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Endogenous Effects of Hubbing on Flow Intensities

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Abstract Location of hub facilities and the allocation decisions in transport networks endogenously affect both the flow intensities and the transportation costs. Since the introduction of the hub location problem to the operations research literature in mid-1980s, many researchers investigated different ways of modelling the effects of hub facilities on the transportation costs. On the other hand, there has been very limited research on their effect on the flow intensities. This study proposes a new approach, inspired by the Bass diffusion model, to forecast the change in the demand patterns generated at different locations as a result of the placement of new hubs. This new model is used in the context of the uncapacitated single allocation p-hub median problem to investigate the effects of endogenous attraction, caused by the spatial interaction of present hubs, on future hub location decisions. Computational results indicate that the location and allocation decisions may be greatly affected when these forecasts are taken into account in the selection of future hub locations.

Keywords Endogenous attraction · Hub location · p-hub median · Flow volumes · Bass model · Diffusion

1 Introduction

In many large logistics networks, transfer of commodities usually involves a consolidation at hub facilities on their routes between origin and destination points. This consolidation helps reduce the transportation costs by taking advantage of the economies of scale. The transportation cost is affected by both the location of the hub facilities and the allocation of other nodes to these facilities. This general problem was first introduced to the Operations Research literature by O'Kelly in 1986 and it has since received a great deal of attention.

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There are several review papers on hub location research (e.g., O'Kelly and Miller 1994; Campbell 1994b; Klincewicz 1998; Campbell et al. 2002; Alumur and Kara 2008; Farahani et al. 2013). Kara and Taner (2011) wrote a book chapter in which they discuss the evolution of the hub location research in relation to the ideas proposed in O'Kelly's seminal paper of 1986 within the framework of a new taxonomy.

Usually, construction of a hub facility presents its surrounding area with an advantage of more economical and convenient connection to farther locations. Thus the allocation sub-problem is not as straightforward as in the classical location theory even after the locations of the hub facilities are selected. The assignment of a node to a hub facility is not only affected by the node's distance to its assigned facility, but also by the ability of that facility to serve the interaction pattern between the other hub facilities. After this property was pointed out by O'Kelly (1986) and further elaborated by O'Kelly (1987), Campbell (1994a) established the logical connections of the hub location problems with the classical location theory. In particular, he defined the following four location analogous hub problems: 1- p-hub median, 2- hub location with fixed costs, 3- p-hub center, and 4- hub covering. In addition to consideration of different objectives, different versions of the hub location problem are presented by adapting different constraint structures such as multi- as opposed to single-allocation (Campbell 1990b), partial versus full interconnectivity in a hub network (Chou 1990), reliability and resilience considerations (Kim and O'Kelly 2009; Parvaresh et al. 2013; O'Kelly 2014), multiple products (Correia et al. 2014), and allowing versus ruling out direct connectivity between non-hub nodes (Aykin 1995). Multi-period or dynamic hub location problems are also studied by several researchers including Campbell (1990a), Contreras et al. (2011) and Alumur et al. (2015). There has been interest, particularly in the airline industry, on developing and using various indices for measuring the degree of hubbing in a network (e.g., Martin and Voltes-Dorta 2008). In fact, the hubbing effect has been observed to cause considerable levels of traffic at the connecting hub facilities, which attracted research interest to optimize even the flows within a hub (O'Kelly 2010).

In the majority of classical hub location literature, the economies-of-scale effect that alters the transfer cost in links with sufficiently large flow volumes is modeled by incorporating a constant discount factor α on the costs of inter-hub transfers. After it was stated by O'Kelly and Bryan (1998) that it may not be reliable to use the total transfer cost of the network estimated based on this common modeling approach in the selection of hub locations and their allocations, several researchers investigated more realistic ways to model the cost efficiencies provided by hub facilities. In their 1998 paper, O'Kelly and Bryan used a piecewise linear concave cost function of the flow intensity in the context of a multi-allocation hub location problem and they provided an example in which the optimum solution with this proposed cost function is different from that obtained by using a constant discount factor, α. Bryan (1998) considered minimum and maximum limits on inter-hub flow volumes, and used this proposed cost function in problems in which transfer costs from and to non-hub locations are also determined based on the respective flow volumes. Klincewicz (2002) proved that when the hub locations are given, the allocation problem with the cost structure proposed by O'Kelly and Bryan (1998) can be solved efficiently as an uncapacitated facility location problem. He presented examples in which the optimum hub locations were different under different cost functions. O'Kelly (1998) discussed the accuracy of the flow economies of scale model for passenger systems in an analysis of hub-and-spoke networks. Horner and O'Kelly (2001) proposed a non-linear cost structure that models the economies-of-scale effect in all links with a sufficient flow volume. They performed a comparative analysis through an experimental study that employs cost functions with this structure under different assumptions. Kimms (2006) developed an MIP formulation for the hub location problem with fixed link usage costs and a piecewise linear function to model the transportation cost in all hub and non-hub links. Wagner (2004) studied single allocation hub covering problems in which the transportation cost is modeled as a non-increasing function of the flow volumes. Racunicam and Wynter (2005) modeled transportation costs as a non-linear concave function of the flow intensity in the inter-hub and hub-to-destination links. While designing a hub network for a less-than-truckload freight company in Brazil, Cunha and Silva (2007) utilized a variable discount factor whose value depends on the total inter-hub flow. More recently, de Camargo et al. (2009), proposed a new and tighter formulation for the uncapacitated hub location problem with multiple allocation and scale economies that outperforms the model of O'Kelly and Bryan (1998). In a recent study, O'Kelly et al. (2014) studied price sensitive flow demands in hub-and-spoke networks to analyze the demand behavior in a quality of service price equilibrium context.

A second type of endogenous attraction also discussed in O'Kelly's seminal paper of 1986, relates to the flow volumes generated in the surrounding area of the sited hub facilities. In particular, the increased connectivity in the form of more frequent and less costly transportation opportunities tends to speed up the local development in the area of a hub facility and results in an increase in the flow intensity at both the hub location and its immediate environment. These effects of consolidation and dissemination points were observed in the literature long before the formal definition of the hub location problems. For example, Taaffe et al. (1963) investigate the formation of transportation infrastructure in third world countries, and note that demand concentration usually occurs at centers of critical importance due to administrative, political, military and/or economic reasons. The development of the network starts with lines of penetration connecting these important centers to each other. These connections create a positive endogenous effect on local development both in the surrounding area along the lines and at the original centers resulting in increased demand at these locations. The development of the network continues until this increase in demand stabilizes. Goodchild (1978) considers an allocation problem to a group of fixed facilities, and models attraction to a facility as a function of its usage in addition to the distances involved. The usage of a facility appears as an endogenous factor while the distances are exogenously determined. Similarly, Ducca and Wilson (1976) model demand attracted by shopping centers based on retail density and employment alongside the distances. Allen and Sanglier (1979) consider a problem that focuses on locating urban centers dynamically interacting with each other. Each center has a certain attraction parameter, and when located, it affects demand at various points due to immigration and emigration due to factors such as employment opportunities.

Kara and Taner (2011) acknowledge that most of the hub location literature stimulated by O'Kelly's seminal paper of 1986 focuses on the effect of selected hubs on the cost of flow. That the endogeneous attraction between interacting facilities would also affect the flow intensity was disregarded in large part. In fact, O'Kelly (1986) is the only researcher who proposed a preliminary model to characterize this latter type of hubbing effect. In essence, selection of hub locations are strategic decisions with potentially important long term effects on regional development as pointed out by the aforementioned authors in the geography literature prior to O'Kelly (1986). An example of such regional development is observed in Memphis, Tennessee after the location of the FedEx super hub in 1973. Thus, expected future flow demands after locating a facility at a particular point may also be of critical importance in the location decision, as disregarding such effects leads to making a strategic decision based on snapshot information about the transfer costs and demand volumes at the time of decision. The purpose of this paper is to further investigate this type of endogenous attraction with a long term perspective. The objective is to construct a model that predicts the long term effects of the hub location on the flow intensities and use this model to help foresee how the future hub locations and allocations may be affected.

Obviously, these endogeneous attraction effects on the flow intensity bear significance primarily on the flow driven objectives. Thus, this study investigates the phenomenon in the context of the uncapacitated single-allocation p-hub-median problem (*pH-median/single/U/full* in Kara and Taner's (2011) notation) in a sufficiently generic manner so as to allow for a straightforward adaptation of the proposed approach and methodology to other problems. The paper continues in Section 2, with a review of the fundamental findings in the p-hub median literature. Then in Section 3, a new model is proposed to forecast future flow intensities between node pairs in a hub network. Section 4 presents an empirical verification of the proposed model. Computational results with this new model are provided in Section 5. Finally, Section 6 concludes the paper with a summary of the major findings and some directions for future research.

2 The p-hub Median Problem

The p-hub median problem on a full network $G(A,N)$ with arc set A and node set N is defined as placing p hubs in some nodes to act as consolidation and dissemination points for the transfer of flows w_{ij} directed from node $i \in N$ to node $j \in N$, and allocating all non-hub nodes to a hub node so as to minimize the total transfer cost in the network. In the classical definition of the problem, no direct transfers are allowed between non-hub nodes. The single allocation version of the problem requires each node to be assigned to exactly one hub, whereas the multiple-allocation version allows a non-hub node to be assigned to more than one hub. In the uncapacitated problem, all node and arc capacities are assumed to be infinite.

Campbell (1994a, 1996) presents the first linear mixed integer formulation for the uncapacitated p-hub median problem with both single- and multiple-allocation. His formulation utilizes four indexed binary integer variables that carry information on all non-hub to hub, hub to non-hub and hub to hub links used for the transfers between every pair of nodes. In particular, variable X_{ijkm} takes on a value of 1 only if the flow between nodes i and j is routed through hubs k and m, respectively. The author observes that the multi-allocation version of the problem is efficiently solved to optimality by allowing a linear relaxation of these binary variables. Furthermore, the multi-allocation solution provides a lower bound to the corresponding single-allocation problem.

Ernst and Krishnamoorthy (1996) developed another formulation with O(n^3) binary integer variables as opposed to the O(n^4) in previous formulations. The decision variables are defined as y^i_{kl} and z_{ik}. Integer variable y^i_{kl} is the amount of flow originated in node i and routed between hubs k and l. Binary variable z_{ik} takes on a value of 1 if node i is assigned to a hub in node k, and it is zero otherwise. When i=k, z_{kk} takes on a value of 1 if a hub is located in node k, and it is zero otherwise. Recently, Correia et al. (2010) published a note on a version of this classical formulation that addresses the capacitated problem with fixed costs for opening hubs, *fixH-cost/single/node/full* (Ernst and Krishnamoorthy 1999), in which they provided an additional set of constraints that also shortens the solution time.

