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Article

A Tale of Two Bargaining Solutions

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Abstract: We set up a rich bilateral bargaining model with four salient points (disagreement point, ideal point, reference point, and tempered aspirations point), where the disagreement point and the utility possibilities frontier are endogenously determined. This model allows us to compare two bargaining solutions that use reference points, the Gupta-Livne solution and the tempered aspirations solution, in terms of Pareto efficiency in a strategic framework. Our main result shows that the weights solutions place on the disagreement point do *not* directly imply a unique efficiency ranking in this bargaining problem with a reference point. In particular, the introduction of a reference point brings one more degree of freedom to the model which requires also the difference in the weights placed on the reference point to be considered in reaching an efficiency ranking.

Keywords: aspiration points; bargaining problem; endogenous disagreement points; reference points

JEL classifications: C72, C78, D63, D74

1. Introduction

In this paper, we present a *first* comparison of the Gupta-Livne solution (see [1]) and the tempered aspirations solution (see [2]) in bargaining problems with a reference point. The test bed we use for this comparison is a bargaining problem with endogenously determined disagreement point and utility possibilities frontier. For this purpose, we extend the model proposed in Anbarcı *et al*. [3]

by incorporating a reference point into the bargaining problem. This allows us to compare the aforementioned solutions in terms of *Pareto efficiency*.[1]

The *disagreement point* is the only salient point in the simple bargaining model proposed by Nash [4]. What individuals would get in case of a disagreement is surely a source of bargaining power. Thus, it has the potential to affect the bargaining outcome. However, some authors later argued that it simply may not be the only salient point with such a potential. Along these lines, Kalai and Smorodinsky [5] extended Nash's bargaining model by incorporating the *ideal (or utopia) point*, whereas Brito *et al.* [6] incorporated the *status quo point* which is different than the disagreement point. Later, Chun and Thomson [7] argued that individuals bring their claims to the negotiation table, and accordingly introduced the *claims point* as a salient point influencing the bargaining agreement.

In the last three decades, there has been an increased interest in the experimental investigation of *reference point* effects in negotiations. In particular, Ashenfelter and Bloom [8], Bazerman [9], Gupta and Livne [10], Blount *et al.* [11], Bohnet and Zeckhauser [12], Gächter and Riedl [13], Bolton and Karagözoğlu [14], Bartling and Schmidt [15], Herweg and Schmidt [16], and Fehr *et al.* [17] all reported that reference points—in the form of existing contracts, expired contracts, historical contractual conditions, informal agreements, norms—significantly influence the whole bargaining process and the negotiated agreement. Accordingly, a strong empirical support for the influence of reference points on bargaining agreements has accumulated in the last three decades. This calls for more theoretical studies modeling the emergence and the influence of reference points in negotiations. The current study aims to contribute to this line of research.

Gupta and Livne [1] is one of the first studies which incorporated the notion of a *reference point* into the simple bargaining model proposed by Nash [4]. These authors argued that the reference point outcome has a potential to influence the negotiated agreement especially if it is found *fair* by all parties. In that case, the reference point can serve as a *starting point* for negotiations. Their main result presents an axiomatic characterization of a bargaining solution later known as the Gupta-Livne solution (GL).

More than two decades after [1], a new bargaining solution was proposed by Balakrishnan *et al.* [2]. The difference between their modeling and that of Gupta and Livne [1] is that they assumed agents who derive aspirations from the reference point outcomes rather than the disagreement point outcomes. This is in line with the argument that when agents come to the negotiation table, their aspirations are likely to be derived from their disagreement outcomes, but then aspirations are updated if a salient reference point that all parties mutually acknowledge emerges/exists.[2] Given that the reference point Pareto dominates the disagreement point (an assumption made both in [1] and [2]), deriving aspirations from the reference point instead of the disagreement point leads to tempered aspirations. Accordingly, Balakrishnan *et al.* [2] called their solution as the tempered aspirations solution (TA). They presented an axiomatic characterization of this solution concept along with a detailed axiomatic analysis. In the current work, we compare GL and TA in terms of Pareto efficiency. Our paper can be seen as a follow-up

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1 There are other ways of comparing bargaining solutions on the basis of efficiency. For instance, if one fixes a social welfare function (e.g., utilitarian or egalitarian), then one can compare two solution concepts on the basis of efficiency *implied by that social welfare function* (see, e.g., [18–20]). We, on the other hand, stick to Pareto efficiency in this paper.

