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EDUCATE OR ADJUDICATE? SOCIOECONOMIC HETEROGENEITY AND WELFARE

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I present a formal framework to explore the welfare and distributional effects of a government's optimal choice over two types of public spending in a closed economy: domestic security (DS) and investment in social capital (SC). Production is characterized as a function of social and physical capital stocks that both vary across the regions. DS stands for total factor productivity, while SC stands for human capital and civic cooperativeness combined. SC accumulates via public spending on universal primary education, cultural, and civic events and such, and is exposed to regional spillover effects. Numerical simulations of the static solution of the government's welfare maximization problem reveal that the optimal rate of spending on SC (m^*) is negatively related with the income share of physical capital, SC spillovers and fiscal decentralization. Simulations also show that SC homogeneity is positively associated with both the level and equitability of aggregate income. The maximum attainable levels of income, welfare and social cohesion and the most equitable incomes are all observed to realize at some intermediate range of m^* values. In case DS augments SC, however, social cohesion improves and welfare declines monotonously in m^*.

Keywords: Social Capital; Economic development; Income equality; Domestic security spending

JEL Codes: E02; E6; H11; H52; I24; I25; I31; Z18

1. INTRODUCTION

> … we have money to give military equipment to police forces, when we don't have money for training and money for public education. (Rev. Al Sharpton at the funeral of James Brown in Ferguson, USA, Aug. 2014)

There is hardly any country that is homogenous in every dimension; regardless of its size, every country portrays some degree of heterogeneity of some sort.[1] Heterogeneity is likely to lead to inefficiency in decision-making and thus retards growth; it may also lead to political conflict that hinders economic development. Therefore, some measure of socioeconomic homogeneity is considered essential for sustainable development. This argument is supported empirically by Knack and Keefer ([1997](#)) and Zak and Knack ([2001](#)), who show that economic performance, formal institutional quality[2], and socioeconomic homogeneity

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[1]Figure 1 in Appendix 1 shows that ethnic fractionalization is not related with land area. Incidentally, of the sample countries, the Republic of Korea seems to be the only country which has no ethnic fractionalization.
[2]measured by protection of property and contract rights.

are positively associated with *social capital* (SC).[3] The term SC has been explained in reference to social cohesion and personal investment in the community (Hanifan 1916), or as encompassing networks of associations and norms that facilitate collective action (Putnam 1995b). As such, SC can be viewed as inclusive informal institutions that facilitate coordination and support among people.

In a society that is heterogeneous in both social and physical capital, public spending on both SC and domestic security (DS) can help achieve socioeconomic cohesion and order, and thus facilitate economic development. This study contributes to the literature in two novel ways: (i) it presents an original formal framework to analyze the government's optimal decision between spending on SC and DS[4] in a heterogeneous society; and (ii) it investigates the outcomes of this decision with regard to the level and the distribution of income, social cohesion, and welfare.

1.1. Heterogeneity and Conflict

Given budget limitations, the greater the differences in social preferences, the greater is the difficulty in taking decisions regarding the type and level of public good provision, as well as the means of its financing. Socioeconomic heterogeneity thus reduces the allocative and the technical efficiency (quality) of public spending, and is therefore often associated with economic and political instability (see e.g. Alesina, Baqir, and Easterly 1999; Alesina and Spolaore 1997; Easterly and Levine 1997; Kujis 2000; Sachs and Warner 1997).[5] Barro (1991) and Montalvoa and Reynal-Querol (2005), among others, have also argued that ethnic heterogeneity or polarization leads to political conflict that increases rent-seeking and reduces investment. Rodrik (1999) supports this view by arguing that even the effect of external shocks is wider in case of social divisions and distributional conflicts.

Ethnic, religious or other forms of social heterogeneity, especially when associated with differential degrees of access to economic opportunities and political means, are likely to result in an increased potential of political conflict.[6] A case in point is the continued political conflict in many former colonies where economic duality, as well as ethnic cleavages, has especially been prevalent (see e.g. Goudie and Neyapti 1999).[7] Horowitz (1985) argue that ethnic diversity and political conflict are related non-monotonically; in a similar vein, Montalvoa and Reynal-Querol (2005) show that it is polarization, rather than fractionalization that matters for the potential of conflict and therefore is more relevant for developmental outcomes.[8] Social polarization occurs when strong *bonding* within groups of a society is

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[3]The discussion of the notion of *SC* dates back to early sociologists: primarily to Tocqueville, Durkheim and Weber.
[4]Examples of SCI and DS are education and adjudication, the latter of which refers to legal and judicial system and enforcement characteristics.
[5]In an international context, Solomon, Seiglie, and Xiang (2007) argue that FDI ties reduce international conflict. Douch and Solomon (2014) also argue that there are significant tradeoffs between military spending, which is found complementary to foreign aid, and non-military spending in middle power nations that face international threat. I refrain, in the interest of keeping the scope clear, from international considerations in this paper.
[6]While several studies (see e.g. Collier and Hoeffler 2004) find no significant correlation between conflict and inequality, Huber and Mayoral (2014) point out that within-group inequality has significant positive effects on conflict.
[7]Lange (2004) notes that dispersed power structure under colonialism reduces governance quality and leads to increased potential of political instability. Moreover, minority direct ruling under colonialism is observed to increase income inequality, as compared to the case where settlers constituted the majority; high inequality remained in those countries even after independence (Angeles 2007).
[8]Figures 1 and 2 in Appendix 1 show that neither land size nor income level show a direct association with ethnic heterogeneity.

accompanied by weak *bridging* relations among them, which implies a weak aggregate SC stock.[9] At the aggregate level, SC manifests itself as ability of consensus building via strong bridging relations. Bluedorn (2001), for example, demonstrates that institutions of governance and democracy ameliorate the negative effects of ethnic diversity.

Olson's (1965, 1982) collective action theory sheds light on how economic inefficiencies result from narrow interest group dynamics that prevail in societies with heterogeneous interests. Similarly, to explain differential growth processes, Acemoglu and Robinson (2012) use the term extractive institutions to describe small groups of elites that redistribute resources towards themselves. Such institutions may be sustained until the technological frontier is pushed via creative destruction; till then, economic success relies mainly on the central delivery of essential public good, which establishes inclusivity to some degree. Olson (1982) argues that special interests may spring from encompassing organizations over time. Acemoglu and Robinson (2012) refer to this as devolution of *inclusive institutions* into extractive ones. Rent-seeking and inefficient institutions that lead to increasing inequality overtime are not phenomena that are exclusive to non-market or non-democratic economies. Neyapti (2013) and Neyapti and Arasil (2016) present a formal framework where the positive linkage between institutional and economic development hinges upon the dynamics between formal institutional reform and socioeconomic heterogeneity.

1.2. SC and DS

Given that heterogeneous interests are potential sources of socio-political conflict and thus may affect developmental prospects of a country negatively, the current paper analyzes optimal public spending on SC and DS to help eliminate these negative effects. Spending on DS, which covers DS forces as well as legal organizational structures and judicial personnel, aims to enforce social order and contributes to total factor productivity. Investment in SC, on the other hand, includes, but is more encompassing than, improvements in human capital; it also, and especially, incorporates efforts to improve trust and capacity of forming civic cooperations.