The relevant parameters are as follows.

d_{ik} Distance between nodes i and k.
w_{ij} Flow from node i to node j.
O_i Total flow generated at node i (i.e., $O_i = \sum_{j\in N} w_{ij}$).
D_i Total flow destined at node i (i.e., $D_i = \sum_{j\in N} w_{ji}$).
p Number of hubs.
χ Unit transfer cost per unit distance from a non-hub to a hub node.
δ Unit transfer cost per unit distance from a hub to a non-hub node.
α Constant discount factor applicable to transfers between two hub nodes.

The formulation proposed by Ernst and Krishnamoorthy (1999) with the amendment developed by Correia et al. (2010) is as follows.

$$\min \sum_{i\in N}\sum_{k\in N} d_{ik}(\chi O_i + \delta D_i) z_{ik} + \sum_{i\in N}\sum_{k\in N}\sum_{l\in N} \alpha d_{kl} y^i_{kl} \tag{1}$$

$$\text{s.t.} \quad \sum_{k\in N} z_{ik} = 1 \quad i\in N \tag{2}$$

$$z_{ik} \le z_{kk} \quad i,k\in N \tag{3}$$

$$\sum_{k\in N} z_{kk} = p \tag{4}$$

$$\sum_{l\in N} y^i_{kl} - \sum_{l\in N} y^i_{lk} = O_i z_{ik} - \sum_{j\in N} w_{ij} z_{jk} \quad i,k\in N \tag{5}$$

$$\sum_{\substack{l\in N\\ l\neq k}} y^i_{kl} \le O_i z_{ik} \quad i,k\in N \tag{6}$$

$$z_{ik}\in\{0,1\} \quad i,k\in N \tag{7}$$

$$y^i_{kl} \ge 0 \quad i,k,l\in N \tag{8}$$

The objective function given in (1) calculates the total network cost composed of three terms: from non-hub to hub, from hub to non-hub and discounted inter hub flow costs. Constraint set (2) ensures that each node is assigned to a single hub. Constraint set (3) restricts assignments only to those nodes in which a hub facility is sited. The number of hubs is set equal to p in constraint (4). The flow balance equations are given in constraint set (5). Constraint set (6) is the amendment proposed by Correia et al. (2010). This amended constraint set avoids potential infeasibilities by ensuring that the flow originated at node i is not routed through hub k unless node i is assigned to hub k. Finally, constraint sets (7) and (8), list the binary and non-negativity restrictions, respectively.

Since four indexed formulations fail to handle the 81-node data set that we analyze in the computational experiments, this three indexed formulation is used in the current study.

As mentioned above, the purpose of this paper is to investigate the effects of hub location decisions on the network in terms of the way that the flow intensities change in the long term. In order to serve this purpose, we define three interrelated uncapacitated single allocation p-hub median problems. In particular, we consider opening p hubs in light of the present data on the flow demands and transfer costs. Then, we consider opening an additional future hub in the same network with the locations of the previously sited p hubs fixed. When opening this additional hub, we first consider the original flow data utilized in determining the locations of the initial p hubs, in which case the endogenous effect of the originally sited hubs is only due to the economies of scale present in the inter-hub transfers. Since the flow routed through these initial hubs can benefit from the economies of scale effect, their assigned nodes enjoy more efficient and less costly transfer opportunities in comparison to the situation in the absence of the hubs. In fact, it is this improved connectivity that leads to the endogenous attraction in the form of increased flow intensities. Next, we consider this endogenous attraction in the locations that gain more efficient connectivity with the installation of the initial set of p hubs, and determine the optimum location of the new hub in light of updated flows. To explore the potential long term effects of this second type of endogenous attraction in designing hub networks, an analysis is performed by comparing the optimum locations of the additional future hub and the resulting allocations based on the original and updated flows. A graphical depiction of this mechanism is given in Appendix 1.

To perform this analysis we need a mechanism to model the flow update process as a function of the hub locations. The next section discusses the groundwork pertinent to this issue and develops a new model to this end.

3 Methodology

To the best knowledge of the authors, the effect of the sited hubs on the flow intensities received no explicit attention by researchers other than O'Kelly (1986). In this seminal work, O'Kelly proposed the following flow update mechanism for an origin–destination pair (i,j):

$$w_{ij} = \frac{O_i D_j \exp\left(-\ddot{\beta} c_{ij}\right)}{\sum_{k \in N} D_k \exp\left(-\ddot{\beta} c_{ik}\right)} \tag{9}$$

where c_{ij} denotes the routing and transportation cost between nodes i and j considering any hubbing effects, O_i and D_j are as defined in Ernst and Krishnamoorthy's (1996) model, and $\ddot{\beta}\geq 0$ is a parameter characterizing the cost elasticity of demand with larger values representing more significant effects. Note that c_{ij} is the cost of transfer routed through the assigned hubs of nodes i and j where the inter-hub cost is discounted due to the scale economies when the two nodes are assigned to different hubs. The paper presents some computational results using the Civil Aeronautics Board data with different α and $\ddot{\beta}$ values. It is important to note that this update mechanism seems to account for several essential properties. Firstly, it considers the current interaction between the two nodes as the product of the flows generated and demanded (i.e., O_iD_j). Then, it employs an exponential term (i.e., exp $\left(-\ddot{\beta}c_{ij}\right)$) to revise the flows as a decreasing function of the transfer costs. Finally, it regards the updated flow between a pair of nodes relative to the total updated flow in the network. Although with its lucid construction this mechanism offers a good starting point, it may be possible to construct a more comprehensive approach to model the endogenous attraction of concern based on various ideas employed to model similar phenomena in the literature.

The marketing and management science literature uses diffusion models as the most prevalent means to forecast the process of adoption of new products in the market. A new hub in a logistics network provides more efficient and less costly connectivity to the assigned nodes. Thus, it provides a new and more efficient transportation service to the market. The first mathematical model for the diffusion of innovations is developed by Bass in 1969 in a classical paper that was reprinted in 2004. The fundamental idea behind this model is that the purchasers can be divided into two broad categories of innovators and imitators. Innovators decide to adopt new products independently of others whereas imitators are affected positively by the number of previous adopters. The accuracy of predictions obtained from the model was empirically verified by using time series data on the sales of eleven different consumer durables. Subsequently, many extensions of this model were developed. A particularly noteworthy one of these is by Robinson and Lakhani (1975) who combined the market penetration idea with the experience curves to construct a dynamic pricing model that represents the volume of new product sales (V) in the following general form.

$$V = \gamma_1[Q_M - Q] + \gamma_2[Q_M - Q]Q \tag{10}$$

where

Q_M Market size

Q Current owners of the product

γ_i Proportionality constants $i = 1,2$

Here, the first and second terms account for the effects of innovators and imitators, respectively. A rearranged version of this model is also given as follows.

$$V = A\left[1 - \frac{Q}{Q_M}\right]\left[\psi + \frac{Q}{Q_M}\right] \tag{11}$$

where

$$A = \gamma_2 Q_M^2 \text{ and } \psi = {}^{\gamma_1}/_{\gamma_2 Q_M}$$

Composite parameter ψ represents the relative effects of innovators and imitators. The innovator forces are more significant than the imitator forces at large values of ψ and the reverse is true at small values of ψ.

These classical models and their various modifications have been used extensively to forecast sale of new products in a wide range of manufacturing and service settings. For instance, Huang et al. (2007) utilized the Bass (1969, 2004) model in combination with the additions proposed by Robinson and Lakhani (1975) to model the sales rate $q(t)$ at time t to investigate the optimum pricing, reliability and warranty strategy of a new product. Their model is relevant to our study in that it explicitly models the time dimension in the process. Also, the presence of innovators and imitators in logistics networks such as passenger and cargo transfer networks seem very realistic, as when a new efficient connection becomes available in such networks a certain group of the passengers/cargo companies immediately start using these new links and remaining companies imitate these practices at a later point in time.

Based on the insights gained from these diffusion studies, we propose a new model to update the flow demand between an origin and destination pair (i, j). In view of the difficulty in estimating the flow intensities and cost rates through a continuous model in a hubbing context, a discrete update mechanism is devised in the following form.

$$W_{ij}^{t+1} = W_{ij}^t + \Delta q_{ij}^t \tag{12}$$

Here, q_{ij}^t stands for the instantaneous rate of change in flow demand between an origin destination pair (i,j) at time t, and parameter Δ is the step size in terms of the time gap between two consecutive discrete points in time indexed as t and $(t+1)$. The rate of flow demand change, q_{ij}^t, in a hub network is calculated as follows.

$$q_{ij}^t = \frac{O_i^t D_j^t}{\sum_{i=1}^{n}\sum_{j=1}^{n} W_{ij}^t} c_{ij}^{-\beta}\left[1 - \frac{W_{ij}^t}{W_{ij}^{\max}}\right]\left[\psi + \frac{W_{ij}^t}{W_{ij}^{\max}}\right] \tag{13}$$

The first quotient term mimics O’Kelly’s (1986) original function in that it accounts for the flow attraction between nodes i and j relative to the total flow in the network. The second term models the elasticity of demand as a function of the transfer cost considering the hubbing effect, with parameter $\beta>1$ to adjust the scale of this elasticity. Finally, the last two terms in square brackets reflect the influence of the innovators and imitators in the market. With the addition of these two terms and the time discretization approach to obtain flow forecasts at a desired future point in time, this new model appears as a more flexible alternative in comparison to that of O’Kelly (1986). Note also that although it does not appear explicitly in the model, discount factor α is another important parameter, as the transportation costs between origin destination pair (c_{ij}) in a hub network are calculated based on α.

4 Empirical Verification of the Model

Although the Bass diffusion model was used to forecast the demand for new products in a wide range of industries, it was not tested particularly in the context of logistics networks. In this section, we present results on forecast accuracy of our proposed model with real data obtained from a major domestic airline company in Turkey.

In year 2013, the airline started offering direct flights between six smaller towns around the country and the major air transit hub of Istanbul. In particular, the company now offers direct connections between Istanbul and the towns located at nodes A, B, C, D, E and F. The available data gives the annual passenger flow between these origin destination pairs in years 2013 and 2014.

