cap-and-trade policy for both parties and the difference in carbon trading prices leads to some novel coordination mechanisms based on carbon credit sharing. We also extend our modeling and analysis to the case of tax policy. A final contribution of our paper is that for both policies, we investigate the impact of coordination on the environment in terms of the resulting carbon emissions. Our numerical analysis for the cap-and-trade policy and our analytical results for the tax policy show that coordination may not always be good for the environment.

In the next section, we begin with the problem definition and formulation under the two policies. In Sect. 3, we present our analytical and numerical results for the cap-and-trade policy. We then continue in Sect. 4 with an analysis for the tax policy. We conclude the paper in Sect. 5 with a discussion of our findings.

2 Problem definition and notation

We consider a system that consists of a buyer (retailer) and a vendor (manufacturer). The buyer and the vendor operate to meet the deterministic demand of a single product in the infinite horizon using a lot-for-lot policy. That is, the quantity produced by the manufacturer at each setup is equal to the retailer's ordering lot size. Shortages are not allowed and the replenishment lead times are zero (or, equivalently, deterministic in this setting). The vendor incurs a setup cost of K_v monetary units at each production run, and the buyer incurs a fixed cost of K_b monetary units at each ordering. The vendor and the buyer are subject to cost rates h_v and h_b, respectively, for each unit held in the inventory for a unit time. It is important to note that the joint replenishment decisions in this setting have been previously studied by Banerjee and Burton (1994) and Lu (1995). In this paper, we model the carbon emissions of the buyer and the vendor resulting from production- and inventory-related activities, and we study how replenishment decisions can be coordinated under a cap-and-trade policy and a tax policy. Table 1 introduces the notation that will be used in our modeling for both policies. Without any loss of generality, the time unit is taken as a year.

In order to arrive at a coordinated solution for the two-echelon system, we study two models under each policy: the decentralized model and the centralized model. In the decentralized model, the buyer's independent replenishment decisions minimizing his/her cost per unit time determine the vendor's replenishment lot size. In the centralized model, the buyer's and vendor's costs and constraints are simultaneously taken into account to find a quantity that minimizes the total system cost per unit time. Using the centralized solution as a benchmark, we develop mechanisms that utilize price discounts, carbon credit sharing, and fixed payments to coordinate the system.

2.1 Modeling of the different solution approaches under the cap-and-trade policy

Under a cap-and-trade policy, both the buyer and the vendor have carbon caps (i.e., a carbon emission quota per unit time). They both emit carbon due to production/ordering setups, inventory holding, and procurement. If the emissions per unit time of one party exceed his/her cap, then he/she buys carbon credits at a rate of p_c^b monetary units for one unit carbon emission. If the emissions per unit time are below the cap, then the excess amount of carbon credit is sold at a rate of p_c^s monetary units for unit carbon emission ($p_c^s \leq p_c^b$). Buying and selling carbon credits can be compared to buying and selling shares in a stock market. The difference $p_c^b - p_c^s$ can be considered as the gap between the bid and asking prices for the allowance of emitting one unit carbon. The particular values of p_c^b and p_c^s are determined by the supply and demand for carbon allowances in the market. Nouira et al. (2014) reports

Table 1 Buyer's and vendor's production/inventory- and emission-related parameters

Buyer's parameters	
D	Annual demand
K_b	Fixed cost of ordering
h_b	Cost of holding one unit inventory for a year
c	Unit purchasing cost
f_b	Fixed amount of carbon emission at each ordering
g_b	Carbon emission amount due to holding one unit inventory for a year
e_b	Carbon emission amount due to unit procurement
Vendor's parameters	
P	Production rate ($P > D$)
K_v	Fixed cost per production run
h_v	Cost of holding one unit inventory for a year
p_v	Unit production cost
f_v	Fixed amount of carbon emission at each production setup
g_v	Carbon emission amount due to holding one unit inventory for a year
e_v	Carbon emission amount due to producing one unit

that in most cases $p_c^b > p_c^s$ due to differences in transaction costs for selling and purchasing allowances. Table 2 summarizes the additional notation specific to our discussion for the cap-and-trade policy.

Under a cap-and-trade policy, the buyer's average annual cost is given by

$$BC\left(Q, X_b\right) = \begin{cases} BC_1\left(Q, X_b\right) & \text{if } \ X_b \leqslant 0 \\ BC_2\left(Q, X_b\right) & \text{if } \ X_b > 0, \end{cases} \tag{1}$$

where

$$BC_1(Q, X_b) = \frac{K_b D}{Q} + \frac{h_b Q}{2} + cD - p_c^b X_b, \tag{2}$$

and

$$BC_2(Q, X_b) = \frac{K_b D}{Q} + \frac{h_b Q}{2} + cD - p_c^s X_b. \tag{3}$$

If the buyer buys carbon credits (i.e., X_b is negative), his/her annual cost function is given by Expression (2). If the buyer sells carbon credits (i.e., X_b is positive), his/her annual cost function is given by Expression (3). Note that if the buyer neither sells nor buys carbon credits (i.e., $X_b = 0$), then $BC_1(Q, X_b) = BC_2(Q, X_b)$.

The buyer's average annual emission when Q units are ordered amounts to

$$\frac{f_b D}{Q} + \frac{g_b Q}{2} + e_b D. \tag{4}$$

When no emission regulation policy is in place, $Q_d^0 = \sqrt{\frac{2K_b D}{h_b}}$ minimizes the buyer's annual costs and $\hat{Q}_d = \sqrt{\frac{2 f_b D}{g_b}}$ minimizes his/her annual emissions.

Table 2 Problem parameters and decision variables under the cap-and-trade policy

Policy parameters	
C_b	Buyer's annual carbon emission cap
C_v	Vendor's annual carbon emission cap
p_c^b	Buying price of unit carbon emission
p_c^s	Selling price of unit carbon emission
Decision variables	
Q	Buyer's order quantity (vendor's production lot size)
X_b	Amount of carbon credit traded by the buyer
X_v	Amount of carbon credit traded by the vendor
X_s	Amount of carbon credit traded by the system in the centralized model with carbon credit sharing
Functions and optimal values of decision variables	
$BC\,(Q, X_b)$	Buyer's average annual costs as a function of Q and X_b
$VC\,(Q, X_v)$	Vendor's average annual costs as a function of Q and X_v
$TC\,(Q, X_b, X_v)$	Total average annual costs as a function of Q, X_b and X_v $(TC\,(Q, X_b, X_v) = BC\,(Q, X_b) + VC\,(Q, X_v))$
$SC\,(Q, X_s)$	Total average annual costs of the buyer–vendor system in the centralized model with carbon credit sharing
Q_d^*	Optimal order quantity as a result of the decentralized model
Q_c^*	Optimal order quantity as a result of the centralized model
Q_s^*	Optimal order quantity as a result of the centralized model with carbon credit sharing

Similar to Expression (1), the vendor's annual cost is given by

$$VC\,(Q, X_v) = \begin{cases} VC_1\,(Q, X_v) & \text{if } X_v \leqslant 0 \\ VC_2\,(Q, X_v) & \text{if } X_v > 0, \end{cases} \tag{5}$$

where

$$VC_1\,(Q, X_v) = \frac{K_v D}{Q} + \frac{h_v D Q}{2P} + p_v D - p_c^b X_v \tag{6}$$

and

$$VC_2\,(Q, X_v) = \frac{K_v D}{Q} + \frac{h_v D Q}{2P} + p_v D - p_c^s X_v. \tag{7}$$

If the vendor buys carbon credits (i.e., X_v is negative), his/her annual cost can be obtained by Expression (6), and if he/she sells carbon credits (i.e., X_v is positive), it can be obtained by Expression (7). If $X_v = 0$, then $VC_1(Q, X_v) = VC_2(Q, X_v)$.

The vendor's average annual emission when he/she produces Q units at each setup is

$$\frac{f_v D}{Q} + \frac{g_v D Q}{2P} + e_v D. \tag{8}$$

The decentralized model and the corresponding centralized model are then as follows:

Decentralized Model:

$$\begin{aligned} Min \quad & BC(Q, X_b) \\ \text{s.t.} \quad & \tfrac{f_b D}{Q} + \tfrac{g_b Q}{2} + e_b D + X_b = C_b, \\ & Q \geq 0. \end{aligned}$$

Centralized Model:

$$\begin{aligned} Min \quad & TC(Q, X_b, X_v) \\ \text{s.t.} \quad & \tfrac{f_b D}{Q} + \tfrac{g_b Q}{2} + e_b D + X_b = C_b, \\ & \tfrac{f_v D}{Q} + \tfrac{g_v D Q}{2P} + e_v D + X_v = C_v, \\ & Q \geq 0. \end{aligned}$$

In the decentralized model presented above, the buyer only considers his/her emission constraint to minimize $BC(Q, X_b)$. In the centralized model, the first and the second constraints belong to the buyer and the vendor, respectively. Since these constraints have to be satisfied at any feasible solution, with a slight change of notation, we will refer to the buyer's and the vendor's traded amounts of carbon credits for replenishing Q units by $X_b(Q)$ and $X_v(Q)$. Note that $X_b(Q) = C_b - \frac{f_b D}{Q} - \frac{g_b Q}{2} - e_b D$ and $X_v(Q) = C_v - \frac{f_v D}{Q} - \frac{g_v D Q}{2P} - e_v D$. The buyer's optimal order quantity in the optimal solution of the decentralized model, Q_d^*, therefore, leads to $X_b(Q_d^*)$ and $X_v(Q_d^*)$ as the traded amounts of carbon credits by the buyer and the vendor. Similarly, in the optimal solution of the centralized model, the traded amounts of carbon credit by the buyer and the vendor are given by $X_b(Q_c^*)$ and $X_v(Q_c^*)$, respectively.

In order for this buyer–vendor system to achieve its maximum supply chain profitability, we propose coordination mechanisms that entail carbon credit sharing. To this end, we introduce a third model, which we refer to as the "centralized model with carbon credit sharing". In this model, it is assumed that if one party has an excess carbon allowance, he/she can make it available to the other party if that party needs it. Therefore, the average annual costs of the buyer–vendor system under carbon credit sharing are given by

$$SC\,(Q, X_s) = \begin{cases} SC_1\,(Q, X_s) & \text{if } X_s \leqslant 0 \\ SC_2\,(Q, X_s) & \text{if } X_s > 0, \end{cases} \tag{9}$$

where

$$SC_1(Q, X_s) = \frac{(K_b + K_v)D}{Q} + \frac{(h_b + \frac{h_v D}{P})Q}{2} + (c + p_v)D - p_c^b X_s, \tag{10}$$

and

$$SC_2(Q, X_s) = \frac{(K_b + K_v)D}{Q} + \frac{(h_b + \frac{h_v D}{P})Q}{2} + (c + p_v)D - p_c^s X_s. \tag{11}$$

Assuming carbon credit sharing is available, the centralized model is as follows:

Centralized Model with Carbon Credit Sharing:

$$\begin{aligned} Min \quad & SC(Q, X_s) \\ \text{s.t.} \quad & \tfrac{(f_b + f_v)D}{Q} + \tfrac{(g_b + \frac{g_v D}{P})Q}{2} + (e_b + e_v)D + X_s = C_b + C_v \\ & Q \geq 0. \end{aligned}$$

Observe that, for any triplet $(Q, X_b(Q), X_v(Q))$, there exists a feasible point $(Q, X_s(Q))$ for the centralized model with carbon credit sharing, where $X_s(Q) = X_b(Q) + X_v(Q)$. Since $p_c^b \leq p_c^s$, $TC\,(Q, X_b(Q), X_v(Q))$ may not be equal to $SC\,(Q, X_s(Q))$. In fact, for any $Q \geq 0$ we have $SC\,(Q, X_s(Q)) \leq TC\,(Q, X_b(Q), X_v(Q))$. More specifically,

$$TC\left(Q, X_b(Q), X_v(Q)\right) - SC\left(Q, X_s(Q)\right) = \begin{cases} (p_c^b - p_c^s)min\{-X_b(Q), X_v(Q)\} & \text{if } X_b(Q) < 0 \text{ and } X_v(Q) > 0, \\ (p_c^b - p_c^s)min\{X_b(Q), -X_v(Q)\} & \text{if } X_b(Q) > 0 \text{ and } X_v(Q) < 0, \\ 0 & \text{o.w.} \end{cases} \quad (12)$$

The above expression implies that when $p_c^b > p_c^s$, there exists a difference between the total average annual costs of the two models (centralized models with or without carbon credit sharing) when one party needs to purchase carbon allowances and the other one requires fewer permits at the traded ordering lot size. If both parties need to purchase carbon allowances, or if both parties have excess allowances to sell, then there is no difference between the objective function values of the two models. It follows due to Expression (12) that we have $SC\left(Q_s^*, X_s(Q_s^*)\right) \leq TC\left(Q_c^*, X_b(Q_c^*), X_v(Q_c^*)\right)$ at the optimal solutions of the two models. Since carbon credit sharing has the potential to increase supply chain profitability further, we consider $SC\left(Q_s^*, X_s(Q_s^*)\right)$ as the least possible cost that the buyer–vendor system can achieve. Therefore, we use the solution of the centralized model with carbon credit sharing as a benchmark to propose a coordinated solution. In the next section, we start with analyzing the decentralized model and the centralized model with carbon credit sharing, and provide solution algorithms.

