# The Value of Multi-Stage Stochastic Programming in Risk-Averse Unit Commitment Under Uncertainty

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***Abstract*—Day-ahead scheduling of electricity generation or unit commitment is an important and challenging optimization problem in power systems. Variability in net load arising from the increasing penetration of renewable technologies has motivated study of various classes of stochastic unit commitment models. In two-stage models, the generation schedule for the entire day is fixed while the dispatch is adapted to the uncertainty, whereas in multi-stage models the generation schedule is also allowed to dynamically adapt to the uncertainty realization. Multi-stage models provide more flexibility in the generation schedule; however, they require significantly higher computational effort than two-stage models. To justify this additional computational effort, we provide theoretical and empirical analyses of the value of multi-stage solution for risk-averse multi-stage stochastic unit commitment models. The value of multi-stage solution measures the relative advantage of multi-stage solutions over their two-stage counterparts. Our results indicate that, for unit commitment models, the value of multi-stage solution increases with the level of uncertainty and number of periods, and decreases with the degree of risk aversion of the decision maker.**

***Index Terms*—Unit commitment, risk-averse optimization, stochastic programming.**

## I. Introduction

*Unit commitment* (UC) is a challenging optimization problem used for day-ahead generation scheduling given net load forecasts and various operational constraints [1]. The output schedule includes on-off status of generators and the production amounts, called *economic dispatch* [2], for every time step.

There has been a great deal of research on deterministic UC models where the problem parameters are assumed to be known exactly [3]. These models cannot capture variability and uncertainty. Common sources of uncertainty are departures from forecasts and unreliable equipment. The departures from forecasts generally stem from the variability in net load and production amounts, whereas unreliable equipment may result in generator and transmission line outages [2], [4]. The penetration of renewable energy has increased the volatility of power systems in recent years. The production amount of energy from wind and solar power are not controllable but can only be forecasted [5].

*Robust optimization* and *stochastic programming* are two common frameworks used to address the uncertainty in UC problems. In robust optimization models, it is assumed that the uncertain parameters take values in some uncertainty sets and the objective is to minimize the worst case cost ([6]–[9] and [10]). In stochastic programming models, the uncertainty is represented by a probability distribution ([11]–[14] and [15]). In *two-stage* stochastic programming UC models, the generation schedule is fixed for the entire day before the beginning of the day while dispatch is adapted to uncertainty as in [16], [17] and [18]. On the other hand, in *multi-stage* stochastic programming UC models both the generation schedule and dispatch are allowed to dynamically adapt to uncertainty realization at each hour (see for example, [15], [19] and [20]). Therefore, they incorporate multistage forecasting information with varying accuracy and express relation between time periods appropriately. However, in general, the multi-stage models are computationally difficult. A detailed comparison of two- and multi-stage models can be found in [21] and [22].

In *risk-neutral* stochastic programming UC models, the objective is to minimize the expected system-wise cost. These models minimize the cost on average as a consequence of the Law of Large Numbers (see, for example, [23]), however, they ignore the risk exposure. In *risk-averse* stochastic programming UC models, in general, both the expected cost and the risk related to the cost are considered. Several risk-averse UC models are presented in [21] and references therein. Risk-averse problems are computationally intractable in existence of random problem parameters with continuous probability distributions. In that case, the original distribution is replaced with an empirical distribution obtained via sampling. The reader is referred to [24] for details. Thus, in this paper, we restrict our attention to the instances with finite number of scenarios in computational experiments even though the theoretical results hold for the general case.

The computational challenge of multi-stage models motivates the question on whether the effort to solve them is worthwhile. In [25], this question is addressed for a risk-neutral stochastic capacity planning problem. In the present paper, we address this question for risk-averse UC (RA-UC) problems where the

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objective is a dynamic measure of risk. We provide theoretical and empirical analysis on the value of the *multi stage solution* (VMS) where VMS measures the relative advantage to solve the multi-stage models over their two-stage counterparts.

The rest of the paper is organized as follows: In Section II, we define the RA-UC problem and present two- and multi-stage stochastic models. In Section III, we define VMS and provide analytical bounds for it. In Section IV, we present results of computational experiments. In Section V, we discuss possible future extensions of the current work.

## II. Risk-averse Unit Commitment Problem

### A. Deterministic UC Formulation

We first present an abstract deterministic formulation of the UC problem. Let $I$ be the number of generators and $T$ be the number of periods. Also, let $\mathcal{I} := \{1, \ldots, I\}$ and $\mathcal{T} := \{1, \ldots, T\}$ be the sets of generators and time periods, respectively. A canonical formulation of the UC problem is as follows:

$$\min \sum_{t=1}^{T} f_t(\boldsymbol{u}_t, \boldsymbol{v}_t, \boldsymbol{w}_t) \tag{1}$$

$$\text{s.t.} \sum_{i=1}^{I} v_{it} \geq d_t, \ \forall t \in \mathcal{T} \tag{2}$$

$$\underline{q}_i u_{it} \leq v_{it} \leq \overline{q}_i u_{it}, \ \forall i \in \mathcal{I}, t \in \mathcal{T} \tag{3}$$

$$(\boldsymbol{u}_1, \boldsymbol{v}_1, \boldsymbol{w}_1) \in \mathcal{X}_1, \tag{4}$$

$$(\boldsymbol{u}_t, \boldsymbol{v}_t, \boldsymbol{w}_t) \in \mathcal{X}_t(\boldsymbol{u}_{t-1}, \boldsymbol{v}_{t-1}, \boldsymbol{w}_{t-1}), \ \forall t \in \mathcal{T} \setminus \{1\} \tag{5}$$

$$\boldsymbol{u}_t \in \{0,1\}^I, \boldsymbol{v}_t \in \mathbb{R}_+^I, \boldsymbol{w}_t \in \mathbb{R}^k, \ \forall t \in \mathcal{T} \tag{6}$$

Decision variables $u_{it}$ and $v_{it}$ represent the binary on/off status and production of generator $i \in \mathcal{I}$ in period $t \in \mathcal{T}$, respectively. The bold symbols $\boldsymbol{u}_t := (u_{1t}, u_{2t}, \ldots, u_{It})$ and $\boldsymbol{v}_t := (v_{1t}, v_{2t}, \ldots, v_{It})$ are the vectors of status and production decisions in period $t \in \mathcal{T}$, respectively. The vector $\boldsymbol{w}_t$ denotes auxiliary variables associated with period $t \in \mathcal{T}$. These variables can be used for modeling various operational constraints. The objective (1) is the sum of production, start-up and shutdown costs in all periods where the function $f_t(\cdot)$ represents the total cost in a period $t \in \mathcal{T}$. Constraint (2) ensures satisfaction of the power demand. Constraint (3) enforces lower and upper production limits on the generators. Other operational restrictions such as transmission capacity constraints are represented by constraints (4) and (5). The temporal relationship between consecutive periods such as start-up, ramp-up, shut-down and ramp-down restrictions can also be included in the set constraint (5). Domain restrictions of the decision variables are given by constraint (6). An explicit deterministic model is given in the Appendix.

### B. Uncertainty and Risk models

In the deterministic formulation (1)–(6), net load values are assumed to be known exactly. This is a restrictive assumption in practice. We assume that the net load is random and denoted by a random variable $\widetilde{d}_t$ in period $t \in \mathcal{T}$ from a probability space $(\Omega, \mathcal{F}, P)$. Here $\Omega$ is a sample space equipped with sigma algebra $\mathcal{F}$ and probability measure $P$. An element of the sample space $\Omega$ is called as a *scenario* (or a sample path) and represents a possible realization of the net load values in all periods. The sequence of sigma algebras $\{\emptyset, \Omega\} = \mathcal{F}_1 \subset \mathcal{F}_2 \subset \cdots \subset \mathcal{F}_T = \mathcal{F}$ is called as a *filtration* and it represents the gradually increasing information through the decision horizon $1, 2, \ldots, T$. The set of $\mathcal{F}_t-$ measurable random variables is denoted by $\mathcal{Z}_t$ for $t \in \mathcal{T}$. The random demand $\widetilde{d}_t$ in period $t$ is $\mathcal{F}_t-$ measurable, that is $\widetilde{d}_t \in \mathcal{Z}_t$ for $t \in \mathcal{T}$. Note that since $\mathcal{F}_1 = \{\emptyset, \Omega\}$ by definition, $\mathcal{Z}_1 = \mathbb{R}$ and the demand in the first period is deterministic.