The proposed model is used to forecast the 2014 flows of the newly added connections based on the 2013 flows, and the forecasted values are compared with the actual flows to observe the forecast accuracy. A ψ value of 0.03 is used in these forecasts as it is the common setting used in other industries to model the relative effect of imitators and innovators. Table 1 presents the actual 2013 and 2014 flows from the newly added towns to Istanbul as well as the forecasted 2014 flows with β values of 1.25, 1.45 and 1.70. We also used the procedure proposed by O'Kelly (1986) to forecast the flows with the $\ddot{\beta}$ values of 0.001 and 0.0001 adopted in the original study. Since the two $\ddot{\beta}$ values produced virtually the same forecasts we report the results with $\ddot{\beta} = 0.001$ in the rightmost column of Table 1. The percentage difference between the actual and forecasted 2014 flows (i.e., 100 × (actual 2014 flow – forecasted 2014 flow) / actual 2014 flow) are also listed in parentheses. The results show that forecasts with the method proposed in this paper are superior to those obtained from O'Kelly's procedure. Note, however, that we only tested the $\ddot{\beta}$ values adopted in O'Kelly's (1986) original study. It may be possible to obtain better forecasts with his procedure if a sensitivity analysis on the $\ddot{\beta}$ value is performed. The results also indicate that price elasticity of demand differs between towns, which should be a natural outcome considering the different alternative air and ground transportation options serving these towns. If the appropriate β value is used in our model to forecast the demand, the absolute value of the forecast error with the proposed technique ranges from 2 to 20 %, which seems fairly satisfactory.

Table 1 Actual 2013 and 2014 and forecasted 2014 flows from towns A-F to Istanbul

Town	2013 flow	2014 flow	2014 forecast, β=1.25 (% Error)	2014 forecast, β=1.45 (% Error)	2014 forecast, β=1.70 (% Error)	2014 forecast with O'Kelly's procedure
A	9236	37317	31155 (−16.5)	17282 (−53.7)	11535 (−69.1)	5510 (−85.2)
B	11617	16461	29458 (79.0)	18167 (10.4)	13489 (−18.1)	4326 (−73.7)
C	38325	50172	164464 (227.8)	84630 (68.7)	51556 (2.8)	35324 (−29.6)
D	9990	30051	27047 (−10.0)	16252 (−45.9)	11779 (−60.8)	3701 (−87.7)
E	15530	31019	46095 (48.6)	26751 (−13.8)	18736 (−39.6)	6055 (−80.5)
F	15380	26802	45546 (69.9)	26454 (−1.3)	18544 (−30.8)	6154 (−77.0)

Next we adopt the best of the three β values tested above for each town, and vary the value of parameter ψ to observe its effect on the forecasts. The results are displayed in Table 2. The forecasts seem to improve slightly on the average as the value of ψ increases, suggesting that innovators play a more significant role in the flow updates. This observation is in agreement with Bass (1969), which suggests that innovators are likely to have a more significant effect on the demand at earlier stages of a product or a service. Nevertheless, the improvement in forecast quality obtained by adopting a larger ψ value does not seem to be very significant. In fact, forecasts deteriorate in some origin destination pairs as ψ increases.

5 Experimental Results

Experiments are performed on the Civil Aeronautics Board (CAB) and Turkish data sets. The CAB data set, on the U.S. intercity air passenger streams recorded in a sample survey of 1970, includes 25 nodes any of which is a potential location for siting a hub. These 25 nodes are shown in Fig. 1. The indices correspond to the following cities: 1- Atlanta, 2- Baltimore, 3- Boston, 4- Chicago, 5- Cincinnati, 6- Cleveland, 7- Dallas FW, 8- Denver, 9- Detroit, 10- Houston, 11- Kansas City, 12- Los Angeles, 13- Memphis, 14- Miami, 15- Minneapolis, 16- New Orleans, 17- New York, 18- Philadelphia, 19- Phoenix, 20- Pittsburg, 21- St. Louis, 22- St. Francisco, 23- Seattle, 24- Tampa, 25- Washington DC. In the larger Turkish data set with 81 nodes, however, there are 22 designated nodes to potentially serve as hub locations. Figure 2 shows the nodes of this data set where the potential hub locations are encircled. The nodes are indexed by the traffic license plate codes of the corresponding cities. For more information about these data sets, the interested reader is referred to the first studies introducing them to the literature, which are O'Kelly (1986) and Tan and Kara (2007). Since the flow demands are of primary concern in this study, the absolute flow demands as opposed to their scaled versions are used in all experiments.

As suggested by Bass (1969) and other researchers including Huang et al. (2007), we expect the innovators to have a significant effect only in the earlier stages after the new product or service enters the market. We follow the common approach in the literature, and set the ψ value at 0.03. All links in the network are assumed to have a 30 % excess capacity (i.e., $W_{ij}/W_{ij}^{\max}=0.7$, $\forall i,j$ with $i\neq j$) in the current situation. The

Table 2 Accuracy of forecasted flows from towns A-F to Istanbul at different ψ values

Town	β	2014 flow	2014 forecast, ψ=0.01 (% Error)	2014 forecast, ψ=0.03 (% Error)	2014 forecast, ψ=0.06 (% Error)	2014 forecast, ψ=0.09 (% Error)
A	1.25	37317	29575 (−20.7)	31155 (−16.5)	33524 (−10.2)	35894 (−3.8)
B	1.45	16461	17989 (9.3)	18167 (10.4)	18434 (12.0)	18701 (13.6)
C	1.70	50172	51223 (2.1)	51566 (2.8)	52056 (3.8)	52556 (4.8)
D	1.25	30051	26106 (−13.1)	27047 (−10.0)	28459 (−5.3)	29871 (−0.6)
E	1.45	31019	26328 (−15.1)	26751 (−13.8)	27385 (−11.7)	28019 (−9.7)
F	1.45	26802	26087 (−2.7)	26454 (−1.3)	27004 (0.8)	27554 (2.8)

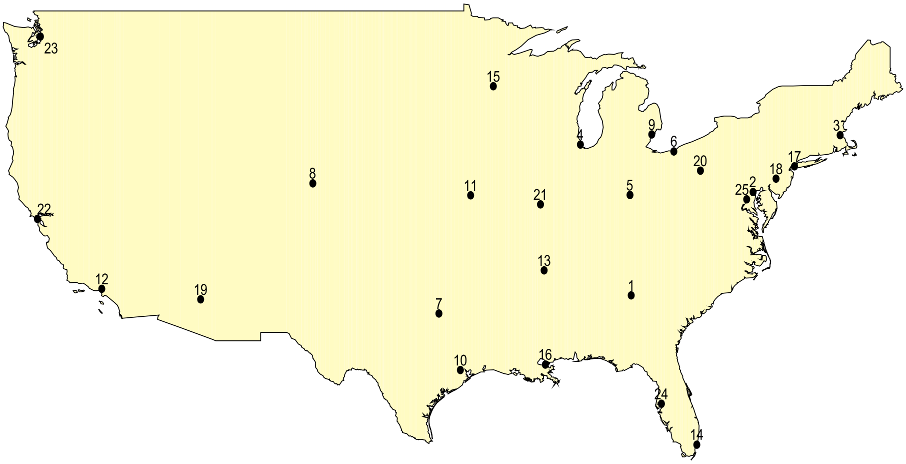


Fig. 1 Nodes of the CAB data set

other parameters in the flow update mechanism are set as β=1.7 and Δ=100 based on an analysis of the convergence properties of the flow update mechanism and the effects of parameter β on network efficiency, which are discussed below.

Since our objective is to investigate the long term effects of the hub location decisions on the flow intensities, we run the update mechanism until the flow demands reach a steady state. In order to establish a compatible stopping criterion, we first analyze the growth of the flow volumes.

Figure 3 shows the percentage change in the average and maximum flow volumes in successive iterations of the proposed update mechanism with two hubs on CAB data using a constant discount factor of α= 0.4. The convex curves indicate that the update process evolves at a decreasing rate with subtle changes in later iterations. Figure 4 presents more detail about the growth patterns in the same experiment by showing the percentage increase in the non-hub and the two hub nodes as a function of the iteration count. The hubs are optimally located in Los Angeles (node 12) and Pittsburg (node 20), respectively. The Pittsburg hub is responsible for routing more traffic than the Los Angeles hub, and hence it is affected more significantly from the endogenous attraction.

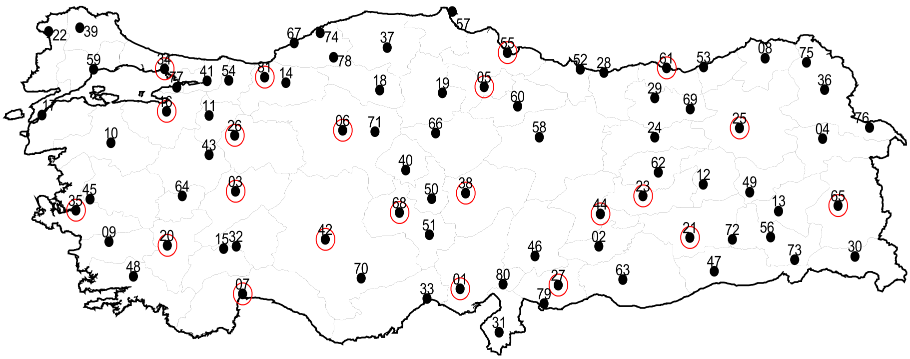


Fig. 2 Nodes of the Turkish data set

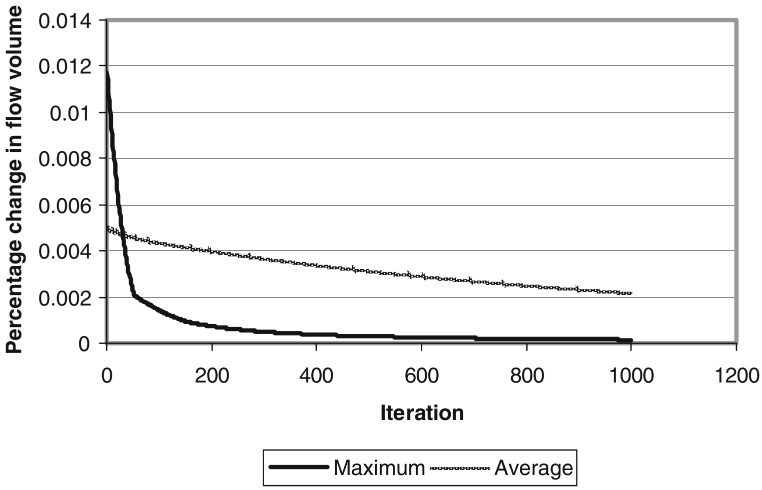


Fig. 3 Growth of the maximum and average flow volumes in the CAB network with p=2 and α=0.4

Using the same parameters, a similar analysis is conducted also on the Turkish data set with two hubs. Figure 5 shows the maximum and average change in flow demands in successive iterations. The two curves exhibit a similar behavior to their CAB counterparts depicted in Fig. 3. Average changes in the hub and non-hub traffic volumes in the Turkish network are plotted as a function of the iteration count in Fig. 6. The hubs are optimally located in Eskisehir (node 26) and Kayseri (node 38), respectively. In parallel with the CAB results, the busier Eskisehir hub is affected more significantly from the endogenous attraction.

