



Manufacturing & Service Operations Management

Publication details, including instructions for authors and subscription information:
<http://pubsonline.informs.org>

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To cite this article:

Osman Alp, Alper Şen (2021) Delegation of Stocking Decisions Under Asymmetric Demand Information. Manufacturing & Service Operations Management 23(1):55-69. <https://doi.org/10.1287/msom.2019.0810>

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Delegation of Stocking Decisions Under Asymmetric Demand Information

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Received: November 22, 2017

Revised: September 10, 2018; January 28, 2019; April 13, 2019

Accepted: April 24, 2019

Published Online in Articles in Advance: March 6, 2020

<https://doi.org/10.1287/msom.2019.0810>

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Abstract. *Problem definition:* We consider the incentive design problem of a retailer that delegates stocking decisions to its store managers who are privately informed about local demand. *Academic/practical relevance:* Shortages are highly costly in retail, but are less of a concern for store managers, as their exact amounts are usually not recorded. In order to align incentives and attain desired service levels, retailers need to design mechanisms in the absence of information on shortage quantities. *Methodology:* The headquarters knows that the underlying demand process at a store is one of J possible Wiener processes, whereas the store manager knows the specific process. The store manager creates a single order before each period. The headquarters uses an incentive scheme that is based on the end-of-period leftover inventory and on a stock-out occasion at a prespecified inspection time *before* the end of a period. The problem for the headquarters is to determine the inspection time and the significance of a stock-out relative to leftover inventory in evaluating the performance of the store manager. We formulate the problem as a constrained nonlinear optimization problem in the single period setting and a dynamic program in the multiperiod setting. *Results:* We show that the proposed “early inspection” scheme leads to perfect alignment when J equals two under mild conditions. In more general cases, we show that the scheme performs strictly better than inspecting stock-outs at the end and achieves near-perfect alignment. Our numerical experiments, using both synthetic and real data, reveal that this scheme can lead to considerable cost reductions. *Managerial implications:* Stock-out-related measures are typically not included in store managers’ performance scorecards in retail. We propose a novel, easy, and practical performance measurement scheme that does not depend on the actual amount of shortages. This new scheme incentivizes the store managers to use their private information in the retailer’s best interest and clearly outperforms centralized ordering systems that are common practice.

Supplemental Material: The online appendix is available at <https://doi.org/10.1287/msom.2019.0810>.

Keywords: incentive alignment • delegation of stocking decisions • asymmetric information

1. Introduction

In this paper, we study incentive issues in an inventory management setting in which attaining high on-shelf availability is crucial. In many industries, ramifications of stock-outs can be costly. For example, in retail, 1 of every 13 items a shopper seeks to buy is out of stock (Ehrental et al. 2014), leading to significant losses for the industry. Walmart estimates that reducing stock-outs may represent a \$3 billion opportunity for the company (Dudley 2014).

A key factor in the severity of the stock-out problem in many retail environments is the contrasting observability of excess inventory and stock-outs. Excess inventory that may boost shrinkages or be salvaged in a secondary market is observable and quantifiable; whereas stock-outs are not easily observable, and their adverse effects on immediate and future revenues are not well understood (Anderson et al. 2006). Therefore, a core

requirement to reduce stock-outs at the retailers is to develop an effective measurement system (Gruen and Corsten 2008), quantify the effect of stock-outs on profitability, and bring the issue to the attention of the management (ECR 2003).

Given a large number of stores a typical retail chain owns, directly managing operations in each store by headquarters is sometimes difficult. Some of the critical operational decisions are delegated to store managers who do not have immediate economic incentives to make better decisions because they are paid a fixed salary. Hence, it is imperative “to design appropriate incentives to motivate store managers to execute activities critical to the performance of the retail store” (DeHoratius and Raman 2007, p. 518). These incentives, including those to reduce stock-outs and excess inventory, can be tied to store managers’ performance scorecards which can be used later for compensation

and promotion decisions. The balanced scorecard is a common tool adopted for this purpose (Kaplan and Norton 1992). Despite the need for incentives, according to a survey by an alliance of food and consumer packaged goods manufacturers and retailers, only 9% of the retailers include stock-outs as a factor in their incentives or rewards (FMI/GMA Trading Partner Alliance 2015). Lowering stock-outs is only implicitly accounted for in the “Sales Revenue” key performance indicator (KPI) that is often used in performance scorecards. On-shelf availability is one of the few factors that store managers can use to influence sales in retail where sales generation is based on self-service (DeHoratius and Raman 2007).

There are two challenges with using incentives directly related to stock-outs in practice. First, stock-outs are not recorded in the transaction systems of most retailers and, therefore, hard to contract on. Second, store employees who are more heavily incentivized on availability explicitly may care less about carrying excess inventories, which are obviously also costly for the retailer. This leads to an incentive misalignment problem (see van Donselaar et al. 2010 for an example of store managers who are only rewarded for on-stock availability and order more or earlier than necessary).

One option is to make the ordering decisions systematic and centralized through the use of a computer-aided ordering (CAO) system. For example, Whole Foods recently moved ownership of its ordering decisions to its corporate headquarters in Austin by implementing a system called “order-to-shelf” across its stores (Peterson 2018). This led to heavy stock-outs and customer dissatisfaction in many stores across the country. One reason is that local information about events such as local events, variations in local demand, local response to promotional activities, or adjustments for spoilage or shrinkage only available to store managers can no longer be used for these critical decisions.