2.2 Modeling of the different solution approaches under the tax policy

An external carbon tax is applied by regulatory agencies, and a linear tax schedule is adopted. That is, the buyer and the vendor pay a monetary amount for each unit of carbon emitted. We consider a general case in which the buyer's and the vendor's tax rates are different, allowing for settings where the parties operate in different geographical locations (e.g., different countries) and/or in different industries. Table 3 summarizes the additional notation specific to our discussion for the tax policy.

Table 3 Problem parameters and decision variables under the tax policy

Policy parameters	
t_b	Carbon tax paid by the buyer for a unit emission
t_v	Carbon tax paid by the vendor for a unit emission
Decision variables	
Q	Buyer's order quantity (vendor's production lot size)
Functions and optimal values of decision variables	
$BC(Q)$	Buyer's average annual costs as a function of Q
$VC(Q)$	Vendor's average annual costs as a function of Q
$TC(Q)$	Total average annual costs as a function of Q ($TC(Q) = BC(Q) + VC(Q)$)
$BT(Q)$:	Average annual tax paid by the buyer as a function of order size Q
$VT(Q)$:	Average annual tax paid by the vendor as a function of order size Q
$TT(Q)$:	Average annual tax paid by the buyer–vendor system as a function of order size Q ($TT(Q) = BT(Q) + VT(Q)$)
Q_d^*	Optimal order quantity as a result of the decentralized model
Q_c^*	Optimal order quantity as a result of the centralized model

In the decentralized model, the buyer solves the following replenishment problem to decide the order quantity that minimizes his/her costs:

$$\begin{aligned} min \quad BC(Q) = & \frac{(K_b + t_b f_b)D}{Q} + \frac{(h_b + t_b g_b)Q}{2} + (c + t_b e_b)D \\ & Q \geq 0, \end{aligned}$$

where $t_b f_b$ is the emission tax paid per replenishment, $t_b g_b$ is the emission tax paid per unit held in inventory per unit time, and $t_b e_b$ is the emission tax paid per unit ordered by the buyer. Since $BT(Q) = \frac{t_b f_b D}{Q} + \frac{t_b g_b Q}{2} + t_b e_b D$, it turns out that $BC(Q) = \frac{K_b D}{Q} + \frac{h_b Q}{2} + cD + BT(Q)$.

The vendor's average annual cost as a function of Q is given by

$$VC(Q) = \frac{(K_v + t_v f_v)D}{Q} + \frac{(h_v + t_v g_v)QD}{2P} + (p_v + t_v e_v)D, \tag{13}$$

where $t_v f_v$ is the emission tax paid per production run, $t_v g_v$ is the emission tax paid per unit held in inventory per unit time, and $t_v e_v$ is the emission tax paid per unit produced by the vendor. Since $VT(Q) = \frac{t_v f_v D}{Q} + \frac{t_v g_v QD}{2P} + t_v e_v D$, it turns out that $VC(Q) = \frac{K_v D}{Q} + \frac{h_v QD}{2P} + p_v D + VT(Q)$.

In the centralized model, the order quantity that minimizes the total cost of the system (i.e, the total cost of the buyer and the vendor) is determined. In mathematical terms, the following problem is solved.

$$\begin{aligned} min \quad TC(Q) = & \frac{(K_b + K_v + t_b f_b + t_v f_v)D}{Q} + \frac{\left[h_b + t_b g_b + \frac{D}{P}(h_v + t_v g_v)\right] Q}{2} \\ & + (c + p_v + t_b e_b + t_v e_v)D \\ & Q \geq 0. \end{aligned}$$

3 Analysis of the solution approaches under the cap-and-trade policy

In this section, we provide an analysis of the decentralized model and the centralized model with carbon credit sharing to find Q_d^* and Q_s^*. Since the objective functions in the two models exhibit piecewise forms, we propose algorithmic solutions based on some structural properties of the two problems. The proofs of all results will be presented in the "Appendix".

3.1 Decentralized model

As implied by Expression (1), $BC(Q, X_b)$ is given by either $BC_1(Q, X_b)$ or $BC_2(Q, X_b)$. In a feasible solution of the decentralized model, the buyer trades $X_b(Q)$ units of carbon credits. Therefore, for a feasible solution pair of Q and $X_b(Q)$, we have

$$BC_1(Q, X_b(Q)) = \frac{(K_b + p_c^b f_b)D}{Q} + \frac{(h_b + p_c^b g_b)Q}{2} + (c + p_c^b e_b)D - p_c^b C_b. \tag{14}$$

Note that $BC_1(Q, X_b(Q))$ is a strictly convex function of Q with a unique minimizer at

$$Q_{d1}^* = \sqrt{\frac{2(K_b + p_c^b f_b)D}{h_b + p_c^b g_b}}. \tag{15}$$

Likewise, for a feasible solution pair of Q and $X_b(Q)$, $BC_2\,(Q, X_b(Q))$ can be rewritten as

$$BC_2\,(Q, X_b(Q)) = \frac{(K_b + p_c^s f_b)D}{Q} + \frac{(h_b + p_c^s g_b)Q}{2} + (c + p_c^s e_b)D - p_c^s C_b. \tag{16}$$

$BC_2\,(Q, X_b(Q))$ is also a strictly convex function with a unique minimizer at

$$Q_{d2}^* = \sqrt{\frac{2(K_b + p_c^s f_b)D}{h_b + p_c^s g_b}}. \tag{17}$$

Lemma 1 *If $(C_b - e_b D) \le \sqrt{2g_b f_b D}$, then the buyer does not sell carbon credits at any order quantity, that is $X_b(Q) \le 0$ for all Q, and $Q_d^* = Q_{d1}^*$.*

Lemma 1 and its proof imply that if the annual cap is smaller than even the minimum annual emission possible by ordering decisions, then regardless of what quantity is ordered, the buyer has to purchase carbon credits. As discussed in Sect. 2, when $X_b(Q) = 0$, the buyer neither purchases nor sells carbon credits. If $(C_b - e_b D)^2 \ge 2g_b f_b D$, there are two order quantities, which we refer to as Q_1 and Q_2, satisfying $X_b(Q) = 0$. In terms of the problem parameters, these quantities are given by

$$Q_1 = \frac{C_b - e_b D - \sqrt{(C_b - e_b D)^2 - 2g_b f_b D}}{g_b} \tag{18}$$

and

$$Q_2 = \frac{C_b - e_b D + \sqrt{(C_b - e_b D)^2 - 2g_b f_b D}}{g_b}. \tag{19}$$

If $(C_b - e_b D)^2 > 2g_b f_b D$, we take Q_2 as the larger root, i.e., $Q_2 > Q_1$.

The results in the seven lemmas (Lemmas 2–8) and the two corollaries (Corollaries 4 and 5) presented in the "Appendix" lead us to the different possible solutions that can happen in case of $(C_b - e_b D) > \sqrt{2g_b f_b D}$. These results, jointly with Lemma 1, yield the optimal solution algorithm, Algorithm 1. Based on Lemmas 2–8 and Corollaries 4–5 we establish the fact that the ordinal relation between $f_b h_b$ and $K_b g_b$ is important. Specifically, we show step by step that if $f_b h_b = K_b g_b$, then $Q_d^* = Q_{d2}^*$, and the optimal solution in the other cases (i.e., $f_b h_b < K_b g_b$ and $f_b h_b > K_b g_b$) depends on the ordering among Q_1, Q_2, Q_{d1}^*, and Q_{d2}^*. We present Algorithm 1 next.

Algorithm 1: Solution of the Decentralized Model

1. If $(C_b - e_b D) \le \sqrt{2g_b f_b D}$, then set $Q_d^* = Q_{d1}^*$.
2. If $(C_b - e_b D) > \sqrt{2g_b f_b D}$, then do the following:
   - (a) If $f_b h_b = K_b g_b$, set $Q_d^* = Q_{d2}^*$.
   - (b) If $f_b h_b < K_b g_b$, and
     - i. if $Q_2 \le Q_{d1}^*$, set $Q_d^* = Q_{d1}^*$,
     - ii. else,
       - A. if $Q_2 \ge Q_{d2}^*$, set $Q_d^* = Q_{d2}^*$,
       - B. if $Q_2 < Q_{d2}^*$, set $Q_d^* = Q_2$.
   - (c) If $f_b h_b > K_b g_b$, and
     - i. if $Q_{d1}^* \le Q_1$, set $Q_d^* = Q_{d1}^*$,
     - ii. else,
       - A. if $Q_{d2}^* \ge Q_1$, set $Q_d^* = Q_{d2}^*$,
       - B. if $Q_{d2}^* < Q_1$, set $Q_d^* = Q_1$.

Theorem 1 *Algorithm 1 gives the optimal solution to the retailer's replenishment problem formulated in the decentralized model.*

Recall from Corollary 4 the three possible orderings among Q_1, Q_2, Q^*_{d1}, and Q^*_{d2} in the case of $(C_b - e_b D) > \sqrt{2g_b f_b D}$ and $f_b h_b < K_b g_b$. Theorem 1 and its proof imply that if $Q_1 < Q_2 \le Q^*_{d1} < Q^*_{d2}$, then $Q^*_d = Q^*_{d1}$; if $Q_1 < Q^*_{d1} < Q^*_{d2} \le Q_2$, then $Q^*_d = Q^*_{d2}$; if $Q_1 < Q^*_{d1} < Q_2 < Q^*_{d2}$, then $Q^*_d = Q_2$. Similarly, in the case of $(C_b - e_b D) > \sqrt{2g_b f_b D}$ and $f_b h_b > K_b g_b$, there are three possible orderings among Q_1, Q_2, Q^*_{d1}, and Q^*_{d2}, as stated in Corollary 5. If $Q_1 \le Q^*_{d2} < Q^*_{d1} < Q_2$, then $Q^*_d = Q^*_{d2}$; if $Q^*_{d2} < Q_1 < Q^*_{d1} < Q_2$, then $Q^*_d = Q_1$; if $Q^*_{d2} < Q^*_{d1} \le Q_1 < Q_2$, then $Q^*_d = Q^*_{d1}$. Theorem 1 has a further implication in terms of the sensitivity of the optimal order quantity to changes in C_b. We present this result in the next corollary.

Corollary 1 *Let us assume that the cap is increased above its current value C_b.*

- *If $f_b h_b = K_b g_b$, then optimal order quantity Q^*_d does not change, and its value is given by Q^*_{d1}.*
- *If $f_b h_b < K_b g_b$, Q^*_d either stays the same or increases (i.e., Q^*_d is nondecreasing in C_b).*
- *If $f_b h_b > K_b g_b$, Q^*_d either stays the same or decreases (i.e., Q^*_d is nonincreasing in C_b).*

The above corollary is presented without a proof. However, a formal proof would be based on Lemma 1, Lemma 5, Corollary 4, Corollary 5, Theorem 1, and the fact that Q_2 is increasing in C_b and Q_1 is decreasing in C_b. Let us define Q'_1 and Q'_2 as the two quantities that satisfy $X_b(Q) = 0$ under the increased value of C_b. We have $Q'_1 < Q_1$ and $Q'_2 > Q_2$. For example, in an instance of the problem where $(C_b - e_b D) > \sqrt{2g_b f_b D}$ and $f_b h_b < K_b g_b$, if $Q_1 < Q^*_{d1} < Q_2 < Q^*_{d2}$ at the current value of C_b, Corollary 4 implies that $Q^*_d = Q_2$ and either one of the following two orderings happens if C_b is increased: $Q'_1 < Q^*_{d1} < Q'_2 < Q^*_{d2}$ or $Q'_1 < Q^*_{d1} < Q^*_{d2} \le Q'_2$. In the former case, the new optimal order quantity is Q'_2, which is greater than Q_2. In the latter case, the new optimal order quantity is Q^*_{d2}, which again is greater than Q_2. Following a similar reasoning for each possible case of the problem leads to Corollary 1.

Corollary 1 is significant for a policy maker to foresee what kind of an effect a change in C_b will have on the quantity traded at each dispatch. It also suggests that knowing how the ratio of fixed ordering cost to inventory holding cost rate (i.e., $\frac{K_b}{h_b}$) compares to the ratio of fixed carbon emission amount at each ordering to carbon emission rate due to inventory holding (i.e., $\frac{f_b}{g_b}$) is sufficient for this prediction. For example, if $\frac{K_b}{h_b} < \frac{f_b}{g_b}$, increasing the cap may result in a fall in the quantity traded at each dispatch.

Next, we proceed with a similar analysis for the centralized model with carbon credit sharing.