Next, we investigate the convergence properties of the flow update mechanism as a function of the cost elasticity of demand, measured by parameter β. Figure 7 shows the percentage change in average flow volume for β values between 0.6 and 1.6 in the CAB network. Since the magnitude of this effect is significantly larger for β values of

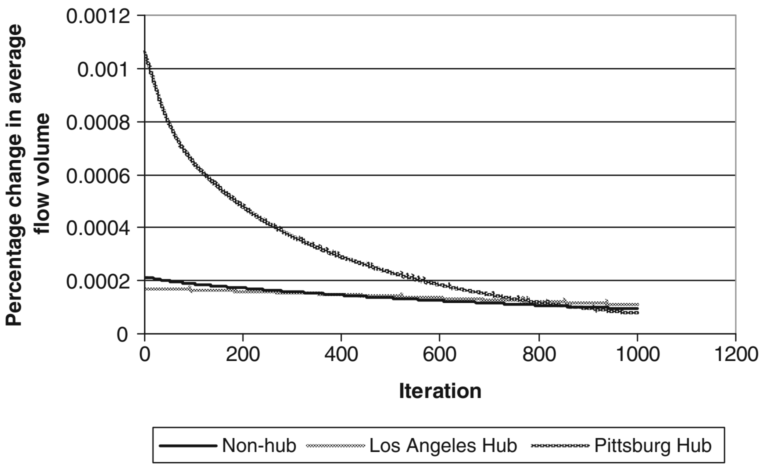


Fig. 4 Percentage change in flow routed through hub and non-hub nodes in the CAB network with p=2 and α=0.4

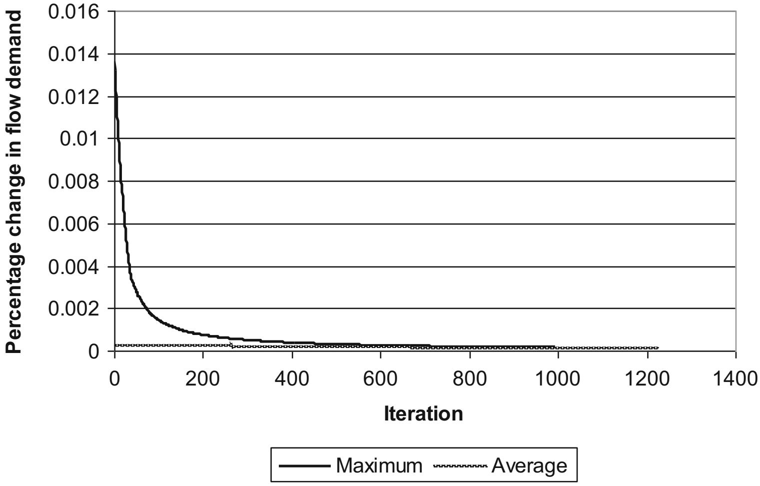


Fig. 5 Growth of the maximum and average flow volumes in the Turkish network with p=2 and α=0.4

1.0 and less, the figure is presented in two parts. We observe that the change on the flow volume in a single iteration decreases when β increases. Consequently, it takes a larger number of iterations for the average flow volumes to stabilize for large β values.

Based on the observations made in Figs. 3, 4, 5, 6 and 7, the update procedure is run in the remaining experiments until the change in the average flow demand of all nodes in two successive iterations drops below 0.01 %, and the network is assumed to have reached at a steady state at that point.

To investigate the effects of changing the value of parameter β used in our proposed update mechanism, we perform experimentation using the CAB data set with the three different β values of 1.1, 1.7 and 2.55. In these experiments, we set parameter ψ=0.03, use the two alternative α values of 0.4 and 0.8, and set the initial number of hubs, p=2, 3, 4 and 5. We update the flows using the procedure outlined in Eqs. (12) and (13)

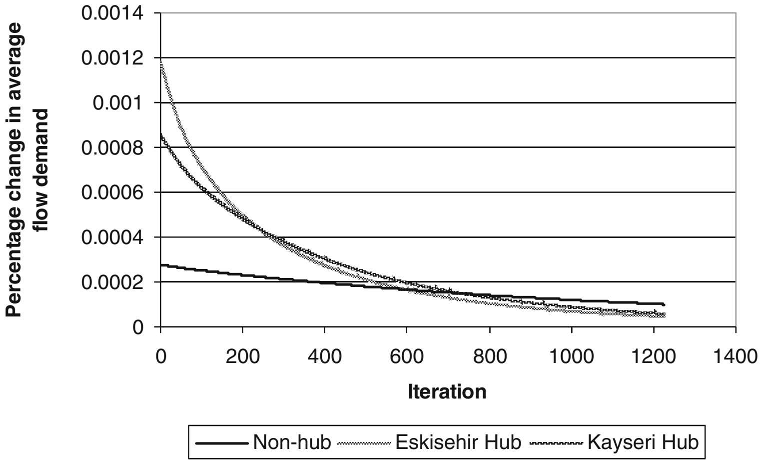
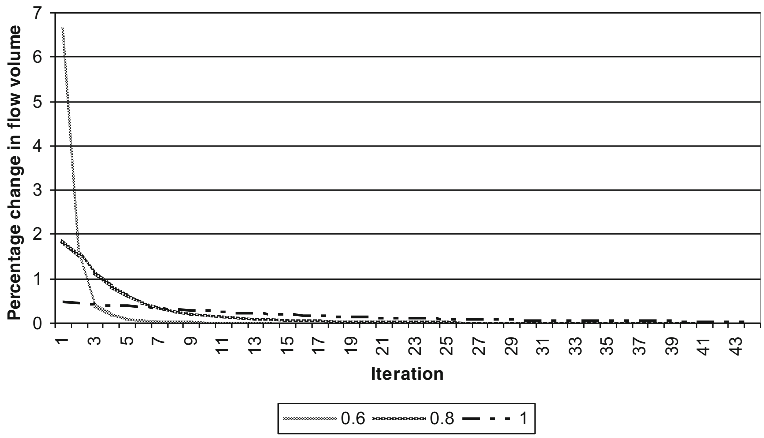
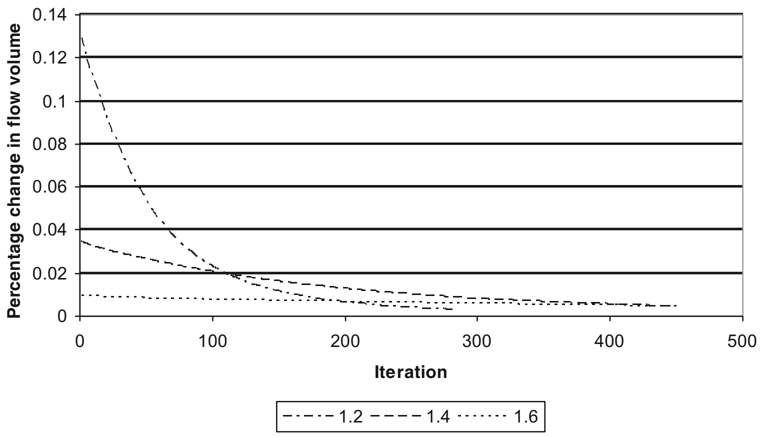


Fig. 6 Percentage change in flow routed through hub and non-hub nodes in the Turkish network with p=2 and α=0.4



a For β between 0.6 and 1.0



b For β between 1.2 and 1.6

Fig. 7 Effect of β on the percentage change in average flow volume in the CAB network

based on the logistics costs derived from the optimum locations of these initial hubs, and then optimally locate $(p+1)$ hubs using the updated flows. The results indicate that the hub locations are very robust to the change in the β value in this range. Thus, we do not report the hub locations. Table 3 shows the costs per flow in each one of the test instances. The costs per flow are smallest when the β value is set at 1.7, resulting in a more efficient network in general. Since it should be expected for the flow structure in the network to evolve in the direction of better efficiency, we believe that the results support the use of $\beta=1.7$ in this context.

We now continue with the main experiments about the effects of endogenous attraction on hub location and allocation decisions by considering three interrelated problems. The first problem is that of optimally locating p hubs based on the original flow and cost/time data. The second problem determines the optimum location of a future $(p+1)^{th}$ hub while taking the locations of the previously sited hubs as fixed and

Table 3 Cost per unit flow in the CAB network with $(p+1)$ hubs sited based on the flows updated using the location of p initial hubs and different β values

p	α	β		
		1.1	1.7	2.55
2	0.4	897	874	902
	0.8	1152	1116	1159
3	0.4	783	750	788
	0.8	1083	1035	1088
4	0.4	706	681	708
	0.8	1031	980	1034
5	0.4	656	628	660
	0.8	987	936	991

considering their economies of scale effect in transfer costs. The third problem is similar to the second with the only exception of using the updated steady state flows obtained from the proposed mechanism given in Eqs. (12) and (13).

Experimentation is performed on both data sets with the same parameter values of β=1.7, ψ=3% and Δ=100. The value of the constant discount factor, α, is varied as 0.2, 0.4, 0.6 and 0.8. For each α value, results are obtained for different numbers of initial hubs by changing p as 2, 3 and 4. In both data sets, the results are particularly interesting when α=0.4 and p=2 (hence the use of these values also in the earlier exposition).

