Sabotage in Team Contests

Serhat Doğan
Bilkent University

Kerim Keskin*
Kadir Has University

Çağrı Sağlam
Bilkent University

September 25, 2017

Abstract

In the contest literature, sabotage is defined as a deliberate and costly activity which damages the opponent's likelihood of winning the contest. The existing results mostly suggest that contestants are discouraged anticipating a possible sabotage. In this paper we investigate the act of sabotage in a team contest where team members exert costly effort as a contribution to their team's aggregate effort which in turn determines the contest outcome. For the baseline model with no sabotage, there exists a corner equilibrium indicating a free-rider problem in each team. As for the model with sabotage, our characterization of Nash equilibrium reveals two important results: (i) There exists a unique interior equilibrium so that the free-rider problem is no longer a concern; and (ii) The discouragement effect of sabotage vanishes for some players. Furthermore, we investigate the team owner's problems of prize allocation and team formation considering the objective to maximize his team's winning probability.

JEL Codes: C72, D74.
Keywords: Team contests, Sabotage, Tullock contests, Free-riding, Discouragement effect, Encouragement effect.

* *Corresponding author*. E-mail: kerim.keskin@khas.edu.tr.

1 Introduction

Contests are strategic interactions in which participants expend costly resources (e.g., effort, time, money, etc.) aiming to win a valuable prize. Perhaps most importantly, all of the resources expended are lost independent of who wins the contest. Many real-life examples can be provided: sports, war, politics, R&D competition, advertising, etc. In all of these examples, contestants exert efforts in order to increase their chances of winning; and in some of them, contestants are also able to take some actions in order to reduce their opponents' winning probabilities, thereby indirectly increasing their own winning probabilities. The latter action is commonly labeled as "sabotage" in the literature.

Real-life examples to sabotage are aggressive play or attempting to provoke illegal responses from competitors in sports, destruction of the rival's weaponry or resources in warfare, negative campaigning in politics, etc.[1] In addition to such real-life observations, many scholars also reported sabotage in laboratory experiments (see Harbring and Irlenbusch, 2005, 2011; Harbring et al., 2007; Vandegrift and Yavas, 2010 among others) and field studies (see Balafoutas et al., 2012; Deutscher et al., 2013; Brown and Chowdhury, 2017 among others). For example, among these papers, Harbring et al. (2007) investigated behavior in experimental corporate contests with heterogeneous players; whereas Vandegrift and Yavas (2010) studied a similar framework preceded by a real-effort task which endogenized the heterogeneity in participants' ability levels. As for the field studies, Balafoutas et al. (2012) and Deutscher et al. (2013) both reported that sabotage is more likely to be used by relatively less qualified parties and to be targeted at more qualified ones, analyzing data from Judo World Championships and German *Bundesliga*, respectively.

To the best of our knowledge, in the literature on contests/tournaments, Dye (1984) is the first to mention the possibility of sabotage. Lazear (1989) presented a theoretical model of sabotage in contests and showed that a larger prize spread between the winner and the loser(s) would lead to an increase in sabotage activity.[2] Later, Konrad (2000) studied the effect of sabotage on equilibrium behavior in an n-player lobbying contest. He showed that sabotage may increase or decrease the total lobbying effort exerted at the

[1] For more real-life examples, see a recent survey by Chowdhury and Gürtler (2015).

[2] A result which is observationally verified by a number of studies (e.g., Garicano and Palacios-Huerta, 2000; Harbring and Irlenbusch, 2004, 2011; Vandegrift and Yavas, 2010).

equilibrium and that the total amount of sabotage decreases in the number of players. Afterwards, Chen (2003) analyzed sabotage in promotion tournaments where relative performances are important and indicated that players with the highest caliber might not have the best chance of being promoted. Münster (2007) studied a case of *directed* sabotage in selection tournaments and showed that contestants who choose a higher productive effort are sabotaged more heavily at the equilibrium. Moreover, he argued that the possibility of sabotage may deter more productive players from entering the tournament. In a related study, Amegashie (2012) analyzed the subgame perfect Nash equilibrium of a two-stage contest in which players choose destructive efforts (i.e., sabotage) in stage 1 and productive efforts in stage 2. He found that players engage in destructive activities only if the winning prize is sufficiently high, and after that threshold point, players' productive efforts remain constant in the prize level.

Despite the fact that the act of sabotage has already been studied in the contest literature for over thirty years, the analysis of sabotage in team contests remains an understudied topic in this literature. To our knowledge, there is only one paper studying sabotage in a team contest. Gürtler (2008) assumed that each contestant —as a member of one of the two teams— chooses a productive effort in order to increase his team's performance, but is also able to sabotage the members of the opposing team. As a main result, it is shown that a team directs all its sabotage activities at the least skilled member of the opposing team. This is rather interesting since it is in stark contrast to the findings that the most skilled players are most likely sabotaged in individual contests (see Chen, 2003; Münster, 2007).

We study sabotage in a one-shot contest between two teams. Each team is divided into two groups which differ in their effectiveness parameters and winning prizes. For this model we can provide several interpretations: (i) Consider two football teams playing a football game. Each football team has attackers and defenders. (ii) Consider two countries in war. Each country has attack forces and defense forces. (iii) Consider two political parties competing over two regions. The representatives in different regions can be interpreted as different groups. In this paper, for the sake of expositional simplicity, we stick to the football game interpretation keeping the alternative interpretations in mind. In a given football team, each group chooses a contest effort which contributes to their team's aggregate effort which in turn determines the contest outcome. Additionally, each group is able to sabotage a particular group in the opposing team, which we call *directionally restricted* sabotage.[3] We then characterize and compare the sets of Nash equilibria in models with no sabotage and with directionally restricted sabotage.

One of the most common results in the literature on sabotage in contests is the *discouragement effect* (see Section 4 by Chowdhury and Gürtler, 2015). For instance, Chen (2003) and Münster (2007) showed that the most skilled players are subject to more sabotage; and Gürtler and Münster (2010) found that players who exert high effort in the first stage of a two-stage contest are sabotaged more than those who exert low effort. These findings imply that if there is a possibility of being sabotaged, then there is less incentive to exert high effort or even to enter the contest. On top of these, Amegashie (2012)'s above-mentioned findings imply that sabotage fully *crowds out* any additional productive effort that would have been exerted by players in the absence of sabotage. By a stark contrast, at the interior equilibrium of our model, we observe a converse effect for one of the groups. More precisely, compared to the equilibrium efforts exerted in the model with no sabotage, there exists a group which exerts higher contest effort once the sabotage option becomes available. This result, which can be labeled as the *encouragement effect*, appears to be related to the collective nature of the model.

Another interesting issue is that in the baseline model with no sabotage, we detect a *free-rider problem*. Depending on the values of effectiveness parameters and winning prizes, either attackers or defenders exert no effort at the equilibrium. Similar findings were previously reported by Baik (1993, 2008) and Baik et al. (2001).[4] This result is also closely related to the seminal work by Holmström (1982) highlighting free-rider problems in a team (i.e., a group of individuals who are organized so that their productive inputs are related). The literature emerged from this seminal paper focuses on optimal contract design to resolve such problems (see McAfee and McMillan, 1991; Itoh, 1991; Vander Veen, 1995; Gershkov et al., 2009 among others). Interestingly, without a referral to an optimal contract, in this paper we show that the option to sabotage works as a *natural* solution to free-riding. More

[3]A sabotage activity is said to be *directed* if a player is facing multiple opponents and is able to choose the victim of his sabotage. Here we *restrict* the possible directions for sabotage, arguing that the defenders/attackers in a football team are facing the attackers/defenders in the opposing team. For the interested reader, we analyze the case of *directed* sabotage in the Appendix and characterize the conditions under which the model reduces to our original model with *directionally restricted* sabotage.

