Collaborative decision making for air traffic management: A generic mathematical program for the rescheduling problem

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ABSTRACT

We propose a model which can be utilized within a collaborative decision making (CDM) framework for re-scheduling of arrivals and departures. The proposed mathematical program is expected to be utilized by major stakeholders, namely airlines and air navigation service providers. After providing the constraints, we list possible performance measures to be used as the objective function by a stakeholder. Performance measures include the ones that represent equity, as well as efficiency. We suggest guidelines to utilize the model for any stakeholder within CDM. Finally, a case study is prepared using publicly available data to demonstrate possible benefits.

1. Introduction

At major airports, air traffic congestion frequently occurs due to various reasons, including weather conditions, mechanical failures, and unavailability or insufficiency of other resources and facilities at the airport. Congestion causes delays in arrival and departure times of the planes.

Delay information is kept by various sources; one such WEB site is flightstats.com (FlightStats, 2016). According to a snap-shoot of 30 days, about 28,000 flights were canceled and 500,000 flights were delayed. According to U.S. Department of Transportation about 20% of arrivals and departures were delayed in 2015 (US Department of Transportation, 2015). Note that a flight is only considered to be delayed if it arrived (or departed) the gate 15 min or more after its scheduled arrival (departure) time.

One can observe that delays are significant. According to 2015 statistics, 26.3% of the delays occur due to the circumstances within the airline's control (maintenance or crew problems, aircraft cleaning, etc.), called "Air Carrier Delay"; 30.9% due to late arriving aircraft, and 36.5% due to National Aviation System (delays that are attributable to airport operations, heavy traffic volume, and air traffic control) (US Department of Transportation, 2015). Extreme weather conditions and security reasons contribute only to a total of 6.4% of the delays during 2015. The main motivation of the current work is to improve the operations under a collaborative decision making environment from the perspective of air navigation service providers (FAA, EuroControl, etc.),

as well as other stakeholders (airlines, passengers) so that delays that are attributable to airport operations, heavy traffic volume, and air traffic control can be managed efficiently, as well as fairly.

Delays affect all the stakeholders of the business (as an example, see the list of stakeholders stated in FAA (2015)). For 2007 the total cost of delays in US domestic flights was estimated to be $32.9, $8.3 billion of this total being for airlines and $16.7 billion for passengers (Ball et al., 2010a). Total cost includes cost of operating air navigation service providers, a cost to the society in general. Hence, one aspect of the rescheduling problem should be concerned with the distribution of additional delay costs to the stakeholders; namely airlines, passengers, and society in general. There are different approaches considered in the literature for equitable allocation of resources in air traffic management considerations, for example see Ball, Donohue, and Hoffman (2006), Hoffman, Ball, and Mukherjee (2007) and Liu and Hansen (2013a, 2013b).

Real-time measures are taken to avoid additional delays. There are many programs available in the US, as well as in Europe all describing what to do in under different circumstances. One such program is GDP, *ground delay program* that reacts when the expected arrivals to an airport exceed current available capacity. Hence delaying a departing flight from another airport in the ground becomes meaningful to decrease the arriving traffic at the airport with insufficient capacity. Other programs include speed control, route adjustments, cancellations and arrival re-sequencing. The FAA and the major airlines in the United States have initiated *collaborative decision making* (CDM) process to

improve the air traffic flow management. Under CDM, FAA repeatedly updates the assignment of arrival flights to the slots for a given set of information of flight delays and cancellations, of course in communication with all stakeholders.

We propose a decision model that can be utilized by any of the stakeholders involved in a CDM environment. The general structure of decision making and decision hierarchy with regards to air traffic management is not unique (Ball et al., 2006). However, the following two levels of decisions can be specified almost as common (though, time horizons utilized in the approach considered may be different):

- Tactical Level: Time-slots are assigned to each flight at each airport; we call these the scheduled arrival/departure times. An early example that discusses various aspects of slot allocation is (Abeyratne, 2000), many others follow.
- Operational Level: The assigned time slots may be changed among flights, sometimes with restrictions, to adapt the changes due to the dynamic nature of the processes in the environment; we call these the re-scheduled arrival/departure times.

One of the most prominent issues regarding airlines at a given airport is the issue of "slot" allocation at the tactical level. As a result of the allocation an airline would know the number of flights in-and-out of the airport, facilitating the airline to plan for service for long time horizons. There are guidelines followed by the air navigation traffic service providers in allocating, and maintaining these slots. Fairness issues and open competition ideas form the backbone of the approaches utilized. Rockefeller and Hutchison (2012), Bachman (2008), Rivers (2013) and Jacquillat and Odoni (2015) are some examples demonstrating the debates on the allocation issue. Zografos, Sakouras, and Madas (2012) develop a single-airport optimization model that implements EU/IATA regulations, operational constraints, and coordination procedures. The ultimate objective is to better accommodate airlines' preferences at coordinated airports by minimizing total displacement. As an extension, Pellegrini, Bolic, Castelli, and Pesenti (2013) propose a decision support tool capable of optimally coordinating the airports' capacity management at the European level. Finally, we refer to Zografos, Madas, and Kandroutsopoulos (2017) for extensive review on slot scheduling for tactical level. Their findings suggest that further research on slot scheduling should explore variations of currently used objectives (e.g., alternative expressions of schedule delay) and most importantly enrich them with fairness and equity, resource utilization and environmental considerations. Our interest in this study is to consider the initial allocation of slots as given, and only consider changes for short-term (daily) operational purposes. We are concerned with efficiency as well as fairness measures while proposing these changes. Note that, we use the word "slot" to denote a time bucket (can be a minute, two minutes, etc.) which will then be the time-unit used to reschedule if necessary.

Operationally, both FAA and EuroControl have their own procedural guidelines for operational schedule changes ((FAA, 2016a), *Part 5. Air Traffic Management System* and (EuroControl, 2012a), Chapter 3 *Implementation for details used by EuroControl*). The idea of collaboration with airlines is utilized in both continents, with service providing organization being the decision maker. These documents contain sufficient detail with respect to how to set the information exchange among the stakeholders, and steps to follow to conclude and execute a decision (see, as an example, EuroControl (2012a), page 3–41). One thing which is not clear is the existence of a tool that will enable the stakeholders (the air navigation traffic service providers, airlines, etc.) to make decisions consistent to their objectives and participate in the collaboration.

The main motivation of this study is to propose an optimization tool that can be utilized by the stakeholders for operational (short-term) purposes within a CDM environment. In other words, we aim to approach the rescheduling problem using a generic optimization tool

based on mixed integer programming. Note that, given the statistics stated above, schedule changes reported make up around 20% of the flights, on the average. Hence, one can see the significance of the re-scheduling problem: on the average in a given time range, at least 20% of the flights will be rescheduled at any given airport. Taking into consideration the frequency of rescheduling efforts and time requirements for obtaining a solution, we offer an approach that can obtain a result in relatively short time and satisfy requirements or any stakeholder to participate in a CDM environment.

1.1. Literature review

An airport schedule has a very dynamic structure. Daily conditions require changes due to many reasons such as weather conditions, possible technical problems, flight emergencies and delays occurred in the arriving flights. In many cases a rescheduling of flight slots is needed. The new schedule, on the other hand, should be as satisfactory as possible for all major stakeholders each having their own set of important measures: central authority (representing the public system with safety and monetary considerations), airlines (representing profit seeking organizations with liabilities, and future expectations), and passengers (individuals affected by changes, but mostly from delays in the flights who can be monetarily compensated to an extent). In rescheduling applications, the existing limitations should not be violated. Programmed turnaround times for the aircraft, programmed passenger connections times and runway capacities are main limitations which should be reconciled.

In this section, we review the related literature under three headings: Rescheduling in general terms, issue of fairness in rescheduling, and market-based approaches for rescheduling.

A number of approaches can be used to efficiently plan for the initial arrival and departure times of aircraft. Two notable examples are Sherali, Staats, and Trani (2003), an approach that includes airspace planning and Bertsimas and Frankovich (2016), a detailed approach on planning all the resources of the airport efficiently. Of course, these are medium term decision tools, and hence within the operational time horizon rescheduling needs may arise.

Under CDM, FAA considers the allocation of arrival slots to airlines instead of assignment of flights to slots. FAA utilizes two algorithms, *Ration by Schedule* (RBS) and *Compression*, both can be seen as incentives for the airlines to reveal truthful information. As explained by Vossen and Ball (2006a) Compression Algorithm aims to provide airlines with an incentive to report flight delays.