With the small (25-node) CAB data set, the location and allocation decisions are unaffected by the endogenous attraction reflected in the flow update mechanism in all but one (i.e., when α=0.4 and p=2) of the tested parameter combinations. When α=0.4 and p=2, Fig. 8 shows the optimum locations of the initial 2 hubs in Pittsburg and Los Angeles and the corresponding allocation structure in the CAB network. The Pittsburg and Los Angeles hubs are marked in a large triangle and a large square, respectively. The nodes assigned to these hubs are also marked in a smaller size of the corresponding shape. When the locations of these two hubs are fixed and a third hub is sited based on

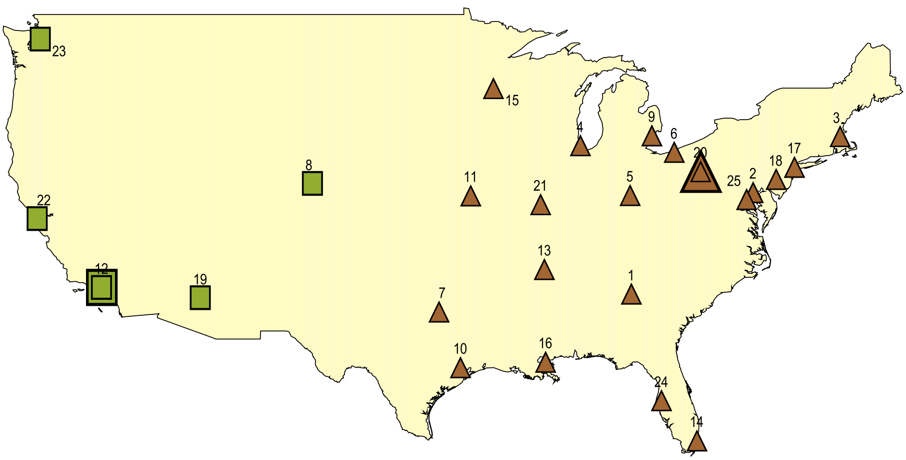


Fig. 8 Hubs and allocations for the CAB data with p=2, α=0.4

the original data, the optimum location of this new hub is determined as St. Louis (node 21). The corresponding allocation structure is as shown in Fig. 9. The new hub in St. Louis is marked in a large circle, and the new allocation structure is superimposed on the same map. The nodes marked twice in the same shape are still allocated to their former hub. Whereas, the nodes marked in two different shapes are now allocated to the hub marked in their newly assigned shape. That is, Nodes 4, 7, 10, 11, 13, 15, 16 and 21 that were previously assigned to the Pittsburg hub are now assigned to the new hub in St. Louis. The St. Louis hub also serves to node 8 (Denver) that was previously assigned to the Los Angeles hub. When the optimum location of the new hub is determined based on the updated steady state flows, it is installed in Chicago (node 4) rather than in St. Louis. The corresponding allocation structure is as shown in Fig. 10, in which the new Chicago hub, marked in a large circle, serves the same nodes as the St. Louis hub in the previous step.

Results with the denser Turkish network are even more interesting. As in the CAB data set, both the hub locations and the allocations are affected by the flow update when α=0.4 and p=2. Although the hub locations are unaffected by the flow updates, the allocation structure changes in three other parameter combinations, when $(\alpha = 0.6, \quad p = 2)$, $(\alpha = 0.4, \quad p = 3)$ and $(\alpha = 0.8, \quad p = 4)$. The hub locations and allocations for these four parameter combinations are tabulated in Appendix 2. When there is a change in the allocation structure, the node numbers with a change in allocation are shown in bold italics.

Returning to the most interesting parameter combination of α=0.4 and p=2, the initial two hubs are optimally sited in nodes 26 (Eskisehir) and 38 (Kayseri). The optimum allocation structure is as shown in Fig. 11, in which the Kayseri and Eskisehir hubs are marked in a large square and a large triangle, respectively. The other nodes are marked in a smaller size of the same shape as the hub to which they are allocated. Since the flow demand in the western regions are heavier, the Eskisehir hub serves only 29 nodes in contrast to the Kayseri hub serving 52. When a third hub is opened while fixing the locations of the initial two, the optimum location for this new hub is determined as node 21 (Diyarbakir). As shown in Fig. 12, 20 of the nodes that were previously served by the Kayseri hub

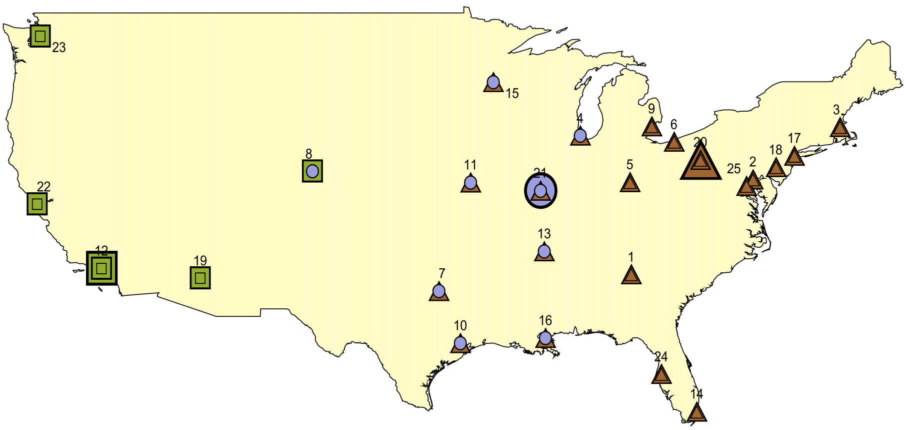


Fig. 9 Introducing a third hub to the CAB network based on the original flow data when α=0.4 and the locations of the first two hubs are fixed

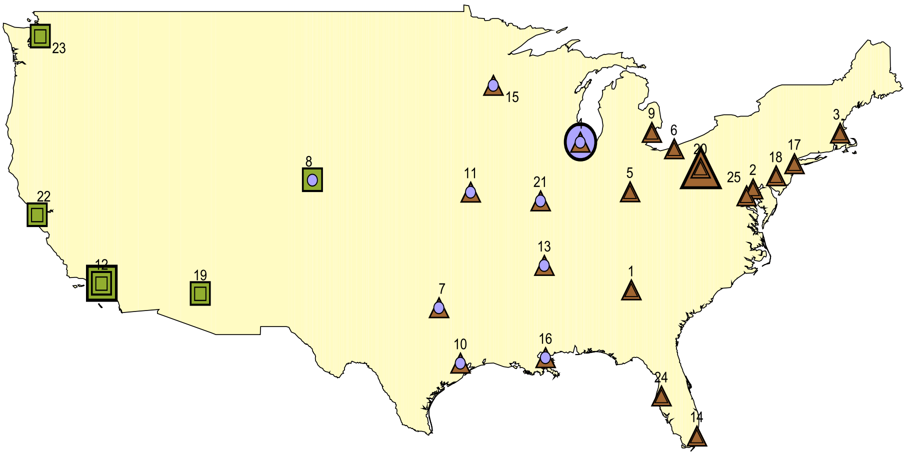


Fig. 10 Introducing a third hub to the CAB network based on the updated flow data when α=0.4,β=1.7 and the locations of the first two hubs are fixed

are now assigned to the new Diyarbakir hub, marked in a large circle, while the other allocations remain unchanged. Considering the updated steady state flow demands in determining the optimum location of the third hub shifts it from Diyarbakir far to the west to node 34 (Istanbul). The corresponding allocation structure is as shown in Fig. 13. The Istanbul hub is marked in a large circle. The nodes marked in two different shapes are the ones whose allocations are changed. All of the 20 nodes that were assigned to the Diyarbakir hub in the previous step are now assigned back to the Kayseri hub. The new hub in Istanbul serves to nodes 17, 22, 39, 41 and 59 that were originally assigned to the Eskisehir hub. This outcome is rather intuitive as it is rational for the already well developed region around Istanbul to generate a sufficiently heavier flow demand to justify opening a new regional hub with the help of more efficient connectivity to the eastern parts of the country.

Next, we compare the effects of our flow update mechanism with the one proposed by O'Kelly (1986), which is the only available alternative in the literature. The comparison is made on the location of future hubs and the costs per unit flow resulting from the associated allocation decisions. We consider

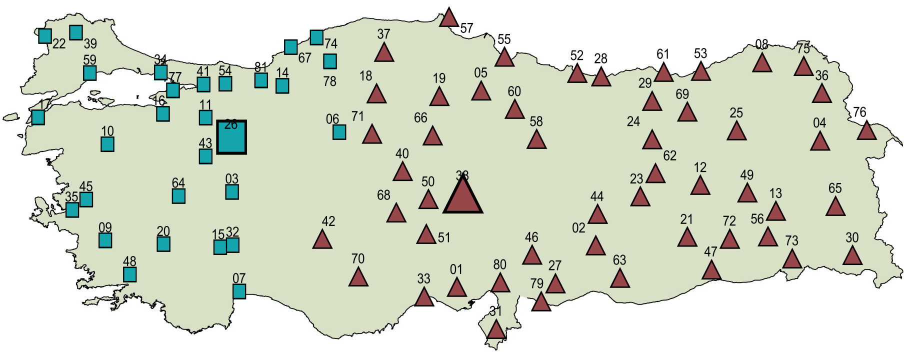


Fig. 11 Hubs and allocations for the Turkish data with p=2, α=0.4

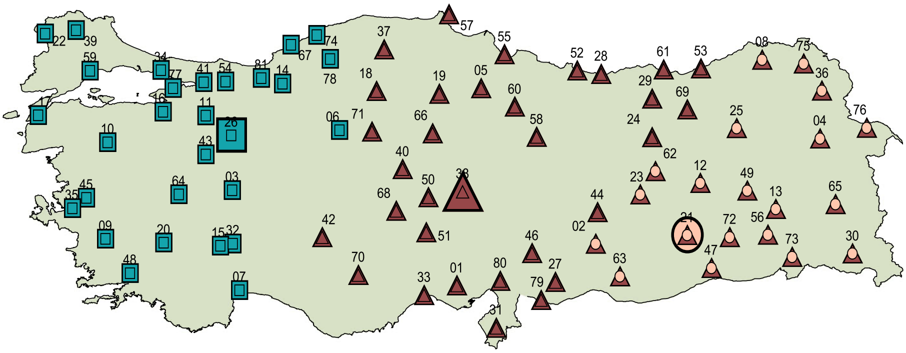


Fig. 12 Introducing a third hub to the Turkish network based on the original flow data when α=0.4 and the locations of the first two hubs are fixed

opening p hubs, fixing their locations, updating the flows based on these initial hubs using both our procedure and that proposed by O'Kelly, and then we open r additional hubs based on the flows obtained from the different update mechanisms. In the use of O'Kelly's update procedure we set the associated $\ddot{\beta}$ value at 0.001, which is identified based on preliminary experimentation as the better of the two settings adopted in his 1986 paper. The number of original hubs, p, is set at 2 and 3, and the number of additional future hubs, r, is set at 1, 2 and 3. Discount factor α is varied between 0.2 and 1.0 in increments of 0.2. Tables 4 and 5 present the results for the CAB and Turkish data sets, respectively. The resulting costs per unit flow are also shown in Appendix 3 for the CAB and Turkish data sets in Tables 11 and 12, respectively. The results indicate that in some cases, using the original flows and the flows updated based on endogenous attraction result in different locations for the future hubs. Although, we do not analyze it here, as observed in our previous results, using the updated flows also alters the allocation decisions. When the update procedure proposed by O'Kelly is used, the hub location decisions change significantly in most problems. This suggests that using an accurate update procedure is particularly important when taking