To extend the deterministic UC model to this uncertainty setting, we have that the decisions in period $t$ to depend on realization of the history of net load process $\widetilde{d}_{[t]} := (\widetilde{d}_1, \ldots, \widetilde{d}_t)$ up to period $t$. Therefore, we use the $\mathcal{F}_t-$ measurable vectors $\widetilde{\boldsymbol{u}}_t(\widetilde{d}_{[t]})$, $\widetilde{\boldsymbol{v}}_t(\widetilde{d}_{[t]})$ and $\widetilde{\boldsymbol{w}}_t(\widetilde{d}_{[t]})$ to represent status, production and auxiliary decisions in period $t \in \mathcal{T}$, respectively. The total cost at period $t$ is also $\mathcal{F}_t-$ measurable, i.e., $f_t(\widetilde{\boldsymbol{u}}_t(\widetilde{d}_{[t]}), \widetilde{\boldsymbol{v}}_t(\widetilde{d}_{[t]}), \widetilde{\boldsymbol{w}}_t(\widetilde{d}_{[t]})) \in \mathcal{Z}_t$. We use conditional risk measures in order to quantify the risk involved in a random cost at period $t+1$ based on the available informations at period $t$ for $t \in \mathcal{T} \setminus \{T\}$. The mapping $\rho_t : \mathcal{Z}_{t+1} \to \mathcal{Z}_t$ is called a *conditional risk measure* if it satisfies the following four axioms of coherent risk measures (the subscript $t$ is suppressed for notational brevity):

A1) *Convexity*: $\rho(\alpha Z + (1-\alpha)W) \leq \alpha\rho(Z) + (1-\alpha)\rho(W)$ for all $Z, W \in \mathcal{Z}$ and $\alpha \in [0,1]$,

A2) *Monotonicity*: $Z \succeq W$ implies $\rho(Z) \geq \rho(W)$ for all $Z, W \in \mathcal{Z}$,

A3) *Translational Equivariance*: $\rho(Z + c) = \rho(Z) + c$ for all $c \in \mathbb{R}$ and $Z \in \mathcal{Z}$,

A4) *Positive Homogeneity*: $\rho(cZ) = c\rho(Z)$ for all $c > 0$ and $Z \in \mathcal{Z}$,

where $Z \succeq W$ indicates point-wise partial ordering defined on set $\mathcal{Z}$. See [23] and [26] for a detailed discussions on coherent and conditional risk measures. An example of a conditional risk measure is the *conditional mean-upper semi deviation*

$$\rho_t(Z_{t+1}) = \mathbb{E}[Z_{t+1}|\mathcal{F}_t] + \lambda\mathbb{E}[(Z_{t+1} - \mathbb{E}[Z_{t+1}|\mathcal{F}_t])_+|\mathcal{F}_t], \tag{7}$$

where $\mathbb{E}[Z_{t+1}|\mathcal{F}_t]$ is the conditional expectation with respect to the sigma algebra $\mathcal{F}_t$, $\lambda \in [0,1]$ is a parameter controlling the degree of risk aversion and $(Z_{t+1})_+$ is the point-wise positive part function for all $Z_{t+1} \in \mathcal{Z}_{t+1}$.

The objective of the risk averse UC (RA-UC) problem is to minimize the risk involved with the cost sequence $\{Z_t\}_{t=1}^T$ where $Z_t := f_t(\widetilde{\boldsymbol{u}}_t(\widetilde{d}_{[t]}), \widetilde{\boldsymbol{v}}_t(\widetilde{d}_{[t]}), \widetilde{\boldsymbol{w}}_t(\widetilde{d}_{[t]}))$ is a shorthand notation for the total cost in period $t \in \mathcal{T}$. Thus, as in [23] and [27], we define the dynamic coherent risk measure $\varrho : \mathcal{Z}_1 \times \mathcal{Z}_2 \times \cdots \times \mathcal{Z}_T \to \mathbb{R}$ by using nested composition of the conditional risk measures $\rho_1(\cdot), \rho_2(\cdot), \ldots, \rho_{T-1}(\cdot)$, that is,

$$\varrho(Z_1, Z_2, \ldots, Z_T) := Z_1 + \rho_1(Z_2 + \cdots \rho_{T-1}(Z_T) \cdots)$$

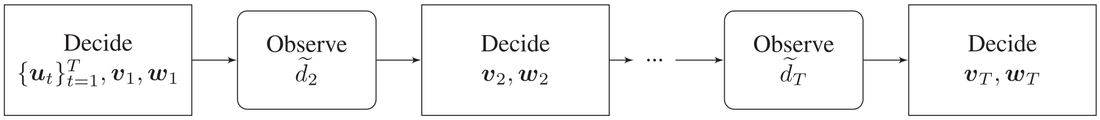



Fig. 1. Order of decisions in the two-stage model.

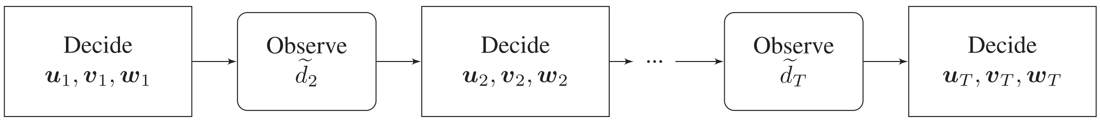



Fig. 2. Order of decisions in the multi-stage model.

is the risk associated with this cost sequence. Due to translational equivariance property of conditional risk measures, we have an alternative representation of the dynamic coherent measure of risk $\varrho(\cdot)$ as

$$\rho\left(\sum_{t=1}^{T} Z_t\right) := \varrho(Z_1, Z_2, \ldots, Z_T) \tag{8}$$

where $\rho = \rho_1 \circ \rho_2 \circ \cdots \circ \rho_{T-1} : \mathcal{Z} \to \mathbb{R}$ is called as a *composite risk measure* and $\mathcal{Z} := \mathcal{Z}_T$. The composite risk measure $\rho(\cdot)$ satisfies the coherence axioms (A1)–(A4). Therefore, $\rho(\cdot)$ is a coherent risk measure as shown in [23, Eqn. 6.234].

### *C. Two-stage and Multi-stage Models*

We consider two different models for the RA-UC problem. In the *two-stage model*, the on/off status decisions are fixed at the beginning of the day and production (or dispatch) decisions are adapted to uncertainty in the random demand. On the other hand, in the *multi-stage model*, both the status and production decisions are fully adapted to uncertainty in net load. In order to clarify the distinction between two models, the decision dynamics in the two- and multi-stage models are depicted as in Fig. 1 and Fig. 2, respectively.

The two-stage model (TS) for the RA-UC problem is given as

$$\min\ \rho\left[\sum_{t=1}^{T} f_t(\boldsymbol{u}_t, \widetilde{\boldsymbol{v}}_t(\widetilde{d}_{[t]}), \widetilde{\boldsymbol{w}}_t(\widetilde{d}_{[t]}))\right] \tag{9}$$

$$\text{s.t.}\ \sum_{i\in\mathcal{I}} \widetilde{v}_{it}(\widetilde{d}_{[t]}) \geq \widetilde{d}_t,\ \forall t \in \mathcal{T} \tag{10}$$

$$\underline{q}_i u_{it} \leq \widetilde{v}_{it}(\widetilde{d}_{[t]}) \leq \overline{q}_i u_{it},\ \forall i \in \mathcal{I}, t \in \mathcal{T} \tag{11}$$

$$(\boldsymbol{u}_1, \boldsymbol{v}_1, \boldsymbol{w}_1) \in \mathcal{X}_1 \tag{12}$$

$$(\boldsymbol{u}_t, \widetilde{\boldsymbol{v}}_t(\widetilde{d}_{[t]}), \widetilde{\boldsymbol{w}}_t(\widetilde{d}_{[t]})) \in \mathcal{X}_t(\boldsymbol{u}_{t-1}, \widetilde{\boldsymbol{v}}_{t-1}(\widetilde{d}_{[t-1]}), \widetilde{\boldsymbol{w}}_{t-1}(\widetilde{d}_{[t-1]}), \widetilde{d}_{[t]}),\ \forall t \in \mathcal{T}\setminus\{1\} \tag{13}$$

$$\boldsymbol{u}_t \in \{0,1\}^I, \widetilde{\boldsymbol{v}}_t(\widetilde{d}_{[t]}) \in \mathbb{R}_+^I, \widetilde{\boldsymbol{w}}_t(\widetilde{d}_{[t]}) \in \mathbb{R}^k,\ \forall t \in \mathcal{T} \tag{14}$$