A short period of congestion can be handled by airborne control such as re-routing and allowing variations in flight speed. However, due to the high fuel cost of airborne delays, most cases with excess demand can be handled by delaying flights at the departure airport which is called as Ground Delay Program. Under CDM, FAA performs updates in arrivals and departures using all the information that flow from the stakeholders. Before CDM, arrival flights were assigned by first come first serve algorithm based on the most recent estimated arrival times, which is called as Grover Jack. Estimated arrival times are private information reported to the mechanism by the airlines themselves. There occurs a drawback of this algorithm as the system would re-project an arrival time with an additional delay on top of its mechanical delay, Shummer and Rakesh (2013). Therefore, airlines would be reluctant to send accurate delay information to FAA, in turn probably resulting in worse economical consequences for the airlines.

In many cases, the air traffic controllers solve the problem of rescheduling using the simple first-scheduled, first-serve (FSFS) rule. Furini, Kidd, Persiani, and Toth (2015) present a rolling horizon approach which partitions a sequence of aircraft into chunks and solves the aircraft sequencing problem (ASP) individually for each of these chunks.

One approach to decrease the need for scheduling is to consider scheduling limits while scheduling an airport for medium-term.

Pyrgiotis and Odoni (2016) propose an integer programming model that generates a new flight schedule in response to scheduling limits at congested airports. For larger problem instances, Bertsimas, Lulli, and Odoni (2011) decide on an optimum combination of flow management actions, including ground-holding, rerouting, speed control and airborne holding on a flight-by- flight basis. Akgunduz and Kazerooni (2018) also introduce a non-time segmented en-route flight plan formulation with rerouting options for aircraft. Moreover, they introduce a linear approximation for the speed dependent fuel consumption. Kim and Hansen (2015) present a game theoretic model that utilizes airborne and ground delay costs. Damgacioglu, Celik, and Guller (2018) analyze the impact of such disruptive events on an airport ground system using a route-based network simulation framework. In the case study, runway closure is investigated with its impact on taxi-in and taxi-out times.

Although first-scheduled, first-served approach of the GDP has been accepted as a fairness standard by the industry, there has been growing literature on the fairness criteria in slot allocation mechanisms to balance equity and efficiency. Manley and Sherry (2008) consider equity issue along with efficiency. More recently, Jacquillat and Vaze (2018) propose a lexicographic modeling architecture based on efficiency, equity, and on-time performance objectives in airport scheduling interventions. Barnhart, Bertsimas, Caramanis, and Fearing (2012) develop an integer programming formulation that minimizes a metric to measure deviation from first-scheduled, first-served in the presence of conflicts. This formulation allows for the flexible trade-off between a delay term and a fairness term. Another group of equity related studies concentrate on the RBS procedure where equitable distribution of delays among airlines is not explicitly incorporated. Bertsimas and Gupta (2016) formulate a discrete optimization model that incorporates an equitable distribution of delays among airlines in Stage I and allow airline collaboration by proposing a network model for slot reallocation in Stage II. Evans, Vaze, and Barnhart (2016) aim is to ask the airlines to set their performance goals and then make trade-offs between different criteria directly, before specific air traffic management strategies are determined. Vossen and Ball (2006a) discuss allocation procedure of GDP through appropriately defined optimization models. They formulate allocation of slots (RBS) as an optimization problem that minimizes the maximum flight delay with respect to the original flight schedule. They claim that the proposed model yields a fair solution and state the example that instead of assigning 30 min of delay to a flight and 90 min of delay to another flight, the model assigns 60 min of delay to both of the flights. In a similar manner, they formulate exchange of slots among the airlines by lexicographically minimizing the maximum delay imposed on a flight. In a more recent study, Ball, Hoffman, and Mukherjee (2010b) develop a new algorithm called as *Ration-by-Distance* (RBD), which maximizes the airport throughput. The algorithm prioritizes the flights by their distance from the destination airport. As RBD may result in assigning flights having a short distance to incur long delays, they propose a new algorithm called as *Equity-based RBD* (E-RBD). The algorithm does not allow the assignment of flights to slots whose time is a set amount after the time of the slot assigned by the RBS algorithm. Glover and Ball (2013) propose a two-stage stochastic, multi-objective integer program for GDP planning. This multi-objective approach considers balancing equity and efficiency by including both components in the objective function. On the other hand, Kuhn (2013) introduces several two-phase methods for ground delay program planning to find unsupported Pareto-optimal policies minimizing delay and inequity. Recently, Sama, D'Ariano, D'Ariaro, and Pacciarelli (2017) proposed real-time optimization models for the problem considered. They propose use of several objective functions in their formulation.

The above approaches consider the equity issue by defining reasonable objective functions, as well as suggest procedures to follow. However, involvement of stakeholders is not necessarily active. In air transportation, there are different stakeholders, including ATC, airlines,

airports and government, who may have their own objectives. Therefore, a simultaneous optimization requires consideration of different objectives that may conflict. There are only a few studies which detail such an interaction explicitly in their modeling framework. Although their focus is runway scheduling, a useful review of the various objectives of different types of stakeholders can be found in Bennel, Mesgarpour, and CNPotts (2013). Ball et al. (2006) is an early example regarding application idea of CDM in the tactical level, a proposal for allocation of slots to airlines. More recent studies, all centered on the idea of CDM, can be mentioned: de Almeida, Weigang, Meinerz, and Li (2016) and Evans et al. (2016). On the other hand, there are studies considering rescheduling problem as a market-based issue. Shummer and Rakesh (2013) state that the Compression Algorithm does not respect the property rights and propose an alternative mechanism that satisfies both incentive compatibility and property rights. Vossen and Ball (2006b) propose new ideas on trading slots enabling airlines to mutually benefit from the slot-exchange process while maintaining the equity among the airlines. They suggest that airlines can make an offer with multiple slots in exchange for multiple slots in return. Sherali, Hill, McCrea, and Trani (2011) extend the airspace planning and collaborative decision making (APCDM) model of Sherali et al. (2003) to incorporate slot-change mechanisms and continuing flight restrictions. The objective of the APCDM is to select an optimal set of flight plans subject to sector workload, collision safety, and airline equity considerations. They integrate a slot exchange mechanism based on a given set of exchange offers within the APCDM model while addressing some equity measures such as relative performance ratio measuring the average delay realized per passenger and on time performance.

Hence, we can say that our approach is consistent with CDM principles and we can operationally describe how to implement. Although FAA utilizes two algorithms, Ration by Schedule (RBS) and Compression, it is not clear how airlines react to those algorithms in their decision making. The proposed model, in that respect is different than the literature as we present a model to be utilized by all stakeholders.

1.2. Motivation, contributions and outline of the study

If we summarize the literature, we observe that there are valuable studies that explain mechanisms that can be applied in different environments. However, it is not clear how different stakeholders will apply them within a collaborative decision framework. Our main motivation in this study is to propose a generic mathematical program to solve the rescheduling problem with CDM framework.

We refer the proposed model "generic" because it provides the least amount of essential constraints and allows stakeholders to expand and use it for several different purposes. Note that any stakeholder can use the basic relations presented, as well as add new constraints or use different objective functions to reflect their purposes. The generic model allows for such expansions and formulations in a straightforward way - like a skeleton ready for add-ons. The problem has a dynamic nature with multiple parties involved. We think that by using such a generic model each stakeholder can better understand the perspective and strategy of other parties so to establish a better ground for collaboration.

The term "generic" is used to express three specific aims:

(1) A stakeholder can use it respecting the limitations in the system as well as only utilizing the data in her knowledge domain or with estimates of the pertinent data(hence constraints, for instance, may be different depending on the stakeholder), as only part of the data is in public domain or within the reach of the stakeholder in question. However, the objective function used can be constructed by the user depending on what is targeted at the end.

To allow the construction of individual objective functions, the generic model should enable mixed integer variables to represents

various performance measures of efficiency as well as equity. We demonstrate that we can explicitly represent fairness or equity performance measures (in some cases with additional decision variables), and hence make it ready for the user to utilize in the objective function formed. Note that if one solves an efficiency-only objective function, it is expected to have many alternative optimal solutions. Hence, even if one utilizes a simple weighted function of one efficiency type and one equity performance measure, the degradation in the efficiency may be negligible.

(2) The given model represents basic specifications as a constraint set. Constraint set of the mathematical program can be extended to represent other specificities of the airport and/or stakeholder or to additionally serve to resolve issues that are needed for an airport.

(3) The given model's common data requirements should be seen as the data which is public among stakeholders (note that this includes cases where stakeholders may agree to share private data with each other) in other to make sure that each stakeholder uses the same information when coming up with a proposal.

It is important to capture the most important limitations, as this problem is an operational one. The frequency of solving the model depends on the size of the airport (or network of airports, although we explicitly consider a single airport) considered. However, we think that a new solution is to be called a few times a day, and hence require that a solution must be obtained in around 20 min including any interactive collaborative efforts.