A form of SC investment is high-quality universal (particularly primary) education, which is non-rivalrous and non-excludable, that endows citizens of all socioeconomic status with the basic knowledge of natural sciences, civics, and rights-consciousness. Notwithstanding the differences in the ability and cognitive capacity of students, such an education system can help generate socially useful ideas, plant and expand civic values in the society, and facilitate the formation of and participation to social and professional networks.[10] Inclusive social norms and formal institutions, in turn, contribute to the ability of citizens of different socioeconomic status, ethnic or cultural backgrounds, and political views to access to the means of expression and markets freely, to form and maintain contracts and to tolerate differences. Facilitating collaborative platforms that increase social cohesion by bridging

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[9]La Porta et al. (1997) argue that trust is lower in hierarchical religions than others.
[10]Putnam (1993) and Heyneman (2000) argue for the positive association between public education and SC and social cohesion; Knack and Keefer (1997) finds positive association between trust and secondary education, and a negative one between trust and primary education. Huang, van den Brink, and Groot (2009) show empirically that while the level of education is positively associated with trust, its relationship with social participation is insignificant. The authors also argue that education has a greater effect on SC in the US than the rest of the sample, explained by the 'melting pot' approach. Imandoust (2011) argues that open distance education is key to building SC in Iran.

the gap between different social groups thus improves the productivity of aggregate human capital and contributes to economic development and growth.[11] Due to increasing online communication technologies that lead people of similar views and backgrounds to get stuck in closed circuits, generating all-encompassing platforms is a growing challenge that is likely to be achieved best by universal public education that emphasizes civic-consciousness. Building on Putnam (1993) and Heyneman (2000), the mega-analysis of Huang, van den Brink, and Groot (2009) present strong and robust evidence on the positive relative effect of education on SC, measured both by trust and civic cooperation.[12]

It is probable, on the other hand, that low quality and/or highly differentiated education systems harm social cohesion.[13] Many countries exhibit a trend towards increasing privatized education and, similarly, increasing political polarization. Religious schools that promote distinct life styles and ideology in heterogenous societies, for example, may perform as a signaling device for employment in bureaucracy, politics, and business. Such identification via exclusion may lead to lack of social cohesion and political conflict over time. Helliwell and Putnam (2007), for example, argue that education systems that are generally used as signaling device for sorting do not associate significantly with large-scale civic participation.[14]

The theoretical nature of the current study permits the assumption of a pure public good version of an education system, however, one that carries the ideal qualities listed above. The study also considers public education spending as only a form of SC investment, however, while keeping the notion of investment in SC general so as to include the funding of scientific, artistic, cultural, and sportive platforms that facilitate civic cooperation and social cohesion.[15]

Besides the potential welfare benefits of SC, increasing complexity of social lives, economies and financial markets necessitate institutional mechanisms to reduce the associated transactions costs. These include increased regulation, supervision, and enforcement, and hence increased spending on auditing, legal personnel, police forces, and related infrastructure: DS, in short. DS helps to achieve social order via wither adjudication or coercion. High stock of SC may reduce the potential of conflict and thus the need for spending on DS, at least the spending on coercive measures. In a non-democratic society where trust and tolerance is lacking, however, settlement of conflicts, which may span from social disagreements to civil war, may necessitate hard power. It is possible that the use of

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[11]Foa (2011) discusses the positive linkage between growth and social cohesion. Bjørnskov and Méon (2013) present evidence that trust increases income both directly and via its positive effects on education and political and economic organization.

[12]Huang, van den Brink, and Groot (2009) consider 65 empirical studies that measure SC by either trust or civic participation, or both. The authors show that the main source of variation across the earlier findings is the problem of endogeneity between the civic participation and education. Correcting for endogeneity, they report positive significant relationship between these variables.

[13]Hanushek and Woessmann (2015) demonstrate that it is not school attainment that is significant in explaining differences in the growth performance across countries, but the quality of education or the *knowledge capital*. Imandoust (2011) argues that open distance education is key to building SC in Iran.

[14]Putnam's (1995a, 1995b) paradox of declining SC despite increasing *average* level of schooling in the US during the past decades are attributed to factors such as women's inclusion in the labor force and electronic revolution that influence all levels of education similarly, as well as diminishing marginal effect of education on SC.

[15]These platforms are different from the horizontal associations that Olson (1982) describes as leading to special interest group formation. Knack and Keefer (1997) provide empirical evidence that contrasts with Putnam (1993) and show that memberships to civic associations are unrelated with or harmful for economic performance. They also argue that the greater is trust the less resources are required for securing property rights and achieving social cohesion; this is the channel that trust enhances efficiency, investor confidence, and economic growth.

hard power to repress opposition and to consolidate the status quo itself generates problems rather than solving them. In addition, spending on DS may be due to external threats.[16]

The empirical analyses of the effects of DS spending are very rare at due to data constraints.[17] In tandem, the literature on the relationship between growth and defense spending is quite inconclusive.[18] The ambiguity arises since, on the one hand, defense spending may have positive effects on investment and growth due via improvement of law enforcement and property rights protection, as well as generating technology spillovers, but it may, on the other hand, also crowd-out private investment.[19] In the interest of presenting a simple model to be able address the basic question of the paper, which is the optimal choice between DS and SC, the current study focuses on a closed economy where a benevolent government's spending on DS has positive effects on social order.

1.3. Contribution of this Paper

Given the above background, public spending on SC and DS can be viewed to be both complementary and, given a limited budget, substitutable. Increasing social cohesion resulting from SC accumulation is likely to lead to a decreasing need for DS, but also a decreasing need for SC investment. The more heterogeneous the society is, on the other hand, the more spending on both DS and SC is needed, the first to cope with potential inefficiencies arising from heterogeneity and the second to eradicate them.[20] It is therefore not clear how a welfare maximizing benevolent government allocates its resources between the two, while it appears that this allocation depends on the level of development.

I consider a society that is heterogeneous in regional incomes due to different levels of social and physical capital stocks, where the cross-regional effects of SC spillovers are not necessarily reciprocal. Heterogeneity leads to an increase in the SC stock of a region in case the sum of spillovers is positive, and to a decrease in the SC stock in case it is negative. While investment in SC benefits the entire society, diminishing returns to SC, and/or their spillover effects, may lead some regions to benefit from it more than others. The government takes these spillovers into account to maximize the welfare by determining the relative provision of two alternative forms of public good spending: SC and DS. Regional heterogeneity in SC thus becomes endogenous to the government's policy decision.