[4]In fact, if one considers our baseline model in the context of public good provision, such results date back to Olson (1965).

precisely, the free-riding group starts contributing to the team's aggregate effort once there is a possibility of sabotage from the opposing team members.

Finally, we are interested in the team owner's problem of optimal design. First, consider the following scenario: If a team wins the contest, the team owner receives a prize; and a certain ratio of this prize will be distributed as a premium among the groups in the team. The team owner's problem is to optimally allocate these prize shares in order to maximize his team's winning probability. Second, consider the following scenario: A team owner has a transfer budget to be spent on attackers and defenders who differ in their effectiveness parameters. Given that hiring more effective players is more costly, the team owner's problem is to optimally form the team in order to maximize his team's winning probability. Here we characterize the team owners' optimal strategies in these two situations.

The rest of the paper is organized as follows: In section 2, we formulate the models with no sabotage and with directionally restricted sabotage. We then characterize and compare the sets of Nash equilibria in these models. In section 3, we investigate the team owner's problems of optimal design by letting the team owner (i) to allocate a given prize among two groups and (ii) to spend a given transfer budget for hiring players with different effectiveness parameters. Section 4 concludes.

2 The Model

As mentioned earlier, for the sake of expositional simplicity, we refer to the football game interpretation in this paper. The alternative interpretations are mentioned when necessary.

2.1 A Team Contest

Consider two football teams playing a football game: team 1 and team 2. Each team consists of two groups: attackers (a) and defenders (d). In this football game, each group decides how much costly effort to exert which contributes to an aggregate effort level for the team. These aggregate efforts determine which team wins the contest. Accordingly, if team $i \in \{1, 2\}$ wins the contest, then group $j \in \{a, d\}$ in team i gets a prize of $V_i^j > 0$, whereas the groups in the losing team do not get any payoff.

Other than winning prizes, the groups also differ in the effectiveness of their contest efforts in their team's aggregate effort function. For every team $i \in \{1,2\}$, the aggregate effort is

$$E_i = \gamma_i^a e_i^a + \gamma_i^d e_i^d$$

where $e_i^j \in [0,\infty)$ is the contest effort exerted by group $j \in \{a,d\}$ and $\gamma_i^j > 0$ is the effectiveness parameter for group $j \in \{a,d\}$. We assume that $\gamma_i^a \neq \gamma_i^d$. The winner is determined by the following Tullock-type contest success function:

$$P_i(E_1,E_2) = \frac{E_i}{E_1+E_2} \ .$$

Finally, for any group $j \in \{a,d\}$ in any team $i \in \{1,2\}$, we consider the same linear cost function

$$C(e_i^j) = e_i^j \ .$$

To sum up, in this contest each group $j \in \{a,d\}$ in each team $i \in \{1,2\}$ maximizes

$$U_i^j(e_i^j,\cdot) = P_i(E_1,E_2)V_i^j - C(e_i^j) = \frac{E_i}{E_1+E_2}V_i^j - e_i^j \ .$$

Below we analyze the Nash equilibrium for this baseline model.

Proposition 1. *In the above-described team contest, assume without loss of generality that for groups $j,j' \in \{a,d\}$ in team 1 and for groups $k,k' \in \{a,d\}$ in team 2:*

$$\gamma_1^j V_1^j > \gamma_1^{j'} V_1^{j'} \quad \text{and} \quad \gamma_2^k V_2^k > \gamma_2^{k'} V_2^{k'}.$$

Then there exists only a corner equilibrium in which

$$e_1^j = \frac{\gamma_1^j \gamma_2^k (V_1^j)^2 V_2^k}{(\gamma_1^j V_1^j + \gamma_2^k V_2^k)^2},\ \ e_1^{j'} = 0,\ \ e_2^k = \frac{\gamma_1^j \gamma_2^k V_1^j (V_2^k)^2}{(\gamma_1^j V_1^j + \gamma_2^k V_2^k)^2},\ \ e_2^{k'} = 0.$$

This leads to the following equilibrium aggregate efforts:

$$E_1^* = \frac{(\gamma_1^j)^2 \gamma_2^k (V_1^j)^2 V_2^k}{(\gamma_1^j V_1^j + \gamma_2^k V_2^k)^2} \quad \text{and} \quad E_2^* = \frac{\gamma_1^j (\gamma_2^k)^2 V_1^j (V_2^k)^2}{(\gamma_1^j V_1^j + \gamma_2^k V_2^k)^2} \ .$$

Furthermore, if $\gamma_i^a V_i^a = \gamma_i^d V_i^d$ for some team $i \in \{1,2\}$, then there exist multiple equilibria such that both groups in team i exert efforts reaching an aggregate effort of E_i^* above.

Proof. Given an aggregate effort E_2 for team 2, the first order condition with respect to e_1^a for group a in team 1 is

$$\frac{\gamma_1^a E_2}{(E_1+E_2)^2} V_1^a - 1 = 0 \ .$$

For group d in team 1, a symmetric first order condition can be written as

$$\frac{\gamma_1^d E_2}{(E_1+E_2)^2} V_1^d - 1 = 0 \ .$$

Accordingly, it must be that

$$\gamma_1^a V_1^a = \gamma_1^d V_1^d \tag{1}$$

at the equilibrium, which is not necessarily true. This leads to a corner solution that if the right-hand-side is greater than the left-hand-side, then only the attackers exert positive effort at the equilibrium; and vice versa. Considering a symmetric result for the other team, and under the assumption that $\gamma_1^j V_1^j > \gamma_1^{j'} V_1^{j'}$ and $\gamma_2^k V_2^k > \gamma_2^{k'} V_2^{k'}$ for groups $j, j' \in \{a, d\}$ in team 1 and for groups $k, k' \in \{a, d\}$ in team 2, the respective equilibrium efforts are

$$\begin{aligned} e_1^j &= \frac{\gamma_1^j \gamma_2^k (V_1^j)^2 V_2^k}{(\gamma_1^j V_1^j + \gamma_2^k V_2^k)^2}, \ \ e_1^{j'} = 0, \\ e_2^k &= \frac{\gamma_1^j \gamma_2^k V_1^j (V_2^k)^2}{(\gamma_1^j V_1^j + \gamma_2^k V_2^k)^2}, \ \ e_2^{k'} = 0. \end{aligned}$$

so that

$$E_1^* = \frac{(\gamma_1^j)^2 \gamma_2^k (V_1^j)^2 V_2^k}{(\gamma_1^j V_1^j + \gamma_2^k V_2^k)^2}, \quad \text{and} \quad E_2^* = \frac{\gamma_1^j (\gamma_2^k)^2 V_1^j (V_2^k)^2}{(\gamma_1^j V_1^j + \gamma_2^k V_2^k)^2} \ . \tag{2}$$

Finally, for the sake of completeness, we must note that if equation (1) indeed holds for team $i \in \{1, 2\}$, then there exist multiple equilibria such that both groups in team i exert efforts reaching an aggregate effort of E_i^*. □

Let $\gamma_i^j V_i^j$ denote a measure for *motivation* of group $j \in \{a, d\}$ in team $i \in \{1, 2\}$. The idea is that an increase in γ_i^j or V_i^j would increase the expected utility of this group which motivates them to contribute more. Notice that in the statement of Proposition 1, group j in team 1 and group k in team 2 are assumed to be relatively more motivated in their teams. And apparently, the equilibrium aggregate efforts only depend on the effectiveness parameters and the winning prizes of these relatively more motivated groups. In particular, E_1^* is increasing in γ_1^j and V_1^j, but decreasing in γ_2^k and V_2^k. The symmetric is true for E_2^*, and the respective interpretations are quite straightforward.