The objective (9) of TS is the composite risk measure defined in (8) applied to the total cost sequence. The inequalities (10) and (11) are analogous to the constraints (2) and (3), respectively. The set constraint (12) is identical to (4) since the net load in the first period is deterministic. In constraint (13), $\mathcal{X}_t$ is an $\mathcal{F}_t-$ measurable feasibility set. The domain constraint (14) states that only production and auxiliary decisions depend on the demand history and the status decisions are deterministic. However, in the multi-stage model of the RA-UC problem, all decisions are made based on the history. Hence, the multi-stage model (MS) can be written as

$$\min\ \rho\left[\sum_{t=1}^{T} f_t(\widetilde{\boldsymbol{u}}_t(\widetilde{d}_{[t]}), \widetilde{\boldsymbol{v}}_t(\widetilde{d}_{[t]}), \widetilde{\boldsymbol{w}}_t(\widetilde{d}_{[t]}))\right] \tag{15}$$

$$\text{s.t.}\ \sum_{i\in\mathcal{I}} \widetilde{v}_{it}(\widetilde{d}_{[t]}) \geq \widetilde{d}_t,\ \forall t \in \mathcal{T} \tag{16}$$

$$\underline{q}_i \widetilde{u}_{it}(\widetilde{d}_{[t]}) \leq \widetilde{v}_{it}(\widetilde{d}_{[t]}) \leq \overline{q}_i \widetilde{u}_{it}(\widetilde{d}_{[t]}),\ \forall i \in \mathcal{I}, t \in \mathcal{T} \tag{17}$$

$$(\boldsymbol{u}_1, \boldsymbol{v}_1, \boldsymbol{w}_1) \in \mathcal{X}_1 \tag{18}$$

$$(\widetilde{\boldsymbol{u}}_t(\widetilde{d}_{[t]}), \widetilde{\boldsymbol{v}}_t(\widetilde{d}_{[t]}), \widetilde{\boldsymbol{w}}_t(\widetilde{d}_{[t]})) \in \mathcal{X}_t(\widetilde{\boldsymbol{u}}_{t-1}(\widetilde{d}_{[t-1]}), \widetilde{\boldsymbol{v}}_{t-1}(\widetilde{d}_{[t-1]}), \widetilde{\boldsymbol{w}}_{t-1}(\widetilde{d}_{[t-1]}), \widetilde{d}_{[t]}),\ \forall t \in \mathcal{T}\setminus\{1\} \tag{19}$$

$$\widetilde{\boldsymbol{u}}_t(\widetilde{d}_{[t]}) \in \{0,1\}^I, \widetilde{\boldsymbol{v}}_t(\widetilde{d}_{[t]}) \in \mathbb{R}_+^I, \widetilde{\boldsymbol{w}}_t(\widetilde{d}_{[t]}) \in \mathbb{R}^k,\ \forall t \in \mathcal{T} \tag{20}$$

Note that the multi-stage model MS is identical with TS except that the status decisions are fully adaptive to the random net load process.

An optimal solution of either TS and MS is a policy that minimizes the value of the dynamic coherent risk measure. Both in TS and MS, the optimality of a policy should only be with respect to possible future realizations given the available information at the time when the decision is made. This principle is called as *time consistency*. In [28, Example 2], it is shown that time consistency enables us to use the composite risk measure in minimization among all possible decisions instead of nested minimizations in a dynamic coherent measure of risk. We prefer conditional risk measures to represent the risk-averse behavior of decision makers since they yield time consistent formulation of the problem and their interpretation is clear.

## III. Value of the Multi-stage Solution

Although an optimal solution of MS provides a more flexible day-ahead schedule with respect to different realizations of parameters, the number of binary variables in MS is proportional to $\mathcal{N} \times I$ where $\mathcal{N}$ is the number of possible demand realizations in all periods if $\Omega$ is finite. However, the number of binary variables in TS is proportional to $T \times I$. Since $\mathcal{N} >> T$ for any non-trivial problem, computational difficulty of MS is significantly more than TS. Therefore, it is important to figure out if the additional effort to solve MS is worthwhile. We define the VMS in order to quantify the relative advantage of the multi-stage solution over their two-stage counterparts.

*Definition 1:* The value of multi-stage solution (VMS) is the difference between the optimal values of TS and MS, that is, VMS $= z^{TS} - z^{MS}$ where $z^{TS}$ and $z^{MS}$ are the optimal values of TS and MS, respectively.

Since an optimal solution of MS provides more flexibility in status decisions with respect to uncertain net load realizations, we have $z^{TS} \geq z^{MS}$ and therefore VMS $\geq 0$. The complex structure of risk-averse UC problem prohibits exact calculation of VMS unless both TS and MS are solved optimally. Even calculation of bounds for VMS is not possible for UC problem. Thus, we provide theoretical bounds on the VMS under some assumptions.

*Assumption 1:* There exists a generator $j^* \in \mathcal{I}$ such that $\underline{q}_{j^*} \leq \widetilde{d}_t \leq \overline{q}_{j^*}$ with probability 1 with no minimum start up and shut down time and no ramping limits for each $t \in \mathcal{T}$.

Assumption 1 ensures that TS and MS always have at least one feasible solution and therefore both problems have *complete recourse*. Assumption 1 holds, for example, when it is possible to outsource the unmet power demand. In that case, decisions $u_{j^*}$ and $v_{j^*}$ represent outsourcing decision and amount of outsourced energy, respectively. Alternatively, $u_{j^*}$ and $v_{j^*}$ can be used to formulate the opportunity cost due to lost demand.

*Assumption 2:* There exists an upper bound $d_t^{\max} \in \mathbb{R}_+$ on the net load values such that $0 \leq \widetilde{d}_t \leq d_t^{\max}$ with probability 1 for each $t \in \mathcal{T}$.

Assumption 2 holds in practice and states that the net load in each period is bounded. We also define $\widetilde{D} := \sum_{t=1}^{T} \widetilde{d}_t$ as the total net load and $D^{\max} := \sum_{t=1}^{T} d_t^{\max}$ as an upper bound on $\widetilde{D}$.

*Assumption 3:* The production cost at each stage is defined as $f_t(\widetilde{\boldsymbol{u}}_t(\widetilde{d}_{[t]}), \widetilde{\boldsymbol{v}}_t(\widetilde{d}_{[t]}), \widetilde{\boldsymbol{w}}_t(\widetilde{d}_{[t]})) = \sum_{i \in \mathcal{I}} g_i(\widetilde{u}_{it}(\widetilde{d}_{[t]}), \widetilde{v}_{it}(\widetilde{d}_{[t]}), \widetilde{w}_{it}(\widetilde{d}_{[t]}))$ where $g_i(\cdot)$ is sum of a fixed commitment cost and a non-decreasing convex dispatch cost for all $i \in \mathcal{I}$.

If Assumption 3 holds, the function $g_i(\cdot)$ can be written as $g_i(\widetilde{u}_{it}(\widetilde{d}_{[t]}), \widetilde{v}_{it}(\widetilde{d}_{[t]}), \widetilde{w}_{it}(\widetilde{d}_{[t]})) = a_i \widetilde{u}_{it}(\widetilde{d}_{[t]}) + h_i(\widetilde{v}_{it}(\widetilde{d}_{[t]}))$ for a coefficient $a_i \geq 0$ and a non-decreasing convex function $h_i(\cdot)$ with $h_i(0) = 0$ for all $i \in \mathcal{I}$. Assumption 3 is somewhat restrictive since it ignores start-up and shut-down costs. However this assumption is necessary for the analytical results. In Section IV, we will provide numerical results showing that the analytical results hold in instances with start-up and shut-down costs as well.

*Theorem 1:* Under Assumptions 1, 2 and 3, we have that

$$\alpha_* D^{\max} - \alpha^* \rho(\widetilde{D}) \leq \text{VMS} \leq \alpha^* D^{\max} - \alpha_* \rho(\widetilde{D}),$$

where

$$\alpha_* := \min_{i \in \mathcal{I}} \left\{ a_i + h_i(\underline{q}_i) \right\} \Big/ \max_{i \in \mathcal{I}} \{\overline{q}_i\} \text{ and}$$

$$\alpha^* := \max_{i \in \mathcal{I}} \{ a_i + h_i(\overline{q}_i) \} \Big/ \min_{i \in \mathcal{I}} \left\{ \underline{q}_i \right\}$$

are cost related problem parameters corresponding to under and over estimations on per unit production costs at each stage, respectively.