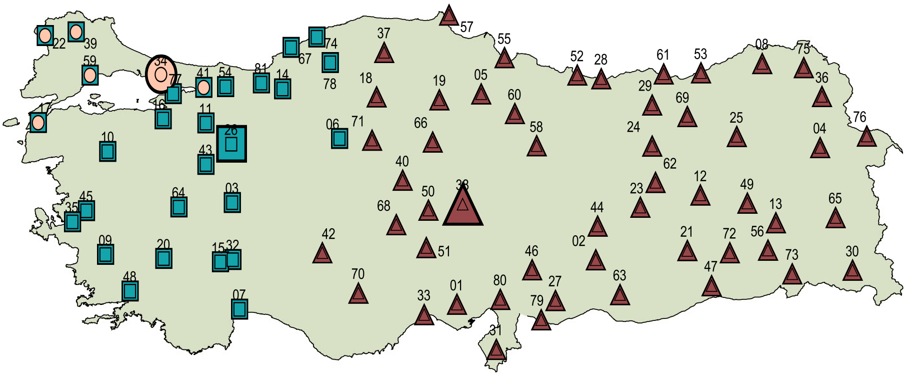


Fig. 13 Introducing a third hub to the Turkish network based on the updated flow data when α=0.4,β=1.7 and the locations of the first two hubs are fixed

Table 4 Locations of the p original and r future hubs in the CAB network when the flows are updated by different procedures

p+r	α	Original hubs	New hubs located based on		
			Original flows	Updated flows our procedure	O'Kelly's update procedure
2+1	0.2	12, 20	21	21	21
	0.4	12, 20	21	4	21
	0.6	12, 20	4	4	21
	0.8	12, 20	4	4	5
	1.0	8, 20	4	4	5
2+2	0.2	12, 20	17, 21	17, 21	17, 21
	0.4	12, 20	17, 21	4, 17	1, 21
	0.6	12, 20	4, 17	4, 17	1, 4
	0.8	12, 20	4, 7	4, 7	1, 4
	1.0	8, 20	4, 7	4, 21	4, 5
2+3	0.2	12, 20	14, 17, 21	14, 17, 21	17, 21, 24
	0.4	12, 20	14, 17, 21	1, 4, 17	1, 17, 21
	0.6	12, 20	1, 4, 17	1, 4, 17	1, 4, 17
	0.8	12, 20	1, 4, 17	1, 4, 17	1, 2, 4
	1.0	8, 20	1, 4, 7	1, 4, 7	1, 4, 21
3+1	0.2	4, 12, 17	24	24	17
	0.4	4, 12, 18	1	1	1
	0.6	2, 4, 12	1	1	1
	0.8	2, 4, 12	1	1	1
	1.0	4, 8, 20	7	21	21
3+2	0.2	4, 12, 17	7, 14	7, 14	7, 17
	0.4	4, 12, 18	7, 14	7, 14	13, 24
	0.6	2, 4, 12	7, 14	7, 14	1, 21
	0.8	2, 4, 12	1, 7	1, 7	1, 21
	1.0	4, 8, 20	1, 7	1, 7	1, 21
3+3	0.2	4, 12, 17	6, 7, 14	6, 7, 14	6, 7, 24
	0.4	4, 12, 18	6, 7, 14	6, 7, 14	6, 13, 24
	0.6	2, 4, 12	7, 14, 17	7, 14, 17	1, 6, 21
	0.8	2, 4, 12	1, 7, 17	1, 7, 17	1, 6, 21
	1.0	4, 8, 20	1, 7, 17	1, 7, 17	1, 8, 21

endogenous attraction into account in the design of a hub network with a long term perspective. In the CAB data set, the average cost per unit flow over all parameter combinations tested is smaller than that with the original flow information by 4 % when our update procedure is used, and by 5 % when O'Kelly's (1986) update procedure is used. In the Turkish data set, the average savings in cost per unit flow are 2.4 % and 0.4 % with our procedure and with O'Kelly's procedure, respectively.

Table 5 Locations of the p original and r future hubs in the Turkish network when the flows are updated by different procedures

Numbers of original+new hubs	α	Original hubs	New hubs located based on		
			Original flows	Updated flows our procedure	O'Kellys update procedure
2+1	0.2	26, 44	34	34	6
	0.4	26, 38	21	34	23
	0.6	26, 38	34	34	23
	0.8	6, 38	26	26	23
	1.0	6, 38	3	3	3
2+2	0.2	26, 44	6, 34	6, 34	6, 25
	0.4	26, 38	21, 34	21, 34	23, 34
	0.6	26, 38	6, 34	6, 34	6, 23
	0.8	6, 38	3, 34	3, 34	3, 34
	1.0	6, 38	3, 34	3, 34	3, 81
2+3	0.2	26, 44	6, 34, 35	6, 34,35	6, 25, 34
	0.4	26, 38	21, 34, 35	21, 34, 35	6, 25, 34
	0.6	26, 38	6, 23, 34	6, 23, 34	6, 25, 34
	0.8	6, 38	1, 3, 34	1, 3, 34	3, 23, 34
	1.0	6, 38	3, 34, 68	3, 34, 68	3, 23, 34
3+1	0.2	3, 34, 44	6	6	6
	0.4	6, 34, 44	35	35	3
	0.6	3, 34, 38	6	6	23
	0.8	6, 26, 38	34	34	23
	1.0	3, 6, 38	34	34	81
3+2	0.2	3, 34, 44	1, 6	1, 6	6, 25
	0.4	6, 34, 44	1, 35	1, 35	3, 25
	0.6	3, 34, 38	6, 23	6, 23	6, 23
	0.8	6, 26, 38	3, 34	3, 34	23, 34
	1.0	3, 6, 38	34, 68	34, 68	68, 81
3+3	0.2	3, 34, 44	1, 6, 61	1, 6, 35	1, 6, 25
	0.4	6, 34, 44	1, 35, 55	1, 35, 55	1, 3, 61
	0.6	3, 34, 38	1, 6, 23	1, 6, 23	6, 25, 27
	0.8	6, 26, 38	1, 3, 34	1, 3, 34	1, 3, 23
	1.0	3, 6, 38	26, 34, 68	26, 34, 68	34, 68, 81

As pointed out in the literature section, some research in multi-period or dynamic hub location problems (e.g., Contreras et al. 2011) allow the possibility of opening and closing hub facilities at every discrete time period. Following this idea, we now consider relocating all hubs at the time an additional hub is to be opened with the updated flow structure. Once again, the experiments are performed on the CAB and Turkish data sets, with the original flows, and the flows updated by both our procedure and the one proposed by O'Kelly (1986). The

Table 6 Locations of the p original and $(p+r)$ future hubs in the CAB network with different update procedures and relocation of all hubs based on the updated flows

$p+r$	α	Original hubs	All future hub locations based on					
			Original flows		Flows updated by our procedure		Flows updated by O'Kelly's procedure	
			Hubs	Cost	Hubs	Cost	Hubs	Cost
3+1	1.0	4, 8, 20	4, 7, 8, 20	1211	4, 8, 20, 21	1159	4, 8, 20, 21	1060
4+1	0.6	1, 4, 12, 17	4, 7, 12, 14, 17	877	1, 4, 7, 12, 17	834	4, 12, 13, 18, 24	863
4+1	1.0	4, 7, 8, 20	1, 2, 4, 7, 8	1173	1, 4, 7, 8, 20	1123	1, 4, 8, 13, 20	1036
6+1	0.6	4, 6, 7, 12, 14, 17	4, 6, 7, 12, 14, 17, 22	795	4, 6, 7, 12, 14, 17, 25	750	1, 4, 6, 7, 12, 18, 24	791
7+1	0.2	4, 6, 7, 12, 14, 17, 22	1, 4, 6, 7, 12, 14, 17, 22	415	4, 6, 7, 12, 14, 17, 22, 25	397	4, 6, 7, 12, 13, 14, 17, 23	488
7+1	0.4	4, 6, 7, 12, 14, 17, 22	1, 4, 6, 7, 12, 14, 17, 22	589	4, 6, 7, 12, 14, 17, 22, 25	562	4, 6, 7, 12, 13, 18, 23, 24	647
7+1	0.6	4, 6, 7, 12, 14, 17, 22	1, 4, 6, 7, 12, 14, 17, 22	764	4, 6, 7, 12, 14, 17, 22, 25	721	1, 4, 6, 7, 12, 18, 23, 24	770
8+1	0.2	1, 4, 6, 7, 12, 14, 17, 22	1, 4, 6, 7, 8, 12, 14, 17, 22	383	1, 4, 6, 7, 12, 14, 17, 22, 25	367	1, 4, 6, 7, 8, 12, 14, 17, 23	454
8+1	0.4	1, 4, 6, 7, 12, 14, 17, 22	1, 4, 6, 7, 8, 12, 14, 17, 22	557	1, 4, 6, 7, 12, 14, 17, 22, 25	531	1, 4, 6, 7, 12, 17, 23, 24, 25	614
8+1	0.6	1, 4, 6, 7, 12, 14, 17, 22	1, 4, 6, 7, 8, 12, 14, 17, 22	733	1, 4, 6, 7, 12, 14, 17, 22, 25	691	1, 4, 6, 7, 12, 18, 21, 23, 24	738
8+1	1.0	1, 4, 6, 7, 8, 17, 21, 25	1, 4, 6, 7, 8, 12, 17, 22, 25	1063	1, 4, 6, 7, 8, 12, 17, 21, 25	999	1, 4, 6, 7, 8, 17, 21, 24, 25	939
10+1	0.2	1, 4, 6, 7, 8, 12, 14, 17, 22, 25	1, 4, 6, 7, 8, 12, 14, 17, 22, 23, 25	328	1, 4, 6, 7, 8, 12, 14, 17, 21, 22, 25	317	1, 4, 6, 7, 8, 12, 14, 17, 21, 23, 25	391
10+1	0.4	1, 4, 6, 7, 8, 12, 14, 17, 22, 25	1, 4, 6, 7, 8, 12, 14, 17, 22, 23, 25	506	1, 4, 6, 7, 8, 12, 14, 17, 21, 22, 25	483	1, 4, 6, 7, 8, 12, 17, 21, 23, 24, 25	553
10+1	1.0	1, 4, 6, 7, 8, 12, 17, 21, 22, 25	1, 4, 6, 7, 8, 12, 17, 21, 22, 23, 25	1026	1, 4, 6, 7, 8, 12, 14, 17, 21, 22, 25	962	1, 4, 6, 7, 8, 12, 17, 19, 21, 23, 25	885