3.2 Centralized model with carbon credit sharing

In a feasible solution of the centralized model with carbon credit sharing, the system trades $X_s(Q)$ units of carbon credits, where $X_s(Q) = C_b + C_v - \frac{(f_b+f_v)D}{Q} - \frac{(g_b + \frac{g_v D}{P})Q}{2} - (e_b + e_v)D$. For this pair of order quantity and traded amount of carbon credits, it turns out that

$$SC_1(Q, X_s(Q)) = \frac{\left(K_b + K_v + p^b_c(f_b + f_v)\right) D}{Q} + \frac{\left(h_b + \frac{h_v D}{Q} + p^b_c\left(g_b + \frac{g_v D}{P}\right)\right) Q}{2} + \left(c + p_v + p^b_c(e_b + e_v)\right) D - p^b_c(C_b + C_v). \tag{20}$$

The above expression is strictly convex in Q with a unique minimizer at

$$Q_{c1}^{*}=\sqrt{\frac{2\left(K_b+K_v+p_c^b(f_b+f_v)\right)D}{h_b+\frac{h_vD}{P}+p_c^b\left(g_b+\frac{g_vD}{P}\right)}}. \tag{21}$$

A similar expression can be derived for $SC_2\left(Q, X_s(Q)\right)$ and is given by

$$SC_2\left(Q, X_s(Q)\right)=\frac{\left(K_b+K_v+p_c^s(f_b+f_v)\right)D}{Q}+\frac{\left(h_b+\frac{h_vD}{Q}+p_c^s\left(g_b+\frac{g_vD}{P}\right)\right)Q}{2} \\ +\left(c+p_v+p_c^s(e_b+e_v)\right)D-p_c^s(C_b+C_v). \tag{22}$$

$SC_2\left(Q, X_b(Q)\right)$ is also a strictly convex function with a unique minimizer at

$$Q_{c2}^{*}=\sqrt{\frac{2\left(K_b+K_v+p_c^s(f_b+f_v)\right)D}{h_b+\frac{h_vD}{P}+p_c^s\left(g_b+\frac{g_vD}{P}\right)}}. \tag{23}$$

Expression (9) is similar to Expression (1) in its structural properties. Therefore, results similar to those proved in Sect. 3.1 for the decentralized model also hold for the centralized model with carbon sharing. If $[C_b+C_v-(e_b+e_v)D]\le\sqrt{2\left(g_b+\frac{g_vD}{P}\right)(f_b+f_v)D}$, then the buyer–vendor system does not sell carbon credits at any order quantity, that is $X_s(Q)\le 0$ for all Q. When $[C_b+C_v-(e_b+e_v)D]\ge\sqrt{2\left(g_b+\frac{g_vD}{P}\right)(f_b+f_v)D}$, we have $X_s(Q)=0$ at the following two values of the order quantity:

$$Q_3=\frac{C_b+C_v-(e_b+e_v)D-\sqrt{[C_b+C_v-(e_b+e_v)D]^2-2(g_b+\frac{g_vD}{P})(f_b+f_v)D}}{g_b+\frac{g_vD}{P}} \tag{24}$$

and

$$Q_4=\frac{C_b+C_v-(e_b+e_v)D+\sqrt{[C_b+C_v-(e_b+e_v)D]^2-2(g_b+\frac{g_vD}{P})(f_b+f_v)D}}{g_b+\frac{g_vD}{P}}. \tag{25}$$

It turns out that the system sells carbon credits only when $[C_b+C_v-(e_b+e_v)D]>\sqrt{2\left(g_b+\frac{g_vD}{P}\right)(f_b+f_v)D}$ and $Q_3<Q<Q_4$.

We propose the following algorithm to obtain the optimal solution of the centralized model with carbon credit sharing. A detailed proof will not be presented because it follows the same lines as Theorem 1's proof and makes use of similar results (i.e., Lemma 1, Lemma 5, Corollary 4, and Corollary 5) that set a foundation for Theorem 1.

Algorithm 2: Solution of the Centralized Model with Carbon Credit Sharing

1. If $[C_b+C_v-(e_b+e_v)D]\le\sqrt{2\left(g_b+\frac{g_vD}{P}\right)(f_b+f_v)D}$, then set $Q_s^*=Q_{c1}^*$.
2. If $[C_b+C_v-(e_b+e_v)D]>\sqrt{2\left(g_b+\frac{g_vD}{P}\right)(f_b+f_v)D}$, then do the following:

   (a) If $(f_b+f_v)(h_b+\frac{h_vD}{P})=(K_b+K_v)(g_b+\frac{g_vD}{P})$, set $Q_s^*=Q_{c2}^*$.

(b) If $(f_b + f_v)(h_b + \frac{h_v D}{P}) < (K_b + K_v)(g_b + \frac{g_v D}{P})$, and
  i. if $Q_4 \leq Q^*_{c1}$, set $Q^*_s = Q^*_{c1}$,
  ii. else,
    A. if $Q_4 \geq Q^*_{c2}$, set $Q^*_s = Q^*_{c2}$,
    B. if $Q_4 < Q^*_{c2}$, set $Q^*_s = Q_4$.

(c) If $(f_b + f_v)(h_b + \frac{h_v D}{P}) > (K_b + K_v)(g_b + \frac{g_v D}{P})$, and
  (i.) if $Q^*_{c1} \leq Q_3$, set $Q^*_s = Q^*_{c1}$,
  (ii.) else,
    A. if $Q^*_{c2} \geq Q_3$, set $Q^*_s = Q^*_{c2}$,
    B. if $Q^*_{c2} < Q_3$, set $Q^*_s = Q_3$.

3.3 Coordination mechanisms

In this section, we present coordination mechanisms that help the buyer–vendor system to arrive at the system optimal solution by making the most efficient use of carbon credits. These coordination mechanisms assume that vendor has full information about the ordering behavior of the buyer, and the buyer orders from the current vendor as long as his/her costs as a result of the coordinated solution are not more than those under the decentralized solution. The novelty of the proposed coordination mechanisms is that they make use of carbon credit sharing. Recall that in this setting, the purchasing price of one unit carbon credit is greater than or equal to its selling price (i.e., $p^b_c \geq p^s_c$). In settings where $p^b_c > p^s_c$, and one party is selling carbon credits while the other party is purchasing them, the system is actually losing an opportunity to profit more due to the monetary value that the purchasing party pays to intermediary agencies (i.e., $p^b_c - p^s_c$ per unit carbon credit purchased). The lost opportunity is quantified in Expression (12). Therefore, the proposed coordination mechanisms, as part of sharing the extra benefits of the centralized solutions, entail the party who has extra carbon credits to pass them to the other party, who would otherwise purchase them at a higher price in the market. This way, we minimize the system's need to purchase carbon credits, and hence, to pay intermediary agencies.

While carbon credit sharing may lead to reduced overall costs, it may increase or decrease the total annual emissions in comparison to a coordinated solution that does not allow carbon credit sharing. The examples in Table 4 are illustrative of these two cases.

In Examples 7 and 8, carbon credit sharing reduces total average annual costs. In Example 7, the optimal order quantity of the centralized model without carbon credit sharing (Q^*_c) is 235.5, and this quantity leads to 705.8425 as the total average annual emissions. The optimal order quantity of the centralized model with carbon credit sharing (Q^*_s) is 251.5, which results in a value of 708.145 as the total average annual emissions. While Example 7 is illustrative of a case in which carbon credit sharing increases the emissions of the buyer–vendor system, Example 8 exemplifies a complementary case. Specifically, in Example 8, the total average annual emissions under the optimal solution of the centralized model without carbon credit sharing is 721.987, which reduces to 715.322 due to carbon credit sharing. These two examples show that the impact of carbon credit sharing on the environment is dependent on the specific setting; however, the total costs either stay the same or reduce due to carbon credit sharing (i.e., $SC\left(Q^*_s, X_s(Q^*_s)\right) \leq TC\left(Q^*_c, X_b(Q^*_c), X_v(Q^*_c)\right)$). Therefore, in the proposed coordination mechanisms, $SC\left(Q^*_s, X_s(Q^*_s)\right)$ will be considered as the minimum system costs that can be achieved. Before we introduce these coordination mechanisms, we present the following result, which is crucial for understanding why these mechanisms work.

Table 4 Numerical instances to illustrate the impact of carbon credit sharing

Example index	Instances with $D = 50$, $h_b = 1$, $c = 12$, $p_c^b = 7.5$, $p_c^s = 6$, $e_b = 5$, $K_v = 1000$, $P = 150$, $h_v = 0.5$, $p_v = 8$, $e_v = 7$										
	K_b	C_b	f_b	g_b	f_v	g_v	C_v	Q_c^*	Q_s^*	$TC\left(Q_c^*, \ldots\right)$	$SC\left(Q_s^*, X_s(Q_s^*)\right)$
7	900	300	40	0.5	135	0.25	450	235.5	251.5	1301.878	1273.314
8	500	528	10	1.5	20	1.25	45	113.5	105.5	3127.106	2839.858

Proposition 1 *$BC\,(Q, X_b(Q))$ is a strictly convex function of Q.*

Observe that if both parties would not sell carbon credits under the system optimal quantity Q_s^* (i.e., $X_b(Q_s^*) \leq 0$ and $X_v(Q_s^*) \leq 0$), or if both parties would not purchase carbon credits under the system optimal quantity Q_s^* (i.e., $X_b(Q_s^*) \geq 0$ and $X_v(Q_s^*) \geq 0$), then carbon credit sharing would not bring any benefit to the system. Therefore, in those cases we rely on traditional coordination mechanisms. In fact, an implication of Proposition 1 is that quantity discounts with economies or diseconomies of scale coordinates the buyer–vendor system. Specifically, if $Q_s^* > Q_d^*$, then under a per unit discount of $d = \frac{BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*))}{D}$ for order quantities greater than or equal to Q_s^*, the buyer would be indifferent to whether Q_d^* or Q_s^* were ordered. If $Q_s^* < Q_d^*$, under a per unit discount of the same amount for order quantities less than or equal to Q_s^*, the buyer would be indifferent to whether Q_d^* or Q_s^* were ordered.

In cases where one party would sell carbon credits while the other party would buy carbon credits under the centralized optimum quantity, we propose the following coordination mechanisms:

CM1: If $X_b(Q_s^*) \leq 0$, $X_v(Q_s^*) \geq 0$, $p_c^b \times \min\left\{-X_b(Q_s^*), X_v(Q_s^*)\right\} \geq BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*))$, and

- if $Q_d^* < Q_s^*$, then for order quantities greater than or equal to Q_s^*,
- if $Q_d^* > Q_s^*$, then for order quantities less than or equal to Q_s^*,

the vendor gives $Y = \min\left\{-X_b(Q_s^*), X_v(Q_s^*)\right\}$ carbon credits for free to the buyer and the buyer makes a fixed payment of $BC(Q_d^*, X_b(Q_d^*)) + p_c^b \times Y - BC(Q_s^*, X_b(Q_s^*))$ to the vendor.

CM2: If $X_b(Q_s^*) \leq 0$, $X_v(Q_s^*) \geq 0$, $p_c^b \times \min\left\{-X_b(Q_s^*), X_v(Q_s^*)\right\} < BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*))$, and

- if $Q_d^* < Q_s^*$, then for order quantities greater than or equal to Q_s^*,
- if $Q_d^* > Q_s^*$, then for order quantities less than or equal to Q_s^*,

the vendor gives $Y = \min\left\{-X_b(Q_s^*), X_v(Q_s^*)\right\}$ carbon credits for free to the buyer and a per unit discount of $d = [BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*)) - p_c^b \times Y]/D$ for all items in the lot.

CM3: If $X_b(Q_s^*) \geq 0$, $X_v(Q_s^*) \leq 0$, and

- if $Q_d^* < Q_s^*$, then for order quantities greater than or equal to Q_s^*,
- if $Q_d^* > Q_s^*$, then for order quantities less than or equal to Q_s^*,

the buyer gives $Y = \min\left\{X_b(Q_s^*), -X_v(Q_s^*)\right\}$ carbon credits for free to the vendor and the vendor gives a per unit discount of $d = [BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*)) + p_c^s \times Y]/D$ to the buyer for all items in the lot.

The first and the second coordination mechanisms (i.e., **CM1** and **CM2**) apply to cases in which the buyer would buy carbon credits while the vendor would sell carbon credits under the centralized optimum solution. The expression $\min\left\{-X_b(Q_s^*), X_v(Q_s^*)\right\}$ refers to the amount of carbon credits the vendor can provide to the buyer. **CM1** and **CM2** differ in whether the monetary value of this amount in the market (i.e., $p_c^b \times \min\left\{-X_b(Q_s^*), X_v(Q_s^*)\right\}$) is greater or less than the buyer's loss from using the centralized solution (i.e., $BC(Q_s^*, X_b(Q_s^*)) - BC(Q_d^*, X_b(Q_d^*))$). In cases where the value of carbon credits given by the vendor to the buyer exceeds the buyer's loss, then **CM1** applies, and the buyer returns the extra value of

Table 5 Numerical instances to illustrate the proposed coordination mechanisms

Example index	Instances with $D = 50$, $c = 12$, $p_v = 8$													
	K_b	h_b	p_c^b	C_b	f_b	g_b	e_b	K_v	P	h_v	f_v	g_v	e_v	C_v
9	900	1	7.5	300	40	0.5	5	1000	150	0.5	135	0.25	7	450
10	90	2	7.5	345	90	1	5	1000	75	0.8	60	1.75	6	400
11	330	3.2	2.5	300	90	0.5	4.5	100	55	3	95	0.25	6	350

Table 6 Solutions of instances in Table 5

Example index	Q_d^*	Q_s^*	$X_b(Q_s^*)$	$X_v(Q_s^*)$	Coordination mechanism
9	158.944	251.425	−20.811	62.677	For $Q \geq 251.425$, the vendor gives 20.811 carbon credits to the buyer, who in return, makes a fixed payment of 75.291
10	89.737	113.186	−1.351	7.470	For $Q \geq 113.186$, the vendor gives 1.351 carbon credits to the buyer and a per unit discount of 0.259
11	110.195	107.345	6.243	−6.448	For $Q \leq 107.345$, buyer gives 6.243 carbon credits to the vendor, who in return, gives a per unit discount of 0.253

carbon credits as a fixed payment to the vendor. If it is less than the buyer's loss, then **CM2** applies, and the vendor further gives an all-units discount to the buyer to compensate his/her remaining loss. **CM3** applies to cases in which the vendor would buy carbon credits while the buyer would sell carbon credits under the centralized optimum solution. In this case, the buyer gives the vendor the carbon credits he/she needs, but in return receives a higher amount of per unit discount. The per unit discount amount is such that it compensates the buyer for his/her losses if he/she orders the centralized optimum quantity in addition to the monetary value of carbon credits he/she agrees to give to the vendor.