We think that this generic model is important in satisfying some of the future needs outlined by the NextGen project (FAA, 2015), as well as (EuroControl, 2012b). Specifically, the proposed model is a tool that can be utilized by any stakeholder in any of the stages of collaborative decision making process.

Although guidelines for collaborative decision making are provided in different sources, the methods and structure of how to form the collaboration is still an open question. In the literature there are proposed methods such as "Ration by Schedule", "Compression" and rolling horizon approach to solve air traffic rescheduling problem; however they are mostly focused on considering only one of the stakeholders at a time. In our study we come up with an environment to realize collaborative decision-making. We propose a novel setting that guides stakeholders to participate in collaborations and enables them to make decisions that are in line with their operational constraints and fairness concerns.

Additionally, the flexible structure of the proposed model helps users to better understand the effect of different objective functions in the outcome. Solving the problem with the possibility of employing alternative objective functions and with different methods will provide valuable information about solutions of the rescheduling problem and finally converging to a solution which is feasible and hopefully acceptable by all stakeholders.

Our main contribution is to suggest a generic model for rescheduling and to show how it can be applied within a collaborative framework. The model is mostly standard though we develop the formulation of a few possible objective measures and constraint types. We also detail how we see the CDM efforts and possible ways of utilizing the proposed model in an environment where CDM plays some role. We finally apply the idea to a case study which was prepared using publicly available data.

In Section 2, we present the generic model. Model assumptions, as well as variable and parameter definitions and constraints are described in detail in Section 2.1. Section 2.2 describes our approach in dealing with performance measures considered for the generic model. In Section 3, we suggest some guidelines to utilize the proposed model. A case study is presented in Section 4 which incorporates results for the case using some of the approaches mentioned in Section 3. We conclude and present suggested future work in Section 5.

2. Rescheduling problem

We consider the rescheduling problem as a tool to bring the stakeholders together in the CDM environment. Loosely speaking, the word CDM considered in this work might correspond to one of the following cases:

(a) The central decision maker includes other stakeholders' concerns without directly consulting them (might be realized by solving a set of problems, each with taking account one or more stakeholders' concerns).

(b) The central decision maker may ask other stakeholders to solve certain problems and state them some constraints. The final set of decisions will likely take some form of these constraints into account.

(c) The stakeholders may interact several times before a conclusive decision is taken by the central organization, a characteristic described in CDM efforts.

We consider a generic Mixed Integer Programming (MIP) model for rescheduling. Our model resembles the available mathematical programming models for arrival-departure scheduling, as the core of the problem is the same. We have a few aspects modeled in different way, as well as we allow for a very flexible selection of objective function, which we assume is a function of performance measures we specify.

The basic decision variable set of the model represents the rescheduled arrival or departure times of flights. We assume that all the performance measures are either a function of these decision variables (or the difference between the rescheduled time and initially scheduled time) or a limited number of additional variables are needed. We have a few additional assumptions which reflect our modeling choice. We consider a single airport; however think that the model can be extended to represent network of airports. Time slots denote a time bucket (can be a minute, two minutes, etc.) which is the time-unit used to reschedule if necessary. The definition becomes important when we consider capacity limitations. We assume that arrival or departure times of some flights may be pre-set, hence not allowing the rescheduling model to change them. In some rescheduling models it is allowed to cancel some of the flights and provide a better schedule for the rest of them. In this paper, we do not allow cancellation as a decision.

We assume that information is available and in the form that can be utilized. Note that estimates or forecasts are also considered as a part of this information set. Each stakeholder will know (or estimate) possible delay situations regarding their own flights, as well as other airlines' flights, and situation regarding common resources (such as runway) which we mentioned as a condition of the generic structure considered. Of course, a stakeholder may add additional constraints or decisions into the model, requiring additional information which may not necessarily be available to different stakeholders. We assume that common (generic) information about near-future operations can be represented in the model; we call such parameters as state-specifying parameters. Note that this definition is relative; we aim to separate parameter values that might change for reasons mentioned to cause delays in some flights from others that are more stable in the short run. Of course, one possibility is to define all parameters as state-specifying.

2.1. Generic module

We now describe the notation: parameters of the model, including state-specifying parameters and decision variables. Note that these might vary depending on the stakeholder.

Parameters

F:	set of arrival flights
F':	set of departure flights
K:	set of airlines

F_k: set of arrival flights of airline $k \in K$

F'_k: set of departure flights of airline $k \in K$

T: set of slots

s_t: actual time of slot $t \in T$

S_{it}^A: 1 if flight $i \in F$ is scheduled to arrive at slot $t \in T$, and 0, otherwise.

S_{it}^D: 1 if flight $i \in F'$ is scheduled to depart at slot $t \in T$, and 0, otherwise.

D_i^A: number of passengers in flight $i \in F$

D_i^D: number of passengers in flight $i \in F'$

AC_{ij}:   1 if flight $j \in F'$ is an immediate successor of $i \in F$ and flown by the same aircraft, and 0, otherwise.

ta_{ij}:   minimum turnaround time for aircraft of flight $i \in F$ and $j \in F'$

PC_{ij}:   1 if there is at least one passenger with connection from flight $i \in F$ to $j \in F'$, and 0, otherwise.

pa_{ij}:   minimum time needed for passenger connections, $i \in F, j \in F'$

In this model, a slot and its actual time can be tracked simultaneously to identify change clearly.

2.1.1. State specifying parameters

State specifying parameters are inputs to the model that reflects changes which occurred in real time. In other words, these parameters reflect the "status" of the system and hence limit or expand the possibilities for rescheduling. Note that it is also possible to set some arrivals (or departures) at desired times and not allow the optimization model change those. There is the list of state-specifying parameters.

Y^A: set of arrival time/flight combinations which are not allowed to change

Y^D: set of departure time/flight combinations which are not allowed to change

C_t^A: arrival capacity of airport at slot $t \in T$

C_t^D: departure capacity of airport at slot $t \in T$

C_t^T: total capacity of airport at slot $t \in T$

E_i^A: earliest arrival time of flight $i \in F$ at airport

E_i^D: earliest departure time of flight $i \in F'$ from airport

R_i^A: realized arrival time of flight $i \in F$ in the initial schedule

R_i^D: realized departure time of flight $i \in F'$ in the initial schedule

UD_i^A: unavoidable delay in flight $i \in F$

UD_i^D: unavoidable delay in flight $i \in F'$

C_t^A and C_t^D are the runway capacities for arrivals and departures, respectively. C_t^T is showing the total available runway capacity. Total capacity is not necessarily sum of arrival and departure capacity, in some slots a runway can be available for both arriving and departing flights or all capacity can be allocated for only departures in some cases.

Unavoidable delay parameters, UD_i^A and UD_i^D, take value zero for the flights that does not incur any delay from the scheduled plan. For the flights with delays, unavoidable delay calculation is presented below.

$$UD_i^A = -E_i^A + R_i^A, \quad i \in F \tag{1}$$

$$UD_i^D = -E_i^D + R_i^D, \quad i \in F'. \tag{2}$$

2.1.2. Decision variables and mathematical model

The main decision variables are X_{it}^A and X_{it}^D, they state the new slot allocation of each flight. According to the difference between the initial and rescheduled plan the delay and the earliness incurred in the flights are represented by; $ID_i^A, IE_i^A, ID_i^D, IE_i^D$ for arrivals and departures respectively.