Because of the nonlinearities that are essential to the model's framework, an analytical solution of the model or even a numerical examination of the long-run equilibrium is not

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[16]A review of defensive and preemptive counter-terrorism studies is provided by Bandyopadhyay and Sandler (2011) and Sandler (2015).
[17]The relationship between citizens' education level and preference for security spending appear ambiguous in a study of US survey (Wong 2010). The US Homeland Security Research shows a great deal of variation in the income share of national security spending across the countries: http://homelandsecurityresearch.com/2008/11/national-security-spending-outlook-in-20-countries-2009-2018/. However, data is too scant for a sensible empirical analysis.
[18]In an international context, Shieh, Lai, and Chang (2002), argue that the relationship between defense spending and growth is nonlinear and optimal defense spending that maximizes economic growth is smaller than the level that maximizes welfare. While that model introduces military spending into both the utility and the production functions, the current model only focuses on the productivity effects of security spending. Similar to Knight, Loayza, and Villanueva (1996), DS is viewed to be positively related with total factor productivity.
[19]F-de-Córdoba and Torres (2016) develop a theoretical framework to investigate the optimal level of military spending. Using defense spending both in the production and the utility functions, the authors analyze the response of military spending to technology and foreign threat shocks for the US economy and report that military spending is complementary to private consumption and is positively associated with the level of income.
[20]Based on 212 countries, it is observed that education spending (as percent of GDP) is correlated negatively with the size of the armed personnel (as percent of total labor) during the past decade, although the correlation is small: -0.10 (source: World Development Indicators).

possible. The set-up of the model is therefore static and presents a descriptive framework, rather than a full-fledged theoretical model. Comparative statics based on the model's static solution indicate that the welfare-optimizing share of SC spending in total public spending decreases in the share of physical capital and the rate of spillovers. It is also observed that the optimal ratio of SC spending that leads to the highest attainable levels of welfare, aggregate income, and most equitable income distribution is close to but less than 0.5.[21]

Two extensions of the benchmark model are presented. The first one demonstrates that the optimal share of SC spending is associated negatively with the rate of fiscal decentralization. That the maximum attainable level of income and welfare are observed to decline in the presence of local spending on SC supports the decentralization theorem, which states that decentralization is not necessarily welfare improving when there are spillovers, which arise from SC differentials in the current model. In the second extension, the production function is modified in such a way that DS augments SC, which is a case that is argued to be more relevant for advanced economies than others. The main observations of the paper remain intact in both extensions, except that in the latter one welfare is observed to decrease strictly in the share of SC.

The rest of the paper is organized as follows. Section 2 presents the model and describes the solution procedure. Section 3 provides the implications of the model and the simulation results. Section 4 concludes.

2. THE MODEL

Consider a country where heterogeneity across its regions arises from differential initial levels of social and physical capital stocks, in addition to the extent that each region is exposed to SC spillovers. Given predetermined level of initial regional incomes (Y_i) and a common income tax rate (τ), initial levels of consumption and physical capital investment in each region are also predetermined; and physical capital stocks follow the usual accumulation rule.

The government is the only optimizing agent and is assumed to be around for two periods: t and $t + 1$.[22] Given the initial (period $t − 1$) stocks of social and physical capital that determine the regional income levels, the government maximizes total welfare by allocating the tax revenue optimally between SC investment and DS in period t. Spending on SC (denoted by) augments each region's SC stock by the same amount, and spending on DS (denoted by G^{DS}) determines the level of total factor productivity in all the regions at time $t + 1$.[23] While individual regions contribute partially to the financing of pure public good via income taxes collected in their regions, they each receive the entire amount of G, where $G = G^{DS} + G^{SC}$.

Formally, the model is as follows. Consumption in region i is given by:

$$C_{i,t} = (1 − \tau)(1 − s_{i,t})Y_{i,t} \qquad (1)$$

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[21]Numerical simulations are based on two regions; hence homogeneity is measured by *relative SC*: A_1/A_2.

[22]One may think of period t as the period of incumbency of a given government.

[23]I later relax this assumption and investigate the case of SC-augmenting DS.

where $i = 1 \ldots n$ stands for n distinct regions, and s_i is the exogenously given saving rate.[24] The part of the disposable income that is not consumed is saved, and savings are assumed to be equal to investment (I):

$$I_{i,t} = s_{i,t}(1 - \tau)Y_{i,t} \qquad (2)$$

From the spending side, Y_{it} consists of local private consumption and investment, and the government spending is financed through taxation of the regional incomes: $G_t = \tau \sum_i Y_{i,t}$,

which states that the government budget balances in all periods. Since G_t is pure public good, locality i receives the transfer given by:

$$\tau \sum_i Y_{i,t} - \tau Y_i. \qquad (3)$$

That is, each region receives transfers by the amount of G_t minus its own tax payments.[25]

In this, the goods market equilibrium condition for any region i is given by:

$$Y_{i,t} = C_{i,t} + I_{i,t} + G_t. \qquad (4)$$

The tax revenue ($G_1 = \tau \sum_i Y_{i,t}$) is allocated between G_t^{SC} and G_t^{DS} as follows:

$$G_t^{SC} = m_t G_t \quad \text{and} \quad G_t^{DS} = (1 - m_t)G_t, \qquad (5)$$

where $m_t \in [0,1]$ stands for the proportion of G_t invested in SC, namely such spending as the promotion of wide-spread basic scientific education and facilitation of social platforms for cultural activities such as arts and sports. $(1 - m_t)$, on the other hand, is the portion of tax revenue that is spent on DS and law enforcement, or *adjudication*. Similar to the effect of private investment on physical capital accumulation, G_t^{SC} augments SC. Following the convention in the literature, G^{DS} is considered as a flow variable, however (see e.g. F-de-Córdoba and Torres 2016).[26]

The two factors of production, *social* and *physical capital* stocks, are denoted by A_i and K_i, respectively, for the region i. The accumulation rules are given by:

$$A_{i,t} = A_{i,t-1}\left(1 + f_{i,t-1}\right) + G_t^{SC} \qquad (6)$$

$$K_{i,t} = K_{i,t-1}(1 - \delta) + I_{i,t-1} \qquad (7)$$

where δ stands for the rate of depreciation of the initial K_i. The accumulation rule in Equation (6) indicates that the prevailing level of region-specific SC ($A_{i,t-1}$) is carried over to period t with the addition of the spillover effects from other regions ($f_{i,t-1}$) received in period ($t - 1$). The regional SC spillovers are expressed formally follows[27]:

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[24]Given the complexity of the model, due to the G^{SC} accumulation of two types of capital over time, as shown below, a joint solution of the household's and the government's problem could not be obtained.

[25]Transfers for all the regions are positive when $Y_{i\backslash t} > 0 \quad \forall i, t..$

[26]While military equipment accumulation can be considered investment, depreciation in defense sector is viewed very high.

[27]It is this via these spillovers that this paper addresses the value of SC as the flow of benefits (see e.g. Imandoust 2011).

$$f_{i,t} = \sum_j h_i(A_{j,t} - A_{i,t}), \text{ where } j = 1....n \, (j \neq i). \tag{8}$$

$h_i \in [0,1]$ is the region specific exposure factor, or a measure of cultural immersion that may result from labor mobility, migration or tourism across regions. Regional spillovers (f_i's) can be negative or positive depending on the relative sizes of A_i and A_j's. In case the regions are not exposed to each other ($h_i = 0$), or when there are no spillover effects (say, when $A_i = A_j$), f_i is zero; otherwise, it increases in h_i and the (negative) distance form other regions' SC stocks. Given this, A_i in Equation (6) stands for the effective units of labor, incorporating the effects of the level of education and thus measures the *human capital* in region *i*-combined with the level of trust and cooperation that contributes to the productivity of human capital.[28] Because G_t^{SC} is a public good, its marginal contribution is greater in regions with lower initial A_i.