In Table 5, parameters of three instances are presented. In Table 6, a summary of their decentralized and centralized solutions as well as the coordinating mechanisms are reported. Examples 9, 10, and 11 are illustrative of **CM1**, **CM2**, and **CM3**, respectively.

3.4 The impact of coordination on the environment under the cap-and-trade policy

To understand the impact of coordination on the environment, we numerically compare the average annual emissions resulting from the decentralized model and the centralized model with carbon credit sharing. For this purpose, the following additional pieces of notation are used.

$TE(Q)$: Average annual emissions of the system if order size is Q units
R: Ratio of average annual system emissions resulting from the two models

The ratio R is a performance measure on the system's environmental quality under the centralized model with carbon credit sharing compared to its environmental performance under the decentralized model. In mathematical terms,

Table 7 Numerical instances to illustrate the impact of coordination on emissions

Example index	Instances with $D = 30$, $p_c^s = 1.5$, $e_b = 1$, $P = 50$, $h_v = 1.2$, $K_b = 40$, $f_b = 20$, $g_b = 0.5$, $e_v = 1.5$, $C_v = 200$								
	K_v	C_b	f_v	g_v	h_b	p_c^b	Q_d^*	Q_s^*	R
12	500	80	120	0.35	1.5	2.5	43.205	117.041	0.813
13	500	80	1800	0.35	1.5	2.5	43.205	276.488	0.274
14	8000	80	120	12	1.5	2.5	43.205	153.123	2.044
15	500	80	120	0.35	10	2.5	19.766	61.793	0.560
16	500	80	120	0.35	10	3.5	19.766	61.793	0.560
17	500	40	120	0.35	1.5	2.5	44.313	117.041	0.822
18	500	120	120	0.35	1.5	2.5	43.205	117.041	0.813

$$R = \frac{TE(Q_s^*)}{TE(Q_d^*)} = \frac{\frac{(f_b+f_v)D}{Q_s^*} + \frac{(g_b+\frac{g_v D}{P})Q_s^*}{2} + (e_b+e_v)D}{\frac{(f_b+f_v)D}{Q_d^*} + \frac{(g_b+\frac{g_v D}{P})Q_d^*}{2} + (e_b+e_v)D}. \tag{26}$$

A value of $R > 1$ would be due to $TE(Q_s^*) > TE(Q_d^*)$, implying that the coordinated solution is not good for the environment. Similarly, a value of $R < 1$ implies that coordination is better for the environment than the uncoordinated solution. In Table 7, we present some instances of the problem to illustrate possible values of R. We would like to note that we have studied the effect of each parameter on R over an extensive numerical analysis; however, there are so many interactions between the problem parameters that there is no generalizable result regarding how R changes with respect to varying values of a certain parameter.

Example 12 in Table 7 can be considered as the base instance around which other examples are generated. Example 13 illustrates an instance in which R is very small (i.e., 0.274) and Example 14 illustrates an instance in which R is very large (i.e., 2.044). In our experimentation, we have identified that most of the instances for which R is very large have extremely high K_v values. Because, when K_v is extremely high, the manufacturer wants to make less frequent setups and produce in larger quantities to save from average annual setup costs, and this results in higher average annual carbon emissions in the centralized model with carbon credit sharing. However, we would like to note that we have also observed instances without extremely high K_v values to also have $R > 1$. In Examples 15 and 16, the inventory holding cost rate of the buyer is so large that in both the decentralized and centralized models, it is only economically appealing to order in small lot sizes and more frequently. It turns out that in both instances, the buyer in the decentralized solution and the system in the centralized solution sell carbon credits. Therefore, even if the buying price of a unit carbon emission is different among these two instances, it does not have an effect on the optimum solutions. Examples 17 and 18 are different than the base instance in the buyer's average annual carbon emission cap, C_b. In Example 17, the buyer's annual cap is so small that he/she has to buy carbon allowances in the decentralized solution. In Example 12, on the other hand, the buyer has extra allowances to sell. Therefore, even if the buyer's annual cap is increased further in Example 18, it does not have an effect on the solution (i.e., the optimal solutions are the same as in the base instance).

4 Analysis of the solution approaches under the tax policy

In this section, we provide an analysis of the decentralized model and the centralized model under the carbon tax mechanism to find the cost-minimizing order quantities Q_d^* and Q_c^*, respectively. We also present some properties related to Q_d^*, Q_c^* and average annual tax amounts of the buyer and the vendor.

The order quantity that minimizes the average annual taxes of the buyer is given by

$$Q_d^t = \sqrt{\frac{2 f_b D}{g_b}}. \tag{27}$$

As the average annual taxes are linearly proportional to the average annual emissions, Q_d^t also minimizes the latter.

Proposition 2 *The buyer's optimal order quantity resulting from the decentralized model is given by*

$$Q_d^* = \sqrt{\frac{2(K_b + t_b f_b)D}{h_b + t_b g_b}}. \tag{28}$$

Observe that as t_b increases, the cost-minimizing order quantity Q_d^* approaches the emission optimal order quantity Q_d^t. We use Proposition 2 to find the minimum average annual cost of the buyer under the decentralized model and present it in the following corollary.

Corollary 2 *The average annual cost of the buyer under the optimal solution of the decentralized model is given by*

$$BC(Q_d^*) = \sqrt{2(K_b + t_b f_b)D(h_b + t_b g_b)} + (c + t_b e_b)D. \tag{29}$$

Similarly, the vendor's average annual cost under the decentralized model ($VC(Q_d^*)$) can be found by plugging Q_d^* into Expression (13). Also, the total average annual cost of the system under the decentralized model is $TC(Q_d^*) = BC(Q_d^*) + VC(Q_d^*)$.

The order quantity that minimizes the average annual taxes of the system (i.e., Q_c^t) is given by

$$Q_c^t = \sqrt{\frac{2(t_b f_b + t_v f_v)D}{t_b g_b + \frac{t_v g_v D}{P}}}. \tag{30}$$

Note that Q_c^t also minimizes the average annual emissions if $t_b = t_v$.

Theorem 2 *The buyer's optimal order quantity resulting from the centralized model is given by*

$$Q_c^* = \sqrt{\frac{2(K_b + K_v + t_b f_b + t_v f_v)D}{h_b + t_b g_b + (h_v + t_v g_v)\frac{D}{P}}}. \tag{31}$$

As t_v gets larger, Q_c^* approaches $\sqrt{\frac{2 f_v D}{g_v}}$, which is the minimizer of $VT(Q)$ (i.e., the vendor's emission optimal order quantity). Similarly, as t_b gets larger, Q_c^* approaches to $\sqrt{\frac{2 f_b D}{g_b}}$, which is the buyer's emission optimal order quantity. In the next corollary, we present the average annual cost of the system resulting from the optimal solution of the centralized model.

Corollary 3 *The total average annual cost of the system under the centralized model is given by*

$$TC(Q_c^*) = \sqrt{2(K_b + K_v + t_b f_b + t_v f_v)D\left[h_b + t_b g_b + (h_v + t_v g_v)\frac{D}{P}\right]} \\ + (c + p_v + t_b e_b + t_v e_v)D. \tag{32}$$

Similarly, the buyer's average annual cost ($BC(Q_c^*)$) and the vendor's average annual cost ($VC(Q_c^*)$) under the centralized model can be found by plugging Q_c^* into $BC(Q)$ and in Expression (13), respectively. In the next proposition, we present a further property of Q_d^* and Q_c^*.

Proposition 3 $Q_d^* \leqslant Q_c^*$ *if and only if* $\frac{K_b+t_b f_b}{h_b+t_b g_b} \leqslant \frac{K_v+t_v f_v}{h_v+t_v g_v}\frac{P}{D}$.

The above proposition implies that any coordination mechanism should take into account both the case of $Q_d^* > Q_c^*$ and the case of $Q_d^* < Q_c^*$. As an example, a per unit discount of $\frac{BC(Q_c^*)-BC(Q_d^*)}{D}$ for order quantities greater than or equal to Q_c^* if $Q_d^* < Q_c^*$, and less than or equal to Q_c^* if $Q_d^* > Q_c^*$ would coordinate the system.

Until this point, we have taken the perspective of the buyer–vendor system in comparing the different solution approaches. We have obtained results on how the buyer's and the vendor's annual costs differ under the decentralized and centralized solutions. In the next two propositions, we take the perspective of the regulator or the government who collects taxes. We compare the average annual amount of taxes collected by the government under the decentralized and centralized solutions.

Proposition 4 *Suppose* $\frac{K_b+t_b f_b}{h_b+t_b g_b} \leqslant \frac{K_v+t_v f_v}{h_v+t_v g_v}\frac{P}{D}$.

(i) If $\frac{t_b f_b+t_v f_v}{t_b g_b+\frac{t_v g_v D}{P}} \leqslant \frac{K_b+t_b f_b}{h_b+t_b g_b}$*, then the government collects no fewer taxes in the centralized solution than it does in the decentralized solution.*
(ii) If $\frac{t_b f_b+t_v f_v}{t_b g_b+\frac{t_v g_v D}{P}} \geqslant \frac{K_b+K_v+t_b f_b+t_v f_v}{h_b+t_b g_b+(h_v+t_v g_v)\frac{D}{P}}$*, then the government collects no fewer taxes in the decentralized solution than it does in the centralized solution.*

Proposition 5 *Suppose* $\frac{K_b+t_b f_b}{h_b+t_b g_b} > \frac{K_v+t_v f_v}{h_v+t_v g_v}\frac{P}{D}$.

(i) If $\frac{t_b f_b+t_v f_v}{t_b g_b+\frac{t_v g_v D}{P}} \geqslant \frac{K_b+t_b f_b}{h_b+t_b g_b}$*, then the government collects more taxes in the centralized solution than it does in the decentralized solution.*
(ii) If $\frac{t_b f_b+t_v f_v}{t_b g_b+\frac{t_v g_v D}{P}} \leqslant \frac{K_b+K_v+t_b f_b+t_v f_v}{h_b+t_b g_b+(h_v+t_v g_v)\frac{D}{P}}$*, then the government collects more taxes in the decentralized solution than it does in the centralized solution.*

Proof The proof follows a similar structure to the proof of Proposition 4 and is omitted. □

Proposition 4 and Proposition 5 imply that there are cases in which coordination of the buyer–vendor system may not be good from the perspective of a government or a regulator who wants to increase total annual average taxes collected. In Table 8, we present some numerical instances to illustrate our analytical results for the buyer–vendor coordination problem under the tax policy. In the last two columns of the table, we report the decentralized and the centralized optimum quantities. In Table 9, we present the buyer's, vendor's, and system's average annual taxes resulting from the decentralized and the centralized solutions to the examples in Table 8. Examples 19, 20, and 21 are to illustrate the first part of Proposition 4.