Table 7 Locations of the p original and $(p+r)$ future hubs in the Turkish network with different update procedures and relocation of all hubs based on the updated flows

$p+r$	α	Original hubs	All future hub locations based on					
			Original flows		Flows updated by our procedure		Flows updated by O'Kelly's procedure	
			Hubs	Cost	Hubs	Cost	Hubs	Cost
2+1	0.4	26, 38	6, 34, 44	771.08	6, 34, 38	754.75	23, 26, 38	783.12
3+1	0.4	6, 34, 44	3, 5, 27, 34	690.28	6, 27, 34, 35	675.86	3, 6, 23, 34	709.55
7+1	0.4	1, 3, 6, 21, 34, 35, 55	1, 3, 6, 21, 25, 34, 35, 55	531.86	1, 3, 6, 16, 21, 34, 35, 55	518.51	1, 3, 5, 6, 21, 25, 34, 35	555.45
8+1	0.4	1, 3, 6, 21, 25, 34, 35, 55	1, 6, 16, 21, 25, 34, 35, 42, 55	511.81	1, 3, 6, 16, 21, 25, 34, 35, 55	499.31	1, 3, 6, 21, 25, 34, 35, 55, 68	535.08
9+1	0.4	1, 6, 16, 21, 25, 34, 35, 42, 55	1, 3, 6, 16, 21, 25, 27, 34, 35, 55	494.42	1, 6, 16, 21, 25, 27, 34, 35, 42, 55	481.83	1, 3, 6, 16, 21, 25, 34, 35, 55, 68	518.59

number of original hubs is varied between 2 and 10 in increments of 1, and opening one additional hub is considered. For the CAB data set, discount factor α is varied between 0.2 and 1.0 in increments of 0.2. Table 6 reports the results in which the hub locations based on our procedure are different from those obtained with the original flows. The costs per unit flow are also reported for O'Kelly (1986) update procedure to allow for comparison. One important observation in the results is that when the flows are updated with our procedure, the locations of the original hubs are retained at the time of opening the additional hub. Since the flow potential of these hubs are further reinforced due to endogenous attraction, this result is what should be expected. Also, when the future hub location decisions are made based on our flow update procedure, the resulting cost per unit flow is smaller than those with O'Kelly's update procedure in 12 of the 14 test instances reported. When our update mechanism is used, the average savings in cost is 5 % in comparison to the baseline scenario of using the original flows.

In the larger Turkish data set, the experimentation is performed in the same way as in the CAB data set with the only exception that the discount factor α is set only at the two levels of 0.4 and 0.8 to reduce the experimental effort. Table 7 reports the results in which the hub locations based on our procedure are different from those obtained with the original flows. No hub changes were observed when α=0.8. Thus, the hub locations seem to be more sensitive to the flow update based on endogenous attraction when the hubbing effect on the costs is more significant. In the reported test instances, when our update mechanism is used, the costs per flow are smaller, and the average savings obtained in cost is 2.3 % in comparison to that based on using the original flows.

To summarize these experimental findings, the location and allocation decisions in the development of a hub network depend on the change in the flow volumes at different points. Since hubs provide more efficient and less costly connectivity to their assigned nodes, a critical part of the flow changes is due to the endogenous attraction resulting from this improved connectivity. Results indicate that this part may be significant enough to alter the future location and allocation decisions in a hub network. Therefore it is important to design hub networks with a long term perspective and perhaps through a dynamic or multi-period analysis. In doing this, of course, the accuracy of the procedure used in modeling flow updates due to endogenous attraction is very important as different methods can lead to very different network designs. The results also suggest that the cost savings that can be obtained by considering the flow updates can be significant, for example, in the neighborhood of 2.4 % and 5 % with the Turkish and CAB data sets in our experiments, which would correspond to very large monetary figures.

6 Conclusion

This paper studied the model behavior and the economic effects of the hub facilities in the context of the single-allocation, uncapacitated p-hub median problem. In particular, effects on flow intensities and future hubbing decisions due to endogenous attraction caused by the spatial interaction of hub facilities are investigated. A new model is developed to forecast the flow demands. This model combines the initial ideas proposed by O'Kelly in his seminal paper of 1986 with the concepts of diffusion, learning, and pricing that are commonly utilized in various marketing contexts to foresee the demand for innovations. The proposed model is tested with empirical data obtained from a domestic airline in Turkey. Computational experiments are performed with this new model on both the CAB and Turkish data sets. First the convergence pattern of the proposed flow update mechanism is observed and a stopping criterion is developed to find the steady state flow demands. Next, a sensitivity analysis is performed on the important model parameters. Then the mechanism using this empirically established stopping criterion is tested with the two data sets. Results indicate that there are cases in which both the locations of future hubs and the corresponding allocation structures using updated flows differ from those produced using only the original flows. Therefore, contrary to the general practice in the existing hub location literature, it may be crucial to use the forecasted future flow demands in making strategic hub location decisions. Such forecasts may also be beneficial in other strategic and policy decisions to various parties such as governments and local authorities, transportation and logistics companies, and real estate investors.

The proposed forecasting mechanism for the flow demands, makes it possible to approach the hub network design problem with an aim to obtain an optimum final configuration at a future point in time. For instance, the decision maker can choose to site some hubs at suboptimal locations in earlier phases of the network construction, allow the flows to update accordingly, and site the future hubs based on the updated flow demands. This type of a long-term, dynamic modeling approach to the hub location decisions could produce a future optimum configuration that may or may not have been possible if all hubs were optimally sited with the available data for the time of their opening. Of course, the suggested long-term, dynamic modeling approach would require an explicit consideration of the discounted logistics costs until the optimum final configuration is obtained. We think that this idea could lead to a very fruitful but potentially challenging future research avenue.

Various other extensions of this study may be possible. The most straightforward one is the use of the proposed model in the context of other hub location problems. Another possibility is the adoption of a continuous discount function that models the cost of flow as a concave function of the flow density and investigating the resulting effects on the forecasted flow volumes within this framework. Since the increased flow volumes are expected to result in congestion at the hub terminals, modeling and finding ways of handling this congestion through capacity planning leads to another research direction. A very important but rather long term avenue of future research is to collect time series data on real life hub location decisions and the evolution of the affected flow volumes to obtain a more elaborate data set that can be used for more extensive empirical tests with the proposed models to find potential ways to improve them.

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Appendixes

Appendix 1

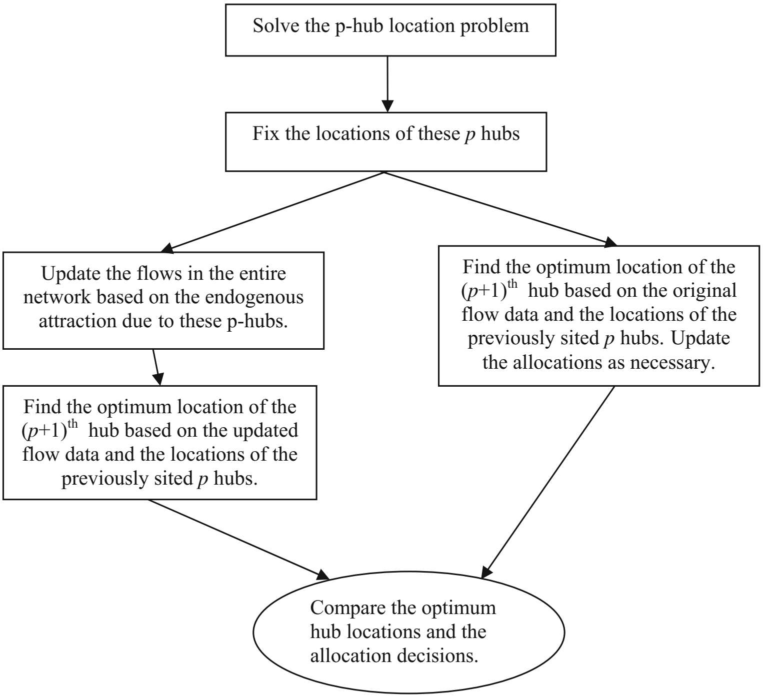

Fig. 14 Procedure for investigating the effect of endogenous attraction on future hub location and allocation decisions

Appendix 2

The scenarios listed in the tables below are defined as follows.

Scenario 1: p-hub location with original flow data
Scenario 2: Siting the $(p+1)^{th}$ hub with the original flow data where the initial p hub's locations are fixed.
Scenario 3: Siting the $(p+1)^{th}$ hub with the updated flow data where the initial p hub's locations are fixed.