Social cohesion, which is denoted by F, is measured by the sum of regional SC stocks and their cross-regional spillovers (see Equation (9)). Given the nature of spillovers that are not necessarily reciprocal to each other, this formulation indicates that positive (negative) net spillovers increases (decreases) social cohesion by adding to aggregate SC:

$$F_t = \sum_i \left(A_i + f_{i,t}\right), \text{ where } , \{f_{i,t}, F\} \in R. \tag{9}$$

(i.e. $\sum_i A_i$). A small F indicates lack of social cohesion, or conflict.[29]

The production function is assumed to exhibit constant returns to scale in each region[30]:

$$Y_{i,t+1} = G_t^{DS} K_{i,t}^{\alpha} A_{i,t}^{1-\alpha} \tag{10}$$

where G_t^{DS} accounts for total factor productivity and α ($0 \leq \alpha \leq 1$) stands for the income share of capital, which indicates diminishing returns to both physical and SC.[31] Output at time ($t + 1$) is produced using the stocks of physical and SC accumulated in period t. It is also assumed that the total factor productivity is determined by the last period's spending on adjudication.

Given Y_{t-1}, the government chooses m_t in period t in order to maximize the social welfare, which is the sum of regional utilities. Using Equations (1) and (10), and assuming a logarithmic form of utility function, the government's problem can thus be written for period t as[32]:

$$\text{Max}_{m_t} \sum_i \log(C_{i,t+1}) = \sum_i \log\left((1 - s_{i,t+1})(1 - \tau)G_t^{DS} K_{i,t}^{\alpha} A_{i,t}^{1-\alpha}\right) \tag{11}$$

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[28]The current model assumes human capital to be identical across the regions, which is constant and equal to 1 for simplicity.

[29]In the simulations below, $|\sum_i f_{i,t}| \leq \sum_i A_i$ holds since $h_i \in [0, 1]$ for all i, which implies that $F \geq 0$.

[30]For simplicity, income shares of physical and SC (α and $1 - \alpha$, respectively) are assumed to be identical across the regions.

[31]Alternatively, G_t^{DS} may be considered to augment the SC stock, in which case it would be modeled as SC-augmenting spending, leading to increasing returns to scale. Section 3.1.2 investigates the implications of this formulation.

[32]No explicit solution to the problem can be obtained when the government chooses τ and m simultaneously to maximize welfare.

TABLE I Ranges of Parameters and Initial Values (Denoted by Subscript 0) Used in Numerical Simulations, Where $i = 1, 2$

α	τ	h_i	s_i	$A_{i,0}$	$K_{i,0}$	G_0^{DS}
[0.1–0.4]	[0.1–0.7]	[0.0–1.0]	[0.1–1.0]	[10–100]	[10–100]	[10–100]

subject to the constraints given by Equations (5) through (8). The non-constrained optimization problem hence becomes:

$$\text{Max}_{m_t} \sum_i \log\left[(1 - s_{i,t+1})(1 - \tau)(1 - m_t)\tau Y_t (K_{i,t-1}(1 - \delta) + I_{i,t-1})^\alpha (A_{i,t-1}(1 + f_{i,t-1}) + m_t \tau Y_t)^{1-\alpha}\right]$$

where $f_{i,t-1}$ is equal to: $\sum_j h_i(A_{j,t-1} - A_{i,t-1})$ according to Equation (6). Given the highly nonlinear nature of the model, indicated by Equations (5)–(9), the problem can only be solved for a given period t.[33]

Lemma: The first-order condition of the problem is:

$$m_t^* - 1 + \frac{n}{(1 - \alpha)\tau Y_t \sum_i (1/(A_{i,t-1}(1 + f_{i,t-1}) + m_t^*\tau Y_t))} = 0 \tag{12}$$

Proof: Appendix 2.

Hence, optimal m_t (m_t^*) depends on the parameters $\{\alpha, h_i, s_i, \tau\}$ and the initial values $\{A_{i,t-1}, K_{i,t-1}, G_{t-1}^{DS}\}$. Assuming $n = 2$ for the tractability of the model, the solution yields two distinct roots of m_t^*. Because only one of these roots meet the feasibility criterion of $0 \le m^* \le 1$, the analysis below uses that as the global maximum.[34]

To recap the timing of the events, the government observes the initial values and optimizes its choice at period t, and its incumbency ends at the end of period $t + 1$.[35] The solution procedure of the model is thus characterized as follows.

(1) Given a set of $\{A_{i,t-1}, K_{i,t-1}, G_{t-1}^{DS}\}$ and α, $Y_{i,t}$ is found.
(2) Given $A_{i \cdot t-1}$ and $h_{i \cdot t-1}$, $f_{i,t-1}$ is found.
(3) The government chooses m_t^* that allocates G_t between G_t^{SC} and G_t^{DS} so as to maximize social welfare in period $t + 1$.
(4) The set of variables $\{A_{i,t}, K_{i,t}, Y_{i,t+1}\}$ is determined.

Table I reports the feasible ranges of parameters and the set of initial values based on which the numerical simulations of the model solution are carried out using the Matlab Program.

Imposing the following additional feasibility constraints: $0 \le m^* \le 1$; $C_i > 0$; $Y_i > 0$; and $A_i > 0$, the simulations yield 901,152 data points.

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[33]This is a reasonable approach given the short-sighted nature of most democratically elected governments.

[34]The roots are obtained using the Matlab program; data generated with one of the roots only match the conditions that $0 \le m^* \le 1$.

[35]A potential extension of the model involves reelection of the government by some exogenous or endogenous probability, which is outside the scope of the current paper.

3. SIMULATION RESULTS

Based on the numerical simulations, the following comparative static analysis can be reported[36]:

$$(\partial m^*/\partial \alpha) \leq 0; \ (\partial m_t^*/\partial f_{i,t-1}) \leq 0; \ (\partial m_t^*/\partial A_{i,t-1}) \gtreqless 0;$$

and

$$(\partial m_t^*/\tau) = \begin{cases} <0 & \text{for} \quad m_t^* > 0.5 \\ ? & \text{for} \quad 0.2 < m^* < 0.5 \\ na & \text{otherwise} \end{cases} , \forall i.$$

The only unambiguous result of the comparative statics is the negative relationship of m_t^* with α and $f_{i,t-1}$. The intuition behind the first of these effects is trivial: as the income share of capital rises, the need for investment in SC to achieve the same level of income declines (substitution effect). The second result indicates that the higher the spillovers, the greater is the aggregate SC and thus the less is the need for further spending on SC. The effects of τ and A_i on m^* are non-uniform and/or ambiguous, however.

Remark 1: m^* decreases in α and $f_{i,t-1}$.

I next turn to investigate the relationship of m^* with the outcomes of particular interest to this paper: the aggregate income level ($Y = Y_1 + Y_2$) and distribution (Y_1/Y_2), social cohesion (F) and welfare, which is measured by the level of utility given in Equation (10). These results are discussed below in references to simulation plots provided in Appendix 3.

A major finding revealed by the numerical simulations is that the range of attainable values F is associated positively with m^* for $m^* \lesssim 0.7$ (see Figure 3). This result is trivial since increasing m_t^* leads A_i's to increase that in turn increase F_t. The nonlinear shape of the plot, where F peaks at $m^* \cong 0.7$, is due to the term $\sum_i f_{i,t}$ whose value depends on other model parameters.

Remark 2: The lower bound of m^* tends to be positively associated with F (for $m^* < 0.7$).

Another trivial observation can be made based on Figure 4 that shows that homogeneity in income shows a positive correlation with that of SC. This is expected given the production function.

Remark 3: Income distribution worsens as SC heterogeneity increases.