X_{it}^A: 1 if flight $i \in F$ is rescheduled to arrive at slot $t \in T$

X_{it}^D: 1 if flight $i \in F'$ is rescheduled to depart at slot $t \in T$

ID_i^A: delay incurred in the arrival flight $i \in F$

IE_i^A: earliness incurred in the arrival flight $i \in F$

ID_i^D: delay incurred in the departure flight $i \in F'$

IE_i^D: earliness incurred in the departure flight $i \in F'$

$avble_i^A$:   avoidable delay incurred in the flight $i \in F$

$avble_i^D$:   avoidable delay incurred in the flight $i \in F'$

The problem is handled as a standard rescheduling problem and there is a core part of the model regardless of the objective function. There are fundamental constraints needed to ensure the feasibility of the problem as provided below;

$$\sum_{t \in T} X_{it}^A = 1, \quad i \in F \tag{3}$$

$$\sum_{t \in T} X_{it}^D = 1, \quad i \in F' \tag{4}$$

$$\sum_{i \in F} X_{it}^A \leqslant C_t^A, \quad t \in T \tag{5}$$

$$\sum_{i \in F'} X_{it}^D \leqslant C_t^D, \quad t \in T \tag{6}$$

$$\sum_{i \in F} \left(X_{it}^A + X_{i(t+1)}^A \right) + \sum_{i \in F'} \left(X_{it}^D + X_{i(t+1)}^D \right) \leqslant C_t^T, \quad t \in T \tag{7}$$

$$\sum_{t \in T} X_{jt}^D \times s_t \geqslant \sum_{t \in T} (X_{it}^A \times s_t \times AC_{ij}) + ta_{ij} \times AC_{ij}, \quad i \in F, \ j \in F' \tag{8}$$

$$\sum_{t \in T} X_{jt}^D \times s_t \geqslant \sum_{t \in T} (X_{it}^A \times s_t \times PC_{ij}) + pa_{ij} \times (1 - AC_{ij}) \times PC_{ij}, \quad i \in F, \ j \in F' \tag{9}$$

$$E_i^A \leqslant \sum_{t \in T} X_{it}^A \times s_t, \quad i \in F \tag{10}$$

$$E_i^D \leqslant \sum_{t \in T} X_{it}^D \times s_t, \quad i \in F' \tag{11}$$

$$\sum_{t \in T} X_{it}^A \times s_t - \sum_{t \in T} S_{it}^A \times s_t = ID_i^A - IE_i^A, \quad i \in F \tag{12}$$

$$\sum_{t \in T} X_{it}^D \times s_t - \sum_{t \in T} S_{it}^D \times s_t = ID_i^D - IE_i^D, \quad i \in F' \tag{13}$$

$$avble_i^A = ID_i^A - UD_i^A, \quad i \in F \tag{14}$$

$$avble_i^D = ID_i^D - UD_i^D, \quad i \in F' \tag{15}$$

$$X_{it}^A \geqslant 0, \quad i \in F, \ t \in T \qquad X_{it}^D \geqslant 0, \quad i \in F', \ t \in T \tag{16}$$

$$X_{it}^A \in \{0, 1\}, \quad i \in F, \ t \in T, \ (i, t) \notin Y^A \tag{17}$$

$$X_{it}^D \in \{0, 1\}, \quad i \in F', \ t \in T, \ (i, t) \notin Y^D \tag{18}$$

$$ID_i^A \geqslant 0, \quad i \in F \qquad ID_i^D \geqslant 0, \quad i \in F' \tag{19}$$

$$IE_i^A \geqslant 0, \quad i \in F \qquad IE_i^D \geqslant 0, \quad i \in F' \tag{20}$$

$$avble_i^A \geqslant 0, \quad i \in F \qquad avble_i^D \geqslant 0, \quad i \in F' \tag{21}$$

Eqs. (3) and (4) are ensuring that each flight is assigned to exactly one slot. None of the flights should be left out or scheduled twice. Constraints (5) and (6) are capacity constraints on arriving and departing runway capacities. Constraint (7) states that total allocated flights to consecutive time slots cannot exceed total capacity of the given slot. Note that the capacity limitation represents infeasibility of scheduling two flights to use the same runway in consecutive time slots, if time slots are short. If necessary more of these constraints can be added to ensure safety. In (8) aircraft connectivity is aimed, if there exists a connection then these constraints force time between those flights to be higher than turnaround time of that aircraft, otherwise it is redundant. (9) indicates the connections for passengers; if there is at least one passenger connected from a flight to another then this

constraint forces duration between them to be not shorter than passenger connection time of that airport. For every arriving and departing flight there is an earliest time for operation. Constraints (10) and (11) make sure that no flight is scheduled earlier than its possible earliest arrival or departure time. Constraints (12) and (13) calculates the amount of delay or earliness incurred. Constraints (14) and (15) are also used to calculate model's achievement in avoiding the delay within the possible limits. Note that these constraints are not actual constraints but our representation of the improvement possibilities when comparing with the real case. Hence, these equations will not be part of the generic model.

Note that if we consider all constraints ((3) thru (13)), a stakeholder different than the central authority will likely have no available exact data needed to write constraint sets (5) thru (9). However, except (9) it is very likely to have good estimates. For (9), on the other hand, depending on the airport being a hub or not, stakeholders may have a good chance of a reasonable estimate. There might be other constraints of concern which are not included in the above formulation. Constraints on gate availability, crew connections, and many more are not included, but one may find how to represent those from the literature. Of course, a stakeholder having information on these may include those constraints into the formulation she uses for herself. For convenience, we do not explicitly state those constraints which very likely require private stakeholder information to set up (and relatively harder to estimate for other stakeholders).

2.2. Possible performance measures

There are various performance measures available depending on the stakeholder solving the problem, as well as the stage in a collaborative decision making process. Please refer to Sama et al. (2017) for a recent review. For instance, for the service provider to minimize average delay, number of changes, cost of delay and cost of change are rather standard performance measures especially if efficiency is the main concern. When equity is the only concern, measures like minimization of maximum delay, or minimization of delay per flight for an airline minus average delay per flight for all airlines make sense. However, in general an objective function defined may utilize both type of performance measures, very similar to coming up with an index to optimize. The formulation given in the previous subsection allows us to model standard performance measures right away, and non-standard ones with additional parameters and decision variables defined. Some examples are presented in Appendix A.

The dimensions of the problem specified by the generic module are provided in Table 1. Note that the additional variable definitions and constraints that we have from objective function definition will be added to these numbers.

3. Suggested guidelines to utilize the rescheduling model

In this section, we consider two broad applications of the generic model: in the first we specify ways of manipulating performance measures (and objective functions) to solve the problem for different stakeholders. In the second, we outline an approach for utilizing the generic model to aid stakeholders for deciding on possible slot exchanges, of course if the specific CDM environment considered allows

Table 1
Size of the generic model.

Number of	
Binary variables	$T * (|F| + |F'|)$
Integer variables	$2 * (|F| + |F'|)$
Continuous variables	$|F| + |F'| + 10 * |K|$
Constraints	$9 * (|F| + |F'|) + 2 * |F| * |F'| + 3 * T + 3 * |K|$

for operational slot exchange among airlines.

The rescheduling model is specified as a flexible tool which can be used by any of the stakeholders involved in such a planning activity. Stakeholders considered are Central Authority (FAA, EuroControl) in charge of all operations, an airline that has a flight in the considered set of airports, and a passenger collective (a hypothetical NGO representing passengers). For each stakeholder, we propose use of several relevant performance measures as objectives. Each measure can be used as is to represent an objective function, or a number of those can be applied lexicographically. Additionally, a number of those can be used together applying an MCDM representation.

3.1. Approaches to solve a problem instance

In this section, we propose three ways to solve the rescheduling problem.

3.1.1. Objective function represented by a single performance measure

The problem can be solved by selecting one of the performance measures and using it as the objective. Performance measure will vary according to the stakeholder and the purpose. This way, problem solver can select the most important measure and see how good it can get and see how applicable this solution is in terms of other measures.

3.1.2. MCDM approach

The multi criteria decision making (MCDM) approach that considers weighted average of relevant performance measures as the objective function is straightforward to apply both in terms of formulation and solution, as long as weights can be determined. Application of this approach requires the decision maker to determine its priorities and calculate the weights of each measure. Note that any stakeholder may be solving this problem. Weights may represent a stakeholder's priorities among several performance measures. This might represent the input of the stakeholder into the CDM process. Another possibility for weights to represent, on the other hand, might be the stakeholder's view of the possible weights of other stakeholders for a set of performance measures. This would still be an input to the CDM process, but this time a compromise is made in obtaining the solution. One would expect the Central Authority to approach the problem in this manner, at least in the final step of the CDM process.

3.1.3. Lexicographic approach

Although it can be considered as a special case of MCDM, we would like to emphasize lexicographic approach as it is natural to use in such CDM environments. We suggest the use of the following approach to exemplify the application of the idea that uses relevant performance measures as objectives in a hierarchical manner. Let there be n performance measures with the most important one being OB_1, the next OB_2, and so on, up to OB_n. This translates to the following set of problems to be solved (without loss of generality, assume that objective functions are of minimization type):

Algorithm 1. Lexicographic Approach Mechanism

Step 0: Let $j = 1$
Step 1: Solve the problem to optimize OB_j. Let the optimal solution be f(OB_j)
Step 2: Add a new constraint that requires the value of the function OB_j to be at most f(OB_j) $\times e_j$, where $e_j \geqslant 1$
Step 3: Increase j by 1. If $j = n + 1$, Stop. Otherwise go to Step 1.

We think that each stakeholder will consider the others, but of course with less priority, meaning that while ranking the objectives the ones that are more important will come first. Note that if all e_j's are selected to be 1, we have a goal programming model with strict priorities. A stakeholder may use this procedure to obtain a solution for her

benefit. On the other hand, at each stage j one stakeholder may be solving the problem, which is then taken as a constraint for the next. In a CDM environment one would expect such an application with the central authority supervising the application.