Table 8 Numerical instances for illustrating analytical results under the tax mechanism ($h_v = 1.5$, $c = 9$, $p_v = 6$, $e_b = 5$ and $e_v = 6$ in all instances)

Example index	D	P	K_b	K_v	h_b	f_b	f_v	g_b	g_v	t_b	t_v	Q_d^*	Q_c^*
19	90	100	200	600	2	30	60	0.2	0.75	2	3	139.642	180.043
20	50	100	700	600	2	60	90	1	0.75	2	3	143.178	169.605
21	50	100	700	600	2	60	90	1	0.6	2	3	143.178	172.949
22	50	100	40	60	2	70	90	1	0.75	2	3	67.082	93.171
23	90	100	200	600	2	100	120	0.15	0.75	2	3	176.930	207.693
24	50	100	40	60	2	30	120	3	2	2	3	35.355	66.525
25	40	60	400	60	2	300	60	0.6	0.2	4	2	170.561	158.523
26	500	600	800	60	1.7	750	310	1	0.75	2	3	788.430	694.299
27	550	600	450	70	2	300	80	1.7	0.2	4	2	454.148	442.915
28	50	60	900	60	1.7	60	90	1	0.75	2	3	166.034	140.642
29	40	90	800	60	1.7	60	90	1	0.7	2	3	141.039	137.361
30	500	600	800	60	1.7	400	90	1	0.75	2	3	657.596	531.774

Table 9 Average annual taxes resulting from the decentralized and the centralized solutions of the instances in Table 8

Example index	$BT\left(Q_d^*\right)$	$VT\left(Q_d^*\right)$	$TT\left(Q_d^*\right)$	$BT\left(Q_c^*\right)$	$VT\left(Q_c^*\right)$	$TT\left(Q_c^*\right)$
19	966.599	1877.399	2843.997	966.001	1892.272	2858.274
20	685.084	1074.826	1759.91	704.982	1075	1779.981
21	685.084	1058.718	1743.802	707.642	1055.885	1763.526
22	671.432	1138.98	1810.412	668.302	1097.303	1765.605
23	1028.275	1982.265	3010.539	1017.82	1986.289	3004.109
24	690.919	1462.15	2153.069	744.670	1270.363	2015.033
25	1286.098	530.884	1816.982	1293.023	531.416	1824.439
26	6739.688	10,328.93	17,068.62	6774.525	10,320.65	17,095.17
27	13,997.37	6877.03	20,874.4	13,996.04	6879.885	20,875.92
28	702.172	1136.966	1839.138	683.304	1127.84	1811.144
29	575.072	862.393	1437.465	572.305	862.727	1435.032
30	6265.872	9281.789	16,087.66	6283.973	9752.405	16,036.38

As it can be observed from Table 9, in these examples, the government collects more taxes in the centralized solution than it does in the decentralized solution (i.e., $TT(Q_c^*) > TT(Q_d^*)$). These examples differ in how the individual parties' average annual taxes change in the two solutions. For example, in Example 19, while $BT(Q_d^*) > BT(Q_c^*)$ and $VT(Q_d^*) < VT(Q_c^*)$, in Example 20, we have $BT(Q_d^*) < BT(Q_c^*)$ and $VT(Q_d^*) < VT(Q_c^*)$. The next three examples (Examples 22, 23, 24) illustrate the second part of Proposition 4. In these examples, the government collects more taxes in the decentralized solution than it does in the centralized solution. Likewise, Examples 25, 26, and 27 illustrate the first part of Proposition 5. As evident in Table 9, in these examples, the government collects more taxes in the centralized solution. Finally, the second part of Proposition 5 is illustrated with

Examples 28, 29, and 30, in which the government collects more average annual taxes in the decentralized solution.

We would like to note that, in Table 8, the instances in which $TT(Q_c^*) > TT(Q_d^*)$ coincide with the cases where coordination is not good for the environment. On the other hand, instances with $TT(Q_c^*) < TT(Q_d^*)$ are illustrative of the cases in which coordination is good for the environment.

5 Conclusion

There is growing recognition of the potential damage of global climate change caused by human activities. Reducing greenhouse gases through some environmental regulations is possible; however, these measures impose costs on the economy, and the efficiency of different policies in the long run is uncertain. In this paper, we investigated the impact of supply chain coordination on environmental measures under two emission-regulation policies: cap-and-trade and tax. We performed our analysis over a buyer–vendor system facing deterministic demand in the infinite horizon. Our findings show that how the buyer and the vendor behave in terms of the contractual agreements they engage in has a significant effect on the resulting emissions under both policies. We conclude that in general, supply chain coordination may or may not be good for the environment, depending on the circumstances, as opposed to having no coordination under a specific policy. Furthermore, the impact of coordinated decisions in comparison to independent decisions depends on the parties' particular production/inventory-related parameters, which means that coordination among firms is a source of unpredictability for the policy maker in designing regulations. In case of cap-and-trade policy, one exception was when the vendor's fixed replenishment cost is extremely high. In our experimentation, we consistently observed that in such cases, coordination between the parties results in more system emissions than the uncoordinated solution does.

We also explored the added flexibility of the cap-and-trade policy for firms to share their carbon credits and we proposed novel coordination mechanisms based on carbon credit sharing. This flexibility has the potential to reduce supply chain costs even further under coordination but may sometimes contribute to higher carbon emissions. Supply chain coordination is an important aid for companies in reducing overall system costs, and carbon credit sharing as part of coordination mechanisms may help companies in reducing the cost of compliance to the cap-and-trade policy. Our results show that whether this comes at the expense of increased carbon emissions in comparison to coordination with no carbon credit sharing, again, depends on the particular parameters of the buyer and the vendor. This result suggests that the benefits of carbon credit sharing in terms of costs should be weighed against a possible increase in carbon emissions, and the policy maker should carefully determine the terms for the private transfer of carbon credits among firms.

Our review of the production/inventory models in the operations research and the management science literature revealed that number of studies considering environmental policies within the context of different problems in multi-echelon settings is limited. We would like to note that our paper is the first one to study coordination in a setting where multiple parties in the supply chain are subject to environmental policies. Furthermore, our modeling for the cap-and-trade policy allows for the purchasing price of unit carbon allowance to be larger than its selling price. The difference in carbon trading prices leads to challenging optimization problems under both independent and integrated decisions. A contribution of this paper is to propose finite-time exact solution procedures for these problems. In our modeling for the tax policy, we also aimed for a generalization by allowing the manufacturer's and the retailer's carbon tax rates to be different, which may happen if the parties are in different industries or in different geographical locations. A future work could be to study other problems such as facility location, transportation mode selection, etc. under the environmental policies modeled herein. We considered a deterministic-demand production-inventory setting with a lot-for-lot policy in place. It is also worthwhile to extend the questions of interest and the analysis in this paper to more complex settings by modeling different dispatch policies or stochasticity of demand.

In case of cap-and-trade policy, our consideration of the difference between the selling and purchasing prices of unit carbon allowance led us to some novel coordination mechanisms based on carbon credit sharing (i.e., carbon-credit sharing along with fixed payments, or carbon credit sharing along with quantity discounts). In case of tax policy, we showed that classical quantity discounts can be used for channel coordination. Our study assumed that retail price of the item is fixed and demand is independent of the retail price. Weng (1995) showed that when the retail price is a decision variable and demand is dependent on the retail price, a quantity discount policy is not sufficient for coordination. A further generalization of our study could be to consider a dependency between demand and retail price, which we believe may necessitate the use of different coordination mechanisms.

In this paper, we also obtained some results which can be helpful for a policy maker in designing environmental regulations. Specifically, in Corollary 1, we showed how the retailer's order quantity changes with his/her cap and how the change depends on the retailer's parameters (i.e., fixed cost and fixed emissions at each ordering, cost rate and emissions rate related to inventory holding). In Propositions 4 and 5, we provided a comparison of the taxes the government collects in case of centralized and decentralized decision-making between the buyer and the vendor, based on a characterization of their parameters. Our objective in this paper was to provide a thorough analysis for the cap-and-trade policy and the tax policy individually. A comparison of these policies within the context of coordination remains a future research. This comparison may investigate how the total emissions change after coordination under a cap-and-trade policy versus under a tax policy. However, we believe obtaining general results requires an extensive numerical study, and the problem instances should be generated carefully to consider equivalent cap-and-trade and tax policies. That is, under the appropriate parameters of the cap-and-trade policy and the corresponding tax policy, the average annual costs and the emissions should be similar in the decentralized solution for a fair comparison.

In analyzing the impact of the coordinated solution on total emissions, we defined a measure which we referred to as R in the paper (ratio of average annual system emissions resulting from the optimal solution of the centralized model to that of the decentralized model). We showed that there are instances of the problem under which $R > 1$ for both the cap-and-trade and the tax policies. The objective functions of the centralized and decentralized models were cost minimization. Therefore, our proposed coordinated solutions aimed for mechanisms by which the manufacturer induces the buyer to order the centralized quantity while having no worse costs than his/her decentralized solution would lead to. As a different and more environmental solution, the integrated model can be studied under the constraint $R \leq 1$ in search for dispatch quantities that have better (not necessarily best) system costs with lesser average annual emissions than the decentralized model does. New mechanisms can then be designed for the retailer to order this environmental quantity.

Appendix

Proof of Lemma 1

For any order quantity Q, the amount of traded carbon credits by the buyer is $X_b(Q) = C_b - \frac{f_b D}{Q} - \frac{g_b Q}{2} - e_b D$. Observe that $\hat{Q}_d$ minimizes $\frac{f_b D}{Q} + \frac{g_b Q}{2}$ with a minimum function value $\sqrt{2 f_b g_b D}$. That is,

$$\frac{f_b D}{Q} + \frac{g_b Q}{2} \geq \sqrt{2 f_b g_b D}$$

for all $Q \geq 0$. This implies

$$X_b(Q) \leq C_b - e_b D - \sqrt{2 f_b g_b D}.$$

Given that $(C_b - e_b D) \leq \sqrt{2 g_b f_b D}$, it turns out that $X_b(Q) \leq 0$ for all $Q \geq 0$. That is, the retailer does not sell carbon credits at any order quantity. In this case, Expression (1) implies that the retailer's inventory replenishment problem reduces to minimizing $BC_1(Q, X_b(Q))$ over $Q \geq 0$. As given by Expression (15), Q^*_{d1} is the optimal solution of this problem. □

Development of the other results for the Proof of Theorem 1

Lemma 2 *The buyer sells carbon credits (i.e., $X_b(Q) > 0$) only when $(C_b - e_b D) > \sqrt{2 g_b f_b D}$ and $Q_1 < Q < Q_2$.*

Proof From Lemma 1, we know that if $(C_b - e_b D) \leq \sqrt{2 g_b f_b D}$, then the buyer does not sell carbon credits. Therefore, selling carbon credits is possible only when $(C_b - e_b D) > \sqrt{2 g_b f_b D}$. Furthermore, under this condition, $X_b(Q) > 0$ should be satisfied. $X_b(Q) = C_b - \frac{f_b D}{Q} - \frac{g_b Q}{2} - e_b D > 0$ holds for order quantities Q such that $Q_1 < Q < Q_2$. Note that, as $(C_b - e_b D) > \sqrt{2 g_b f_b D}$, both Q_1 and Q_2 are defined and $Q_1 < Q_2$. □

Lemma 2 implies that in addition to the case of $(C_b - e_b D) \leq \sqrt{2 g_b f_b D}$ suggested by Lemma 1, there are two cases in which the retailer does not sell carbon credits: if $(C_b - e_b D) > \sqrt{2 g_b f_b D}$ and $Q \leq Q_1$, or if $(C_b - e_b D) > \sqrt{2 g_b f_b D}$ and $Q \geq Q_2$.

Lemma 3 *Depending on how $f_b h_b$ compares to $K_b g_b$, the following ordinal relations exist between Q^*_{d1} and Q^*_{d2}:*

- *If $f_b h_b > K_b g_b$, then $Q^*_{d1} > Q^*_{d2}$.*
- *If $f_b h_b = K_b g_b$, then $Q^*_{d1} = Q^*_{d2}$.*
- *If $f_b h_b < K_b g_b$, then $Q^*_{d1} < Q^*_{d2}$.*

Proof We will prove the first part of the lemma. The proofs of the remaining two parts are similar.

Since $p^b_c \geq p^s_c$, $f_b h_b > K_b g_b$ implies that $(p^b_c - p^s_c) f_b h_b > (p^b_c - p^s_c) K_b g_b$. Adding $K_b h_b + p^b_c p^s_c f_b g_b$ to both sides of this inequality, and after some rearrangement of terms, we have

$$(K_b + p^b_c f_b)(h_b + p^s_c g_b) > (K_b + p^s_c f_b)(h_b + p^b_c g_b).$$

The above expression can be rewritten as

$$\frac{(K_b + p^b_c f_b)}{(h_b + p^b_c g_b)} > \frac{(K_b + p^s_c f_b)}{(h_b + p^s_c g_b)},$$

which further implies

$$\sqrt{\frac{2(K_b + p_c^b f_b)D}{(h_b + p_c^b g_b)}} > \sqrt{\frac{2(K_b + p_c^s f_b)D}{(h_b + p_c^s g_b)}}.$$

Observe that the left-hand side of the above inequality is Q_{d1}^* and the right-hand side is Q_{d2}^*, and therefore, $Q_{d1}^* > Q_{d2}^*$. □

In the next lemma, we present further properties of the retailer's problem in the case of $(C_b - e_b D) > \sqrt{2g_b f_b D}$.

Lemma 4 *When* $(C_b - e_b D) > \sqrt{2g_b f_b D}$*, the following cases cannot be observed.*

- $Q_1 < Q_2 \le Q_{d2}^* \le Q_{d1}^*$.
- $Q_{d1}^* \le Q_{d2}^* \le Q_1 < Q_2$.