2 For theoretical models where this phenomenon is modeled, the reader is referred to [21,22].

to Balakrishnan *et al*. [2] in that these authors presented the comparison of these two solution concepts as an open question for future research.

In many real-life negotiations, the disagreement outcomes and the utility possibilities set are not exogenously given. Rather, they are endogenously and sometimes strategically determined.[3] Recently, Anbarcı *et al*. [3] set up a model where the disagreement point and the utility possibilities frontier are endogenously determined in the equilibrium of a strategic game. This allows them to compare three bargaining solution concepts in terms of Pareto efficiency. These solution concepts are the *split-the-surplus solution* (SS) (see [23]), the *Kalai-Smorodinsky solution* (KS) (see [5]), and the *equal sacrifice solution* (ES) (see [24]). These solutions differ on the basis of the weights they place on the disagreement point. The authors showed that, under the assumption of symmetry, the relative weights solution concepts place on the disagreement point directly imply an efficiency ranking. More precisely, ES is the solution which places the least weight on the disagreement point, and it Pareto dominates the others; whereas SS is the one which places the greatest weight on the disagreement point, and it is Pareto dominated by the others.

We enrich Anbarcı *et al*. [3]'s model with a reference point.[4] We show that the efficiency ranking of GL and TA does *not* solely depend on the relative weights they place on the disagreement point. In particular, TA more heavily depends on the disagreement point than GL does, but there are instances when it Pareto dominates GL. The intuition for this result is as follows: Among the solution concepts studied in [3], KS uses both the disagreement point and the ideal point, whereas SS uses only the disagreement point and ES uses only the ideal point. Hence, (i) only one solution uses more than one salient point; and (ii) whenever a salient point other than the disagreement point is used, it is common among the solutions using it. On the other hand, in the current work, the introduction of a reference point brings one more degree of freedom. In particular, TA uses the tempered aspirations point whereas GL uses the reference point. Hence, not only do they differ in their use of the disagreement point but also in their use of the reference point. Therefore, their efficiency ranking depends on multiple factors: (i) the weights placed on the disagreement point; (ii) the weights placed on the reference point; and (iii) the (relative) importances of the disagreement point and the reference point, which are influenced by other model parameters. Below we will explain the factors affecting the Pareto ranking of GL and TA in greater detail.

The paper is organized as follows. Section 2 introduces the two bargaining solution concepts studied in the paper. Section 3 introduces the economic environment building on [3] and presents the main result along with a discussion. Section 4 concludes.

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[3] See [25] for a comprehensive survey of the experimental literature on bargaining problems in which the bargaining pie is endogenously determined.

[4] Anbarcı *et al*. [3] presented a reasonable way of endogenizing feasible sets for the purpose of efficiency comparison. It is a tractable model and it captures essential characteristics of conflict situations such as investment in tools/technology/weapons that would provide power/advantage and the efficiency losses such an investment may cause. These factors make their model a natural one to be utilized for the comparison of GL and TA, as well.

2. Two Bargaining Solutions

In this section, we introduce the two solution concepts employed in this paper. While doing this, we follow the notation used in [2] for reader friendliness. Later, in Section 3, when we follow the model notation used by Anbarcı *et al*. [3], we will make the necessary changes.

The *feasible set* is a *non-empty*, *convex*, *comprehensive*, and *closed* set $S \subset \mathbb{R}^n$ bounded from above. The feasible set consists of all the utility vectors that can be achieved by agents. The *disagreement point* $d \in S$ represents the utility levels obtained by agents if no agreement is reached. A *bargaining problem* is a pair (S, d) such that there exists $x \in S$ with $x \gg d$.[5] For every $S \subset \mathbb{R}^n$, its weakly Pareto optimal set is defined as $WPO(S) = \{y \in S \mid x \gg y \text{ implies } x \notin S\}$. A *bargaining problem with a reference point* is a triple (S, d, r) where the reference point $r \in S \setminus WPO(S)$ satisfies $r \geq d$.

Gupta and Livne [1] (p. 1304) interpreted the reference outcome as "... *an intermediate agreement which, although nonbinding, facilitates the conflict resolution process*". The source of reference outcome can be culture, tradition, norms, historical precedents, focal outcomes, values of relevant economic parameters, *etc.*[6] As it is reported in many experimental studies, whatever the source is, a salient reference outcome has a strong potential to influence bargaining agreements.