*Proof:* Assumption 1 implies that both TS and MS are feasible. Since the net loads are bounded due to Assumption 2, both models have at least one optimal solution.

Let $\{\widetilde{\boldsymbol{u}}_t^*, \widetilde{\boldsymbol{v}}_t^*, \widetilde{\boldsymbol{w}}_t^*\}_{t \in \mathcal{T}}$ be an optimal policy obtained by solving the multi-stage model MS. By Assumption 3, we have $\sum_{t \in \mathcal{T}} f_t(\widetilde{\boldsymbol{u}}_t^*(\widetilde{d}_{[t]}), \widetilde{\boldsymbol{v}}_t^*(\widetilde{d}_{[t]}), \widetilde{\boldsymbol{w}}_t^*(\widetilde{d}_{[t]})) = \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} g_i(\widetilde{u}_{it}^*(\widetilde{d}_{[t]}), \widetilde{v}_{it}^*(\widetilde{d}_{[t]}), \widetilde{w}_{it}^*(\widetilde{d}_{[t]}))$.

For a realization $d_1, d_2, \ldots, d_T$ of the random net load process $\widetilde{d}_1, \widetilde{d}_2, \ldots, \widetilde{d}_T$, let $[u_t^*, v_t^*, w_t^*] := [\widetilde{\boldsymbol{u}}_t^*, \widetilde{\boldsymbol{v}}_t^*, \widetilde{\boldsymbol{w}}_t^*](d_{[t]})$ be the optimal status and production decisions for $t \in \mathcal{T}$. Then, we have

$$\begin{aligned}
\sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} g_i(u_{it}^*, v_{it}^*, w_{it}^*) &= \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} a_i u_{it}^* + h_i(v_{it}^*) \\
&\geq \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} a_i u_{it}^* + h_i(\underline{q}_i u_{it}^*) = \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} a_i u_{it}^* + h_i(\underline{q}_i) u_{it}^* \\
&= \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} [a_i + h_i(\underline{q}_i)] u_{it}^* \geq \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} [a_i + h_i(\underline{q}_i)] \frac{v_{it}^*}{\overline{q}_i} \\
&\geq \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} \frac{\min_{i \in \mathcal{I}} \left\{ a_i + h_i(\underline{q}_i) \right\}}{\max_{i \in \mathcal{I}} \{\overline{q}_i\}} v_{it}^* \\
&= \alpha_* \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} v_{it}^* = \alpha_* \sum_{t \in \mathcal{T}} d_t,
\end{aligned}$$

where the first inequality holds due to feasibility and non-decreasing monotonicity of $h_i(\cdot)$. The second inequity also follows from feasibility. The second equality holds since $h(qu) = h(q)u$ for any function $h : \mathbb{R} \to \mathbb{R}$ with $h(0) = 0$ where $q \in \mathbb{R}_+$ and $u \in \{0, 1\}$.

Since $\sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} g_i(u_{it}^*, v_{it}^*, w_{it}^*) \geq \alpha_* \sum_{t \in \mathcal{T}} d_t$ for any sample path $d_1, d_2, \ldots, d_T$, we have $\sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} g_i(\widetilde{u}_{it}^*(\widetilde{d}_{[t]}), \widetilde{v}_{it}^*(\widetilde{d}_{[t]}), \widetilde{w}_{it}^*(\widetilde{d}_{[t]})) \succeq \alpha_* \sum_{t \in \mathcal{T}} \widetilde{d}_t = \alpha_* \widetilde{D}$. Due to the monotonicity axiom (A2) and positive homogeneity axiom (A4), we get

$$\begin{aligned}
z^{MS} &= \rho \left( \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} g_i(\widetilde{u}_{it}^*(\widetilde{d}_{[t]}), \widetilde{v}_{it}^*(\widetilde{d}_{[t]}), \widetilde{w}_{it}^*(\widetilde{d}_{[t]})) \right) \\
&\geq \rho(\alpha_* \widetilde{D}) = \alpha_* \rho(\widetilde{D}).
\end{aligned}$$

Next, we consider a feasible policy $\{\widehat{\boldsymbol{u}}_t, \widehat{\boldsymbol{v}}_t, \widehat{\boldsymbol{w}}_t\}_{t \in \mathcal{T}}$ to the multi-stage model where $\widehat{u}_{j^*t}(\widetilde{d}_{[t]}) = 1$, $\widehat{v}_{j^*t}(\widetilde{d}_{[t]}) = \widetilde{d}_t$ and all other status and generation variables are set to zero for a sample path $d_1, d_2, \ldots, d_t$. The feasibility of the solution is guaranteed by Assumption 1. Then,

$$
\begin{aligned}
z^{MS} &\leq \rho\left(\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}} g_i(\widehat{u}_{it}(\widetilde{d}_{[t]}), \widehat{v}_{it}(\widetilde{d}_{[t]}), \widehat{w}_{it}(\widetilde{d}_{[t]}))\right) \\
&= \rho\left(\sum_{t\in\mathcal{T}} a_{j^*} + h_{j^*}(\widetilde{d}_t)\right) = \rho\left(\sum_{t\in\mathcal{T}} \frac{a_{j^*} + h_{j^*}(\widetilde{d}_t)}{\widetilde{d}_t}\widetilde{d}_t\right) \\
&\leq \rho\left(\sum_{t\in\mathcal{T}} \frac{a_{j^*} + h_{j^*}(\overline{q}_{j^*})}{\underline{q}_{j^*}}\widetilde{d}_t\right) \leq \frac{\max_{i\in\mathcal{I}}\{a_i + h_i(\overline{q}_i)\}}{\min_{i\in\mathcal{I}}\left\{\underline{q}_i\right\}}\rho\left(\sum_{t\in\mathcal{T}}\widetilde{d}_t\right) \\
&= \alpha^*\rho\left(\sum_{t\in\mathcal{T}}\widetilde{d}_t\right) \leq \alpha^*\rho(\widetilde{D}),
\end{aligned}
$$

where the first inequality follows from feasibility, the second inequality follows from Assumption 1 and the third equality follows from axiom (A4) and the definition of $\alpha^*$. Thus, we get lower and upper bounds for the MS problems, that is,

$$\alpha_*\rho(\widetilde{D}) \leq z^{MS} \leq \alpha^*\rho(\widetilde{D}). \tag{21}$$

Note that in the TS model, the status decisions in period $t \in \mathcal{T}$ are identical for all realizations of problem parameters in that period and satisfies $\max\{\widetilde{v}_{it}(\widetilde{d}_{[t]})\} \leq \overline{q}u_{it}$ and $D^{max} \leq \sum_{t\in\mathcal{T}}\max\{\widetilde{v}_{it}(\widetilde{d}_{[t]})\}$. Moreover, the policy $\{\widehat{\boldsymbol{u}}_t, \widehat{\boldsymbol{v}}_t, \widehat{\boldsymbol{w}}_t\}_{t\in\mathcal{T}}$ is also feasible for the TS model and $\rho(\widetilde{D}) \leq D^{max}$. Using these facts, a similar analysis can be used to obtain lower and upper bounds for the two-stage model and we get

$$\alpha_* D^{\max} \leq z^{TS} \leq \alpha^* D^{\max}. \tag{22}$$

Hence, the claim of the theorem follows from (21) and (22). ■

The inequalities given in (21) and (22) relate the optimal values of MS and TS, respectively, to the under and over estimations on per unit production costs.

If the generators are almost identical and lower and upper production limits are close enough, we have $\alpha_* \approx \alpha \approx \alpha^*$. Then, we have

$$\text{VMS} \approx \alpha(D^{\max} - \rho(\widetilde{D})). \tag{23}$$

Note that $0 \leq \rho(\widetilde{D}) \leq D^{\max}$ and the approximation (23) implies that the VMS increases with $D^{\max}$ and therefore variability in the net load. However, for fixed variability, the VMS decreases with $\rho(\widetilde{D})$ and therefore the degree of risk aversion.