Proof Let us start with the first part of the lemma. Using Expression (17) and Expression (19), $Q_2 \le Q_{d2}^*$ implies that

$$\frac{C_b - e_b D + \sqrt{(C_b - e_b D)^2 - 2g_b f_b D}}{g_b} \le \sqrt{\frac{2(K_b + p_c^s f_b)D}{h_b + p_c^s g_b}}.$$

Since $(C_b - e_b D) > \sqrt{2g_b f_b D}$, the left-hand side is positive. Therefore, taking the square of both sides leads to

$$\frac{(C_b - e_b D)^2 + (C_b - e_b D)\sqrt{(C_b - e_b D)^2 - 2g_b f_b D} - g_b f_b D}{g_b} \le \frac{(K_b g_b + p_c^s f_b g_b)D}{h_b + p_c^s g_b}.$$

Due to Lemma 3, we know that having $Q_{d2}^* \le Q_{d1}^*$ is possible only when $f_b h_b \ge K_b g_b$, which implies

$$\frac{(f_b h_b + p_c^s f_b g_b)D}{h_b + p_c^s g_b} \ge \frac{(K_b g_b + p_c^s f_b g_b)D}{h_b + p_c^s g_b}.$$

Combining the last two inequalities, we obtain

$$\frac{(C_b - e_b D)^2 + (C_b - e_b D)\sqrt{(C_b - e_b D)^2 - 2g_b f_b D} - g_b f_b D}{g_b} \le \frac{(f_b h_b + p_c^s f_b g_b)D}{h_b + p_c^s g_b} = f_b D.$$

Multiplying both sides of the above expression by g_b and after some rearrangement of terms, it follows that

$$(C_b - e_b D)^2 - 2g_b f_b D \le -(C_b - e_b D)\sqrt{(C_b - e_b D)^2 - 2g_b f_b D}.$$

Recall that, Q_1 and Q_2 were formed by considering the positive square root of the discriminant in $X_b(0)$, and Q_2 was defined as the larger root. Since $(C_b - e_b D) > \sqrt{2g_b f_b D}$, the above inequality cannot hold for the positive square root of $(C_b - e_b D)^2 - 2g_b f_b D$. Therefore, we cannot have $Q_1 < Q_2 \le Q_{d2}^* \le Q_{d1}^*$.

Now, let us continue with the second part of the lemma. Using Expression (17) and Expression (18), $Q_{d2}^* \le Q_1$ implies that

$$\sqrt{\frac{2(K_b + p_c^s f_b)D}{h_b + p_c^s g_b}} \le \frac{C_b - e_b D - \sqrt{(C_b - e_b D)^2 - 2g_b f_b D}}{g_b}.$$

Taking the square of both sides of this inequality leads to

$$\frac{(K_b + p_c^s f_b)D}{h_b + p_c^s g_b} \leq \frac{(C_b - e_b D)^2 - (C_b - e_b D)\sqrt{(C_b - e_b D)^2 - 2g_b f_b D} - g_b f_b D}{(g_b)^2},$$

which is equivalent to

$$\frac{(K_b g_b + p_c^s f_b g_b)D}{h_b + p_c^s g_b} \leq \frac{(C_b - e_b D)^2 - (C_b - e_b D)\sqrt{(C_b - e_b D)^2 - 2g_b f_b D} - g_b f_b D}{g_b}.$$

Based on Lemma 3, having $Q_{d2}^* \geq Q_{d1}^*$ suggests that $f_b h_b \leq K_b g_b$, which implies

$$\frac{(f_b h_b + p_c^s f_b g_b)D}{h_b + p_c^s g_b} \leq \frac{(C_b - e_b D)^2 - (C_b - e_b D)\sqrt{(C_b - e_b D)^2 - 2g_b f_b D} - g_b f_b D}{g_b}.$$

Observe that the left-hand side of the above inequality reduces to $f_b D$. Therefore, after some rearrangement of terms, it can be rewritten as

$$(C_b - e_b D)^2 - 2g_b f_b D \geq (C_b - e_b D)\sqrt{(C_b - e_b D)^2 - 2g_b f_b D}.$$

Again, the above inequality cannot hold for the positive square root of $(C_b - e_b D)^2 - 2g_b f_b D$. Therefore, we cannot have $Q_{d1}^* \leq Q_{d2}^* \leq Q_1 < Q_2$. □

The first part of Lemma 4 implies that when $(C_b - e_b D) > \sqrt{2g_b f_b D}$, the case of $Q_1 < Q_2 \leq Q_{d2}^* = Q_{d1}^*$ cannot occur. Likewise, the second part implies that when $(C_b - e_b D) > \sqrt{2g_b f_b D}$, the case of $Q_{d1}^* = Q_{d2}^* \leq Q_1 < Q_2$ cannot take place. Combining this result with Lemma 3 further leads to the following implication: If $(C_b - e_b D) > \sqrt{2g_b f_b D}$ and $f_b h_b = K_b g_b$, the only possible ordering of Q_1, Q_2, Q_{d1}^* and Q_{d2}^* is $Q_1 < Q_{d1}^* = Q_{d2}^* < Q_2$, because having $(C_b - e_b D) > \sqrt{2g_b f_b D}$ implies $Q_2 > Q_1$, and it follows due to Lemma 3 and the fact that $f_b h_b = K_b g_b$ that $Q_{d1}^* = Q_{d2}^*$. Under these conditions, excluding the cases covered in Lemma 4 from further consideration, the only possible ordering that remains is $Q_1 < Q_{d1}^* = Q_{d2}^* < Q_2$.

Lemma 5 *If $(C_b - e_b D) > \sqrt{2g_b f_b D}$ and $f_b h_b = K_b g_b$, then $Q_d^* = Q_{d1}^* = Q_{d2}^*$.*

Proof Under the conditions of the lemma, the only possible ordering of Q_1, Q_2, Q_{d1}^*, and Q_{d2}^* is $Q_1 < Q_{d1}^* = Q_{d2}^* < Q_2$. To prove the lemma, we will consider three regions of Q separately: $Q \leq Q_1$, $Q_1 < Q < Q_2$, and $Q \geq Q_2$. Expression (1) and Lemma 2 together imply that if $(C_b - e_b D) > \sqrt{2g_b f_b D}$, for order quantities Q such that $Q_1 < Q < Q_2$, we have $BC\,(Q, X_b(Q)) = BC_2\,(Q, X_b(Q))$; for order quantities Q such that $Q \leq Q_1$, we have $BC\,(Q, X_b(Q)) = BC_1\,(Q, X_b(Q))$; for order quantities Q such that $Q \geq Q_2$, we have $BC\,(Q, X_b(Q)) = BC_1\,(Q, X_b(Q))$.

Let us start with Q such that $Q_1 < Q < Q_2$ and $Q \neq Q_{d2}^*$. Since Q_{d2}^* is the unique minimizer of $BC_2(Q, X_b(Q))$ and $BC\,(Q, X_b(Q)) = BC_2\,(Q, X_b(Q))$, it follows that

$$BC\,(Q, X_b(Q)) = BC_2\,(Q, X_b(Q)) > BC_2\left(Q_{d2}^*, X_b(Q_{d2}^*)\right) = BC\left(Q_{d2}^*, X_b(Q_{d2}^*)\right),$$
$$\forall Q \text{ s.t. } Q_1 < Q < Q_2 \text{ and } Q \neq Q_{d2}^*. \tag{33}$$

Now, let us continue with $Q \leq Q_1$. Recall that at Q_1, we have $BC_1\,(Q_1, X_b(Q_1)) = BC_2\,(Q_1, X_b(Q_1))$. Since $BC_1\,(Q, X_b(Q))$ is a strictly convex function with a unique minimizer Q_{d1}^*, and $Q \leq Q_1 < Q_{d1}^*$, it follows that

$$BC_1\,(Q, X_b(Q)) \geq BC_1\,(Q_1, X_b(Q_1)) = BC_2\,(Q_1, X_b(Q_1)).$$

Using the fact that $BC_2\,(Q, X_b(Q))$ is a strictly convex function with a unique minimizer Q^*_{d2}, and $Q_1 \neq Q^*_{d2}$, we further have

$$BC_2\,(Q_1, X_b(Q_1)) > BC_2\left(Q^*_{d2}, X_b(Q^*_{d2})\right).$$

Combining the last two inequalities leads to

$$BC_1\,(Q, X_b(Q)) > BC_2\left(Q^*_{d2}, X_b(Q^*_{d2})\right),$$

which is equivalent to

$$BC\,(Q, X_b(Q)) > BC\left(Q^*_{d2}, X_b(Q^*_{d2})\right), \qquad \forall Q \text{ s.t. } Q \leq Q_1. \tag{34}$$

Finally, let us consider order quantities Q such that $Q \geq Q_2$. Recall that at Q_2, we have $BC_1\,(Q_2, X_b(Q_2)) = BC_2\,(Q_2, X_b(Q_2))$. Since $BC_1\,(Q, X_b(Q))$ is a strictly convex function with a unique minimizer Q^*_{d1}, and $Q^*_{d1} < Q_2 \leq Q$, it follows that

$$BC_1\,(Q, X_b(Q)) \geq BC_1\,(Q_2, X_b(Q_2)) = BC_2\,(Q_2, X_b(Q_2)).$$

Using the fact that $BC_2\,(Q, X_b(Q))$ is a strictly convex function with a unique minimizer Q^*_{d2}, and $Q_2 \neq Q^*_{d2}$, we further have

$$BC_2\,(Q_2, X_b(Q_2)) > BC_2\left(Q^*_{d2}, X_b(Q^*_{d2})\right).$$

Combining the last two inequalities leads to

$$BC_1\,(Q, X_b(Q)) > BC_2\left(Q^*_{d2}, X_b(Q^*_{d2})\right),$$

which, is also equivalent to

$$BC\,(Q, X_b(Q)) > BC\left(Q^*_{d2}, X_b(Q^*_{d2})\right), \qquad \forall Q \text{ s.t. } Q \geq Q_2. \tag{35}$$

Based on Expressions (33), (34), and (35), we conclude that $Q^* = Q^*_{d2}$. □

Lemma 1 and Lemma 5 constitute parts of our solution algorithm for the retailer's decentralized replenishment problem. Lemma 1 suggests the solution in the case of $(C_b - e_b D) \leq \sqrt{2 g_b f_b D}$, and Lemma 5 provides the solution in the case of $(C_b - e_b D) > \sqrt{2 g_b f_b D}$ and $f_b h_b = K_b g_b$. At this point, there is one more case to be considered, that is, $(C_b - e_b D) > \sqrt{2 g_b f_b D}$ and $f_b h_b \neq K_b g_b$. Before proceeding with a detailed analysis of this case, let us present another result that applies to the case of $(C_b - e_b D) > \sqrt{2 g_b f_b D}$ in general.

Lemma 6 *When $(C_b - e_b D) > \sqrt{2 g_b f_b D}$, we have $BC_1\,(Q, X_b(Q)) \leq BC_2\,(Q, X_b(Q))$ for all Q such that $Q_1 \leq Q \leq Q_2$, and $BC_1\,(Q, X_b(Q)) > BC_2\,(Q, X_b(Q))$ for all Q such that $Q < Q_1$ or $Q > Q_2$.*

Proof Recall that $X_b(Q) = C_b - \frac{f_b D}{Q} - \frac{g_b Q}{2} - e_b D$, and $X_b(Q) = 0$ when $Q = Q_1$ and $Q = Q_2$. Furthermore, we have $X_b(Q) > 0$ for all Q s.t. $Q_1 < Q < Q_2$, and we have $X_b(Q) < 0$ for all Q s.t. $Q < Q_1$ and for all Q s.t. $Q > Q_2$. We will show that $BC_1\,(Q, X_b(Q)) \leq BC_2\,(Q, X_b(Q))$ if $Q \in [Q_1, Q_2]$. The proofs of the other parts of the lemma, which are omitted, follow in a similar fashion.

Since $p_c^b \geq p_c^s$, it follows that

$$(p_c^b - p_c^s)\left(C_b - \frac{f_b D}{Q} - \frac{g_b Q}{2} - e_b D\right) \geq 0.$$

After adding $\frac{K_b D}{Q} + \frac{h_b Q}{2} + cD$ to both sides of the above inequality and rearranging the terms, we have

$$\begin{aligned}
&\frac{K_b D}{Q} + \frac{h_b Q}{2} + cD - p_c^b \left(C_b - \frac{f_b D}{Q} - \frac{g_b Q}{2} - e_b D \right) \\
&\quad \leq \frac{K_b D}{Q} + \frac{h_b Q}{2} + cD - p_c^s \left(C_b - \frac{f_b D}{Q} - \frac{g_b Q}{2} - e_b D \right),
\end{aligned}$$

which implies $BC_1\,(Q, X_b(Q)) \leq BC_2\,(Q, X_b(Q))$. □

The above lemma will be used in the proofs of the next two results.

Lemma 7 *When* $(C_b - e_b D) > \sqrt{2 g_b f_b D}$ *and* $f_b h_b < K_b g_b$, *the following orderings among* Q_1, Q_2, Q_{d1}^*, *and* Q_{d2}^* *cannot take place:*

- $Q_{d1}^* \leq Q_1 < Q_2 \leq Q_{d2}^*$,
- $Q_{d1}^* \leq Q_1 < Q_{d2}^* < Q_2$, *and*
- $Q_{d1}^* < Q_{d2}^* \leq Q_1 < Q_2$.

Proof We will prove the first two parts of the lemma. Note that the third part is a special case of $Q_{d1}^* \leq Q_{d2}^* \leq Q_1 < Q_2$ and is covered in Lemma 4.