Data plots in Figure 5 show that extreme values (both high and low) of m^* are associated with increasing income inequality.[37] The explanation is as follows. Increasing m^* contributes to A_i in both regions by the same amount, hence marginally more in the region where it is lower. Hence, SC homogeneity,[38] and given the above remark also income equality, improves as m^* increases so long as $m^* \lesssim 0.35$. Above that threshold, however, increasing m^* is associated with worsening income and SC distribution also. This is due to the fact that the output impact of falling G^{DS} starts to dominate the gain in SC as m^* increases. Thus, based on the plots of the numerical simulations, $m^* \cong 0.35$ appears to be the rate at which most equitable income distribution is likely to be achieved.

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[36]The partial derivatives are too long to report here. It is possible to identify their signs of the derivatives through numerical simulations, however, which are definitive given the feasible ranges of the model parameter and the feasibility constraints. The analysis is available in Matlab Program files and can be provided upon request.
[37]Same result is obtained for SC:
[38]measured by A_1/A_2.

Remark 4: Income distribution improves as m^* approaches to 0.35.

Next, Figure 6 shows that the range of income distribution figures corresponding to increasing levels of F gets smaller, indicating less income inequality as F gets higher. This supports the initial thesis of this paper regarding the positive developmental effects of social cohesion.

Remark 5: Income distribution is likely to improve with F.

Figure 7 shows that the more equitable is SC distribution, that is the closer the ratio A_1/A_2 is to one, the higher is the level of aggregate income. Along with the earlier remark, this suggests that societies that have both SC homogeneity and high level of social cohesion can achieve both high level of income and an equitable distribution. This observation is consistent with Neyapti and Arasil (2016), who show theoretically that welfare improves with cultural homogeneity, which also confirms the empirical findings of Easterly, Ritzen, and Woolcock (2006), Boettke, Coyne, and Leeson (2008) and Williamson (2009).

Remark 6: The greater is the homogeneity in SC, the higher is the range of aggregate income that can be attained.

It is interesting to note in Figure 8, however, that the rate of m^* that corresponds to that highest attainable level of aggregate income is an intermediate level of m^* (similar to, but somewhat higher than the rate at which income distribution gets to be most equitable), namely $m^* \cong 0.45$. Consistent with Remark 6, it is observed in Figure 9 that welfare reaches its maximum levels at $m^* \cong 0.45$.

3.1. Extensions

In this section, the benchmark model is modified to analyze two additional issues. Section 3.1.1 investigates the coexistence of local and pure public good provisions and hence the effect of fiscal decentralization on optimal m. Section 3.1.2 explores the implications of a change in the production technology where, similar to the case of labor-augmenting technology, DS augments SC.[39]

3.1.1. Decentralization of the public good provision

Consider that fd, the extent of fiscal decentralization (where $0 \leq \text{fd} \leq 1$), stands for the share of local public good provision vis-à-vis the pure public good. Accordingly, the central government's tax revenue becomes a fraction $(1 - \text{fd})$ of total tax revenue: $G_t^{\text{fd}} = (1 - fd)\tau \sum_i Y_{i,t}$, where G_t^{fd} is the pure public good in this regime, and $G_t^{\text{fd,SC}}$ and $G_t^{\text{fd,DS}}$ are similarly obtained by multiplying G_t^{SC} and G_t^{DS} that were given in Equation (5) by the factor of $(1 - \text{fd})$.[40] Hence, if fd > 0, the amount of pure public good provision is less than the case of the above benchmark model. *Local public good*, denoted by $G_{i,t}$, in turn, is financed by the amount of local tax revenues: $\text{fd}(\tau Y_{i,t})$, which is also used to augment A_i. Equation (6) is therefore modified as follows:

$$A_{i,t} = A_{i,t-1}\left(1 + f_{i,t-1}\right) + (m_t G_t) + G_{i,t}, \tag{6'}$$

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[39]In addition, I investigated a case where the central government redistributes part of the tax revenue to one of the districts for, say, political considerations. As is the case for the tax rate, no explicit solution could be obtained for the optimal rate of such transfers when the government optimally selects m^* at the same time.
[40]I refrain from the political economy aspects of decentralization and assume that fd is given level the central government.

where the last two terms stand for the central and local provision of SC under decentralization. Hence, the unconstrained maximization problem becomes:

$$\text{Max}_{m_t} \sum_i \log\big[(1 - s_{i,t+1})(1 - \tau)(1 - m_t)\tau Y_t(1 - fd)(K_{i,t-1}(1-\delta) + I_{i,t-1})^\alpha (A_{i,t-1}(1 + f_{i,t-1})$$

$$+ m_t(1 - fd)\tau Y_t + fd\tau Y_{i,t})^{1-\alpha}] \tag{11$'$}$$

The optimum value of m_t that the solution of this modified version of the model yields is given by:

$$m_t^{\text{fd}*} - 1 + \frac{n}{(1 - fd)(1 - \alpha)\tau Y_t \sum_i (1/(A_{i,t-1}(1 + f_{i,t-1}) + m_t^{\text{fd}*}\tau Y_t + fd\tau Y_{i,t})} = 0, \tag{12$'$}$$

which is different from m^* in Equation (12) only by the term $(1 - fd)$, besides the change in A_i's, in the denominator as given by (6$'$). The key observation is that $\partial m_t^{\text{fd}*}/\partial fd < 0$.[41] Hence, ceteris paribus, increasing fd is associated with an increase in the share of adjudication spending: $\partial G^{\text{fd,DS}}/\partial fd > 0$.

This is an expected result given that local public good spending on SC ($G_{i,t}$) is modeled as a substitute for $G_t^{\text{fd,SC}}$.[42] Furthermore, this relationship is observed to be more pronounced the higher is fd. Hence, as fd increases, the resulting decrease in $m^{\text{fd}*}$ leads to a slowdown in the increase in SC stocks. As a result, the highest attainable values of Y and F decrease as fd increases.[43] This is consistent with the decentralization literature that states that fd may not be welfare improving in case there are spillovers or when regions do not internalize externalities, which arise from differences in regional SC in the current model. The important policy implication, therefore, is that, assuming the existence of a benevolent government assumed earlier, large scale fiscal decentralization is not advisable for countries that have not yet reached a high level of income and/or are highly heterogeneous with respect to SC.

Remark 7: $m^{\text{fd}*}$ is associated negatively with fd.

Simulations also reveal that the addition of decentralization dimension to the model leaves the main findings of the benchmark case intact; particularly, the highest levels of welfare and income are also achieved when $0.3 < m^{\text{fd}*} < 0.5$ in this case. Additionally, the highest level of F is observed at $m^{\text{fd}*} = 0.8$ and the highest level of Y is observed at $m^{\text{fd}*} = 0.5$, which are slightly higher values observed in the benchmark case.

3.1.2. SC-augmenting adjudication

The foregoing analysis rests on the assumption that DS provides social order and an environment that makes both physical and SC more productive. As is well known, arms industry may have significant technological spillover effects, which explains why DS would improve the productivity of physical capital. It is also predictable that high technology and the organization structure of the security-related industries would have positive effects on SC, which is a combined measure of human capital and civic cooperation. In the absence of

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[41]The derivative of (the second root of) $m_t^{\text{fd}*}$ is not given here to save space, but can be obtained from the author upon request. Because the expression is too long to observe the sign, the sign is confirmed via simulations that cover all the possible ranges of the model parameters.
[42]It is straightforward to predict that substituting G^{DS} by its local version would produce an opposite effect.
[43]Simulation plots are available upon request.

an external threat and strategic armament, and in a relatively secure socio-political environment where tolerance and trust is high and the rule of law is strong, however, it is expected that soft regulations replace hard power, and adjudication replaces coercion, to achieve DS. As distinct from spending on repressive measures and police forces, DS spending that is geared to property rights protection and improved social guidelines contributes to social trust and civic cooperation and is therefore likely to improve particularly the productivity of SC, rather than total factor productivity. Whether spending on DS should concentrate on coercive measures or softer regulations is therefore related to the level of social development.