Table 8 Experiments with Turkish data in which optimum location and/or allocations in opening a third hub are affected from the flow updates when the locations of the first two hubs are fixed

α	Scenario	Hub	Allocations
0.4	1	26	3, 6, 7, 9, 10, 11, 14, 15, 16, 17, 20, 22, 26, 32, 34, 35, 39, 41, 43, 45, 48, 54, 59, 64, 67, 74, 77, 78, 81
		38	1, 2, 4, 5, 8, 12, 13, 18, 19, 21, 23, 24, 25, 27, 28, 29, 30, 31, 33, 36, 37, 38, 40, 42, 44, 46, 47, 49, 50, 51, 52, 53, 55, 56, 57, 58, 60, 61, 62, 63, 65, 66, 68, 69, 70, 71, 72, 73, 75, 76, 79, 80
	2	21	2, 4, 8, 12, 13, 21, 23, 25, 30, 36, 47, 49, 56, 62, 63, 65, 72, 73, 75, 76
		26	3, 6, 7, 9, 10, 11, 14, 15, 16, 17, 20, 22, 26, 32, 34, 35, 39, 41, 43, 45, 48, 54, 59, 64, 67, 74, 77, 78, 81
		38	1, 5, 18, 19, 24, 27, 28, 29, 31, 33, 37, 38, 40, 42, 44, 46, 50, 51, 52, 53, 55, 57, 58, 60, 61, 66, 68, 69, 70, 71, 79, 80
	3	26	3, 6, 7, 9, 10, 11, 14, 15, 16, 20, 26, 32, 35, 43, 45, 48, 54, 64, 67, 74, 77, 78, 81
		34	17, 22, 34, 39, 41, 59
		38	1, 2, 4, 5, 8, 12, 13, 18, 19, 21, 23, 24, 25, 27, 28, 29, 30, 31, 33, 36, 37, 38, 40, 42, 44, 46, 47, 49, 50, 51, 52, 53, 55, 56, 57, 58, 60, 61, 62, 63, 65, 66, 68, 69, 70, 71, 72, 73, 75, 76, 79, 80
0.6	1	26	3, 6, 7, 9, 10, 11, 14, 15, 16, 17, 18, 20, 22, 26, 32, 34, 35, 37, 39, 41, 43, 45, 48, 54, 59, 64, 67, 74, 77, 78, 81
		38	1, 2, 4, 5, 8, 12, 13, 19, 21, 23, 24, 25, 27, 28, 29, 30, 31, 33, 36, 38, 40, 42, 44, 46, 47, 49, 50, 51, 52, 53, 55, 56, 57, 58, 60, 61, 62, 63, 65, 66, 68, 69, 70, 71, 72, 73, 75, 76, 79, 80
	2	26	3, 6, 7, 9, 10, 11, 14, 15, 16, 20, 26, 32, 35, 43, 45, 48, 54, 64, 67, 74, 77, 78, 81
		34	17, 22, 34, 39, 41, 59
		38	1, 2, 4, 5, 8, 12, 13, ***18***, 19, 21, 23, 24, 25, 27, 28, 29, 30, 31, 33, 36, ***37***, 38, 40, 42, 44, 46, 47, 49, 50, 51, 52, 53, 55, 56, 57, 58, 60, 61, 62, 63, 65, 66, 68, 69, 70, 71, 72, 73, 75, 76, 79, 80
	3	26	3, 6, 7, 9, 10, 11, 14, 15, 16, ***18***, 20, 26, 32, 35, ***37***, 43, 45, 48, 54, 64, 67, 74, 77, 78, 81
		34	17, 22, 34, 39, 41, 59
		38	1, 2, 4, 5, 8, 12, 13, 19, 21, 23, 24, 25, 27, 28, 29, 30, 31, 33, 36, 38, 40, 42, 44, 46, 47, 49, 50, 51, 52, 53, 55, 56, 57, 58, 60, 61, 62, 63, 65, 66, 68, 69, 70, 71, 72, 73, 75, 76, 79, 80

Table 9 Experiments with Turkish data in which optimum location and/or allocations in opening a fourth hub are affected from the flow updates when the locations of the first three hubs are fixed

α	Scenario	Hub	Allocations
0.4	1	6	1, 3, 5, 6, 7, 9, 11, 14, 15, 18, 19, 20, 26, 28, 32, 33, 35, 37, 38, 40, 42, 43, 45, 48, 50, 51, 52, 55, 57, 60, 64, 66, 67, 68, 70, 71, 74, 78, 81
		34	10, 16, 17, 22, 34, 39, 41, 54, 59, 77
		44	2, 4, 8, 12, 13, 21, 23, 24, 25, 27, 29, 30, 31, 36, 44, 46, 47, 49, 53, 56, 58, 61, 62, 63, 65, 69, 72, 73, 75, 76, 79, 80

Table 9 (continued)

α	Scenario	Hub	Allocations
	2	6	1, 3, 5, 6, 7, ***11***, 14, 15, 18, 19, 26, 28, 32, 33, 37, 38, 40, 42, 43, 50, 51, 52, 55, 57, 60, 66, 67, 68, 70, 71, 74, 78, 81
		34	16, 17, 22, 34, 39, 41, 54, 59, 77
		35	9, 10, 20, 35, 45, 48, 64
		44	2, 4, 8, 12, 13, 21, 23, 24, 25, 27, 29, 30, 31, 36, 44, 46, 47, 49, 53, 56, 58, 61, 62, 63, 65, 69, 72, 73, 75, 76, 79, 80
	3	6	1, 3, 5, 6, 7, 14, 15, 18, 19, 26, 28, 32, 33, 37, 38, 40, 42, 43, 50, 51, 52, 55, 57, 60, 66, 67, 68, 70, 71, 74, 78, 81
		34	***11***, 16, 17, 22, 34, 39, 41, 54, 59, 77
		35	9, 10, 20, 35, 45, 48, 64
		44	2, 4, 8, 12, 13, 21, 23, 24, 25, 27, 29, 30, 31, 36, 44, 46, 47, 49, 53, 56, 58, 61, 62, 63, 65, 69, 72, 73, 75, 76, 79, 80

Table 10 Experiments with Turkish data in which optimum location and/or allocations in opening a fifth hub are affected from the flow updates when the locations of the first four hubs are fixed

α	Scenario	Hub	Allocations
0.8	1	1	1, 2, 12, 13, 21, 23, 27, 30, 31, 33, 44, 46, 47, 49, 56, 62, 63, 65, 72, 73, 79, 80
		3	3, 7, 9, 10, 11, 15, 16, 20, 26, 32, 35, 42, 43, 45, 48, 64, 70
		6	4, 5, 6, 8, 14, 18, 19, 24, 25, 28, 29, 36, 37, 38, 40, 50, 51, 52, 53, 54, 55, 57, 58, 60, 61, 66, 67, 68, 69, 71, 74, 75, 76, 78, 81
		34	17, 22, 34, 39, 41, 59, 77
	2	1	1, 2, 27, 31, 33, 46, 63, 79, 80
		3	3, 7, 9, 10, 11, 15, 16, 20, 26, 32, 35, 43, 45, 48, 64
		6	5, 6, 14, 18, 19, 28, 37, 38, 40, ***42***, 50, 51, 52, 53, ***54***, 55, 57, 58, 60, 61, 66, 67, 68, ***70***, 71, 74, 78, 81
		23	4, 8, 12, 13, 21, 23, 24, 25, 29, 30, 36, 44, 47, 49, 56, 62, 65, 69, 72, 73, 75, 76
		34	17, 22, 34, 39, 41, 59, 77
	3	1	1, 2, 27, 31, 33, 46, 63, 79, 80
		3	3, 7, 9, 10, 11, 15, 16, 20, 26, 32, 35, ***42***, 43, 45, 48, 64, ***70***
		6	5, 6, 14, 18, 19, 28, 37, 38, 40, 50, 51, 52, 53, 55, 57, 58, 60, 61, 66, 67, 68, 71, 74, 78, 81
		23	4, 8, 12, 13, 21, 23, 24, 25, 29, 30, 36, 44, 47, 49, 56, 62, 65, 69, 72, 73, 75, 76
		34	17, 22, 34, 39, 41, ***54***, 59, 77

Appendix 3

Table 11 Cost per unit flow in the CAB network with p original and r future hubs when the flows are updated by different procedures (locations of the original hubs fixed)

$p+r$	α	Original cost per flow	Cost per unit flow with new hubs located based on		
			Original flows	Updated flows our procedure	O'Kellys update procedure
2+1	0.2	1000.91	836.87	813.18	825.75
	0.4	1101.62	949.62	920.71	928.58
	0.6	1201.21	1059.91	1020.10	996.15
	0.8	1294.08	1166.78	1118.95	1031.08
	1.0	1359.19	1256.63	1207.49	1096.86
2+2	0.2	1000.91	702.04	677.95	729.08
	0.4	1101.63	842.18	812.52	849.96
	0.6	1201.20	978.80	939.23	927.32
	0.8	1294.08	1105.60	1064.25	984.20
	1.0	1359.19	1211.23	1166.67	1072.53
2+3	0.2	1000.91	584.18	569.98	637.71
	0.4	1101.63	744.61	721.25	772.35
	0.6	1201.20	897.59	862.76	867.56
	0.8	1294.08	1047.30	1004.30	943.63
	1.0	1359.19	1174.49	1131.93	1041.86
3+1	0.2	767.35	629.63	599.23	678.11
	0.4	901.70	794.56	757.12	813.60
	0.6	1033.56	952.46	907.45	912.70
	0.8	1158.83	1091.83	1037.22	978.13
	1.0	1256.63	1211.23	1158.92	1059.47
3+2	0.2	767.35	538.37	519.67	615.07
	0.4	901.70	715.37	688.23	753.08
	0.6	1033.56	890.17	852.59	863.40
	0.8	1158.83	1038.62	989.65	931.77
	1.0	1256.63	1174.49	1125.44	1029.42
3+3	0.2	767.35	491.03	471.50	563.21
	0.4	901.70	670.74	643.25	705.85
	0.6	1033.56	839.63	801.39	821.30
	0.8	1158.83	1000.27	950.93	893.73
	1.0	1256.63	1137.01	1089.87	1007.97

Table 12 Cost per unit flow in the Turkish network with p original and r future hubs when the flows are updated by different procedures (locations of the original hubs fixed)

p+r	α	Original cost per unit flow	Cost per unit flow with new hubs located based on		
			Original flows	Updated flows our procedure	O'Kellys update procedure
2+1	0.2	784.84	696.36	684.60	714.38
	0.4	860.75	788.80	771.23	783.12
	0.6	916.90	863.76	842.75	854.43
	0.8	961.83	923.14	900.51	910.58
	1.0	992.72	973.49	949.54	948.82
2+2	0.2	784.84	618.93	608.16	648.48
	0.4	860.76	717.65	704.93	738.05
	0.6	916.90	813.03	792.76	813.58
	0.8	961.83	873.16	852.54	879.81
	1.0	992.72	946.94	924.15	931.38
2+3	0.2	784.84	555.22	545.40	589.87
	0.4	860.76	667.58	655.59	693.04
	0.6	916.90	770.16	753.06	778.44
	0.8	961.83	845.25	825.21	849.98
	1.0	992.72	926.61	904.60	946.61
3+1	0.2	672.02	596.74	582.41	631.21
	0.4	771.08	693.57	676.26	712.79
	0.6	850.31	798.40	776.46	809.34
	0.8	923.14	883.71	857.25	871.06
	1.0	973.49	946.95	922.62	926.48
3+2	0.2	672.02	540.31	528.10	565.90
	0.4	771.08	639.35	624.56	664.03
	0.6	850.31	755.53	737.31	763.81
	0.8	923.14	857.15	831.43	846.15
	1.0	973.49	926.61	903.21	913.57
3+3	0.2	672.02	491.85	481.90	523.79
	0.4	771.08	597.39	585.04	624.86
	0.6	850.31	714.33	697.13	730.01
	0.8	923.14	831.34	806.52	821.38
	1.0	973.49	910.56	886.89	844.60

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