The following salient points are employed by the two solution concepts studied in this paper. For every $x \in S$, an *aspiration vector* $a(S, x)$ is defined in such a way that for every $i \in \{1, \ldots, n\}$: $a_i(S, x) = \max\{t \in \mathbb{R} \mid (t, x_{-i}) \in S\}$. Accordingly, the *ideal (or utopia) point* introduced by Kalai and Smorodinsky [5] is defined as $a(S, d) \notin S$. The ideal point gives, for each agent, the maximum utility level that can be reached in an individually rational agreement. Similarly, the *tempered aspirations point* introduced by Balakrishnan *et al*. [2] is defined as $a(S, r) \notin S$. These aspirations are *tempered* since $r \geq d$ implies that aspirations will be lower compared to the aspirations derived from the ideal point (formally, $a(S, r) \leq a(S, d)$).

Let Σ^n be the family of all n-person bargaining problems with a reference point. A solution concept on Σ^n is a function ϕ that associates with each triple $(S, d, r) \in \Sigma^n$ a unique outcome $\phi(S, d, r) \in S$.

Now, the Gupta-Livne solution (GL) is formally defined as follows.

Definition 1. *For every* $(S, d, r) \in \Sigma^n$*,*

$$GL(S, d, r) = \lambda^* a(S, d) + (1 - \lambda^*) r$$

where $\lambda^* = \max\{\lambda \in [0, 1] \mid \lambda a(S, d) + (1 - \lambda) r \in S\}$.

GL proposes the maximum point of the feasible set on the line segment connecting the ideal point, $a(S, d)$, and the reference point, r (see Figure 1). This is equivalent to saying that GL chooses the maximum individually rational utility profile at which each agent's utility gain from his/her reference point has the same proportion to the utility difference between his/her ideal point and his/her reference point. Accordingly, the disagreement point does not have a *direct* influence on the bargaining outcome

5 For $x, y \in R^n$, the vector inequalities are given as: $x \geq y$, $x > y$, and $x \gg y$.

6 Note that reference points in these cooperative bargaining models are different in nature than the mental/cognitive reference points in [26–28].

proposed by GL. Yet, it has an indirect influence on the proposed outcome as it is used to derive agents' aspirations (*i.e.*, the ideal point).

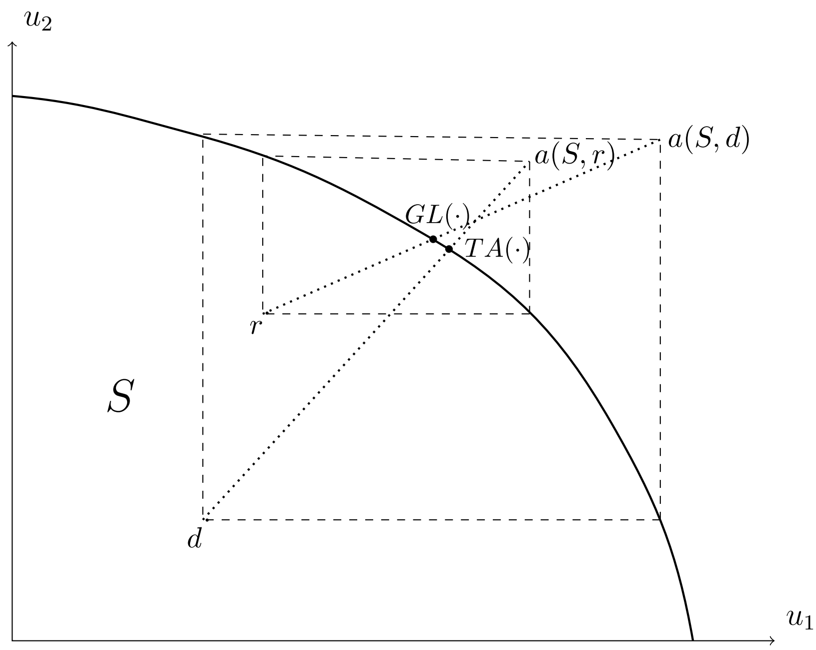

Figure 1. GL *vs*. TA.

The tempered aspirations solution (TA) is formally defined as follows.

Definition 2. *For every* $(S,d,r) \in \Sigma^n$,

$$TA(S,d,r) = \lambda^* a(S,r) + (1-\lambda^*)d$$

where $\lambda^* = \max\{\lambda \in [0,1] \mid \lambda a(S,r) + (1-\lambda)d \in S\}$.