We complete this section by emphasizing the following remark.

Remark 1. *As stated in Proposition 1, if the equation (1) does not hold for some team $i \in \{1, 2\}$, then there exists a free-riding group in team i exerting no effort in the team contest.*

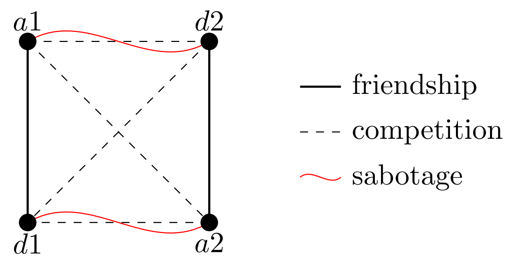
This observation becomes particularly important when the option to sabotage is introduced in the following section.

2.2 Introducing Sabotage

In this section, we introduce an additional choice variable for the groups in both teams: *sabotage*. Any act that reduces the effectiveness of the opposing team members can be classified as a sabotage activity. For instance, given the football game interpretation, possible sabotage activities are playing more aggressively and attempting to provoke illegal responses from the opposing team members. With such actions one may inflict injuries or may cause the opponent players to receive yellow/red cards. These would decrease the effectiveness of the opposing team members, thereby indirectly creating an advantage for one's team.[5]

We consider a situation where group a/d in one team can *only* sabotage group d/a in the opposing team. To put it differently, group a/d in one team *cannot* sabotage group a/d in the opposing team. The intuition is as follows: In a football game, the attackers in a team are faced with the defenders in the opposing team; so that if the opposing defenders exert some sabotage effort, then the attackers should be the ones suffering from this act. This is what we call *directionally restricted* sabotage.

[5]Following the war interpretation, the destruction of rival's weaponry or resources can be labeled as a sabotage act. Or, following the political party interpretation, a possible sabotage activity is negative campaigning.



In that sense, one can argue that we study a small network with (i) four nodes representing the groups and (ii) three types of links representing group interactions. In particular, $a1$ has a friendship link with $d1$; $a2$ has a friendship link with $d2$; $a1$ and $d1$ have competition links with both $a2$ and $d2$; $a1$ has a sabotage link with $d2$; and $d1$ has a sabotage link with $a2$.[6]

Accordingly, the aggregate effort functions become

$$E_1 = \frac{\gamma_1^a}{1+s_2^d} e_1^a + \frac{\gamma_1^d}{1+s_2^a} e_1^d$$

and

$$E_2 = \frac{\gamma_2^a}{1+s_1^d} e_2^a + \frac{\gamma_2^d}{1+s_1^a} e_2^d$$

where $s_i^j \in [0, \infty)$ denotes the sabotage effort exerted by group $j \in \{a, d\}$ in team $i \in \{1, 2\}$ at the respective group in the opposing team. As it can be seen, we assume for concreteness that if group $j \in \{a, d\}$ in team $i \in \{1, 2\}$ is sabotaged in the amount of s, then the effectiveness level of this group reduces from γ_i^j to $\gamma_i^j/(1+s)$.

The contest success function is preserved in terms of the aggregate efforts E_1 and E_2. Finally, the cost function of group $j \in \{a, d\}$ in team $i \in \{1, 2\}$ is updated to

$$C_i^j(e_i^j, s_i^j) = e_i^j + \mu_i^j s_i^j$$

where $\mu_i^j > 0$ denotes the cost of exerting one unit of sabotage effort.

[6] In a similar manner, consider the war interpretation of our model. For each country, the defense forces are located within the country, whereas the attack forces are moved to the opposition territory for battle. It seems clear that it would be impossible for the defense forces of a country to sabotage the defense forces of the opposing country, since they are located within different territories. Also, consider the political party interpretation of our model. Party 1's representatives in one region can only sabotage party 2's representatives in the same region; and so on. Notice that this situation can be captured by the network above following a simple replacement of the nodes $a2$ and $d2$.

In this contest each group $j \in \{a, d\}$ in each team $i \in \{1, 2\}$ maximizes

$$U_i^j\left((e_i^j, s_i^j), \cdot\right) = P_i(E_1, E_2)V_i^j - C_i^j(e_i^j, s_i^j) = \frac{E_i}{E_1 + E_2}V_i^j - e_i^j - \mu_i^j s_i^j \ .$$

Below we characterize the unique interior Nash equilibrium of this model.

Proposition 2. *In this model with directionally restricted sabotage, the aggregate efforts at the unique interior equilibrium are given by*

$$E_1^* = \gamma_1^d \mu_2^a \frac{V_1^d}{V_2^a} + \gamma_1^a \mu_2^d \frac{V_1^a}{V_2^d} \quad \text{and} \quad E_2^* = \gamma_2^d \mu_1^a \frac{V_2^d}{V_1^a} + \gamma_2^a \mu_1^d \frac{V_2^a}{V_1^d} \ .$$

Furthermore, the respective equilibrium contest and sabotage efforts are

$$s_1^{d*} = \frac{\gamma_2^a E_1^* V_2^a}{(E_1^* + E_2^*)^2} - 1, \quad s_1^{a*} = \frac{\gamma_2^d E_1^* V_2^d}{(E_1^* + E_2^*)^2} - 1 \ ,$$

$$s_2^{d*} = \frac{\gamma_1^a E_2^* V_1^a}{(E_1^* + E_2^*)^2} - 1, \quad s_2^{a*} = \frac{\gamma_1^d E_2^* V_1^d}{(E_1^* + E_2^*)^2} - 1 \ ,$$

$$e_1^{d*} = \frac{\mu_2^a \gamma_1^d E_2^* (V_1^d)^2}{V_2^a (E_1^* + E_2^*)^2}, \quad e_1^{a*} = \frac{\mu_2^d \gamma_1^a E_2^* (V_1^a)^2}{V_2^d (E_1^* + E_2^*)^2} \ ,$$

$$e_2^{d*} = \frac{\mu_1^a \gamma_2^d E_1^* (V_2^d)^2}{V_1^a (E_1^* + E_2^*)^2}, \quad e_2^{a*} = \frac{\mu_1^d \gamma_2^a E_1^* (V_2^a)^2}{V_1^d (E_1^* + E_2^*)^2} \ .$$

Such an interior equilibrium exists if and only if

$$\frac{\gamma_i^j E_{-i}^* V_i^j}{(E_1^* + E_2^*)^2} > 1$$

for every $i \in \{1, 2\}$ and $j \in \{a, d\}$; and this inequality is satisfied as long as all of the winning prizes are sufficiently large.