Finally note that there can be many different objectives defined as a function of delay and different measures can be created, so that the consistency with the approach is maintained.

3.2. Usage within the slot trading mechanism

Consider an airline having usage rights over any slot assigned to it. More emphasis on the property rights is stated by Vossen and Ball (2006b), who claim that notion of slot ownership is one of the main tenets of the CDM paradigm. Moreover, they add the fact that it is a general consensus among airlines as well.

Under the CDM program, slot exchange mechanism is conducted by Compression Algorithm Vossen and Ball (2006a). More recently, Vossen and Ball (2006b) suggest that airlines can make an offer with multiple slots in exchange for multiple slots in return. The mechanism is called as at-most, at-least (AMAL) offer such that an airline proposes to move a flight at most t_1 minutes later slot in exchange for moving its another flight to at least t_2 minutes earlier slot. This mechanism results in more flexible exchange opportunities. This offer mechanism can be used for transferring a delay from critical to less critical flights. If the main concern for an airline is to minimize the weighted passenger delay, an airline can prefer moving the flight operated by a smaller aircraft in a later slot in exchange for moving the flight operated by a larger aircraft to earlier time in return. Alternatively, this trade offer mechanism can be utilized to increase the opportunity of satisfying the vulnerable connections. Therefore, each airline can make an AMAL offer based on own preferences (or performance measures), leading a trade which can be mutually beneficial.

In Algorithm 2, we describe how we can use the generic model to Slot Trade Offer Mechanism. Firstly, each airline solves the rescheduling model proposed in the previous sections using any approach described in Section 3.1 together with any set of performance measures. Without loss of generality, we can assume that the objective of the airline is to minimize an objective function. Once a solution is reached, rescheduled flights obtained can be listed such that the top change has the largest contribution to decrease the objective function value, and the list contains all the proposals in a decreasing order of contribution. The airline wishes to keep the flight on top of the list in the slot same as her rescheduling solution, since its contribution to the objective value is maximum. Therefore, airlines make an offer to move the flight at the bottom of the interval to a later acceptable time period in exchange for realizing the flight on the top of the list as rescheduled (or within an acceptable interval around the rescheduled time). $t(f)$ is the slot of the flight f determined by the rescheduling model and ϵ can be specified by each airline.

Algorithm 2. Slot Trade Offer Mechanism

---

for each airline $k \in K$ **do**
  Airline k solves rescheduling model with respect to his/her objective preference and generates a list (L_k) involving her flights $f \in F_k$ in decreasing order of *objective value deduction*
  index $m = 1$
  while $L_k = \varnothing$ **do**
    $f_m^1 :=$ flight is on the top of the list
    $f_m^2 :=$ flight is on the bottom of the list
    Airline k states:
    "Flight f_m^2 can be moved to any time within the interval $(t(f_m^2) - \epsilon_m^2, t(f_m^2) + \epsilon_m^2)$
    in exchange for moving f_m^1 to any time within $(t(f_m^1) - \epsilon_m^1, t(f_m^1) + \epsilon_m^1)$"
    Delete flights f_m^1 and f_m^2 from the list
    $m = m + 1$
  end while
end for

Note that if these proposals are to be used centrally, an approach described in Sherali et al. (2011) can be applied.

4. A case study

This section reports the efforts spent to validate the model and verify that it can bring benefits, as prescribed. We follow the following steps in reporting the case.

(a) We state the preliminaries (basic assumptions, data sources, etc.) in Section 4.1.
(b) As we only use publicly available data, there is no possibility for us to estimate or use data privately available to some stakeholders. To come up with a reasonable case, we calibrate the data using and comparing the model results and actual (realized) schedule reported. As a result of this step, we obtain a data set that we can further analyze using the approach presented in the paper. Calibration efforts are explained in Section 4.2.
(c) We analyzed what we called the "base case" in Section 4.3. In this base case, we implemented a lexicographic approach presented in Section 3.1.3.
(d) In Section 4.4, we consider the computational issues and duality gap for the set of problems solved in Section 4.3. This section is important as to make sure that the proposed approach obtains a reasonable solution within the time limitations required for a real application.
(e) Finally, in Section 4.5, we make a number of experiments, including more runs using lexicographic approach (Section 3.1.3), MCDM approach (Section 3.1.2) and three scenarios where in two of them the central decision maker implements a more interactive CDM approach by considering constraints established by the airlines.

4.1. Preliminaries

We selected Minneapolis-St. Paul International Airport (MSP) as it is a hub airport and 46% of the passengers are connecting. United States Department of Transportation publishes actual schedule of the airports on their web page. Information is available about all landings and takeoff times, delays or earliness made from the original schedule, in terms of flight number, tail number of the aircraft; airline ids, all provided in tables. We used data for 1.March 2015 time between 10.00 am and 16.00 pm, Bureau of Transportation Statistics (2015). There are 151 arrivals and 155 departures reported in the 6-h period. The selected date represents a standard day, not very much affected by Minnesota's possible extreme weather conditions, causing exceptional delays for very likely biased results. There was not any specific information about connections but by using the tail numbers we found the aircraft connections in the given day. For the turnaround times we used tail number to find the manufacturer and model of the planes. FAA has a tail number inquiry service that gives specific information about aircraft (FAA, 2016b). We used information from Boeing for turnaround times and estimated those times for other aircraft by property comparison. Boeing has the Airplane Characteristics for Airport Planning Reports published on its website where average turnaround times for each model are provided (Boeing, 2011).

Data provided the airline of each flight and it was found that majority of the flights belong to Delta Airlines, 148 out of 304 flights. There are 78 SkyWest flights. Rest of the flights belonged to several airlines; we assumed that they formed a third airline group.

4.2. Calibration of the data

We have the information about aircraft connection using tail number of the aircraft; however there is no info regarding the passenger connections. We initially assumed that there is passenger connection between all the flights which have aircraft connection. Moreover, as

MSP is a hub, we further assumed that there is a connection between all the flights of an airline where there is minimum of 30 min gap between one arrival and another departure in the original schedule. Purpose of calibration is to revise these assumptions so that we have pertinent data for a more close-to-real case.

One complicated set of input is related to the earliest departure and arrival times of flights. We have the initial schedule and realized schedule information on hand; however we don't know how early a flight can be shifted. For the arrival information we took the minimum of initial and realized landing time of the aircraft. If it arrived before its original schedule time, we assumed we can also use that limit. For arrivals it is easier since that flight's departure is planned in another airport and it can be on air while we are solving the problem, we cannot interfere its timing too much.

For departures we have the chance to delay the flight on the ground. However we do not know the exact earliest departure time. If a departing aircraft operated earlier than the initial schedule we took the realized time as the earliest departure time of that flight, but if it had a delay we need to consider different cases. Since there is no information for the cause of the delay we cannot know whether the delay could have been avoided by rescheduling or not. Delay could have been caused by a natural or a mechanical problem, which makes it unavoidable. If we take all of that delay as avoidable it would not be fair to the realized schedule. On the other hand if we take it all as unavoidable it would not be realistic and there won't be any room for improvement. As a result, we use different scenarios as 100%–70%–30%–0% unavoidable and compare the schedules. Note that constraints (14) and (15) are used to measure the effect of these scenarios.

If we had all the correct connection and capacity information the total delay and the schedule we obtained by 100% unavoidable scenario would be very close, if not equal to the realized schedule. Using this property we are able to calibrate the set of state parameters which will induce this equality condition to be satisfied. In other words, this enables us to validate the parameters selection of our model and hence later comparisons will yield more reliable results.

Starting from the highest delay, the cause of each delay obtained by the model has been examined. If the limitation is caused only by a passenger connection, that connection is eliminated (hence relaxed). For each delay, this method has been used to identify the reasons causing it.

After this stage, we are ready to test the model for further issues, as we now can claim that the set of state-defining variables we are using, together with the scenarios are validated. A copy of the finalized input data set is provided at an accompanying WEB site. Note that, in the experiments we assume that this data is available and/or correctly estimated and hence available for use to all stakeholders, following the third aim for the model being "generic" as introduced in Section 1. We summarized the realized values for the performance measures used in the study in Tables 2 and 3.

4.3. Base case and comparison with the actual outcome

We first consider the application of a simple lexicographic approach to the available data. Our first objective function is minimizing average delay which is taken as the main efficiency measure in the rescheduling problem. As we decrease the amount of unavoidable delay we get better

Table 2
Realized schedule system measures.

Performance measures per flight	Min
Average delay	6.33
Average arrival delay	6.79
Average departure delay	5.88
Max delay	194.00
Average delay deviation	3.032

Table 3
Realized schedule airline measures.