Assume that the net load in period $t \in \mathcal{T}$ is $\widetilde{d}_t = \overline{d}_t + \mathcal{U}[-\Delta, \Delta]$ where $\overline{d}_t$ is a deterministic value and $\mathcal{U}[-\Delta, \Delta]$ is an error term uniformly distributed between $-\Delta$ and $\Delta$ for some $\Delta \in \mathbb{R}_+$. Also, assume that the composite risk measure $\rho(\cdot)$ is obtained using conditional mean-upper semi deviation as given in (7) for simplicity. Then,

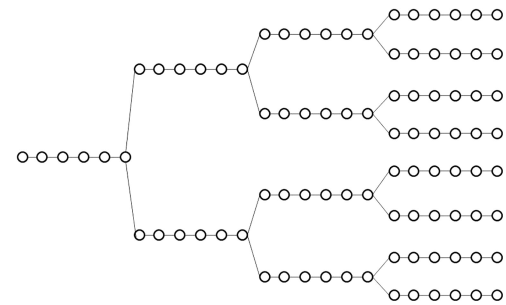


Fig. 3. Scenario tree for the data set in [1].

$$
\begin{aligned}
\text{VMS} &\approx \alpha(D^{\max} - \rho(\widetilde{D})) \\
&= \alpha\left(\sum_{t=1}^{T} d_t^{\max} - \rho\left(\sum_{t=1}^{T}\widetilde{d}_t\right)\right) \\
&= \alpha T\left(1 - \frac{\lambda}{4}\right)\Delta
\end{aligned} \tag{24}
$$

where the second equality follows from definitions of $d_t^{\max}$, $\widetilde{d}_t$ and evaluation of mean-upper semi deviation risk measure $\rho(\cdot)$. The approximation in (24) suggests that the VMS increases with the number of periods $T$ and the variability in the net load $\Delta$. However, VMS decreases with the degree of risk aversion $\lambda$.

The inverse relation between VMS and the degree of risk aversion may seem counter-intuitive at first glance. However, as the degree of risk aversion increases, $\rho(\widetilde{D})$ gets closer to $D^{\max}$, that is, the decision maker tries to minimize the cost in the most pessimistic scenarios. In that case, MS sacrifices its adaptivity in order to put emphasis to the most pessimistic scenarios. Thus, optimal values of TS and MS get closer.

## IV. Computational Experiments

The analytical results of the previous section rely on restrictive assumptions to simplify the structure of the RA-UC problem. In order to see how the VMS behave in the absence of these assumptions, we conduct two sets of computational experiments.

We first consider a power system with 10 generators in the computational experiments. We use the data set presented in [1] with some modifications. We also consider a random net load process with eight scenarios where the power demand at each hour is subject to uncertainty. The scenario tree depicting the random process is given Fig. 3. A similar scenario tree structure is used in [29].

The test data is presented in the Appendix. We use the base net load values presented in Table VI to generate random net load values. A variability parameter $\epsilon$ is used to control the dispersion of net load across all scenarios. Net load values for each scenario are presented in Table VII. All other parameters except the production limits are set to the values given in [1]. The lower and upper production limits are increased by a half in order to avoid infeasibility in case of large variability in the

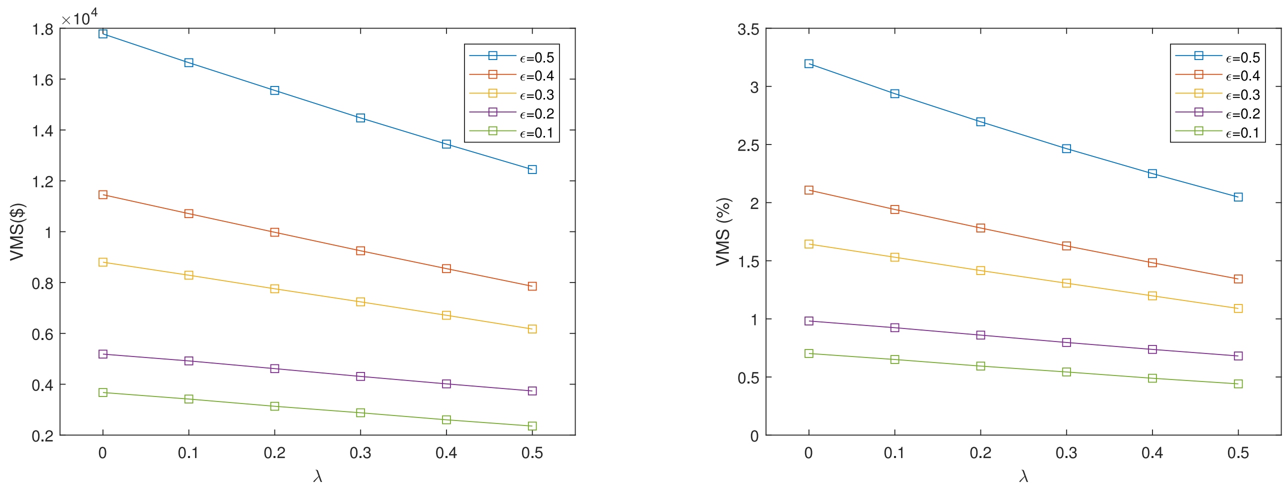



Fig. 4. Results of the computational experiments on the VMS($) and VMS(%) for the data set [1] with respect to different variability ($\epsilon$) and degree of risk aversion levels ($\lambda$).

net load amount. Start/shut-up/down limits are calculated as in [20]. A PC with two 2.2 GHz processors and 6 GB of RAM is used in the computational experiments.

The quadratic production cost functions $\{h_i(\cdot)\}_{i\in\mathcal{I}}$ are approximated by a piecewise linear cost function with four pieces of equal lengths. This approximation of convex cost functions enables us to have a linear model for RA-UC problem and yields near-optimal solutions (see, for example, [18]). We also use a conditional mean-upper semi deviation risk measure (7) in each period. The conditional risk measures $\rho_1(\cdot), \rho_2(\cdot), \ldots, \rho_{T-1}(\cdot)$, the dynamic coherent risk measure $\varrho(\cdot)$ and the composite risk measure $\rho(\cdot)$ are defined accordingly.

We model and solve the two-stage model TS and the multi-stage model MS for five different values of variability parameter $\epsilon$ and six different values of the penalty parameter $\lambda$. For each $\epsilon$ and $\lambda$ pair, we calculate VMS in terms of difference of optimal values, that is,

$$\mathrm{VMS}(\$) = z^{TS} - z^{MS},$$

and in terms of percentage

$$\mathrm{VMS}(\%) = \frac{z^{TS} - z^{MS}}{z^{MS}}.$$

The results on the VMS are presented in Fig. 4.

Fig. 4 verifies our analytical findings on VMS. We observe an increase in VMS with the uncertainty in net load values. The VMS and hence importance of the multi-stage model increases as the dispersion among the scenarios increases. As expected, the day-ahead schedule obtained by solving the multi-stage model is more adaptive and provides more flexibility in case of high variability of problem parameters. We also observe decrease in the VMS with the level of risk aversion. In parallel with the analytical results in Theorem 1, higher risk aversion leads lower VMS. Hence, the importance of the multi-stage model decreases as risk aversion increases.

We also consider a rolling horizon policy obtained by solving two-stage approximations to the multi-stage problem in each period and fixing the decisions at that stage with respect to the optimal solution of the two-stage model. In order to the measure the quality of the rolling horizon policy, we calculate the gap between the value of the rolling horizon policy and the optimal value of MS. The gap value GAP is calculated in terms of difference of objective values

$$\mathrm{GAP}(\$) = z^{RH} - z^{MS},$$

and in terms of percentage

$$\mathrm{GAP}(\%) = \frac{z^{RH} - z^{MS}}{z^{MS}}$$

where $z^{RH}$ is the value of the rolling horizon policy. Note that since rolling horizon provides a feasible policy to the multistage problem that is at least as good as that of TS, we have that $0 \le \mathrm{GAP} \le \mathrm{VMS}$. The results are presented in Fig. 5.

We present the solution times for each TS and MS instance at Table I and Table II, respectively. The required time to obtain the rolling horizon policy is also presented in Table III.