To incorporate this argument in the analysis, the benchmark model is modified by changing the production function into one that G^{DS} augments SC: $Y_t = K_{i,t-1}^\alpha (G_{t-1}^{\mathrm{DS}} A_{i,t-1})^{1-\alpha}$. Hence, the objection function becomes:

$$\mathrm{Max}_{m_t} \sum_i \log(C_{i,t+1}) = \sum_i \log((1 - s_{i,t+1})(1 - \tau)K_{i,t}^\alpha (G_t^{\mathrm{DS}} A_{i,t})^{1-\alpha}).$$

The solution of the optimization problem, subject to the same set of constraints as in the benchmark model, yields the following first-order condition:

$$m_t^{**} - 1 + \frac{n}{\sum_i (1/(A_{i,t-1}(1 + f_{i,t-1}) + m_t^{**}\tau Y_t))} = 0, \tag{12''}$$

which differs from the solution of the benchmark case by the absence of the term $(1 - \alpha)\tau Y_t$ in the denominator. The solution of Equation (12'') for m^{**} yields two roots, but only one of them yields non-negative values while the other root is infeasible.

Numerical simulations of this modified model reveal that the main implications of the benchmark model remain mostly intact: (i) the extent of equitability of income and SC across regions is associated positively; (ii) the highest attainable values of aggregate income shows an inverted U-relationship with m^*; (iii) the highest attainable aggregate income level is observed when SC is homogeneous; and (iv) and income distribution is likely to first worsen and then improve with social cohesion. There are a couple of modifications to these observations, however. For the second observation, the maximum attainable level of Y is lower and is also obtained for a lower optimal m. Likewise, for the last observation, it is observed that income distribution is worse and starts to improve at much higher levels of F than the benchmark case.

Observations that differ significantly from the benchmark case are as follows. First, the relationship between m^{**} and socioeconomic heterogeneity, measured by the equity of both incomes and SCs across the regions, now increases monotonically in m^{**}; that is, the flat bottom portion of Figure 3 disappears in this scenario. In other words, low values of m^{**} is no more associated with high inequality as was observed in the case of m^*. What this implies is that, when DS augments SC, decreasing the investment on SC or increasing DS, is clearly associated with an improvement in the distributions of income and of SC. Second, the falling portion of welfare associated with low m^* (Figure 9) is not observed for m^{**}. In addition, the highest levels of welfare is obtained for much lower levels of m^{**} ($m^{**} \cong 0.05$) than for the benchmark case of $m^* \cong 0.45$. However, the highest attainable level of welfare is observed to be lower in this scenario than in the benchmark scenario.

Remark 8: SC-augmenting DS slows down the accumulation of SC.

These observations indicate that as DS augments SC, it also leads a lower accumulation in SC, welfare and Y than otherwise. The economic intuition for this is similar to the

implications of labor-augmenting technological change that causes displacement of labor. As discussed above, however, the nature of spending on DS that mainly contributes to the productivity of SC, rather than to physical capital, is likely to be soft regulations rather than repressive measures that are more likely to be needed in unstable economies than others. The form of the production function utilized for modeling the optimal division of the tax revenue between SC and DS, or between education and adjudication, therefore depends on the type of DS spending, which is in turn likely to depend on the socioeconomic characteristics of an economy. That is, the economies that may be suitable to study under this particular extension are likely to have greater welfare than others. Given the static nature of the model solution currently employed, the implications of this section are therefore not directly comparable to those of the benchmark case.

4. CONCLUSIONS

This paper presents an original framework to analyze the optimum allocation of public spending between SC investment and DS spending by a central government that faces heterogeneous regions with respect to their initial social and physical capital. The government decides on the allocation of its budget across these two types of pure public goods to maximize welfare. The model's solution indicates that the optimal share of SC investment decreases in income and fiscal decentralization, while it increases in social cohesion, measured by the total spillover effects of social-capital across the regions. The aggregate income level increases in cultural homogeneity that in turn increases with public education investment. Hence, the findings of this paper support several earlier empirical studies that point at the importance of social cohesion for economic development. The findings also indicate that increasing fiscal decentralization lowers optimal investment in SC; this observation cautions policy makers against decentralization in countries in that exhibit low levels and high variability of SC.

The benchmark model is also investigated under an alternative specification of the production function, where DS is assumed to augment SC, similar to labor-augmenting technology. While this extension leads to lower income, SC and wealth than the benchmark case, the different specifications of technology in the two models essentially makes them incomparable as they refer to different underlying economic realities. A potential future extension of this study includes adding political economy features to the model, with regional competition or international dynamics. Additionally, exploring the implications of fiscal rules constitutes a potentially interesting future direction.

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DISCLOSURE STATEMENT

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APPENDIX 1. ETHNIC HETEROGENEITY, LAND AREA AND INCOME (SOURCE: THE WORLD BANK)
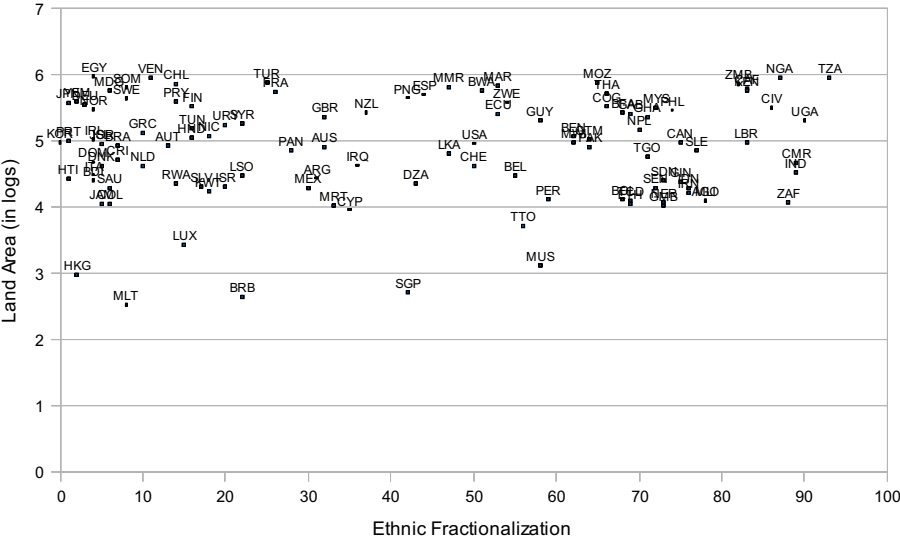
FIGURE 1 Country size and ethnic fractionalization
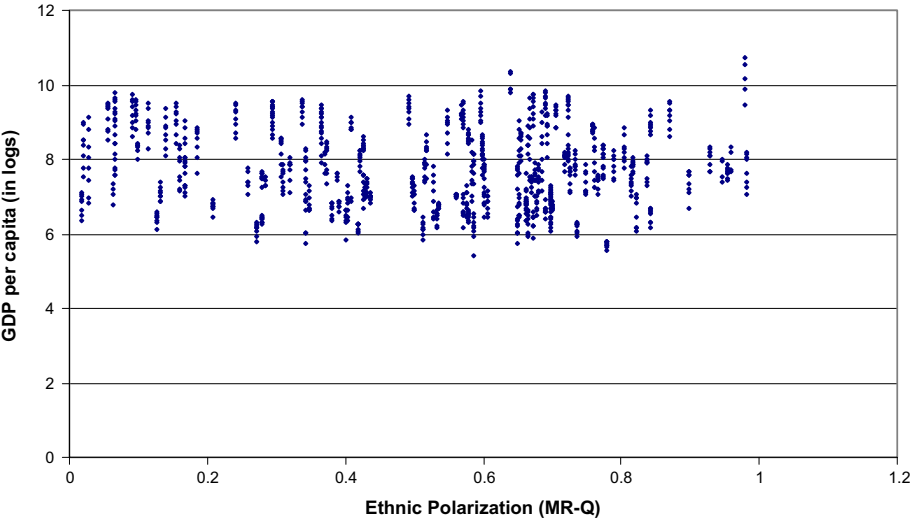
FIGURE 2 Income and ethnic polarization