Proof. Consider the maximization problems for group d in team 1 and group a in team 2. The corresponding first order conditions are

$$\frac{\partial U_1^d}{\partial e_1^d} = \frac{\gamma_1^d}{1 + s_2^a} \frac{E_2}{(E_1 + E_2)^2} V_1^d - 1 = 0 \tag{3}$$

$$\frac{\partial U_1^d}{\partial s_1^d} = \frac{\gamma_2^a}{(1 + s_1^d)^2} \frac{e_2^a E_1}{(E_1 + E_2)^2} V_1^d - \mu_1^d = 0 \tag{4}$$

$$\frac{\partial U_2^a}{\partial e_2^a} = \frac{\gamma_2^a}{1 + s_1^d} \frac{E_1}{(E_1 + E_2)^2} V_2^a - 1 = 0 \tag{5}$$

$$\frac{\partial U_2^a}{\partial s_2^a} = \frac{\gamma_1^d}{(1 + s_2^a)^2} \frac{e_1^d E_2}{(E_1 + E_2)^2} V_2^a - \mu_2^a = 0 \tag{6}$$

From (3) we get

$$\frac{\gamma_1^d E_2 V_1^d}{(E_1+E_2)^2} = 1 + s_2^a \ ;$$

and from (6) we get

$$\frac{\gamma_1^d E_2 V_2^a}{(E_1+E_2)^2} = \mu_2^a \frac{(1+s_2^a)^2}{e_1^d} \ .$$

Thus

$$e_1^d = \frac{\mu_2^a \gamma_1^d E_2 (V_1^d)^2}{V_2^a (E_1+E_2)^2} \ .$$

In a similar manner, we can write all equilibrium contest and sabotage efforts in terms of each γ_i^j, V_i^j, μ_i^j, and E_i. These values are

$$1 + s_1^d = \frac{\gamma_2^a E_1 V_2^a}{(E_1+E_2)^2}, \quad 1 + s_1^a = \frac{\gamma_2^d E_1 V_2^d}{(E_1+E_2)^2} \ , \tag{7}$$

$$1 + s_2^d = \frac{\gamma_1^a E_2 V_1^a}{(E_1+E_2)^2}, \quad 1 + s_2^a = \frac{\gamma_1^d E_2 V_1^d}{(E_1+E_2)^2} \ , \tag{8}$$

$$e_1^d = \frac{\mu_2^a \gamma_1^d E_2 (V_1^d)^2}{V_2^a (E_1+E_2)^2}, \quad e_1^a = \frac{\mu_2^d \gamma_1^a E_2 (V_1^a)^2}{V_2^d (E_1+E_2)^2} \ , \tag{9}$$

$$e_2^d = \frac{\mu_1^a \gamma_2^d E_1 (V_2^d)^2}{V_1^a (E_1+E_2)^2}, \quad e_2^a = \frac{\mu_1^d \gamma_2^a E_1 (V_2^a)^2}{V_1^d (E_1+E_2)^2} \ . \tag{10}$$

Notice that we also have

$$\frac{e_2^a}{(1+s_1^d)} = \mu_1^d \frac{V_2^a}{V_1^d};$$

and by symmetry,

$$\frac{e_1^a}{(1+s_2^d)} = \mu_2^d \frac{V_1^a}{V_2^d} \ ; \quad \frac{e_2^d}{(1+s_1^a)} = \mu_1^a \frac{V_2^d}{V_1^a} \ ; \ \text{ and } \frac{e_1^d}{(1+s_2^a)} = \mu_2^a \frac{V_1^d}{V_2^a} \ .$$

Thus

$$E_1^* = \gamma_1^d \frac{e_1^d}{1+s_2^a} + \gamma_1^a \frac{e_1^a}{1+s_2^d} = \gamma_1^d \mu_2^a \frac{V_1^d}{V_2^a} + \gamma_1^a \mu_2^d \frac{V_1^a}{V_2^d}$$

and

$$E_2^* = \gamma_2^d \frac{e_2^d}{1+s_1^a} + \gamma_2^a \frac{e_2^a}{1+s_1^d} = \gamma_2^d \mu_1^a \frac{V_2^d}{V_1^a} + \gamma_2^a \mu_1^d \frac{V_2^a}{V_1^d} \ .$$

By replacing these E_1^* and E_2^* values into the equations (7)–(10), we can write all equilibrium contest and sabotage efforts.

Finally, from the equations (7)–(10), we can derive the necessary and sufficient conditions for the existence of an interior equilibrium: Given positive aggregate efforts for both teams, an interior equilibrium exists if and only if for every $i \in \{1, 2\}$ and $j \in \{a, d\}$: we have $s_i^j > 0$, i.e.,

$$\frac{\gamma_i^j E_{-i} V_i^j}{(E_1 + E_2)^2} > 1.$$

And these conditions are satisfied when all of the winning prizes are sufficiently large.[7] This completes the proof. □

For comparative statics results, we focus on team 1. The equilibrium aggregate effort E_1^* increases in the effectiveness parameters and winning prizes for both groups in team 1, as well as in the cost of sabotage for both groups in team 2. Furthermore, E_1^* decreases when the winning prize for either of the groups in team 2 increases.

Comparative statics for the equilibrium contest and sabotage efforts are not straightforward. Accordingly, we concentrate on the ratio of equilibrium efforts. For instance, considering the ratio

$$\frac{e_1^{d*}}{e_1^{a*}} = \frac{\mu_2^a \gamma_1^d V_2^d (V_1^d)^2}{\mu_2^d \gamma_1^a V_2^a (V_1^a)^2},$$

we see that the *relative* contest effort of group d in team 1 increases when there is an increase in the effectiveness parameter for this group, or in the cost of sabotage for the respective saboteur, or in the winning prize for group d in either of the teams. The converse would be true for the parameters in the denominator. As for the sabotage activity, considering the ratio

$$\frac{1 + s_1^{d*}}{1 + s_1^{a*}} = \frac{\gamma_2^a V_2^a}{\gamma_2^d V_2^d},$$

we see that the *relative* sabotage effort of group d in team 1 increases when there is an increase either in the effectiveness parameter or the winning prize

[7] Notice that if all winning prizes are multiplied by the same scalar, then the equilibrium values for E_1 and E_2 remain unchanged. Accordingly, for any quadruple of winning prizes, there exists a scalar above which the respective winning prizes lead to positive sabotage efforts for all players.

for the respective victim: group a in team 2. The converse would be true for the parameters in the denominator.

The current model yields an interesting insight regarding free-riding. As highlighted in Remark 1, our result in the baseline model is related to the findings in the optimal contract literature following the seminal work by Holmström (1982). This literature focuses on free-rider problems among team members and studies optimal contract design to resolve such problems (see McAfee and McMillan, 1991; Itoh, 1991; Vander Veen, 1995; Gershkov et al., 2009 among others). By contrast, in the current paper, we show that an optimal contract analysis may not be necessary in the sense that none of the groups free-rides once they are given an option to sabotage their opponents.

Remark 2. *Although there exits a free-rider problem in the baseline model with no sabotage, our model with directionally restricted sabotage has a unique interior equilibrium for which none of the groups free-rides. This indicates that the possibility of sabotage turns out to be a natural solution to free-riding.*

Recall that $\gamma_i^j V_i^j$ denotes a measure for *motivation* of group $j \in \{a, d\}$ in team $i \in \{1, 2\}$. Then our equilibrium analysis for the baseline model suggests that the relatively less motivated group in a team free-rides if there is no option to sabotage. However, when sabotage becomes available, the *new* motivation of group d in team 1 reduces to

$$\frac{\gamma_1^d}{1 + s_2^{a*}} V_1^d = \frac{(E_1^* + E_2^*)^2}{E_2^*}.$$

at the unique interior equilibrium. The new motivation of group a in team 1 after being sabotaged turns out to be the same. Since both groups are now equally motivated, both groups are willing to exert positive contest efforts at the equilibrium. It is worth noting that this does not lead to a multiplicity of equilibria since equilibrium contest efforts should be chosen in such a way that they are consistent with the equilibrium sabotage efforts.

From an efficiency perspective, this result deserves a further discussion. Exerting more effort in a contest with an exogenously given winning prize is arguably inefficient, since exerting effort has some cost, but has no influence on the winning prize. This implies that solving the free-rider problem would be undesirable. Yet, it may be desirable in certain situations where the third parties (e.g., spectators in sport contests, consumers in a competitive market, etc.) benefit from increased contest efforts. These issues aside, one should keep in mind that the sabotage act is not proposed as a rule or mechanism in order to solve the free-rider problem in this paper, but rather it turns out to be a natural solution after being introduced as a realistic extension to the baseline model with no sabotage. This highlights that a designer who has efficiency concerns should be extra careful studying such team contests and should avoid using an oversimplified model[8] which might be very misleading.