TA proposes the maximum point of the feasible set on the line segment connecting the tempered aspirations point, $a(S,r)$, and the disagreement point, d (see Figure 1). This is equivalent to saying that TA chooses the maximum individually rational utility profile at which each agent's utility gain from his/her disagreement point has the same proportion to the utility difference between his/her tempered aspirations point and his/her disagreement point. Accordingly, the disagreement point has a *direct* influence on the bargaining outcome proposed by TA. This time, the reference point is used to derive agents' aspirations (*i.e.*, the tempered aspirations point).

As it is put by Balakrishnan *et al*. [2], the majority of the axioms used in the characterizations of these two solutions overlap. As a matter of fact, it is easy to see from the statements above that GL is *dual* to TA in that the roles played by the reference point and the disagreement point are switched (see [2], p. 145). More precisely, in GL (TA), the disagreement (reference) point is used to derive aspirations and the reference (disagreement) point is used as a benchmark for establishing the proportionality of payoffs at the solution. As mentioned above, this difference in their reliance on the disagreement point and the duality result make the Anbarcı *et al*. [3]'s framework a natural test bed for these two solutions.

3. The Model

In this section, we introduce a reference point into the model studied by Anbarcı *et al*. [3]. Other than the introduction of a reference point, all the model assumptions are kept identical to those in [3] to be able to compare the results in two papers.

There are two agents who have claims to T_0 units of a productive resource (say *land*, as in [3]). Agents can either divide the contested land according to some division rule they both find acceptable, or they can engage in an open conflict.

Each agent $i \in \{1, 2\}$ has T_i units of secure land and R_i units of secure human capital. Human capital can be used for guns G or productive labor L. For simplicity, it is assumed that one unit of human capital can produce one unit of gun or be converted into one unit of labor. Thus, each agent $i \in \{1, 2\}$ faces the following resource constraint:

$$R_i = L_i + G_i. \tag{1}$$

Labor and land are used to produce a final good. The production technology is described by $F^i \equiv F(T_i, L_i)$. We assume that $F(T_i, L_i)$ is twice continuously differentiable and that it is increasing and strictly concave in T_i and L_i. Moreover, F_L/F_T is assumed to be nondecreasing in T_i and nonincreasing in L_i, where a subscript G, L, or T denotes the partial derivative with respect to the corresponding variable.

A conflict between both agents, if exists, is resolved by a winner-take-all contest in which $p^1 \equiv p(G_1, G_2) = 1 - p^2$ represents the winning probability of agent 1. It is assumed that p is symmetric for every (G_1, G_2),[7] twice continuously differentiable, increasing and concave in G_1, and decreasing and convex in G_2. Given the winner-take-all contest, each agent's payoff function is the expected consumption of the final good he/she produces. This gives us the threat/disagreement point payoff for $i \in \{1, 2\}$ as

$$U^i(G_1, G_2) = p^i(G_1, G_2)F(T_i + T_0, R_i - G_i) + (1 - p^i(G_1, G_2))F(T_i, R_i - G_i).$$

For a given (G_1, G_2), the strict concavity of $F(\cdot)$ in T implies the following inequality for every $i \in \{1, 2\}$:

$$U^i(G_1, G_2) < F(T_i + p^i T_0, R_i - G_i).$$

When there is no conflict, that is, when agents *agree* to divide the land according to some division rule,

$$U^i(G_1, G_2) = F(T_i + \tilde{\alpha}^i T_0, L_i)$$

is the least payoff amount that agent i would accept; so $\tilde{\alpha}^i$ is the smallest share of T_0 that he/she would accept. We also define

$$\tilde{\tilde{\alpha}}^i = 1 - \tilde{\alpha}^j$$

[7] That is, $p(G, G') = 1 - p(G', G)$.

as the largest share of T_0 agent i would receive under the condition that agent $j \neq i$ gets an individually rational share. Therefore, the following expression gives the ideal point payoff for agent $i \in \{1, 2\}$:

$$W^i(G_1, G_2) = F(T_i + \tilde{\tilde{\alpha}}^i T_0, L_i).$$

Considering the solution concepts studied here, we assume an exogenously given reference point (in terms of payoffs) for agents. We describe the reference point as follows:

$$\mathcal{R}^i = F(T_i + \bar{\beta}^i T_0, L_i).$$

Here, $\bar{\beta}^i$ denotes the share of T_0 that yields to agent i his/her reference point. Accordingly, $\bar{\bar{\beta}}^i$ is the share agent i would receive under the condition that agent $j \neq i$ gets at least as much as his/her reference point. Hence,

$$\bar{\bar{\beta}}^i = 1 - \bar{\beta}^j.$$

Therefore, the following expression gives the tempered aspirations point payoff for agent $i \in \{1, 2\}$:

$$\mathcal{A}^i(G_1, G_2) = F(T_i + \bar{\bar{\beta}}^i T_0, L_i).$$

Considering that the reference point is given exogenously and that the disagreement point and the feasible set will be determined endogenously, there may occur instances where $\mathcal{R} \not\geq U$ or instances where $\mathcal{R} \notin S$. These cases are not covered by the two solution concepts studied here.[8] However, in order to have a well-defined game, payoffs should be assigned for these cases as well. Concerning this matter, we make the following assumptions: (i) if $\mathcal{R} \not\geq U$, then agents treat U as the reference point; and (ii) if $\mathcal{R} \notin S$, then agents receive their disagreement point payoffs.

These assumptions may appear technical, but they are also reasonable. The disagreement point is considered to provide a *hard* power (*i.e.*, it can be exercised unilaterally), whereas the reference point is a source of *soft* power (*i.e.*, it needs to be mutually acknowledged). Hence, any agent whose payoff from $\mathcal{R}$ is worse than his/her disagreement point payoff can block the use of $\mathcal{R}$ as a reference point, implying that a reference point that is not Pareto superior to the disagreement point would never be employed. For the latter assumption, we refer to the *intermediate agreement* (or starting point) interpretation of reference points offered by Gupta and Livne [1]. Along these lines, one can argue that if an intermediate agreement is not reached (or if there is no agreement on the starting point), a final agreement cannot be reached either. Accordingly, agents end up with their disagreement point payoffs.

It is good to note here that the cases mentioned above will not be decisive in the efficiency ranking of GL and TA. For the former case, the disagreement point is treated as the reference point which implies that the ideal point is equivalent to the tempered aspirations point. Hence, GL and TA coincide. Naturally, they lead to the same equilibrium and the same allocation. For the latter case, it is sufficient to recall that the disagreement point is not Pareto optimal. Hence, agents would prefer to choose their

[8] Balakrishnan *et al*. [29] extended TA to bargaining problems in which the reference point is not restricted to lie in the interior of the feasible set. We do not utilize this extended solution concept in the paper, since GL does not have such an extension.

strategies in such a way that the corresponding feasible set includes the reference point. As a result, the latter case will never be realized in any equilibrium. Keeping these in mind, the current problem turns out to be interesting only when $\mathcal{R} \geq U$ and $\mathcal{R} \in S$. Therefore, the following analysis will focus on this particular case.

From now on, for notational simplicity, we suppress the dependence of functions on (G_1, G_2) wherever it will not cause confusion. Each solution concept we consider is defined for a pre-determined number of guns and, consequently, for pre-determined winning probabilities, threat/disagreement point payoffs, ideal point payoffs, and tempered aspirations point payoffs. First, for given guns (G_1, G_2), let

$$V^1(\alpha) = F(T_1 + \alpha T_0, R_1 - G_1) \quad \text{and} \quad V^2(\alpha) = F(T_2 + (1-\alpha)T_0, R_2 - G_2)$$

represent a Pareto efficient division of T_0 where $\alpha \in [\bar{\beta}^1, \tilde{\tilde{\alpha}}^1] \cup [\tilde{\alpha}^1, \bar{\bar{\beta}}^1]$.

Let (G_1, G_2) be any combination of guns which uniquely define the threat point payoffs U^1 and U^2, the tempered aspirations point payoffs $\mathcal{A}^1$ and $\mathcal{A}^2$, and the Pareto efficient pairs $(V^1(\alpha), V^2(\alpha))$ for $\alpha \in [\tilde{\alpha}^1, \bar{\bar{\beta}}^1]$. Accordingly, TA satisfies the following equation:

$$\frac{V^2(\alpha_{TA}) - U^2}{V^1(\alpha_{TA}) - U^1} = \frac{\mathcal{A}^2 - V^2(\alpha_{TA})}{\mathcal{A}^1 - V^1(\alpha_{TA})}$$

where α_{TA} is the allocation proposed by TA. Similarly, let (G_1, G_2) be any combination of guns which uniquely define the ideal point payoffs W^1 and W^2, and the Pareto efficient pairs $(V^1(\alpha), V^2(\alpha))$ for $\alpha \in [\bar{\beta}^1, \tilde{\tilde{\alpha}}^1]$. Accordingly, GL satisfies the following equation:

$$\frac{V^2(\alpha_{GL}) - \mathcal{R}^2}{V^1(\alpha_{GL}) - \mathcal{R}^1} = \frac{W^2 - V^2(\alpha_{GL})}{W^1 - V^1(\alpha_{GL})}$$

where α_{GL} is the allocation proposed by GL. Letting $k \in \{TA, GL\}$, the payoff functions are

$$V_k^1(G_1, G_2) = F(T_1 + \alpha_k T_0, R_1 - G_1)$$
$$V_k^2(G_1, G_2) = F(T_2 + (1-\alpha_k)T_0, R_2 - G_2).$$