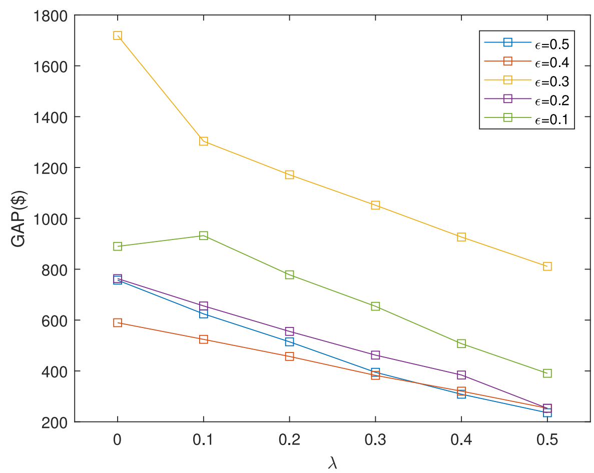
In all instances, the rolling horizon policy performs much better than the policy obtained by solving the two-stage problem with a small increase in computational effort. The GAP (%) of rolling horizon policy is 0.12% on average (with max-

TABLE I
SOLUTION TIMES OF TS (IN SECONDS)

| $\epsilon\backslash\lambda$ | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
|---|---|---|---|---|---|---|
| 0.1 | 7.5 | 10.4 | 9.6 | 7.7 | 7.2 | 7.2 |
| 0.2 | 4.2 | 3.8 | 3.5 | 4.0 | 3.7 | 3.2 |
| 0.3 | 12.2 | 10.9 | 9.5 | 8.1 | 7.8 | 6.0 |
| 0.4 | 7.9 | 3.8 | 4.1 | 4.0 | 3.3 | 2.7 |
| 0.5 | 8.8 | 5.4 | 6.3 | 4.8 | 4.8 | 4.6 |

TABLE II
SOLUTION TIMES OF MS (IN SECONDS)

| $\epsilon\backslash\lambda$ | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
|---|---|---|---|---|---|---|
| 0.1 | 1004.2 | 1280.0 | 1255.2 | 1489.7 | 1789.6 | 2009.1 |
| 0.2 | 328.3 | 381.6 | 400.4 | 444.7 | 324.6 | 393.8 |
| 0.3 | 480.0 | 1042.4 | 435.8 | 780.0 | 453.8 | 358.5 |
| 0.4 | 192.9 | 674.5 | 529.4 | 323.0 | 328.6 | 279.8 |
| 0.5 | 85.7 | 147.5 | 116.6 | 119.0 | 118.5 | 113.1 |



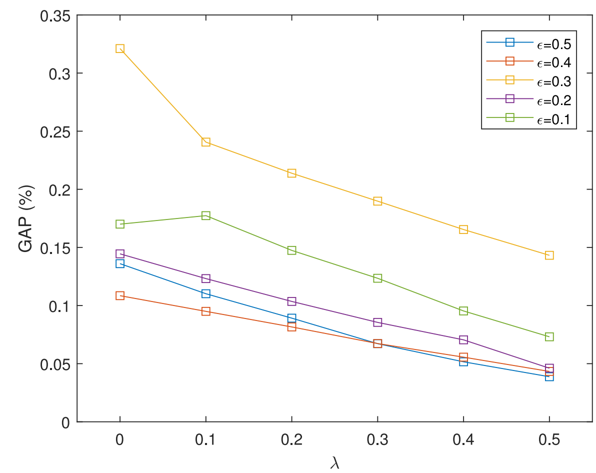




Fig. 5. Results of the computational experiments on GAP($) and GAP(%) for the data set [1] with respect to different variability ($\epsilon$) and degree of risk aversion levels ($\lambda$).

TABLE III
REQUIRED TIME TO OBTAIN THE ROLLING HORIZON POLICY (IN SECONDS)

| $\epsilon\backslash\lambda$ | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
|---|---|---|---|---|---|---|
| 0.1 | 16.6 | 15.1 | 14.7 | 13.6 | 14.9 | 12.8 |
| 0.2 | 8.0 | 9.0 | 9.0 | 8.7 | 8.0 | 8.5 |
| 0.3 | 15.1 | 17.3 | 15.1 | 15.2 | 14.6 | 11.4 |
| 0.4 | 9.0 | 10.4 | 8.3 | 9.1 | 7.7 | 7.8 |
| 0.5 | 10.2 | 9.6 | 9.0 | 12.3 | 9.7 | 9.5 |

imum 0.32%) whereas the VMS (%) is 1.42% on average (with maximum 3.20%). Thus, the rolling horizon policy obtained by using two-stage approximations to the multi-stage solution can provide enough flexibility in generation schedule to obtain a near-optimal schedule in RA-UC problems with a reasonable computational effort.

The computational effort to solve the MS model is much larger than that of the TS model and the rolling horizon policy in all instances. The higher the demand variability leads higher VMS while decreasing the solution times as an additional benefit.

In the data given in [1], transmission capacity constraints are missing. Therefore, we conduct another set of experiments on the IEEE reliability test system [31] where transmission capacity constraints are also included in the model. In these experiments, we use the parameters presented in [32] and we consider $T = 6, 7, 8, 9$ and 10 stage problems with mean-upper semi deviation risk measure (7). As in the previous set of experiments, the quadratic cost functions are replaced by a piece-wise linear approximation. We assume that the net load value at each stage can take values $(1-\epsilon)\overline{d}_t$ and $(1+\epsilon)\overline{d}_t$ with equal probabilities where $\overline{d}_t$ is the deterministic net load value at stage $t \in \mathcal{T}$ in the original data set. Thus, the resulting scenario tree is a binary tree where the number of scenarios is $2^{T-1}$ in a $T-$ stage problem. Some instances of MS require long CPU times or cannot be solved optimally due to memory limitations. For these instances, the exact value of VMS cannot be calculated, however, we use the best objective value after two hours in calculation of an approximate VMS. The results of these experiments are presented in Fig. 6 and Table IV.

Results in Table IV reveal that even in existence of transmission capacity constraints, our findings on the relationship between VMS and degree of risk aversion, level of uncertainty of net load values and number of periods hold, in general. For the instances that cannot be solved within the time limit, the average optimality gap values are 0.03% and 0.07% for $T = 8$ and 9, respectively. Therefore, we obtain a good approximation of VMS and our findings are consistent in this approximation as well. However, for $T = 10$, the average optimality gap for the instances that cannot be solved within the time limit is 0.32%. Because of this relatively poor approximation of VMS, we observe that approximate VMS fluctuates as $\lambda$ increases for the instances $T = 10, \epsilon \in \{0.3, 0.4\}$. However, even $T = 10$, the results of the instances with $\epsilon \in \{0.1, 0.2, 0.5\}$ confirm our findings.

Especially for, $T = 9$ and 10, MS cannot be solved within two hours of time limit, on the other hand, the longest running time for TS is 1724.41 seconds. The average CPU times of TS and MS for the data set in [32] are given in Table V. The CPU time for MS, compared to TS, increases very rapidly as $T$ increases even though the additional computational effort brings a benefit in the objective less than 1% in all instances. Therefore, implementing the policy obtained by solving TS can be a promising alternative under industry time constraints.

## V. CONCLUSION

Recent improvements in the renewable power production technologies have motivated the stochastic unit commitment problems, since these models can explicitly address the variability in net load. Multi-stage models provide completely flexible schedules where all decisions are adapted to the uncertainty. However, these models require high computational effort, and therefore, their two-stage counterparts are used to obtain approximate policies. In order to justify the additional effort to solve the multi-stage model rather than its two-stage counterpart, we define the VMS and provide analytical and computational results on it. These results reveal that, for RA-UC problems, the