Furthermore, our findings also oppose a common result in the literature on sabotage in contests/tournaments. More precisely, it is commonly argued in this literature that the prospect of being sabotaged has a discouragement effect in the sense that contestants exert lower equilibrium efforts in comparison to what they would choose in case of no sabotage. By contrast, we show here that both directions are possible. For instance, in our baseline model with no sabotage, *only* the relatively more motivated groups exert positive contest efforts. When there is a possibility of sabotage, those groups *might* exert lower contest efforts at the equilibrium depending on the model parameters, meaning that they *might* be discouraged. On the other hand, the free-riding groups start contributing to their teams' aggregate efforts, meaning that they are not discouraged, but are even *encouraged.* We relate this result to the collective nature of our model.

Remark 3. *For the free-riding group in the baseline model, a possibility of sabotage has an encouragement effect.*

It is worth mentioning that even when the model is extended to include teams with n groups, we would obtain qualitatively similar results. To be more precise, if there exist k groups with the highest motivation in a team, then the remaining $n-k$ groups would free-ride in the baseline model; and after a sabotage option is introduced, some of these groups would start exerting positive contest efforts at the equilibrium given that their respective interior equilibrium conditions are satisfied.

Finally, in this paper we study a particular team contest with *directionally restricted* sabotage which is played on a network with given sabotage links. We could also consider a case in which a group in one team can sabotage any group in the opposing team. In order to provide some insights to the interested reader, such an analysis is given in the Appendix.

[8]An oversimplified model refers to a model that disregards sabotage although the real-life scenario to be explained includes a sabotage act. Apparently, such an oversimplified model makes significantly different predictions.

3 Team Owner's Problems

In this section we study the team owner's problems of optimal design. Throughout this section, we restrict ourselves to cases in which there exists an interior equilibrium. As specified earlier, this can easily be achieved if the winning prize V_i^j is sufficiently high for every $i \in \{1, 2\}$ and $j \in \{a, d\}$.

3.1 Prize Allocation

Assume that the owner of team i decides on how to distribute a total prize of V_i among the groups a and d in case of winning. Accordingly, the respective constraints are $V_1^a + V_1^d = V_1$ and $V_2^a + V_2^d = V_2$. Obviously, given a strategy for the opposing team owner, the team owner's objective is to maximize his team's winning probability. More precisely, the owner of team 1 maximizes

$$P_1(E_1, E_2) = \frac{E_1}{E_1 + E_2} = \frac{\gamma_1^d \mu_2^a \frac{V_1^d}{V_2^a} + \gamma_1^a \mu_2^d \frac{V_1^a}{V_2^d}}{\left(\gamma_1^d \mu_2^a \frac{V_1^d}{V_2^a} + \gamma_1^a \mu_2^d \frac{V_1^a}{V_2^d}\right) + \left(\gamma_2^d \mu_1^a \frac{V_2^d}{V_1^a} + \gamma_2^a \mu_1^d \frac{V_2^a}{V_1^d}\right)} .$$

The following proposition shows the optimal allocation of prize shares.

Proposition 3. *In the model with directionally restricted sabotage, the owner of team 1 should allocate a total prize of V_1 according to*

$$V_1^{d*} = \frac{V_1}{2}\left(\frac{\gamma_1^d \mu_2^a}{\gamma_1^d \mu_2^a + \gamma_1^a \mu_2^d} + \frac{\gamma_2^a \mu_1^d}{\gamma_2^d \mu_1^a + \gamma_2^a \mu_1^d}\right)$$

and

$$V_1^{a*} = \frac{V_1}{2}\left(\frac{\gamma_1^a \mu_2^d}{\gamma_1^d \mu_2^a + \gamma_1^a \mu_2^d} + \frac{\gamma_2^d \mu_1^a}{\gamma_2^d \mu_1^a + \gamma_2^a \mu_1^d}\right)$$

in order to maximize his team's winning probability.

Proof. See the Appendix. □

For comparative statics, we concentrate on V_1^{d*}. It is easy to see that if the expression in parenthesis is greater than 1, then $V_1^{d*} > V_1/2 > V_1^{a*}$, i.e., the owner of team 1 prefers to allocate a higher prize share to group d. This occurs when γ_1^d, γ_2^a, μ_1^d, and μ_2^a are sufficiently high. This is quite intuitive, since these parameters represent the significance of group d in team 1: γ_1^d is the effectiveness parameter for this group; γ_2^a is the effectiveness parameter for the group in the opposing team which has a sabotage link with this group; and μ_1^d and μ_2^a are the respective costs of sabotage for these two groups. Accordingly, if γ_1^d increases, the contest effort by group d in team 1 should be incentivized more; if γ_2^a increases, the sabotage effort at group a in team 2 should be incentivized more; if μ_1^d increases, the sabotage effort by group d in team 1 should be compensated; and if μ_2^a increases, the contest effort by group d in team 1 should be incentivized even further since group a in team 2 is to be compensated for its sabotage at the equilibrium.

Let $\gamma_1^d\mu_2^a$ be defined as the *weighted effectiveness* of group d in team 1. As μ_2^a decreases, this group is apt to be sabotaged more, which in turn decreases the group's effectiveness; so that the weighted effectiveness somehow captures the effectiveness of a group depending on its adversary. Now, returning back to V_1^{d*}, we can reinterpret our result: The owner of team 1 allocates the half of the prize V_1 proportional to the weighted effectiveness of the groups in his team and the rest of the prize proportional to the weighted effectiveness of their respective adversaries.

Obviously, the optimal prize shares for team 2 can be expressed and interpreted in a symmetric manner.

3.2 Team Formation

Here we model the transfer market. Consider the situation where a team owner has a choice of forming the team under a given budget constraint. In particular, we let the team owner choose any effectiveness level for each group in his team: γ^a and γ^d. Since μ_i^j is not a choice variable for the owner of team $i \in \{1,2\}$, in order for this analysis to be meaningful, we assume that $\mu_1^a = \mu_2^a = \mu^d$ and $\mu_1^d = \mu_2^d = \mu^a$.[9]

Under the assumption that the cost of hiring a player with an effectiveness parameter γ is γ^α where $\alpha > 1$, the owner of team 1 aims to maximize

$$P_1(E_1, E_2) = \frac{E_1}{E_1 + E_2} = \frac{\gamma_1^d\mu^a\frac{V_1^d}{V_2^a} + \gamma_1^a\mu^d\frac{V_1^a}{V_2^d}}{\left(\gamma_1^d\mu^a\frac{V_1^d}{V_2^a} + \gamma_1^a\mu^d\frac{V_1^a}{V_2^d}\right) + \left(\gamma_2^d\mu^a\frac{V_2^d}{V_1^a} + \gamma_2^a\mu^d\frac{V_2^a}{V_1^d}\right)}$$

[9] Suppose that the effectiveness parameters could be different for teams and assume without loss of generality that $\mu_1^d > \mu_2^d$. This implies that team 1 cannot hire a defender with a sabotage cost lower than that of the defenders in team 2. This surely sounds odd. Here we simply assume that μ^j is a property of a group, but not a team.

subject to the budget constraint $(\gamma_1^a)^\alpha + (\gamma_1^d)^\alpha = \Gamma_1$. We note here that E_2 is independent of γ_1^a and γ_1^d; and as a result, this maximization problem corresponds to the maximization of E_1.[10]

The following proposition shows the optimal effectiveness parameters.