APPENDIX 2. OPTIMALITY CHECKS

After substituting for G^{SC} and f_i in Equation (6) from Equations (5) and (8), respectively, Equation (6) becomes:

$$A_{i,t} = A_{i,t-1}\left[1 + \sum_j h(A_j - A_i)_{t-1}\right] + m_t \tau Y_t$$

Hence, the expression (11) becomes:

$$\text{Max}_{m_t} \sum_i \log\left[(1-m_t)\tau Y_t \cdot \left[A_{i,t-1}\left(1 + \sum_j h(A_j - A_i)_{t-1}\right) + m_t\tau Y_t\right]^\beta \cdot\right.$$
$$\left. \left[K_{i,t-1}(1-\delta) + s_{i,t-1}(1-\tau)Y_{i,t-1}\right]^\alpha (1-s_{i,t})(1-\tau)\right]$$

The partial derivative of the above unconstrained optimization with respect to m_t is given by: $\frac{-n}{(1-m_t^*)} + (1-\alpha)\tau Y_t \sum_i 1/A_{i,t} = 0$, which can be rewritten as Equation (12). The solution of this expression for optimal m_t yields two roots; we use only one of these roots generates the feasible values of the state variables.[44]

The *second-order condition* for a maximum requires that:

$$\frac{\partial}{\partial m_t^*}\left[\frac{-n}{(1-m_t^*)} + (1-\alpha)\tau Y_t \sum_i 1/A_{i,t}\right] < 0.$$

The derivative yields:
$-\frac{n}{(1-m_t^*)^2} - (1-\alpha)(\tau Y_t)^2\left[1/A_{1t}^2 + 1/A_{2t}^2\right]$, which is definitively negative and ensures that m_t^* is at its maximum.
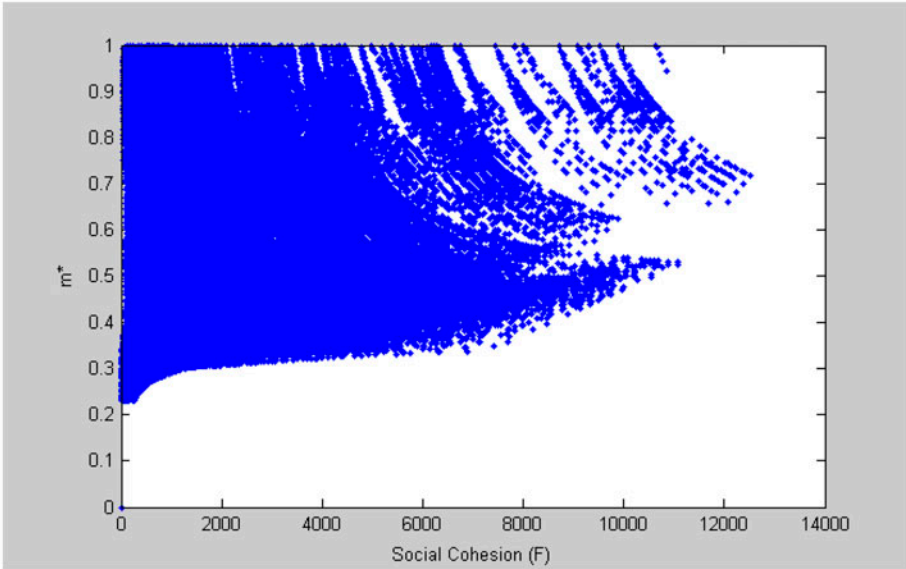
APPENDIX 3. SIMULATION RESULTS

FIGURE 3 Social cohesion (F) and m^*

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[44]It is observed that the ranges the two roots span are: $0 < m_1 < 0.1764$ vs. $0.375 < m_2 < 0.9508$; of these, m_2 produces the feasible values of the model variables.
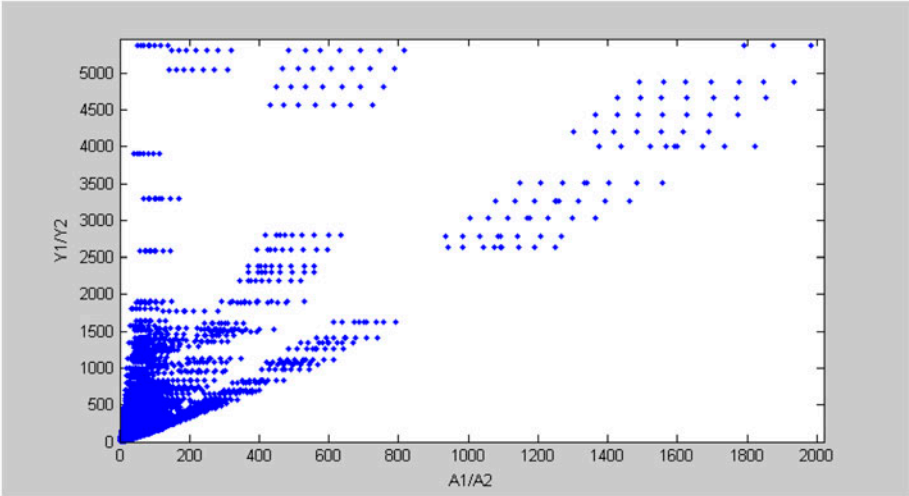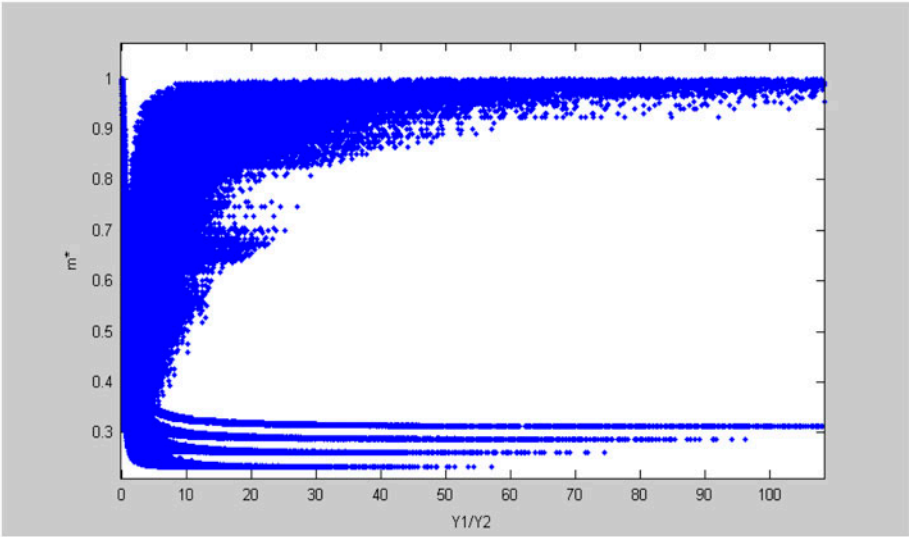
FIGURE 4 Heterogeneity in SC and income distribution