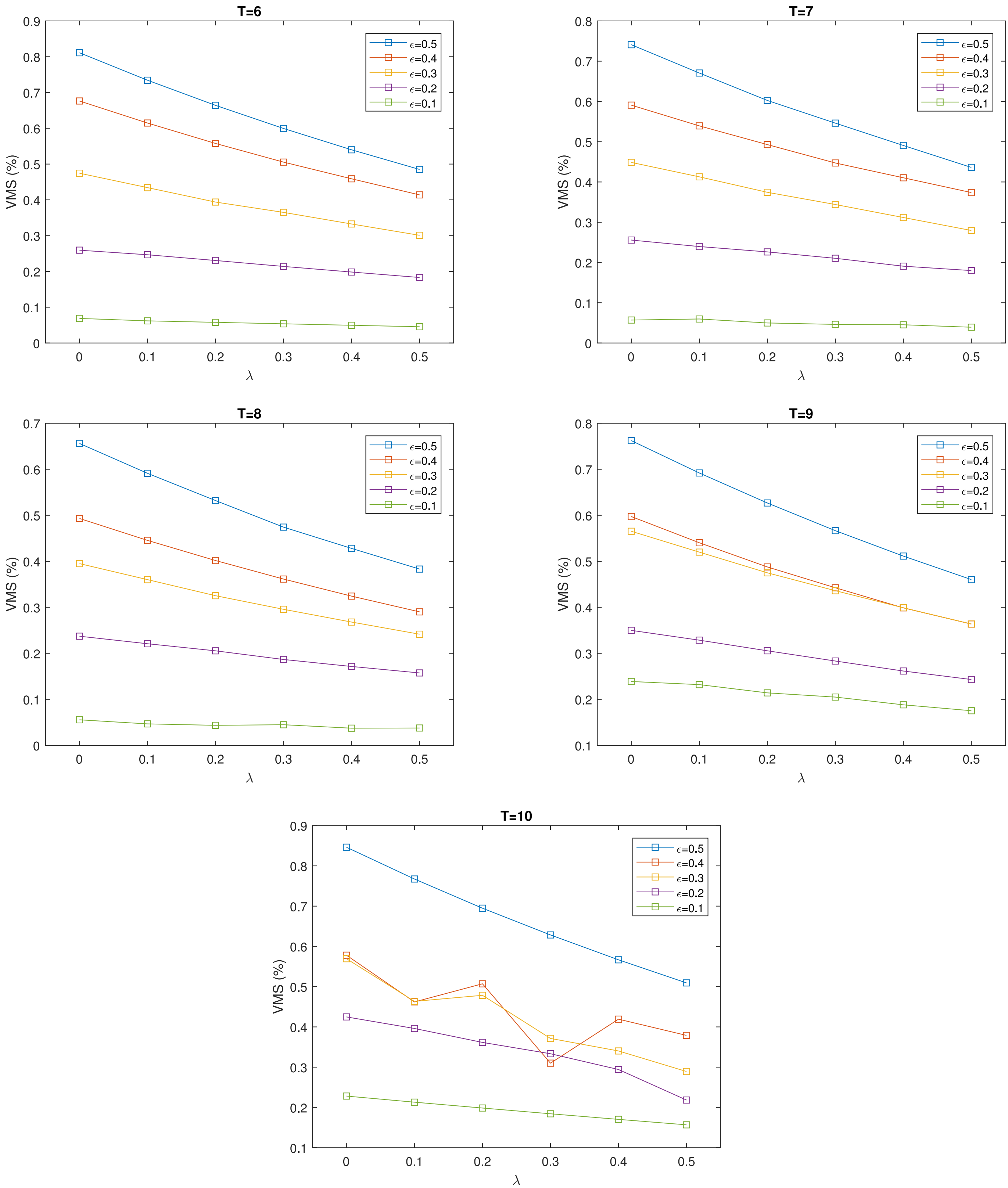



Fig. 6. Results of the computational experiments on the VMS(%) for the data set in [32] with respect to different variability ($\epsilon$) and degree of risk aversion levels ($\lambda$).

TABLE IV
VMS($) AND VMS(%) FOR THE DATA SET IN [32] WITH RESPECT TO DIFFERENT VARIABILITY ($\epsilon$) AND DEGREE OF RISK AVERSION LEVELS ($\lambda$)

| $T$ | $\epsilon \backslash \lambda$ | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
|---|---|---|---|---|---|---|---|
| 6 | 0.1 | 107.86 | 97.51 | 91.31 | 85.11 | 78.91 | 72.73 |
| | | 0.07% | 0.06% | 0.06% | 0.05% | 0.05% | 0.05% |
| | 0.2 | 410.26 | 393.11 | 370.31 | 346.26 | 323.32 | 300.70 |
| | | 0.26% | 0.25% | 0.23% | 0.21% | 0.20% | 0.18% |
| | 0.3 | 755.90 | 699.71 | 641.85 | 601.45 | 554.11 | 506.83 |
| | | 0.47% | 0.43% | 0.39% | 0.36% | 0.33% | 0.30% |
| | 0.4 | 1086.54 | 1002.56 | 923.08 | 848.99 | 781.85 | 715.17 |
| | | 0.67% | 0.61% | 0.55% | 0.50% | 0.46% | 0.41% |
| | 0.5 | 1313.64 | 1211.58 | 1115.92 | 1025.51 | 939.97 | 858.95 |
| | | 0.80% | 0.73% | 0.66% | 0.60% | 0.54% | 0.48% |
| 7 | 0.1 | 103.72 | 108.93 | 91.31 | 85.11 | 83.68 | 72.73 |
| | | 0.06% | 0.06% | 0.05% | 0.05% | 0.05% | 0.04% |
| | 0.2 | 468.64 | 442.63 | 421.27 | 394.76 | 360.73 | 342.83 |
| | | 0.26% | 0.24% | 0.23% | 0.21% | 0.19% | 0.18% |
| | 0.3 | 828.53 | 771.33 | 707.70 | 657.86 | 602.85 | 546.62 |
| | | 0.45% | 0.41% | 0.37% | 0.34% | 0.31% | 0.28% |
| | 0.4 | 1100.65 | 1020.47 | 946.89 | 871.97 | 812.28 | 750.36 |
| | | 0.59% | 0.54% | 0.49% | 0.45% | 0.41% | 0.37% |
| | 0.5 | 1391.73 | 1283.89 | 1175.38 | 1085.07 | 992.53 | 897.97 |
| | | 0.74% | 0.67% | 0.60% | 0.55% | 0.49% | 0.44% |
| 8 | 0.1 | 115.55 | 97.51 | 91.31 | 94.47 | 78.91 | 80.03 |
| | | 0.06% | 0.05% | 0.04% | 0.04% | 0.04% | 0.04% |
| | 0.2 | 497.70 | 467.20 | 438.09 | 401.12 | 371.40 | 343.64 |
| | | 0.24% | 0.22% | 0.21% | 0.19% | 0.17% | 0.16% |
| | 0.3 | 835.65 | 771.17 | 704.73 | 648.16 | 594.22 | 541.80 |
| | | 0.39% | 0.36% | 0.33% | 0.30% | 0.27% | 0.24% |
| | 0.4 | 1052.62* | 966.11* | 884.84* | 808.41* | 736.43 | 668.59* |
| | | 0.49% | 0.45% | 0.40% | 0.36% | 0.32% | 0.29% |
| | 0.5 | 1411.91 | 1297.33 | 1189.99 | 1080.89 | 993.60 | 905.40 |
| | | 0.66% | 0.59% | 0.53% | 0.47% | 0.43% | 0.38% |
| 9 | 0.1 | 569.40 | 555.62 | 514.87 | 494.91 | 456.22 | 426.90 |
| | | 0.24% | 0.23% | 0.21% | 0.20% | 0.19% | 0.18% |
| | 0.2 | 840.97 | 796.39 | 746.61 | 697.92 | 649.63 | 608.69 |
| | | 0.35% | 0.33% | 0.31% | 0.28% | 0.26% | 0.24% |
| | 0.3 | 1370.20* | 1275.94* | 1180.00* | 1096.45* | 1015.09* | 936.09* |
| | | 0.57% | 0.52% | 0.48% | 0.44% | 0.40% | 0.36% |
| | 0.4 | 1460.80* | 1343.19* | 1232.00* | 1135.35* | 1039.17* | 961.80* |
| | | 0.60% | 0.54% | 0.49% | 0.44% | 0.40% | 0.36% |
| | 0.5 | 1881.41 | 1742.39 | 1609.45 | 1483.10 | 1363.85 | 1250.51 |
| | | 0.76% | 0.69% | 0.63% | 0.57% | 0.51% | 0.46% |
| 10 | 0.1 | 620.71* | 581.99* | 544.72* | 507.70* | 471.40* | 435.80* |
| | | 0.23% | 0.21% | 0.20% | 0.18% | 0.17% | 0.16% |
| | 0.2 | 1164.00* | 1094.52* | 1007.44* | 936.85* | 833.71* | 624.08* |
| | | 0.42% | 0.40% | 0.36% | 0.33% | 0.29% | 0.22% |
| | 0.3 | 1574.31* | 1298.25* | 1356.30* | 1066.33* | 988.83* | 850.78* |
| | | 0.57% | 0.46% | 0.48% | 0.37% | 0.34% | 0.29% |
| | 0.4 | 1615.47* | 1313.53* | 1463.80* | 910.46* | 1249.11* | 1146.56* |
| | | 0.58% | 0.46% | 0.51% | 0.31% | 0.42% | 0.38% |
| | 0.5 | 2387.27 | 2209.31 | 2040.97 | 1882.50 | 1730.43 | 1584.75 |
| | | 0.85% | 0.77% | 0.69% | 0.63% | 0.57% | 0.51% |

* VMS is calculated with the best objective value obtained after two hours.