Due to the strict convexity of $BC_2\,(Q, X_b(Q))$ and the fact that Q_{d2}^* is its minimizer, having $Q_1 < Q_2 \leq Q_{d2}^*$ implies

$$BC_2\,(Q_1, X_b(Q_1)) > BC_2\,(Q_2, X_b(Q_2))\,.$$

At $Q = Q_1$ and $Q = Q_2$, we have $BC_1\,(Q, X_b(Q)) = BC_2\,(Q, X_b(Q))$. Therefore, the above inequality is equivalent to the following:

$$BC_1\,(Q_1, X_b(Q_1)) > BC_1\,(Q_2, X_b(Q_2))\,. \tag{36}$$

However, due to the strict convexity of $BC_1\,(Q, X_b(Q))$ and Q_{d1}^* being its unique minimizer, having $Q_{d1}^* \leq Q_1 < Q_2$ would imply

$$BC_1\,(Q_1, X_b(Q_1)) < BC_1\,(Q_2, X_b(Q_2))\,. \tag{37}$$

Expression (36) and (37) contradict, therefore, it is not possible to have $Q_{d1}^* \leq Q_1 < Q_2 \leq Q_{d2}^*$.

Now, let us continue with the proof of the second part. Note that Lemma 6 and its proof imply

$$BC_1\left(Q_{d2}^*, X_b(Q_{d2}^*)\right) < BC_2\left(Q_{d2}^*, X_b(Q_{d2}^*)\right)$$

in case of $Q_1 < Q_{d2}^* < Q_2$. Furthermore, having $Q_1 < Q_{d2}^*$ leads to $BC_2\left(Q_{d2}^*, X_b(Q_{d2}^*)\right) < BC_2\,(Q_1, X_b(Q_1))$ due to the strict convexity of $BC_2\,(Q, X_b(Q))$ and the fact that Q_{d2}^* is its minimizer. Combining this with the above inequality implies

$$BC_1\left(Q_{d2}^*, X_b(Q_{d2}^*)\right) < BC_2\,(Q_1, X_b(Q_1))\,.$$

At $Q = Q_1$, we have $BC_2\,(Q_1, X_b(Q_1)) = BC_1\,(Q_1, X_b(Q_1))$. Therefore, the above expression is equivalent to

$$BC_1\left(Q_{d2}^*, X_b(Q_{d2}^*)\right) < BC_1\,(Q_1, X_b(Q_1))\,. \tag{38}$$

However, due to the strict convexity of $BC_1\,(Q, X_b(Q))$ and Q_{d1}^* being its unique minimizer, having $Q_{d1}^* \leq Q_1 < Q_{d2}^*$ would imply

$$BC_1\left(Q_{d2}^*, X_b(Q_{d2}^*)\right) > BC_1\,(Q_1, X_b(Q_1))\,. \tag{39}$$

Table 10 Numerical illustrations of Corollaries 4 and 5 given $D = 50$, $c = 12$ and $g_b = 0.5$

Example index	K_b	h_b	f_b	e_b	p_b^c	p_s^c	C_b	Q_{d1}^*	Q_{d2}^*	Q_1	Q_2
1	900	1	40	5	7.5	6	300	158.944	168.819	55.279	144.721
2	500	1	90	5	7.5	6	350	157.28	161.245	51.676	348.324
3	900	1	40	5	7.5	6	303	158.944	168.819	49.114	162.886
4	100	1.2	90	5	2.5	2	320	115.175	112.815	100	180
5	40	3.2	90	4.5	2.5	2	304	77.169	72.375	74.549	241.451
6	40	3.2	90	4.5	2.5	2	300	77.169	72.375	82.918	217.082

As Expressions (38) and (39) contradict, it is not possible to have $Q_{d1}^* \leq Q_1 < Q_{d2}^* < Q_2$. □

Notice that, since $(C_b - e_b D) > \sqrt{2 g_b f_b D}$ and $f_b h_b < K_b g_b$ are the two conditions of Lemma 7, two common properties of the cases considered are $Q_1 < Q_2$ and $Q_{d1}^* < Q_{d2}^*$. Lemma 7 further leads to the result in Corollary 4.

Corollary 4 *When $(C_b - e_b D) > \sqrt{2 g_b f_b D}$ and $f_b h_b < K_b g_b$, the following orderings are possible:*

- $Q_1 < Q_2 \leq Q_{d1}^* < Q_{d2}^*$,
- $Q_1 < Q_{d1}^* < Q_{d2}^* \leq Q_2$, *and*
- $Q_1 < Q_{d1}^* < Q_2 < Q_{d2}^*$.

Numerical instances to illustrate the cases in Corollary 4 (and Corollary 5) are presented in Table 10. The first three examples of Table 10 correspond to the different cases of the corollary in the order they are presented.

In the next lemma, we provide a similar result to Lemma 7, now for the case of $(C_b - e_b D) > \sqrt{2 g_b f_b D}$ and $f_b h_b > K_b g_b$.

Lemma 8 *When $(C_b - e_b D) > \sqrt{2 g_b f_b D}$ and $f_b h_b > K_b g_b$, the following orderings among Q_1, Q_2, Q_{d1}^*, and Q_{d2}^* cannot take place:*

- $Q_{d2}^* < Q_1 < Q_2 \leq Q_{d1}^*$,
- $Q_1 \leq Q_{d2}^* < Q_2 \leq Q_{d1}^*$, *and*
- $Q_1 < Q_2 \leq Q_{d2}^* < Q_{d1}^*$.

Proof Similar to the proof of Lemma 7, we will prove the first two parts of the lemma. The third part is a special case of $Q_1 < Q_2 \leq Q_{d2}^* \leq Q_{d1}^*$ and is covered in Lemma 4.

Let us assume that the ordering in the first part of the lemma takes place. Due to Lemma 6, having $Q_2 \leq Q_{d1}^*$ implies $BC_1\left(Q_{d1}^*, X_b(Q_{d1}^*)\right) \geq BC_2\left(Q_{d1}^*, X_b(Q_{d1}^*)\right)$. Furthermore, it follows from the strict convexity of $BC_2\left(Q, X_b(Q)\right)$ that having $Q_{d2}^* < Q_1 < Q_2 \leq Q_{d1}^*$ leads to

$$BC_2\left(Q_{d1}^*, X_b(Q_{d1}^*)\right) \geq BC_2\left(Q_2, X_b(Q_2)\right) > BC_2\left(Q_1, X_b(Q_1)\right),$$

and hence, $BC_1\left(Q_{d1}^*, X_b(Q_{d1}^*)\right) > BC_2\left(Q_1, X_b(Q_1)\right)$. At $Q = Q_1$, we have $BC_2\left(Q, X_b(Q)\right) = BC_1\left(Q, X_b(Q)\right)$. Therefore, if the ordering is true, as it is assumed, it would follow that $BC_1\left(Q_{d1}^*, X_b(Q_{d1}^*)\right) > BC_1\left(Q_1, X_b(Q_1)\right)$, which contradicts with Q_{d1}^* being the minimizer of $BC_1\left(Q, X_b(Q)\right)$. Therefore, it is not possible to have $Q_{d2}^* < Q_1 < Q_2 \leq Q_{d1}^*$.

Let us continue with the proof of the second part by assuming that there exists an instance with this ordering. Due to Lemma 6, having $Q_1 \leq Q^*_{d2} < Q_2$ implies $BC_2\left(Q^*_{d2}, X_b(Q^*_{d2})\right) \geq BC_1\left(Q^*_{d2}, X_b(Q^*_{d2})\right)$. Furthermore, it follows from the strict convexity of $BC_1\left(Q, X_b(Q)\right)$ that having $Q^*_{d2} < Q_2 \leq Q^*_{d1}$ leads to

$$BC_1\left(Q^*_{d2}, X_b(Q^*_{d2})\right) > BC_1\left(Q_2, X_b(Q_2)\right) \geq BC_1\left(Q^*_{d1}, X_b(Q^*_{d1})\right),$$

and hence, $BC_2\left(Q^*_{d2}, X_b(Q^*_{d2})\right) > BC_1\left(Q^*_{d1}, X_b(Q^*_{d1})\right)$. Using Lemma 6 once again and the fact that $Q^*_{d1} \geq Q_2$, we must have $BC_1\left(Q^*_{d1}, X_b(Q^*_{d1})\right) \geq BC_2\left(Q^*_{d1}, X_b(Q^*_{d1})\right)$, which would imply $BC_2\left(Q^*_{d2}, X_b(Q^*_{d2})\right) > BC_2\left(Q^*_{d1}, X_b(Q^*_{d1})\right)$. However, this contradicts with the fact that Q^*_{d2} is the minimizer of $BC_2\left(Q, X_b(Q)\right)$. Therefore, it is not possible to have $Q_1 \leq Q^*_{d2} < Q_2 \leq Q^*_{d1}$. □

Note that under the two conditions of Lemma 8, two common properties of the cases considered are $Q_1 < Q_2$ and $Q^*_{d1} > Q^*_{d2}$. Lemma 8 further leads to the result in the next corollary.

Corollary 5 *When $(C_b - e_b D) > \sqrt{2g_b f_b D}$ and $f_b h_b > K_b g_b$, the following orderings are possible:*

- $Q_1 \leq Q^*_{d2} < Q^*_{d1} < Q_2$,
- $Q^*_{d2} < Q_1 < Q^*_{d1} < Q_2$, *and*
- $Q^*_{d2} < Q^*_{d1} \leq Q_1 < Q_2$.

Numerical instances to illustrate the cases in Corollary 5 are also presented in Table 10. The last three examples of Table 10 correspond to the different cases of the corollary in the order they are presented.

Proof of Theorem 1

The proof will follow based on considering the cases presented in Lemma 1, Lemma 5, Corollary 4, and Corollary 5.

Case 1: $(C_b - e_b D) \leq \sqrt{2g_b f_b D}$
It follows due to Lemma 1 that in this case $Q^*_d = Q^*_{d1}$.
Case 2: $(C_b - e_b D) > \sqrt{2g_b f_b D}$
We have the following three subcases ($f_b h_b = K_b g_b$, $f_b h_b < K_b g_b$, and $f_b h_b > K_b g_b$):
Case 2.1: $(C_b - e_b D) > \sqrt{2g_b f_b D}$ and $f_b h_b = K_b g_b$
It follows due to Lemma 5 that in this case $Q^*_d = Q^*_{d2}$.
Case 2.2: $(C_b - e_b D) > \sqrt{2g_b f_b D}$ and $f_b h_b < K_b g_b$

Corollary 4 implies the following three subcases: $Q_1 < Q_2 \leq Q^*_{d1} < Q^*_{d2}$, $Q_1 < Q^*_{d1} < Q^*_{d2} \leq Q_2$, and $Q_1 < Q^*_{d1} < Q_2 < Q^*_{d2}$. We present a detailed proof for the first subcase. Since the proofs of the other subcases are similar, we present sketches of the proofs for those.

- *Case 2.2.1: $Q_1 < Q_2 \leq Q^*_{d1} < Q^*_{d2}$*
  Note that the subcase of $Q_1 < Q_2 \leq Q^*_{d1} < Q^*_{d2}$ is distinguished from the other two by the fact that $Q_2 \leq Q^*_{d1}$. The proof will follow by considering three different regions of Q ($Q > Q_2$, $Q_1 \leq Q \leq Q_2$, $Q < Q_1$), and in each case by showing that $BC\left(Q^*_{d1}, X_b(Q^*_{d1})\right) \leq BC\left(Q, X_b(Q)\right)$. Let us start with Q values such that $Q > Q_2$. Expression (1) and Lemma 2 imply that $BC\left(Q, X_b(Q)\right) = BC_1\left(Q, X_b(Q)\right)$. By definition, Q^*_{d1} is the minimizer of $BC_1\left(Q, X_b(Q)\right)$, therefore, $BC_1\left(Q, X_b(Q)\right) \geq BC_1\left(Q^*_{d1}, X_b(Q^*_{d1})\right)$. Since Q^*_{d1} is also in the region of Q values considered (i.e., $Q^*_{d1} \geq Q_2$), this, in turn, is equivalent to $BC\left(Q, X_b(Q)\right) \geq$

$BC\left(Q^*_{d1}, X_b(Q^*_{d1})\right)$. Now, let us consider Q values such that $Q_1 \le Q \le Q_2$. Expression (1) and Lemma 2 imply that $BC\,(Q, X_b(Q)) = BC_2\,(Q, X_b(Q))$. Since $BC_2\,(Q, X_b(Q))$ is a strictly convex function with a unique minimizer Q^*_{d2} and $Q < Q^*_{d2}$, $BC_2\,(Q, X_b(Q))$, and hence $BC\,(Q, X_b(Q))$, is decreasing in this region. Therefore, $BC\,(Q, X_b(Q)) \ge BC\,(Q_2, X_b(Q_2))$ for all Q such that $Q_1 \le Q \le Q_2$. Furthermore, we have $BC\,(Q_2, X_b(Q_2)) = BC_2\,(Q_2, X_b(Q_2)) = BC_1\,(Q_2, X_b(Q_2))$ and $BC_1\,(Q_2, X_b(Q_2)) \ge BC_1\left(Q^*_{d1}, X_b(Q^*_{d1})\right) = BC\left(Q^*_{d1}, X_b(Q^*_{d1})\right)$. Hence, $BC\,(Q, X_b(Q)) \ge BC\left(Q^*_{d1}, X_b(Q^*_{d1})\right)$. Finally, let us consider Q values such that $Q < Q_1$. Again, due to Expression (1) and Lemma 2, we know that $BC\,(Q, X_b(Q)) = BC_1\,(Q, X_b(Q))$. Since $BC_1\,(Q, X_b(Q))$ is a strictly convex function with a unique minimizer Q^*_{d1} and $Q < Q^*_{d1}$, $BC_1\,(Q, X_b(Q))$, and hence $BC\,(Q, X_b(Q))$, is decreasing in this region. Therefore, $BC\,(Q, X_b(Q)) > BC\,(Q_1, X_b(Q_1))$ for all Q such that $Q < Q_1$. We have discussed above that $BC\,(Q, X_b(Q))$ is decreasing over $Q_1 \le Q \le Q_2$, hence $BC\,(Q_1, X_b(Q_1)) > BC\,(Q_2, X_b(Q_2))$. Combining the last two results implies $BC\,(Q, X_b(Q)) > BC\,(Q_2, X_b(Q_2))$. We have also argued above that $BC\,(Q_2, X_b(Q_2)) \ge BC\left(Q^*_{d1}, X_b(Q^*_{d1})\right)$. Therefore, we conclude $BC\,(Q, X_b(Q)) > BC\left(Q^*_{d1}, X_b(Q^*_{d1})\right)$.