Performance measures	Delta Airlines	Skywest Airlines	Rest
Average delay (min)	4.74	8.60	7.09
Max delay (min)	194.00	80.00	137.00
Average delay deviation (min)	−1.586	2.273	0.759

results in terms of all the performance measures used in this example. One of the critical measures is the maximum delay a flight faces. Note that this is important as a fairness measure; even if the overall delay is comparatively short, if a small group of flights has to live with all the total delay, the result would not be fair. Therefore one of our fairness measures is to minimize the maximum delay in the schedule. Since this measure only deals with the highest delay it will not search for better results in other flights, as long as the maximum delay is minimized other flights will be allocated randomly each with shorter than maximum delay. Note that unless one uses a lexicographic approach or something similar, the results of solving minimization of maximum delay problem will not be reasonable. Finally, third objective is placed so that possibility of a significant difference between airlines will be avoided. Note that this time fairness is considered among different airlines. This fairness measure should be used together with an efficiency measure (like minimization of maximum delay) as it may result in very poor delays values if applied by itself. These arguments support the fact that we have the lexicographic approach outlined in Section 3.1.3.

The solution we obtained when average delay per flight is minimized is given in Table B.1, under the column which says "100% unavoidable delay". The other two measures used were maximum delay and average delay deviation of airlines per flight. Note that these results correspond to the calibrated case and value of the average delay is, for all practical purposes, very close to the actual delay observed. To be more specific, at the end of calibration there is still a difference of 0.4 min of delay per flight remaining (compare Table 2, Table 3 numbers, with the "100% unavoidable delay" column of Table B.1). By comparison with the realized data it is found that those delays can be attributed to some case specific situations and hence we decided not to change the constraints further.

Table 4 summarizes the results for the 70% unavoidable delay scenario (Details of computations can be found in Appendix B. The first column stands for the performance measure which is used as the objective (main objective)). The other columns record the values of the three performance measures obtained.

Note that, any row can be considered as a candidate non-dominating solution in terms of the objectives used. In this case, we observe that the first row represents values which are at most the values obtained by the realized schedule, in other words we obtained a dominating (re) schedule in terms of all performance measures considered.

All the numerical results are summarized in three tables presented in Appendix B.

4.4. Computational issues and duality gap

For the above Base-Case described, including the additional variables and constraints needed for the three performance measures

Table 4
Performance measure results of 70% unavoidable delay scenario.

OPTIMIZED FOR	Average delay (min)	Max delay (min)	Average delay deviation (min)
Average delay	4.94	136.00	2.542
Max delay	6.71	136.00	5.298
Average delay deviation	7.50	136.00	0.000

considered, there were 202,210 binary, 608 integer and 354 continuous variables and 50,956 constraints in the model. We attempted to solve this problem with 304 flights with CPLEX 12.6 solver.

While running the experiments we had used a few very basic remedies to decrease or eliminate the duality gap when we have limited the run times to at most 20 min, which we think is an acceptable time for a real-time application. Note that, one can utilize more sophisticated approaches (cuts, reformulations, etc.) if 20-min time limit is to be further decreased. These remedies are,

1. Related performance measures are placed in the objective function with a very small weight so that a solution among the alternatives which produces better performance measure values (other than the one we actually want to optimize) is selected. This in turn lowers the chances of having a duality gap, for example when applying several solutions lexicographically.
2. In the generic formulation described in Section 2.1, we used equation sets (12) and (13) with the standard logic to represent the fact that either earliness or delay is supposed to get a positive value and the other should be zero. This way, model works when objective function includes minimizing delay directly. However, when objective function is different, both earliness and delay values can get positive. Even though delay is zero, since they both get positive values, misleads other performance measures. In Appendix C, we present additional decision variables and constraints to ensure that this does not happen. Additionally, we need to treat some other variables in the generic model similarly. Please see Appendix C for details. Note that the updated model has more binary variables. However computation times were not affected significantly.
3. To prevent numerical instability, one needs to normalize the values of performance measures when a weighted sum of different measures is optimized.
4. When solving problems lexicographically, one may need to put a bound on the worst solution of a performance measure which is not optimized simply to eliminate some of the undesired alternative solutions. This is not straightforward; it may require solving the problem with different bounds.

As a result of these modifications, we obtained an optimal solution for 178 of the 184 problems within 2–4 min. We have only encountered duality gaps within 20 min of CPU time allowed which are less than 3% for the remaining 6 problems. Of course, if time allowed is less, duality gap for such instances (in our experiments 6 out of 184) will be more.

4.5. Description of the experiments and overall results

We used the input data under various scenarios. Our aim in selecting these computational setting scenarios is simply to show that the model presented works as desired. The following runs were carried out:

1. *Generalized Base Case Scenarios:* We used all the three objectives functions in the base case but changed the order of solutions. As a result, we had 6 different solutions for each of the unavoidable levels. In these experiments we used $e_i = 1$. We call these scenarios as Scenario O (Scenarios O1, O2, O3, O4, O5, O6 are corresponding to; (Average delay, Max delay, Average delay deviation), (Average delay, Average delay deviation, Max delay), (Max delay, Average delay, Average delay deviation), (Max delay, Average delay deviation, Average delay), (Average delay deviation, Max delay, Average delay), (Average delay deviation, Average delay, Max delay) respectively, each triplet is solved lexicographically in the provided order).
2. *Scenarios based on simple MCDM approach - Weighted Sum of Measures:* In these problems we used weighted average of average delay, maximum delay and average delay deviation per flight. However, each measure has a different range and when weights are

Table 5
Scenario O with $e = 1$ and weighted objective solutions.

Scenarios	Performance measure per flight in minutes		Domination
Scenario O1	Average delay	0.50	
	Max delay	36.00	No
	Average delay deviation	0.288	
Scenario O2	Average delay	0.50	
	Max delay	36.00	No
	Average delay deviation	0.288	
Scenario O3	Average delay	0.50	
	Max delay	36.00	No
	Average delay deviation	0.288	
Scenario O4	Average delay	0.63	
	Max delay	36.00	No
	Average delay deviation	0.131	
Scenario O5	Average delay	0.63	
	Max delay	36.00	No
	Average delay deviation	0.131	
Scenario O6	Average delay	0.63	
	Max delay	36.00	No
	Average delay deviation	0.131	
Scenario O7 (0.33,0.33.34)	Average delay	0.53	
	Max delay	36.00	No
	Average delay deviation	0.230	
Scenario O8 (0.5,0.3,0.2)	Average delay	0.51	
	Max delay	36.00	No
	Average delay deviation	0.269	
Scenario O9 (0.3,0.5,0.2)	Average delay	0.52	
	Max delay	36.00	No
	Average delay deviation	0.236	
Scenario 10 (0.3,0.2,0.5)	Average delay	0.76	
	Max delay	36.00	No
	Average delay deviation	0.000	
Scenario 11 (0.9,0.05,0.05)	Average delay	0.50	
	Max delay	36.00	No
	Average delay deviation	0.288	
Scenario 12 (0.05,0.9,0.05)	Average delay	0.53	
	Max delay	36.00	No
	Average delay deviation	0.230	
Scenario 13 (0.05,0.05,0.9)	Average delay	0.76	
	Max delay	36.00	No
	Average delay deviation	0.000	

assigned directly to these measures the ones with the higher values will have priority and weights are not reflected realistically. Therefore, we normalized these values to put them in a common scale. While doing this we used the results of Scenario O. We solved seven different weight combinations. We categorize these scenarios as part of Scenario O and call them Scenario O7 through O13.

3. *Scenarios to represent a possible CDM process:* Here we aim to show how the model can be used by different stakeholders and by using epsilon constraints non-dominated solutions can be generated. Several scenarios are created to show possible solutions to be obtained by a collaborative decision making process. Two values of

Table 6
Scenarios for 0% unavoidable delay.

| Scenarios	Performance measure per flight in minutes $e = 1.1$		Domination	Performance measure per flight in minutes $e = 1.4$		Domination
Scenario 1	Average delay	0.55		Average delay	0.70	
	Max delay	36.00	Non dominated	Max delay	36.00	Non dominated
	Average delay deviation	0.220		Average delay deviation	0.075	
Scenario 2	Average delay	0.50		Average delay	0.55	
	Max delay	36.00	Non dominated	Max delay	36.00	Non dominated
	Average delay deviation	0.301		Average delay deviation	0.209	
Scenario 3	Average delay	1.50		Average delay	9.00	
	Max delay	39.00	Non dominated	Max delay	351.00	Non dominated
	Average delay deviation	0.000		Average delay deviation	0.000	

constant e_j are used: 1.1 and 1.4. Here are the details:

- Scenario 1
    - Step 1: Minimize average system delay.
    - Step 2: Minimize average delay deviation of airlines per flight with the epsilon constraint of average delay.
    - Step 3: Minimize maximum system delay with the epsilon constraints of average delay and average delay deviation.
- Scenario 2
    - Step 1: Delta Airline solves for minimizing own average delay.
    - Step 2: SkyWest Airline solves for minimizing own average delay.
    - Step 3: Central Authority uses Delta and SkyWest average delays as epsilon constraints and minimizes over average delay deviation.
- Scenario 3
    - Step 1: Delta Airline solves to minimize its own maximum delay.
    - Step 2: SkyWest Airline solves to minimize its own maximum delay.
    - Step 3: Central Authority uses Delta and SkyWest maximum delays as epsilon constraints and minimizes over average delay deviation.