TABLE V
AVERAGE CPU TIMES (IN SECONDS) OF TS AND MS FOR THE DATA SET IN [32] (FOR THE INSTANCES THAT CANNOT BE SOLVED IN TWO HOURS, THE CPU TIMES ARE TAKEN AS 7200 SECONDS.)

| $T$ | TS | MS |
|---|---|---|
| 6 | 2.75 | 10.25 |
| 7 | 6.60 | 25.64 |
| 8 | 18.98 | 1551.31 |
| 9 | 116.83 | 4187.73 |
| 10 | 703.59 | 6445.46 |

VMS decreases with the degree of risk aversion, and increases with the level of uncertainty and number of time periods.

Performance of the rolling horizon polices obtained by two-stage approximations of the multi-stage models are promising. As a future research direction, it would be interesting to consider the rolling horizon policies in instances with more complicated random net load processes. However, in that case, the number of two-stage models to be solved would be large and their solution would require significant computation time. Theoretical analysis of the value of rolling horizon policies is also an important future step.

## APPENDIX
## DETERMINISTIC UNIT COMMITMENT FORMULATION

### NOMENCLATURE

*Indexes and Sets*

- $t$ Period index,
- $T$ Number of periods,
- $\mathcal{T}$ Set of periods,
- $l$ Line index,
- $i$ Generator index,
- $I$ Number of generators,
- $\mathcal{I}$ Set of generators,
- $\mathcal{L}$ Set of lines,

*Parameters*

- $a_i$ Fixed cost of running generator $i \in \mathcal{I}$,
- $h_i(\cdot)$ Production cost function of running generator $i \in \mathcal{I}$, specifically, $h_i(v) = b_i v + c_i v^2$ for $v \geq 0$ with parameters $b_i, c_i \in \mathbb{R}_+$,
- $SU_i$ Start-up cost of generator $i \in \mathcal{I}$,
- $SD_i$ Shut-down cost of generator $i \in \mathcal{I}$,
- $\underline{q}_i$ Minimum production amount of generator $i \in \mathcal{I}$,
- $\overline{q}_i$ Maximum production amount of generator $i \in \mathcal{I}$,
- $d_t$ Net load in period $t \in \mathcal{T}$,
- $M_i$ Minimum up time of generator $i \in \mathcal{I}$,
- $L_i$ Minimum down time of generator $i \in \mathcal{I}$,
- $V_i'$ Start up rate of generator $i \in \mathcal{I}$,
- $V_i$ Ramp up rate of generator $i \in \mathcal{I}$,
- $B_i'$ Shut down rate of generator $i \in \mathcal{I}$,
- $B_i$ Ramp down production limit of generator $i \in \mathcal{I}$,
- $C_l$ Capacity of transmission line $l \in \mathcal{L}$,
- $K$ Flow line distribution matrix.

*Variables*

- $u_{it}$ Status of generator $i \in \mathcal{I}$ in period $t \in \mathcal{T}$, (1 if generator $i$ is ON in period $t$; 0 otherwise),
- $v_{it}$ Production amount of generator $i \in \mathcal{I}$ in period $t \in \mathcal{T}$,
- $y_{it}$ Start up decision of generator $i \in \mathcal{I}$ in period $t \in \mathcal{T}$, (1 if $u_{i(t-1)} = 0$ and $u_{it} = 1$ ; 0 otherwise),
- $z_{it}$ Shut down decision of generator $i \in \mathcal{I}$ in period $t \in \mathcal{T}$, (1 if $u_{i(t-1)} = 1$ and $u_{it} = 0$ ; 0 otherwise).

*Model*

$$\min_{u,v,y,z} \sum_{t=1}^{T} \sum_{i=1}^{I} a_i u_{it} + h_i(v_{it}) + SU_i y_{it} + SD_i z_{it}, \quad (25)$$

s.t. (2), (3)

$$u_{it} - u_{i(t-1)} \leq u_{i\tau}, \ \forall t \in \mathcal{T}, \forall i \in \mathcal{I}, \ \forall \tau \in \{t+1, \ldots, \min\{t + M_i, T\}\} \quad (26)$$

$$u_{i(t-1)} - u_{it} \leq 1 - u_{i\tau}, \ \forall t \in \mathcal{T}, \forall i \in \mathcal{I}, \ \forall \tau \in \{t+1, \ldots, \min\{t + L_i, T\}\} \quad (27)$$

$$u_{it} - u_{i(t-1)} \leq y_{it}, \ \forall t \in \mathcal{T}, \forall i \in \mathcal{I} \quad (28)$$

$$u_{i(t-1)} - u_{it} \leq z_{it}, \ \forall t \in \mathcal{T}, \forall i \in \mathcal{I} \quad (29)$$

$$v_{it} - v_{i(t-1)} \leq V_i' y_{it} + V_i u_{i(t-1)},$$

TABLE VI
DEMAND DATA (MW = MEGAWATT)

| $t$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $\overline{d}_t$ (MW) | 700 | 750 | 850 | 950 | 1000 | 1100 |
| $t$ | 7 | 8 | 9 | 10 | 11 | 12 |
| $\overline{d}_t$ (MW) | 1150 | 1200 | 1300 | 1400 | 1450 | 1500 |
| $t$ | 13 | 14 | 15 | 16 | 17 | 18 |
| $\overline{d}_t$ (MW) | 1400 | 1300 | 1200 | 1050 | 1000 | 1100 |
| $t$ | 19 | 20 | 21 | 22 | 23 | 24 |
| $\overline{d}_t$ (MW) | 1200 | 1400 | 1300 | 1100 | 900 | 800 |

TABLE VII
SCENARIO DATA

| | | Period (or hour) $t$ | | | |
|---|---|---|---|---|---|
| Scenario | Probability | 1-6 | 7-12 | 13-18 | 19-24 |
| 1 | 0.125 | $\overline{d}_t$ | $(1-\epsilon)\overline{d}_t$ | $(1-\epsilon)\overline{d}_t$ | $(1-\epsilon)\overline{d}_t$ |
| 2 | 0.125 | $\overline{d}_t$ | $(1-\epsilon)\overline{d}_t$ | $(1-\epsilon)\overline{d}_t$ | $(1+\epsilon)\overline{d}_t$ |
| 3 | 0.125 | $\overline{d}_t$ | $(1-\epsilon)\overline{d}_t$ | $(1+\epsilon)\overline{d}_t$ | $(1-\epsilon)\overline{d}_t$ |
| 4 | 0.125 | $\overline{d}_t$ | $(1-\epsilon)\overline{d}_t$ | $(1+\epsilon)\overline{d}_t$ | $(1+\epsilon)\overline{d}_t$ |
| 5 | 0.125 | $\overline{d}_t$ | $(1+\epsilon)\overline{d}_t$ | $(1-\epsilon)\overline{d}_t$ | $(1-\epsilon)\overline{d}_t$ |
| 6 | 0.125 | $\overline{d}_t$ | $(1+\epsilon)\overline{d}_t$ | $(1-\epsilon)\overline{d}_t$ | $(1+\epsilon)\overline{d}_t$ |
| 7 | 0.125 | $\overline{d}_t$ | $(1+\epsilon)\overline{d}_t$ | $(1+\epsilon)\overline{d}_t$ | $(1-\epsilon)\overline{d}_t$ |
| 8 | 0.125 | $\overline{d}_t$ | $(1+\epsilon)\overline{d}_t$ | $(1+\epsilon)\overline{d}_t$ | $(1+\epsilon)\overline{d}_t$ |

$$\forall t \in \mathcal{T}, \forall i \in \mathcal{I} \tag{30}$$

$$v_{i(t-1)} - v_{it} \leq B'_i z_{it} + B_i u_{it},$$

$$\forall t \in \mathcal{T}, \forall i \in \mathcal{I} \tag{31}$$

$$- C_l \leq K v_t \leq C_l,$$

$$\forall t \in \mathcal{T}, \forall l \in \mathcal{L}$$

$$u_{it}, y_{it}, z_{it} \in \{0,1\}, v_{ti} \geq 0, \ \forall t \in \mathcal{T}, \forall i \in \mathcal{I}. \tag{32}$$

The objective (25) is total fixed, production, start up and shut down costs. Constraints (26), (27), (28) and (29) are minimum up time, minimum down time, start up and shut down constraints, respectively. The ramp/start up rate constraint is given in (30). Similarly, (31) is the ramp/shut down rate constraint. Constraints (32) are the flow balance constraints in linear form as given in [20].

Computational experimental data are provided in Tables VI and VII.

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