- *Case 2.2.2:* $Q_1 < Q^*_{d1} < Q^*_{d2} \le Q_2$
  We have $BC\,(Q, X_b(Q)) \ge BC\left(Q^*_{d2}, X_b(Q^*_{d2})\right)$ for all $Q \in [Q_1, Q_2]$, because, $Q_1 < Q^*_{d2} \le Q_2$ and $BC\,(Q, X_b(Q)) = BC_2\,(Q, X_b(Q))$ in this region of Q values. Next, we use the facts that $BC\,(Q, X_b(Q)) = BC_1\,(Q, X_b(Q))$ for all $Q \in (Q_2, \infty)$, $BC_1\,(Q, X_b(Q))$ is increasing in this region, and $BC_1\,(Q_2, X_b(Q_2)) = BC_2\,(Q_2, X_b(Q_2))$ to conclude that $BC\,(Q, X_b(Q)) > BC\,(Q_2, X_b(Q_2))$. This further implies $BC\,(Q, X_b(Q)) > BC\left(Q^*_{d2}, X_b(Q^*_{d2})\right)$ for all $Q \in (Q_2, \infty)$. Finally, using the facts that $BC\,(Q, X_b(Q)) = BC_1\,(Q, X_b(Q))$ for all Q such that $Q < Q_1$, $BC_1\,(Q, X_b(Q))$ is decreasing in this region, and $BC_1\,(Q_1, X_b(Q_1)) = BC_2\,(Q_1, X_b(Q_1))$ to conclude that $BC\,(Q, X_b(Q)) > BC\,(Q_1, X_b(Q_1))$. This further implies $BC\,(Q, X_b(Q)) > BC\left(Q^*_{d2}, X_b(Q^*_{d2})\right)$ for all Q such that $Q < Q_1$.
- *Case 2.2.3:* $Q_1 < Q^*_{d1} < Q_2 < Q^*_{d2}$
  We have $BC\,(Q, X_b(Q)) \ge BC\,(Q_2, X_b(Q_2))$ for all $Q \in [Q_1, Q_2]$, because, $BC\,(Q, X_b(Q)) = BC_2\,(Q, X_b(Q))$ and $Q_1 < Q_2 < Q^*_{d2}$ (implying that $BC_2\,(Q, X_b(Q))$ is decreasing in this region of Q values). Next, we use the facts that $BC\,(Q, X_b(Q)) = BC_1\,(Q, X_b(Q))$ for all $Q \in (Q_2, \infty)$ and $Q^*_{d1} < Q_2$ (implying that $BC_1\,(Q, X_b(Q))$ is increasing in this region) to conclude that $BC\,(Q, X_b(Q)) > BC\,(Q_2, X_b(Q_2))$. Finally, using the facts that $BC\,(Q, X_b(Q)) = BC_1\,(Q, X_b(Q))$ for all Q such that $Q < Q_1$, and $Q_1 < Q^*_{d1}$ (implying that $BC_1\,(Q, X_b(Q))$ is decreasing in this region), we conclude that $BC\,(Q, X_b(Q)) > BC\,(Q_1, X_b(Q_1))$. Combining this with the fact that $BC\,(Q_1, X_b(Q_1)) > BC\,(Q_2, X_b(Q_2))$ further leads to $BC\,(Q, X_b(Q)) > BC\,(Q_2, X_b(Q_2))$ for all Q such that $Q < Q_1$.

Case 2.2: $(C_b - e_b D) > \sqrt{2 g_b f_b D}$ and $f_b h_b > K_b g_b$

Corollary 5 implies the following three subcases: $Q^*_{d2} < Q_1 < Q_2 \le Q^*_{d1}$, $Q_1 \le Q^*_{d2} < Q_2 \le Q^*_{d1}$, $Q_1 < Q_2 \le Q^*_{d2} < Q^*_{d1}$. A detailed proof will be omitted for this case as it follows by analyzing the different subcases, as in the proof of Case 2.1. □

Proof of Proposition 1

Suppose $C_b - e_b D \le \sqrt{2 g_b f_b D}$. Using Expression (1) and Lemma 1, $BC(Q, X_b(Q)) = BC_1(Q, X_b(Q))$ for all Q. Since $BC_1(Q, X_b(Q))$ is a strictly convex function of Q, $BC(Q, X_b(Q))$ is also a strictly convex function of Q.

Now, suppose that $C_b - e_b D > \sqrt{2g_b f_b D}$. The proof will follow by showing that $BC(\alpha Q_a + (1-\alpha)Q_b, X_b(\alpha Q_a + (1-\alpha)Q_b)) < \alpha BC(Q_a, X_b(Q_a)) + (1-\alpha)BC(Q_b, X_b(Q_b))$ for all $Q_a \geqslant 0$, $Q_b \geqslant 0$ ($Q_a \neq Q_b$), and $\alpha \epsilon (0, 1)$. First, observe from Expression (1), Lemma 2, and Lemma 6 that $BC(Q, X_b(Q)) = \max\{BC_1(Q, X_b(Q)), BC_2(Q, X_b(Q))\}$ when $C_b - e_b D > \sqrt{2g_b f_b D}$. Since $BC_1(Q, X_b(Q))$ and $BC_2(Q, X_b(Q))$ are strictly convex functions of Q, it follows that $BC_1(\alpha Q_a + (1-\alpha)Q_b, X_b(\alpha Q_a + (1-\alpha)Q_b)) < \alpha BC_1(Q_a, X_b(Q_a)) + (1-\alpha)BC_1(Q_b, X_b(Q_b))$ and $BC_2(\alpha Q_a + (1-\alpha)Q_b, X_b(\alpha Q_a + (1-\alpha)Q_b)) < \alpha BC_2(Q_a, X_b(Q_a)) + (1-\alpha)BC_2(Q_b, X_b(Q_b))$ for all $Q_a \geqslant 0$, $Q_b \geqslant 0$ ($Q_a \neq Q_b$) and $\alpha \epsilon (0, 1)$. Combining this with $BC(Q, X_b(Q)) = \max\{BC_1(Q, X_b(Q)), BC_2(Q, X_b(Q))\}$ leads to $BC_1(\alpha Q_a + (1-\alpha)Q_b, X_b(\alpha Q_a + (1-\alpha)Q_b)) < \alpha BC(Q_a, X_b(Q_a)) + (1-\alpha)BC(Q_b, X_b(Q_b))$ and $BC_2(\alpha Q_a + (1-\alpha)Q_b, X_b(\alpha Q_a + (1-\alpha)Q_b)) < \alpha BC(Q_a, X_b(Q_a)) + (1-\alpha)BC(Q_b, X_b(Q_b))$. Hence, $\max\{BC_1(\alpha Q_a + (1-\alpha)Q_b, X_b(\alpha Q_a + (1-\alpha)Q_b)), BC_2(\alpha Q_a + (1-\alpha)Q_b, X_b(\alpha Q_a + (1-\alpha)Q_b))\} < \alpha BC(Q_a, X_b(Q_a)) + (1-\alpha)BC(Q_b, X_b(Q_b))$. Note that the left-hand side of this inequality is $BC(Q, X_b(Q))$. Thus, $BC(Q, X_b(Q))$ is also a strictly convex function of Q if $C_b - e_b D > \sqrt{2g_b f_b D}$. □

Proof of Proposition 3

Using Equations (28) and (31), we have $Q_d^* \leqslant Q_c^*$ if and only if

$$\sqrt{\frac{2(K_b + t_b f_b)D}{h_b + t_b g_b}} \leqslant \sqrt{\frac{2(K_b + K_v + t_b f_b + t_v f_v)D}{h_b + t_b g_b + (h_v + t_v g_v)\frac{D}{P}}}.$$

Taking the square of both sides leads to

$$\frac{2(K_b + t_b f_b)D}{h_b + t_b g_b} \leqslant \frac{2(K_b + K_v + t_b f_b + t_v f_v)D}{h_b + t_b g_b + (h_v + t_v g_v)\frac{D}{P}}.$$

This, in turn, implies

$$\begin{aligned} &K_b h_b + K_b t_b g_b + K_b h_v \tfrac{D}{P} + K_b t_v g_v \tfrac{D}{P} + t_b f_b h_b + t_b^2 f_b g_b + h_v t_b f_b \tfrac{D}{P} + t_b f_b t_v g_v \tfrac{D}{P} \\ &\leqslant K_b h_b + K_v h_b + h_b t_b f_b + h_b t_v f_v + K_b t_b g_b + K_v t_b g_b + t_b^2 f_b g_b + t_b g_b t_v g_v. \end{aligned}$$

After some cancellations and rearrangement of terms, we get

$$(K_b + t_b f_b)(h_v + t_v g_v)\frac{D}{P} \leqslant (K_v + t_v f_v)(h_b + t_b g_b).$$

This results in

$$\frac{K_b + t_b f_b}{h_b + t_b g_b} \leqslant \frac{K_v + t_v f_v}{h_v + t_v g_v}\frac{P}{D}.$$

□

Proof of Proposition 4

(i) It follows from Proposition 3 that $\frac{K_b + t_b f_b}{h_b + t_b g_b} \leqslant \frac{K_v + t_v f_v}{h_v + t_v g_v}\frac{P}{D}$ is equivalent to $Q_d^* \leq Q_c^*$. Multiplying both sides of the inequality $\frac{t_b f_b + t_v f_v}{t_b g_b + \frac{t_v g_v D}{P}} \leqslant \frac{K_b + t_b f_b}{h_b + t_b g_b}$ with $2D$ and taking the square root of both sides, we obtain

$$\sqrt{\frac{2(t_b f_b + t_v f_v)D}{t_b g_b + \frac{t_v g_v D}{P}}} \leqslant \sqrt{\frac{2(K_b + t_b f_b)D}{h_b + t_b g_b}},$$

which implies $Q_c^t \leq Q_d^*$. Combining this result with the fact that $Q_d^* \leq Q_c^*$ implies that $Q_c^t \leq Q_d^* \leq Q_c^*$. Since $TT(Q)$ is a strictly convex function, this implies that $TT(Q_c^*) \geq TT(Q_d^*)$. That is, in the centralized solution, the government collects at least the same amount of taxes as it collects in the decentralized solution.

(ii) Again, due to Proposition 3, we know that $\frac{K_b+t_b f_b}{h_b+t_b g_b} \leqslant \frac{K_v+t_v f_v}{h_v+t_v g_v}\frac{P}{D}$ implies $Q_d^* \leq Q_c^*$. Multiplying both sides of the inequality $\frac{t_b f_b+t_v f_v}{t_b g_b+\frac{t_v g_v D}{P}} \geqslant \frac{K_b+K_v+t_b f_b+t_v f_v}{h_b+t_b g_b+(h_v+t_v g_v)\frac{D}{P}}$ with $2D$ and taking the square root of both sides, we obtain

$$\sqrt{\frac{2(t_b f_b + t_v f_v)D}{t_b g_b + \frac{t_v g_v D}{P}}} \geqslant \sqrt{\frac{2(K_b + K_v + t_b f_b + t_v f_v)D}{h_b + t_b g_b + (h_v + t_v g_v)\frac{D}{P}}},$$

which is equivalent to $Q_c^t \geq Q_c^*$. Combining this result with the fact that $Q_d^* \leq Q_c^*$ implies $Q_c^t \geq Q_c^* \geq Q_d^*$. Since $TT(Q)$ is a strictly convex function, this implies $TT(Q_d^*) \geq TT(Q_c^*)$. That is, in the decentralized solution, the government collects at least the same amount of taxes as it collects in the centralized solution. □

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