The detailed results obtained can be made available at an accompanying WEB site. Here we present some sample results: In Table 5, we summarize the solutions for Scenario O, for 0% unavoidable delay and $e_i = 1$. In Table 6, we summarize the results for Scenarios 1, 2 and 3 for 0% unavoidable delay and $e_i = 1.1$ and 1.4. Note that none of the solutions are dominated by another. Fig. 1 demonstrates one other possible way of displaying results. Note that with these examples we demonstrate the capabilities of the generic model if applied in a CDM environment.

Finally, we carry out an experiment to compare our results with the Ration-by-schedule (RBS) which is practiced by FAA. We adopted the RBS algorithm to our problem setting. In the adaptation of RBS (we name it as A-RBS) instead of using earliest arrival/departure times, we used initial scheduled slots of flights as described in Vossen and Ball (2006a). Table 7 indicates the comparison of performance measures presented in "100% unavoidable delay" column of Table B.1 in Appendix B and obtained with A-RBS method. A-RBS algorithm preserves the initial order of flights. From the results it can be seen that when focus is on the order of flights, average delay is highest among all four solution methods. In terms of average delay deviation A-RBS gives more balanced results than the second column where main objective function is minimizing average delay. Here, the important take-away is to understand the trade-off between performance measures. Concentrating on certain fairness or efficiency criteria yields different assignments that are not necessarily compatible. That shows the importance of incorporating different performance measures in rescheduling problem. By employing different combinations of performance measures in the
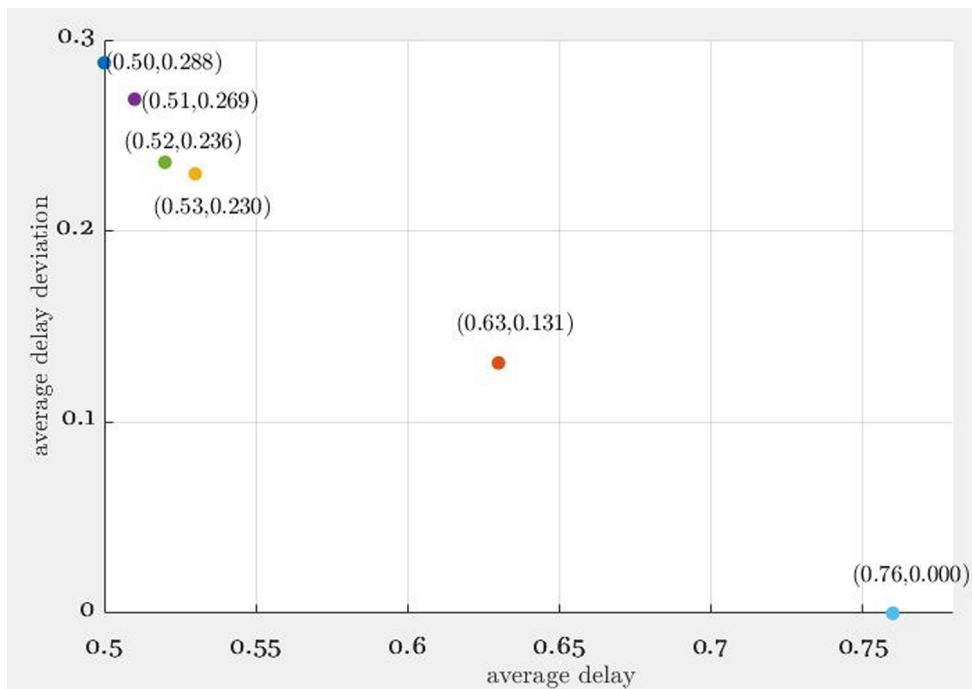
Fig. 1. 0% unavoidable delay Pareto solutions, in all cases maximum delay is 36.00 min.

Table 7
Comparison of solution methods with 100% unavoidable delay and adopted RBS.

Performance measures	Average delay (min)	Max delay (min)	Average delay deviation (min)	A-RBS
Average delay	6.73	8	9	9.92
Max Delay	194	194	240	196
Ave. Delay Devi.	3.411	2.449	0	2.779

proposed generic model, stakeholders can understand the trade-offs clearly and make better decisions in the CDM environment.

5. Conclusions and future research

As popularity of travel is increasing all around the world, airport congestions are more apparently observed. This study aims at supporting the collaborative decision making (CDM) framework proposed by navigation service providers. Specifically, we propose a generic mathematical model which will support rescheduling decisions made on day-to-day basis. The proposed model can be utilized by any stakeholder; model construction for a stakeholder is finalized by selection of an appropriate objective function. A stakeholder will consider the available information (data or estimates), and information is likely to be asymmetric. CDM may require a single or repeated number of inputs from the stakeholders and the main use of the proposed model is to aid a stakeholder in the preparation of the input to CDM.

We discuss how the model can be applied. Lexicographically applying several reasonable performance measures or an MCDM type approach will likely fulfill most of the needs of a stakeholder. In case of a more sophisticated CDM environment where airline slots can be traded to realize a reschedule, we propose an algorithm that utilizes the proposed generic model to generate exchange possibilities for the airlines.

A case study was prepared where we were able to calibrate data (and in a way validate the model) to compensate for the unknown parameters. We then used the case to show that our approach generates results which dominates the realized reschedule under three performance measures, two of them being equity based. More examples to represent possible CDM processes are specified and shown that the generic model can be utilized in the related environments.

The model can be extended to cover more than one airport, or to model other limitations at an airport. This work did not concentrate on the solution time, as we were able to solve the problem we proposed in a reasonable time. However, if one would like to extend and detail the generic model, attention should be spend for the solution efficiency, as well.

Another possible extension is to come up with algorithms which will cover other ways of fulfilling slot trading mentioned in the literature. Note that information asymmetry on the state-defining parameters we use in the model is an obstacle for a more thorough cooperation in the process of operational slot trading among airlines.

Appendix A. Examples for performance measures

The following are examples of performance measures that can complement the model presented. There are numerous possibilities and all the measures that we found in the literature can be formulated in a similar manner, of course some requiring additional decision variables.

A.1. Maximum change

Additional Variables:

$maxdelay$: maximum delay occurred in the schedule
$maxchange$: maximum change occurred in the schedule
$maxdelay_k$: maximum delay occurred in flights of airline $k \in K$
$passmaxdelay$: maximum passenger weighted delay in the schedule
$passmaxdelay_k$: maximum passenger weighted delay in flights of airline $k \in K$

$$\min \quad maxchange \tag{22}$$

$$maxdelay \geqslant TD_i^A, \quad i \in F \tag{22a}$$

$$maxdelay \geqslant TD_i^D, \quad i \in F' \tag{22b}$$

$$maxearly \geqslant TA_i^A, \quad i \in F \tag{22c}$$

$$maxearly \geqslant TA_i^D, \quad i \in F' \tag{22d}$$

$$maxchange \geqslant maxdelay \tag{22e}$$

$$maxchange \geqslant maxearly \tag{22f}$$

$$maxdelay_k \geqslant TD_i^A, \quad i \in F_k \tag{22g}$$

$$maxdelay_k \geqslant TD_i^D, \quad i \in F'_k \tag{22h}$$

$$passmaxdelay \geqslant D_i^A \times TD_i^A, \quad i \in F \tag{22i}$$

$$passmaxdelay \geqslant D_i^D \times TD_i^D, \quad i \in F' \tag{22j}$$

$$passmaxdelay_k \geqslant D_i^A \times TD_i^A, \quad i \in F_k \quad k \in K \tag{22k}$$

$$passmaxdelay_k \geqslant D_i^D \times TD_i^D, \quad i \in F'_k \quad k \in K \tag{22l}$$

A.2. Average passenger delay

Additional Variables:

APD: average passenger delay in the schedule
APD_k: average passenger delay for airline $k \in K$

$$\min \quad APD \tag{23}$$

$$APD = \frac{\sum\limits_{i \in F} ID_i^A \times D_i^A + \sum\limits_{i \in F'} ID_i^D \times D_i^D}{\sum\limits_{i \in F} D_i^A + \sum\limits_{i \in F'} D_i^D} \tag{23a}$$