Given the solution concept that will be implemented, agents simultaneously choose the amount of endowments they invest in guns. This creates a trade-off between the investment in labor (directly increasing the production level) and the investment in guns (increasing the disagreement point payoffs and the share of land received, hence indirectly increasing the production level).

Among these two solution concepts, the one that leads to a higher level of production is considered to be *Pareto superior* to the other. The following proposition compares GL and TA in terms of Pareto efficiency under symmetry assumptions on secure land, human capital, and reference point payoffs. We focus on symmetric, pure strategy Nash equilibrium.[9]

[9] Since the model environment is completely symmetric, we think focusing on the symmetric equilibrium is reasonable. Moreover, note that the assumptions on functions guarantee that the equilibrium will be interior.

Proposition 1. *Assume that agents* 1 *and* 2 *have identical endowments of secure land, human capital, and reference point payoffs. Letting* G_k *and* V_k *denote the representative agent's (symmetric) equilibrium guns and payoffs under each bargaining solution* $k \in \{TA, GL\}$*, respectively, we have*

$$G_{TA} > G_{GL}$$
$$V_{TA} < V_{GL}$$

if and only if

$$\gamma_{GL}\left(W^1_{G_1} - W^2_{G_1}\right) < \gamma_{TA}\left(\mathcal{A}^1_{G_1} - \mathcal{A}^2_{G_1}\right) + (1-\gamma_{TA})\left(U^1_{G_1} - U^2_{G_1}\right)$$

where

$$\gamma_{TA} \equiv \frac{V^1_{TA} - U^1}{\mathcal{A}^1 - U^1} \qquad \gamma_{GL} \equiv \frac{V^1_{GL} - \mathcal{R}^1}{W^1 - \mathcal{R}^1} .$$

Proof. The proof technique is similar to that of Anbarcı *et al.* [3]. First, for simplicity and without loss of generality (by symmetry), we present arguments only for agent 1. Letting α_k denote the share of agent 1 under the bargaining solution $k \in \{TA, GL\}$, the derivative of V^1_k with respect to G_1 is

$$\frac{\partial V^1_k}{\partial G_1} = T_0\, F^1_T\, \frac{\partial \alpha_k}{\partial G_1} - F^1_L$$

where $k \in \{TA, GL\}$. Since the symmetry assumptions in [3] are preserved, and since $\mathcal{R}$ is symmetric, we have

$$\alpha_k = 1/2$$

for every $k \in \{TA, GL\}$. Moreover, agents' marginal products of land and labor will not differ across $k \in \{TA, GL\}$. Then, the comparison of the derivatives above is equivalent to the comparison of $\partial\alpha_k/\partial G_1$. This implies that the solution concept that gives a smaller derivative will Pareto dominate the other.

Now, we define the following:

$$\Theta \equiv \frac{\mathcal{A}^2 - V^2(\alpha)}{\mathcal{A}^1 - V^1(\alpha)} \qquad \Phi \equiv \frac{V^2(\alpha) - U^2}{V^1(\alpha) - U^1}$$

$$\Omega \equiv \frac{W^2 - V^2(\alpha)}{W^1 - V^1(\alpha)} \qquad \gamma \equiv \frac{V^2(\alpha) - \mathcal{R}^2}{V^1(\alpha) - \mathcal{R}^1}$$

Considering the solution concepts mentioned above, we have

$$\Theta\left(\alpha_{TA}, G_1, G_2\right) = \Phi\left(\alpha_{TA}, G_1, G_2\right)$$

$$\Omega\left(\alpha_{GL}, G_1, G_2\right) = \gamma\left(\alpha_{GL}, G_1, G_2\right)$$