Proposition 4. *In the model with directionally restricted sabotage, given the budget constraint $(\gamma_1^a)^\alpha + (\gamma_1^d)^\alpha = \Gamma_1$, the owner of team 1 should form the team in such a way that*

$$\gamma_1^{a*} = \left(\frac{\Gamma_1}{1 + \left(\frac{\mu^a V_1^d V_2^d}{\mu^d V_2^a V_1^a} \right)^{\frac{\alpha}{\alpha-1}}} \right)^{\frac{1}{\alpha}} \quad \textit{and} \quad \gamma_1^{d*} = \left(\frac{\Gamma_1}{1 + \left(\frac{\mu^d V_2^a V_1^a}{\mu^a V_1^d V_2^d} \right)^{\frac{\alpha}{\alpha-1}}} \right)^{\frac{1}{\alpha}}$$

in order to maximize his team's winning probability.

Proof. See the Appendix. □

We see that the optimal choice of effectiveness parameter for group d in team 1 increases in μ^a, V_1^d, V_2^d, and Γ_1; whereas it decreases in μ^d, V_1^a, V_2^a, and α. Here we omit the interpretations for Γ_1 and α, as they seem to be straightforward. The owner of team 1 invests more on the defensive side, in case there is (i) an increase in μ^a which makes the sabotage by group a in team 2 more costly; (ii) an increase in V_1^d which motivates group d in team 1 to exert a higher contest effort; and (iii) an increase in V_2^d which deters group a in team 2 from exerting a higher contest effort so that group d in team 1 can further concentrate on its contest effort rather than sabotage. The converse interpretations follow for μ^d, V_1^a, and V_2^a.

Given Proposition 4, we also have

$$\frac{\gamma_1^{d*}}{\gamma_1^{a*}} = \left(\frac{\mu^a V_1^d V_2^d}{\mu^d V_2^a V_1^a} \right)^{\frac{1}{\alpha-1}}.$$

This means that when $\mu^a V_1^d V_2^d > \mu^d V_2^a V_1^a$, the owner of team 1 focuses relatively more on the defensive side. This happens when the cost of sabotage is higher for the respective saboteurs and/or when the winning prize for

[10]This eliminates the strategic interaction between the team owners. Independent of what the owner of team $-i$ does, the owner of team i would always choose the same values of effectiveness parameters for groups a and d.

group d in either of the teams is higher. On top of these, the difference between γ_1^{d*} and γ_1^{a*} decreases as the hiring cost parameter α increases.

In the analysis above, we assume that winning prizes V_i^j are exogenously given. Instead, if we consider endogenous prizes (and refer to the analysis in Subsection 3.1), since we would have $V_1^d V_2^d = V_2^a V_1^a$, the new ratio between γ_1^{d*} and γ_1^{a*} would become

$$\frac{\gamma_1^{d*}}{\gamma_1^{a*}} = \left(\frac{\mu^a}{\mu^d}\right)^{\frac{1}{\alpha-1}}.$$

This observation leads to the following remark.

Remark 4. *In the model with directionally restricted sabotage, given the budget constraint* $(\gamma_1^a)^\alpha + (\gamma_1^d)^\alpha = \Gamma_1$, *if the team owners strategically choose the allocation of prize shares, then the owner of team 1 should form the team in such a way that*

$$\gamma_1^{a*} = \left(\frac{\Gamma_1}{1+\left(\frac{\mu^a}{\mu^d}\right)^{\frac{\alpha}{\alpha-1}}}\right)^{\frac{1}{\alpha}} \quad \text{and} \quad \gamma_1^{d*} = \left(\frac{\Gamma_1}{1+\left(\frac{\mu^d}{\mu^a}\right)^{\frac{\alpha}{\alpha-1}}}\right)^{\frac{1}{\alpha}}$$

in order to maximize his team's winning probability.

It appears that when winning prizes for both groups are endogenously determined by the team owners, the effects of winning prizes on the optimal choices of effectiveness parameters are suppressed. However, the remaining part of the aforementioned interpretations are preserved.

4 Conclusion

In this study we have contributed to the burgeoning literature on team contests by introducing sabotage as an additional dimension of the contestants' strategy space. The members of a team chooses not only contest efforts which contribute to their team's aggregate effort, but also sabotage efforts directed at a particular group in the opposing team. Our analysis unveils two fundamental differences in the equilibrium strategies: (i) the discouragement effect of sabotage commonly reported in individual contests does not appear for some players in this team contest; and even more interestingly, (ii) the free-rider problem inherent in team contests disappears with the added option to sabotage.

These results highlight the undesired consequences of ignoring a factor that could be involved in the strategic trade-offs of players for the sake of simplicity. For instance, in this paper we have observed that analyzing a strategic interaction between teams that naturally includes a sabotage act via an oversimplified model in which the team members are only allowed to choose their contest efforts may create a free-rider problem that *in fact* does not exist. This indicates that a designer who is concerned about free-riding or who values the intensity of competition between teams might be misled by an oversimplified model, and therefore should not disregard the effect of sabotage on the effort choices of players. Furthermore, as sabotage turns out to be a natural solution to the free-rider problem, our model allows us to investigate two different design problems of a team owner: (i) allocation of winning prizes among team members and (ii) team formation under a given transfer budget.[11]

Finally, our results are also of interest from an experimental design perspective. The theoretical predictions of our analysis will be of practical value to experimental economists who investigate team contests in the lab. Future work may elaborate on this issue.

[11] In the baseline model with no sabotage, the free-riding result makes the team owner's problems trivial since the team owner would always concentrate on the most motivated group in his team.

References

[1] Amegashie, J. A. (2012). Productive versus destructive efforts in contests. *European Journal of Political Economy*, 28, 461–468.

[2] Baik, K. H. (1993). Effort levels in contests: The public-good prize case. *Economics Letters*, 41, 363–367

[3] Baik, K. H. (2008). Contests with group-specific public-good prizes. *Social Choice and Welfare*, 30, 103–117.

[4] Baik, K. H., Kim, I. G., & Na,S. (2001). Bidding for a group-specific public-good prize. *Journal of Public Economics*, 82, 415–429.

[5] Balafoutas, L., Lindner, F., & Sutter, M. (2012). Sabotage in tournaments: Evidence from a natural experiment. *Kyklos*, 65, 425–441.

[6] Brown, A., & Chowdhury, S. M. (2017). The hidden perils of affirmative action: Sabotage in handicap contests. *Journal of Economic Behavior and Organization*, 133, 273–284.

[7] Chen, K. P. (2003). Sabotage in promotion tournaments. *Journal of Law, Economics and Organization*, 19, 119–140.

[8] Chowdhury, S. M. & Gürtler, O. (2015). Sabotage in contests: A survey. *Public Choice*, 164: 135–155.

[9] Deutscher, C., Frick, B., Gürtler, O., & Prinz, J. (2013). Sabotage in tournaments with heterogeneous contestants: Empirical evidence from the soccer pitch. *Scandinavian Journal of Economics*, 115, 1138–1157.

[10] Dye, R. A. (1984). The trouble with tournaments. *Economic Inquiry*, 22, 147–149.

[11] Garicano, L., & Palacios-Huerta, I. (2014). Making the beautiful game a bit less beautiful. In *Beautiful Game Theory — How Soccer can Help Economics* (pp. 124–150). Princeton: Princeton University Press.

[12] Gershkov, A., Li, J., Schweinzer, P. (2009). Efficient tournaments within teams. *The RAND Journal of Economics*, 40, 103-119.

[13] Gürtler, O. (2008). On sabotage in collective tournaments. *Journal of Mathematical Economics*, 44, 383–393.

[14] Gürtler, O. & Münster, J. (2010). Sabotage in dynamic tournaments. *Journal of Mathematical Economics*, 46: 179-190.