FIGURE 5 Income distribution and m^*
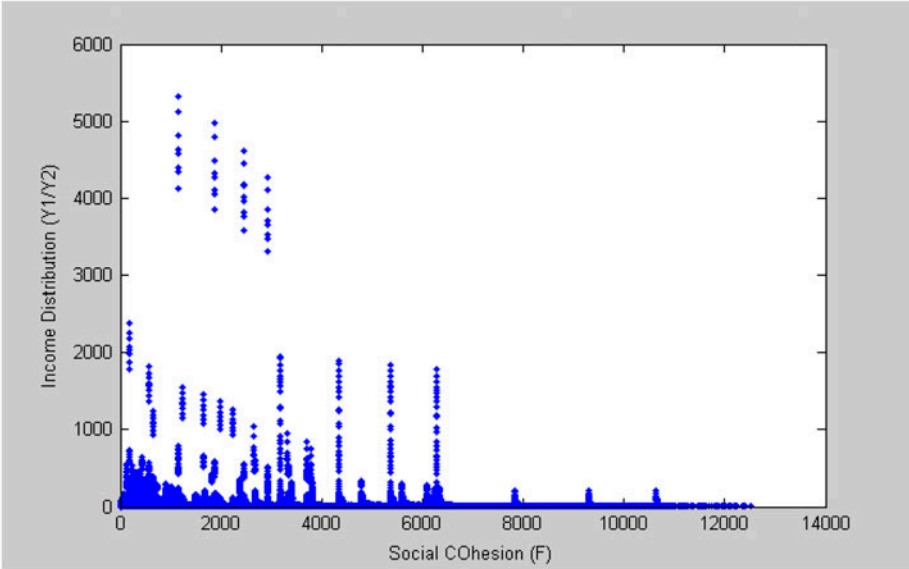
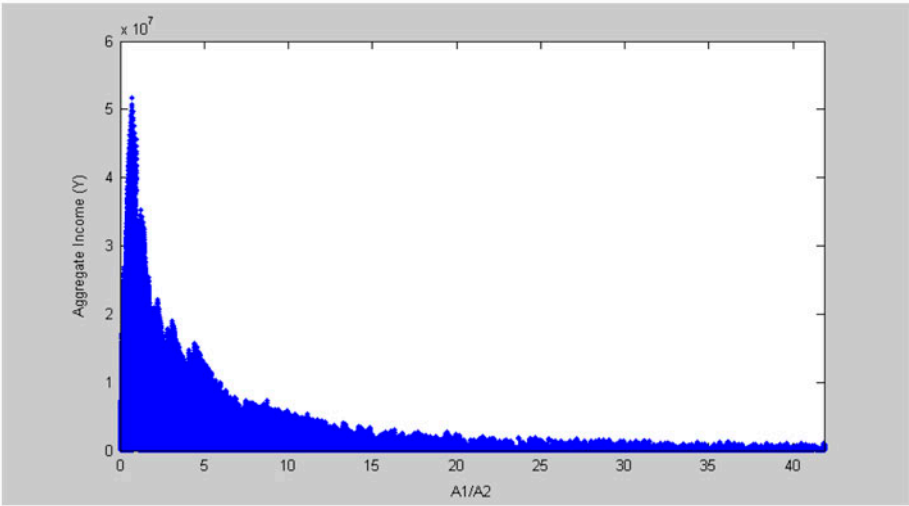FIGURE 6 Social cohesion and income distribution

FIGURE 7 Aggregate income and distribution of SC

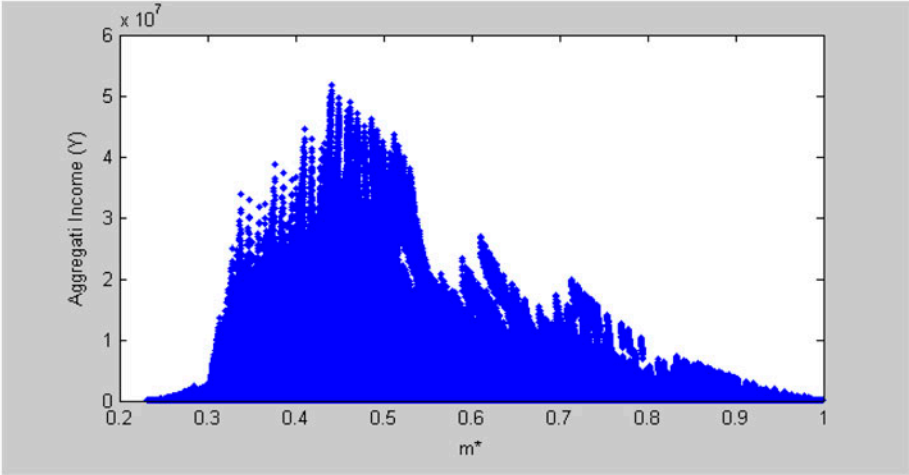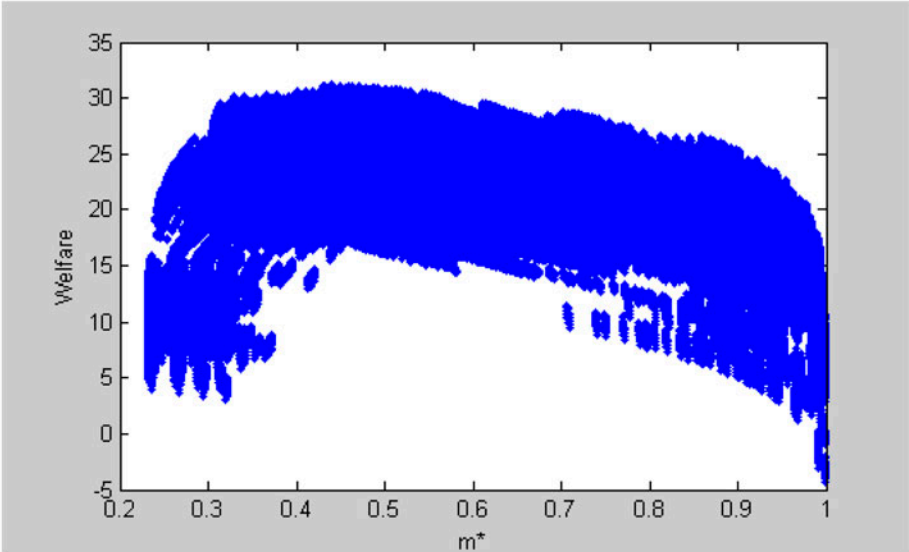FIGURE 8 Aggregate income (Y) and m^*

FIGURE 9 Welfare and m^*