Therefore,

$$\frac{\partial \alpha_{TA}}{\partial G_1} = \gamma_{TA}\frac{\Phi(\mathcal{A}^1_{G_1} + F^1_L) - \mathcal{A}^2_{G_1}}{T_0(\Phi F^1_T + F^2_T)} + (1-\gamma_{TA})\frac{\Phi(U^1_{G_1} + F^1_L) - U^2_{G_1}}{T_0(\Phi F^1_T + F^2_T)}$$

and

$$\frac{\partial\alpha_{GL}}{\partial G_1} = \gamma_{GL}\frac{\Omega(W^1_{G_1}+F^1_L)-W^2_{G_1}}{T_0(\Omega F^1_T+F^2_T)} + (1-\gamma_{GL})\frac{\Omega(\mathcal{R}^1_{G_1}+F^1_L)-\mathcal{R}^2_{G_1}}{T_0(\Omega F^1_T+F^2_T)}$$

Under the symmetric case, we also have $G_1 = G_2$ and $\Phi = \Omega = 1$. Recalling that we assume an exogenously given reference point, we have $\mathcal{R}^i_{G_j} = 0$ for every $i, j \in \{1, 2\}$. It then follows that $\mathcal{R}^1_{G_1} - \mathcal{R}^2_{G_1} = 0$. Thus, if

$$\begin{aligned}\gamma_{TA}\frac{(\mathcal{A}^1_{G_1}+F^1_L)-\mathcal{A}^2_{G_1}}{T_0(F^1_T+F^2_T)} + (1-\gamma_{TA})\frac{(U^1_{G_1}+F^1_L)-U^2_{G_1}}{T_0(F^1_T+F^2_T)} &> \\ \gamma_{GL}\frac{(W^1_{G_1}+F^1_L)-W^2_{G_1}}{T_0(F^1_T+F^2_T)} &+ (1-\gamma_{GL})\frac{F^1_L}{T_0(F^1_T+F^2_T)};\end{aligned}$$

that is, if

$$\gamma_{GL}(W^1_{G_1} - W^2_{G_1}) < \gamma_{TA}(\mathcal{A}^1_{G_1} - \mathcal{A}^2_{G_1}) + (1-\gamma_{TA})(U^1_{G_1} - U^2_{G_1}),$$

then we have

$$\frac{\partial\alpha_{TA}}{\partial G_1} > \frac{\partial\alpha_{GL}}{\partial G_1},$$

which implies that

$$G_{TA} > G_{GL} \quad \text{and} \quad V_{TA} < V_{GL} \ .$$

Conversely, if

$$\gamma_{GL}(W^1_{G_1} - W^2_{G_1}) > \gamma_{TA}(\mathcal{A}^1_{G_1} - \mathcal{A}^2_{G_1}) + (1-\gamma_{TA})(U^1_{G_1} - U^2_{G_1}),$$

then we have

$$\frac{\partial\alpha_{TA}}{\partial G_1} < \frac{\partial\alpha_{GL}}{\partial G_1},$$

which implies that

$$G_{TA} < G_{GL} \quad \text{and} \quad V_{TA} > V_{GL} \ . \quad \square$$

As it can be seen from the proof, factors other than the weights placed on the disagreement point play a role in our comparison. These are due to the introduction of a reference point into the economic environment.

At first sight, it may sound very natural—and hence not surprising—that when the reference point is introduced to the model, the efficiency ranking does not only depend on the weights placed on the disagreement point but also the weights placed on the reference point. This first impression is somewhat misleading. The reason is that it is the incentive to invest in the disagreement point which creates a trade-off; however, there is no such trade-off concerning the reference point since there is no investment in the reference point.

In the model, investments in the disagreement point (*i.e.*, guns) influence the utility possibilities frontier. Hence, despite the fact that the reference point remains constant, the tempered aspirations point (derived from the reference point and the utility possibilities frontier) is indirectly affected by these investments. In addition to the term $U^1_{G_1} - U^2_{G_1}$ (directly affected by the investments) and the term $W^1_{G_1} - W^2_{G_1}$ (both directly and indirectly affected), which are the critical terms leading to the main result in [3], the efficiency comparison of GL and TA depends on the term $\mathcal{A}^1_{G_1} - \mathcal{A}^2_{G_1}$ (only indirectly affected). Therefore, the relative magnitudes of direct and indirect effects influence the efficiency ranking in our model. These effects are summarized in Table 1. While reading the table, remember that direct effects depend on the marginal return from investing in guns on the probability of winning the contest (*i.e.*, p_{G_1}) and the marginal productivity of labor (*i.e.*, F_L), whereas indirect effects depend on the marginal productivities of labor or land (*i.e.*, F_L or F_T).

Table 1. Direct and Indirect Effects.