[15] Harbring, C., & Irlenbusch, B. (2005). Incentives in tournaments with endogenous prize selection. *Journal of Institutional and Theoretical Economics*, 161, 636–663.

[16] Harbring, C., & Irlenbusch, B. (2011). Sabotage in tournaments: Evidence from a laboratory experiment. *Management Science*, 57, 611–627.

[17] Harbring, C., Irlenbusch, B., Kräkel, M., & Selten, R. (2007). Sabotage in corporate contests—an experimental analysis. *International Journal of the Economics of Business*, 14, 367–392.

[18] Holmström, B. (1982). Moral hazard in teams. *The Bell Journal of Economics*, 13, 324–340.

[19] Itoh, H. (1991). Incentives to help in multi-agent situations. *Econometrica*, 59, 611–636.

[20] Konrad, K. A. (2000). Sabotage in rent-seeking contests. *Journal of Law, Economics and Organization*, 16, 155–165.

[21] Lazear, E. P. (1989). Pay equality and industrial politics. *Journal of Political Economy*, 97, 561–580.

[22] McAfee, R. P. & McMillan, J. (1991). Optimal contracts for teams. *International Economic Review*, 32, 561–577.

[23] Münster, J. (2007). Selection tournaments, sabotage, and participation. *Journal of Economics and Management Strategy*, 16, 943–970.

[24] Olson, M. (1965). The logic of collective action: public goods and the theory of groups. Harvard University Press, Cambridge.

[25] Vandegrift, D., & Yavas, A. (2010). An experimental test of sabotage in tournaments. *Journal of Institutional and Theoretical Economics*, 166, 259–285.

[26] Vander Veen, T. D. (1995). Optimal contracts for teams: A note on the results of McAfee and McMillan. *International Economic Review* 36, 1051–1056.

Appendix

Team owner's problem of prize allocation:
For given values of γ_i^j and μ_i^j for every $i \in \{1,2\}$ and $j \in \{a,d\}$, the owner of team 1 aims to maximize[12]

$$P_1(E_1,E_2) = \frac{E_1}{E_1+E_2} = \frac{\gamma_1^d\mu_2^a\frac{V_1^d}{V_2^a} + \gamma_1^a\mu_2^d\frac{V_1^a}{V_2^d}}{\left(\gamma_1^d\mu_2^a\frac{V_1^d}{V_2^a} + \gamma_1^a\mu_2^d\frac{V_1^a}{V_2^d}\right) + \left(\gamma_2^d\mu_1^a\frac{V_2^d}{V_1^a} + \gamma_2^a\mu_1^d\frac{V_2^a}{V_1^d}\right)} \quad .$$

The first order condition for team 1 yields

$$\frac{\partial E_1}{\partial V_1^d}E_2 = \frac{\partial E_2}{\partial V_1^d}E_1 \quad .$$

Now, considering that V_1^a and V_1^d are dependent variables, we have

$$\left(\gamma_1^d\mu_2^a\frac{1}{V_2^a} - \gamma_1^a\mu_2^d\frac{1}{V_2^d}\right)E_2 = \left(\gamma_2^d\mu_1^a\frac{V_2^d}{(V_1^a)^2} - \gamma_2^a\mu_1^d\frac{V_2^a}{(V_1^d)^2}\right)E_1 \quad .$$

Similarly, for team 2 we get

$$\left(\gamma_2^d\mu_1^a\frac{1}{V_1^a} - \gamma_2^a\mu_1^d\frac{1}{V_1^d}\right)E_1 = \left(\gamma_1^d\mu_2^a\frac{V_1^d}{(V_2^a)^2} - \gamma_1^a\mu_2^d\frac{V_1^a}{(V_2^d)^2}\right)E_2 \quad .$$

Multiplying these two equations side by side, canceling out E_1E_2 from both sides, and then multiplying both sides with $(V_1^dV_1^aV_2^dV_2^a)^2$ we obtain

$$\begin{aligned}&\left(\gamma_1^d\mu_2^aV_2^d - \gamma_1^a\mu_2^dV_2^a\right)\left(\gamma_2^d\mu_1^aV_1^d - \gamma_2^a\mu_1^dV_1^a\right)V_1^dV_1^aV_2^dV_2^a = \\ &\qquad \left[\gamma_2^d\mu_1^aV_2^d(V_1^d)^2 - \gamma_2^a\mu_1^dV_2^a(V_1^a)^2\right]\left[\gamma_1^d\mu_2^aV_1^d(V_2^d)^2 - \gamma_1^a\mu_2^dV_1^a(V_2^a)^2\right] \quad .\end{aligned}$$

[12] Note that

$$\begin{aligned}\left(\frac{f(x)}{f(x)+g(x)}\right)' &= \frac{f'(x)(f(x)+g(x)) - f(x)(f(x)'+g(x)')}{(f(x)+g(x))^2} \\ &= \frac{f'(x)g(x) - f(x)g'(x)}{(f(x)+g(x))^2}\end{aligned}$$

so that in order for the above expression to be equal to zero, it must be that

$$f'(x)g(x) = f(x)g'(x).$$

After arranging terms, we are left with

$$\gamma_1^d\gamma_2^d\mu_1^a\mu_2^a(V_1^d)^2V_1^a(V_2^d)^2V_2^a+\gamma_1^a\gamma_2^a\mu_1^d\mu_2^dV_1^d(V_1^a)^2V_2^d(V_2^a)^2=$$
$$\gamma_1^d\gamma_2^d\mu_1^a\mu_2^a(V_1^d)^3(V_2^d)^3+\gamma_1^a\gamma_2^a\mu_1^d\mu_2^d(V_1^a)^3(V_2^a)^3 \ .$$

And this equality can be rewritten as

$$\left[\gamma_1^d\gamma_2^d\mu_1^a\mu_2^a(V_1^dV_2^d)^2+\gamma_1^a\gamma_2^a\mu_1^d\mu_2^d(V_1^aV_2^a)^2\right](V_1^dV_2^d-V_1^aV_2^a)=0 \ .$$

Since the first term is positive, it must be that $V_1^dV_2^d=V_1^aV_2^a$. From this equation we find

$$\frac{V_1^d}{V_2^a}=\frac{V_1^a}{V_2^d}=\frac{V_1^d+V_1^a}{V_2^d+V_2^a}=\frac{V_1}{V_2} \ .$$

Then, for the sake of expositional simplicity, we set

$$\rho_1^d=\gamma_1^d\mu_2^a,\quad \rho_2^d=\gamma_2^d\mu_1^a,\quad \rho_1^a=\gamma_1^a\mu_2^d,\quad \text{and } \rho_2^a=\gamma_2^a\mu_1^d \ .$$

Now we can write

$$E_1=\gamma_1^d\mu_2^a\frac{V_1^d}{V_2^a}+\gamma_1^a\mu_2^d\frac{V_1^a}{V_2^d}=\frac{V_1}{V_2}(\rho_1^d+\rho_1^a)$$

and

$$E_2=\gamma_2^d\mu_1^a\frac{V_2^d}{V_1^a}+\gamma_2^a\mu_1^d\frac{V_2^a}{V_1^d}=\frac{V_2}{V_1}(\rho_2^d+\rho_2^a) \ .$$

Following some algebraic operations, the first order condition for team 1 can be rewritten as

$$\left(\rho_1^d\frac{V_1}{V_2V_1^d}-\rho_1^a\frac{V_1}{V_2V_1^a}\right)\frac{V_2}{V_1}(\rho_2^d+\rho_2^a)=\left(\rho_2^d\frac{V_2}{V_1V_1^a}-\rho_2^a\frac{V_2}{V_1V_1^d}\right)\frac{V_1}{V_2}(\rho_1^d+\rho_1^a) \ .$$