A.3. Performance measures based on differences among airlines

Additional Variables:

AD: average delay occurred in the schedule
AD_k: average delay occurred in the flights of airline $k \in K$
AD_k^+: average delay of airline $k \in K$ if it is than average delay
AD_k^-: average delay of airline $k \in K$ if it is lower than average delay

$$\min \quad \sum\limits_{k \in K} AD_k^+ \tag{24}$$

$$AD = \frac{\sum\limits_{i \in F} ID_i^A + \sum\limits_{i \in F'} ID_i^D}{|F| + |F'|} \tag{24a}$$

$$\text{where } |\cdot| \text{ denotes cardinality of set } \{\times\}$$

$$AD_k = \frac{\sum\limits_{i \in F} ID_i^A + \sum\limits_{i \in F'} ID_i^D}{|F_k| + |F_k'|} \quad k \in K \tag{24b}$$

$$AD_k - AD = AD_k^+ - AD_k^- \quad k \in K \tag{24c}$$

A.4. A cost related performance measure

Additional Variables:

M: set of intervals for delays
Z_{im}^A: 1 if flight $i \in F$'s delay is in interval $m \in M$, and 0, otherwise.
Z_{im}^D: 1 if flight $i \in F'$'s delay is in interval $m \in M$, and 0, otherwise.
$\overline{L}_m$: upper bound of interval $m \in M$
$\underline{L}_m$: lower bound of interval $m \in M$
$cost_m$: cost of delay in interval $m \in M$
θ: operation cost of delay
$Cost^A$: total operational and passenger cost of delayed flights $i \in F$ and
$Cost^D$: total operational and passenger cost of delayed flights $i \in F'$
$Cost_k^A$: total operational and passenger cost of delayed flights $i \in F_k$ of airline $k \in K$
$Cost_k^D$: total operational and passenger cost of delayed flights $i \in F_k'$ of airline $k \in K$

$$\min \quad Cost^A + Cost^D \tag{25}$$

$$\sum\limits_{m \in M} Z_{im}^A = 1, \quad i \in F \tag{24a}$$

$$\sum\limits_{m \in M} Z_{im}^D = 1, \quad i \in F' \tag{24b}$$

$$\sum\limits_{m \in M} Z_{im}^A \times \underline{L}_m \leqslant ID_i^A, \quad i \in F \tag{24c}$$

$$\sum\limits_{m \in M} Z_{im}^A \times \overline{L}_m \geqslant ID_i^A, \quad i \in F \tag{24d}$$

$$\sum_{m \in M} Z_{im}^D \times \underline{L}_m \leqslant ID_i^D, \quad i \in F' \tag{24e}$$

$$\sum_{m \in M} Z_{im}^D \times \bar{L}_m \geqslant ID_i^D, \quad i \in F' \tag{24f}$$

$$Cost^A = \sum_{i \in F} \sum_{m \in M} Z_{im}^A \times D_i^A \times cost_m + \sum_{i \in F} \sum_{2}^{M} Z_{im}^A \times \theta \tag{24g}$$

$$Cost^D = \sum_{i \in F'} \sum_{m \in M} Z_{im}^D \times D_i^D \times cost_m + \sum_{i \in F'} \sum_{2}^{M} Z_{im}^D \times \theta \tag{24h}$$

$$Z_{im}^A \geqslant 0, \quad i \in F, \ m \in M \qquad Z_{im}^D \geqslant 0, \quad i \in F', \ m \in M \tag{24i}$$

$$Z_{im}^A \in \{0, 1\} \quad i \in F, \ m \in M \qquad Z_{im}^D \in \{0, 1\} \quad i \in F', \ m \in M \tag{24j}$$

$$Cost^A \geqslant 0, \qquad Cost^D \geqslant 0 \tag{24k}$$

Appendix B. Case study results

Table B.1
Model results with the objective of minimizing average delay.

	Minimize Average Delay			
	100%unavoidable delay (min)	70%unavoidable delay (min)	30%unavoidable delay (min)	0%unavoidable delay (min)
Average delay	6.73	4.84	2.38	0.50
Avoidable average delay	0.40	0.31	0.36	0.50
Average arrival delay	6.79	4.84	2.16	0.01
Average departure delay	6.68	4.85	2.60	0.99
Delta average delay	4.95	3.61	1.74	0.35
SkyWest average delay	8.84	6.50	3.21	0.77
Rest average delay	8.03	5.66	2.74	0.53
Max delay	194.00	136.00	59.00	36.00
Delta max delay	194.00	136.00	59.00	32.00
SkyWest max delay	80.00	56.00	37.00	36.00
Rest max delay	137.00	96.00	42.00	28.00
Average delay deviation	3.411	2.542	1.189	0.295
Delta average deviation	-1.784	-1.232	-0.640	-0.148
SkyWest average deviation	2.116	1.660	0.825	0.269
Rest average deviation	1.295	0.883	0.364	0.025

Table B.2
Model results with the objective of minimizing maximum delay.

	Minimize Maximum Delay			
	100%unavoidable delay (min)	70%unavoidable delay (min)	30%unavoidable delay (min)	0%unavoidable delay (min)
Average delay	8.00	6.71	2.72	0.95
Avoidable average delay	1.67	2.14	0.70	0.95
Average arrival delay	8.43	4.99	2.37	0.13
Average departure delay	7.59	8.38	3.06	1.74
Delta average delay	6.72	3.92	1.99	0.86
SkyWest average delay	9.56	9.72	3.64	1.03
Rest average delay	8.88	9.00	3.18	0.99
Max delay	194.00	136.00	59.00	36.00
Delta max delay	194.00	136.00	59.00	36.00
SkyWest max delay	80.00	136.00	53.00	36.00
Rest max delay	139.00	136.00	42.00	36.00
Average delay deviation	2.449	5.298	1.378	0.113
Delta average deviation	-1.277	-2.791	-0.727	-0.065
SkyWest average deviation	1.564	3.008	0.921	0.076
Rest average deviation	0.885	2.290	0.459	0.037

Table B.3

Model results with the objective of minimizing average delay deviation among airlines per flight.

	Minimize average delay deviation of airlines per flight			
	100%unavoidable delay (min)	70%unavoidable delay (min)	30%unavoidable delay (min)	0%unavoidable delay (min)
Average delay	9.00	7.50	3.50	1.50
Avoidable average delay	2.66	2.93	1.48	1.50
Average arrival delay	7.06	6.53	2.57	0.34
Average departure delay	10.88	8.45	4.41	2.63
Delta average delay	9.00	7.50	3.50	1.50
SkyWest average delay	9.00	7.50	3.50	1.50
Rest average delay	9.00	7.50	3.50	1.50
Max delay	240.00	136.00	59.00	60.00
Delta max delay	240.00	136.00	59.00	47.00
SkyWest max delay	80.00	56.00	38.00	60.00
Rest max delay	140.00	96.00	42.00	45.00
Average delay deviation	0.000	0.000	0.000	0.000
Delta average deviation	0.000	0.000	0.000	0.000
SkyWest average deviation	0.000	0.000	0.000	0.000
Rest average deviation	0.000	0.000	0.000	0.000

Appendix C. Mathematical model update

In the calculation of delay and earliness both $ID_i^{(A,D)}$, $IE_i^{(A,D)}$ values may have positive values depending on the objective function; therefore we introduced $|F| + |F'|$ new binary variables and add the following set of constraints;

y_i^A: 1 if flight $i \in F$ incurred any delay, and 0 otherwise
y_i^D: 1 if flight $i \in F'$ incurred any delay, and 0 otherwise

$$ID_i^A \leqslant y_i^A \times t, \quad i \in F$$

$$IE_i^A \leqslant (1 - y_i^A) \times t, \quad i \in F$$

$$ID_i^D \leqslant y_i^D \times t, \quad i \in F'$$

$$IE_i^D \leqslant (1 - y_i^D) \times t, \quad i \in F'$$

$$y_i^A \in \{0, 1\}, \quad i \in F$$

$$y_i^D \in \{0, 1\}, \quad i \in F'$$

In the calculation of average delay deviation both AD_k^+, AD_k^- values may have positive values depending on the objective function; therefore we introduced $|K|$ new binary variables and add the following set of constraints;

p_k: 1 if deviation of airline $k \in K$ positive, and 0 otherwise

$$AD_k^+ \leqslant p_k \times a, \quad k \in K$$

$$AD_k^- \leqslant (1 - p_k) \times a, \quad k \in K$$

$$p_k \in \{0, 1\}, \quad k \in K$$

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