	Relevant Derivatives	$U^1_{G_1}$	$U^2_{G_1}$	$W^1_{G_1}$	$W^2_{G_1}$	$\mathcal{A}^1_{G_1}$	$\mathcal{A}^2_{G_1}$
Direct Effect †	p_{G_1} and F_L	$+/-$	$-$	$+$	$-/+$	N.A.	N.A.
Indirect Effect ‡	F_L or F_T *	N.A.	N.A.	$-$	$-$	$-$	$-$

† Through investment in the threat point; ‡ Through the utility possibilities frontier; * F_L is effective for agent 1 and F_T is effective for agent 2.

4. Concluding Remarks

We enrich the economic environment introduced in Anbarcı *et al.* [3] by incorporating a reference point into the bargaining problem. In our bilateral bargaining model, the disagreement point, the ideal point, the tempered aspirations point, as well as the utility possibilities frontier are all endogenously determined in a simultaneous-form strategic game. In this strategic game, agents decide on their investment in threat/disagreement point payoffs, which also influence their utility possibilities set. There exists a trade-off in this decision: larger investment in the threat point leads to inferior utility possibilities set.

Anbarcı *et al.* [3] compared three solution concepts in terms of Pareto efficiency: (i) the split-the-surplus solution; (ii) the Kalai-Smorodinsky solution; and (iii) the equal sacrifice solution. These solutions differ on the basis of the weights they place on the disagreement point, and the authors showed that these weights directly imply an efficiency ranking. Their intuition is simple: A solution that places a greater weight on the disagreement point will be Pareto inferior compared to the one that places less weight since it gives stronger incentives to invest in guns (*i.e.*, the disagreement outcome).

We compare two solution concepts, the Gupta-Livne solution and the tempered aspirations solution, both of which incorporate a reference point outcome into the simple model of bargaining. These concepts differ in the weights they place on the disagreement point. In particular, the tempered aspirations solution directly depends on the disagreement point, whereas the Gupta-Livne solution depends only indirectly on the disagreement point. Interestingly, we show that the intuition mentioned above is not necessarily valid for our comparison. If the same intuition was valid, this would have implied that the Gupta-Livne solution is Pareto superior to the tempered aspirations solution. However, we show that the efficiency ranking of the two solutions we study depends on the following factors: (i) marginal return from investing in guns on the probability of winning the contest; (ii) marginal productivities of land and labor; (iii) the weights assigned to the disagreement point by the two solutions; and (iv) the weights assigned to the reference point point by the two solutions. The main reason for this result is the additional degree of freedom brought by the introduction of a reference point and how the reference point is employed by the two solution concepts (*i.e.*, directly by GL and indirectly by TA). The solution concepts we compare do not only differ in terms of the weights they place on the disagreement point but also in terms of the weights they place on the reference point. In addition to this, the tempered aspirations point employed by TA is endogenously determined whereas the reference point employed by GL is exogenously given. Combination of these factors make our comparison depend on—essentially—all factors influencing an agent's optimal investment in guns: a more instructive but less straightforward exercise compared to that in [3].

As we mentioned above, this is a *first* attempt to compare two bargaining solutions which use reference points. Our comparison is based on Pareto efficiency, which is surely an important criterion in comparing solution concepts. Future research may compare these solutions on the basis of different criteria (e.g., axiomatic, strategic, or experimental). We build on the model proposed in [3] as (i) it is a simple and natural model to perform a Pareto efficiency comparison and (ii) it allows us to incorporate the reference point in a reasonable fashion. Obviously, future research may design new/alternative economic environments for this purpose. Finally, we take the reference point as exogenously given in order to keep the model tractable and the result transparent.[10] A natural agenda for future research would be to perform various comparisons of solution concepts in models with endogenous reference points.

Author Contributions

All authors contributed equally to this article.

Conflicts of Interest

The authors declare no conflict of interest.

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[10] Shalev [27] is the first to study endogenous reference points in cooperative bargaining. However, in [27], the reference point is a *cognitive* one. It only affects the feasible set of the bargaining problem and does not have a direct influence on the bargaining outcome. In that sense, it is different than the reference point described by Gupta and Livne [1] and Balakrishnan *et al*. [2]. On the other hand, to the best of our knowledge, Karagözoğlu and Keskin [30] is the first study endogenizing the reference point in the Gupta-Livne framework. These authors introduced a pre-bargaining game through which agents' reference points are endogenously determined. The cognitive meaning of the reference point is preserved as in [27], but now the reference point also has a direct influence on the bargaining outcome.

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