Canceling out all V_1 and V_2, we have

$$\frac{\rho_1^d(\rho_2^d+\rho_2^a)+\rho_2^a(\rho_1^d+\rho_1^a)}{V_1^d}=\frac{\rho_2^d(\rho_1^d+\rho_1^a)+\rho_1^a(\rho_2^d+\rho_2^a)}{V_1^a} \ .$$

From this equality we find

$$V_1^d=\frac{V_1\left[\rho_1^d(\rho_2^d+\rho_2^a)+\rho_2^a(\rho_1^d+\rho_1^a)\right]}{2(\rho_2^d+\rho_2^a)(\rho_1^d+\rho_1^a)}=\frac{V_1}{2}\left(\frac{\rho_1^d}{\rho_1^d+\rho_1^a}+\frac{\rho_2^a}{\rho_2^d+\rho_2^a}\right) \ .$$

Finally, returning back to the standard notation, we have

$$V_1^{d*} = \frac{V_1}{2}\left(\frac{\gamma_1^d\mu_2^a}{\gamma_1^d\mu_2^a + \gamma_1^a\mu_2^d} + \frac{\gamma_2^a\mu_1^d}{\gamma_2^d\mu_1^a + \gamma_2^a\mu_1^d}\right).$$

The optimal share for the other group is simply $V_1^{a*} = V_1 - V_1^{d*}$. $\square$

Team owner's problem of team formation:
Now that the strategic interaction is absent, the analysis turns out to be much simpler. The first order condition with respect to γ_1^d is

$$\mu^a \frac{V_1^d}{V_2^a} + \mu^d \frac{V_1^a}{V_2^d}\frac{\partial \gamma_1^a}{\partial \gamma_1^d} = 0 \ .$$

Moreover, from the derivative of the budget constraint, it follows that

$$\alpha(\gamma_1^a)^{\alpha-1}\frac{\partial \gamma_1^a}{\partial \gamma_1^d} + \alpha(\gamma_1^d)^{\alpha-1} = 0$$

which implies

$$\frac{\partial \gamma_1^a}{\partial \gamma_1^d} = -\left(\frac{\gamma_1^d}{\gamma_1^a}\right)^{\alpha-1} \ .$$

Using this information in the first order condition above, we have

$$\mu^a \frac{V_1^d}{V_2^a} - \mu^d \frac{V_1^a}{V_2^d}\left(\frac{\gamma_1^d}{\gamma_1^a}\right)^{\alpha-1} = 0$$

so that

$$\frac{\gamma_1^d}{\gamma_1^a} = \left(\frac{\mu^a V_1^d V_2^d}{\mu^d V_2^a V_1^a}\right)^{\frac{1}{\alpha-1}} \ .$$

Then putting this finding into the budget constraint, we find

$$\gamma_1^{a*} = \left(\frac{\Gamma_1}{1 + \left(\frac{\mu^a V_1^d V_2^d}{\mu^d V_2^a V_1^a}\right)^{\frac{\alpha}{\alpha-1}}}\right)^{\frac{1}{\alpha}} \quad \text{and} \quad \gamma_1^{d*} = \left(\frac{\Gamma_1}{1 + \left(\frac{\mu^d V_2^a V_1^a}{\mu^a V_1^d V_2^d}\right)^{\frac{\alpha}{\alpha-1}}}\right)^{\frac{1}{\alpha}} \ . \quad \square$$

The alternative model with restricted sabotage:
In this paper we have considered *directionally restricted* sabotage allowing each group in a team to sabotage a certain group in the opposing team. Here we relax this assumption and analyze the case with *directed* sabotage: Any group in a team can sabotage any group in the opposing team. Similar to our original model, e_i^j denotes the contest effort by group $j \in \{a, d\}$ in team $i \in \{1, 2\}$. As for the sabotage efforts we need a new notation including information regarding the origin and the destination of sabotage. Let s_i^{jk} denote the sabotage made by group $j \in \{a, d\}$ in team $i \in \{1, 2\}$ towards group $k \in \{a, d\}$ in the opposing team. Also let μ_i^{jk} denote the corresponding cost of this sabotage activity.

Then we can write the aggregate effort functions as follows:

$$E_1 = \frac{\gamma_1^a}{1 + s_2^{aa} + s_2^{da}} e_1^a + \frac{\gamma_1^d}{1 + s_2^{ad} + s_2^{dd}} e_1^d$$

and

$$E_2 = \frac{\gamma_2^a}{1 + s_1^{aa} + s_1^{da}} e_2^a + \frac{\gamma_2^d}{1 + s_1^{ad} + s_1^{dd}} e_2^d \ .$$

Letting

$$C_i^j(e_i^j, s_i^{ja}, s_i^{jd}) = e_i^j + \mu_i^{ja} s_i^{ja} + \mu_i^{jd} s_i^{jd},$$

we say that group $j \in \{a, d\}$ in team $i \in \{1, 2\}$ maximizes

$$U_i^j \left((e_i^j, s_i^{ja}, s_i^{jd}), \cdot \right) = \frac{E_i}{E_1 + E_2} V_i^j - e_i^j - \mu_i^{ja} s_i^{ja} - \mu_i^{jd} s_i^{jd} \ .$$

Utilizing the first order conditions with respect to s_1^{aa} and s_1^{da}, we get

$$\frac{\partial U_1^a}{\partial s_1^{aa}} = \frac{\gamma_2^a}{(1 + s_1^{da} + s_1^{aa})^2} \frac{e_2^a E_1}{(E_1 + E_2)^2} V_1^a - \mu_1^{aa} = 0$$

$$\frac{\partial U_1^d}{\partial s_1^{da}} = \frac{\gamma_2^a}{(1 + s_1^{da} + s_1^{aa})^2} \frac{e_2^a E_1}{(E_1 + E_2)^2} V_1^d - \mu_1^{da} = 0 \ .$$

In order for these first order conditions to be satisfied simultaneously, it must be that

$$\frac{V_1^a}{\mu_1^{aa}} = \frac{V_1^d}{\mu_1^{da}}.$$

Otherwise, we must have a corner solution. To put it differently, unless this equality is satisfied, a directionally restricted sabotage would be observed in the equilibrium. Below we further elaborate on this issue.

Note that if any of the derivatives with respect to s_1^{aa} and s_1^{da} are positive, then a marginal increment in the corresponding variable would be a possible deviation. Therefore none can be positive at an equilibrium. This implies that if one of the first order conditions is satisfied, then the derivative with respect to the other variable should be negative, which corresponds to a corner solution for that variable. For more concrete arguments, assume without loss of generality that

$$\frac{V_1^a}{\mu_1^{aa}} < \frac{V_1^d}{\mu_1^{da}}.$$

If the former first order condition is satisfied, then the derivative with respect to s_1^{da} would be positive. This cannot happen at an equilibrium. Then, it must be that the latter first order condition is satisfied, meaning that there will be a corner solution for s_1^{aa}, which is $s_1^{aa} = 0$.

Finally, given our model interpretation, it is reasonable to assume that $\mu_1^{aa} > \mu_1^{da}$. This is because the defenders in team 1 are located closer to the attackers in team 2 in comparison to the attackers in team 1, so that if the attackers in team 2 are to be sabotaged, then the defenders in team 1 should have a cost lower than that of the attackers in team 1.[13] Accordingly, for a wide range of V_1^a, V_1^d, V_2^a, and V_2^d values, the current model would reduce to our model with directionally restricted sabotage at the equilibrium. $\square$

[13]We are referring to the football game or war interpretation here. If we consider the political party interpretation, we would expect that $\mu_1^{aa} < \mu_1^{da}$ since group a in team 1 is now closer to group a in team 2 than group d in team 1 is.