We propose an incentive and measurement scheme that attempts to include stock-outs in scorecards with an easy-to-implement measure and address information asymmetry issues mentioned above. We assume that a “principal” (a retailer or a manufacturer) needs to satisfy uncertain periodic demand over a finite horizon. The principal incurs the typical underage and overage costs: for every unit of demand that is not satisfied in a period there is a shortage cost; any inventory left over at the end of a period incurs a holding cost per unit. Replenishment can take place only before each period, and that decision is delegated to an agent (such as a store manager or an inventory manager) who is better informed about the demand process. The principal can observe the inventory and sales, but lost sales are not recorded in the

retailer’s transaction system. Therefore, an incentive scheme based on the amount of shortages, such as penalizing the agent for underage and overage in the same proportion of the principal’s underage and overage costs, is not possible.

The challenge for the principal is to design an incentive mechanism that induces the agent to make an ordering decision that minimizes the principal’s expected overage and underage costs under incomplete demand information. We suggest incorporating an inventory management performance (IMP) score into the store manager’s scorecard. The maximum possible value of the IMP score, W_{IMP} , represents the weight of the IMP in the manager’s balanced scorecard, relative to his performance in other elements of store management, such as customer retention, shrinkage, and employee turnover. A particular store manager’s IMP score is calculated by deducting points from W_{IMP} : a lump-sum deduction if a stock-out is observed at a prespecified instant in the period or a deduction proportional to the remaining stock at the end of the period. The manager’s IMP score together with his scores from other elements then can be used to calculate his bonus compensation.

We study the case in which the underlying demand process is a Wiener process, and the principal only knows that the process is one of a finite number of such processes. We show that when the possible Wiener processes share the same variance, it is possible to induce the agent to order the optimal quantity without revealing the exact demand information to the principal, when the stock-out measure is enforced at the end of the period. Even more interesting is that checking and penalizing the stock-outs at an optimal time inside the period (i.e., an “early inspection” scheme), rather than at the end, leads to perfect or near-perfect alignment in more general cases. In particular, we show that the early inspection scheme can outperform the scheme that penalizes the stock-outs at the end of the period. Our results also show that the proposed incentive schemes can be substantially more effective than solely relying on a centralized CAO system.

The problem and our models are partially motivated by our interactions with a major European discount grocer with over 1,000 stores that offer an assortment of approximately 1,000 SKUs. Products are shipped to stores on a weekly schedule (certain products once a week, others two or three times a week) from company-owned distribution centers using the company’s own fleet of trucks. The company uses a centralized CAO system to create replenishment orders for its stores. Store managers have limited authority to override the system. Actual order quantity recommendations of the system can be changed for only a fixed percentage of the SKUs. The company pulls

data from its ERP system and reports the number of SKUs without stocks at its stores and distribution centers to its senior management at the end of each day. This is used to estimate the stock-outs and lost sales at the store level, and root causes are sought if the levels are unexpectedly high.

The company acknowledges the fact that store employees may have local information that could lead to improved forecasts and replenishment—and perhaps to improved stock availability—but is unwilling to completely delegate the replenishment decision to its store employees. There are several reasons. First, shortages are not recorded, and quantifying their effect on sales is difficult. Second, the company perceives that stock-outs in its stores are not primarily due to store operations and that they should focus more on problems at its distribution center operations, logistics, and procurement. The company does not use stock-outs explicitly in its performance measurement of store employees. Similar to many retailers, “sales revenue” and “inventory shrinkage” KPIs are used to assess performance on a monthly basis. In Section 6.3, we performed an initial analysis for a limited number of SKUs and store locations at this grocer. Our experiments with actual demand data show that the retailer may improve its profitability considerably by using the incentive schemes suggested in this paper.

The rest of the paper is organized as follows. In Section 2, we review the literature. In Section 3, we introduce the proposed incentive schemes. In Section 4, we analyze the problem in a single period setting and show conditions where perfect alignment is possible. The analysis is extended to multiple periods in Section 5. In Section 6, we conduct a numerical study to quantify the benefits of the proposed incentive mechanisms. We conclude the paper in Section 7.

2. Literature Survey

This paper is related to the literature on incentive alignment problems in supply chains. These problems arise mainly due to hidden actions by the players in the chain, information asymmetries, or badly designed incentive schemes (Narayanan and Raman 2004). Incentive problems are relevant even for vertically integrated firms, as decisions at different echelons are often delegated to individuals whose performance measurement schemes are not well aligned with the overall profitability of the firm (Lee and Whang 1999). Aligning or redesigning incentive schemes may yield significant increases in profitability of the supply chain, whether it is within the boundaries of a single firm or consists of multiple independent firms. See Chen (2001) for a general review of the earlier literature in this area. Information asymmetry can exist for cost parameters and/or demand process. Asymmetric demand information is frequently observed in

practice, as the party closer to the customer will have more information about localities and past sales. Recent examples of asymmetric demand information considered in a supply chain context include papers by Akan et al. (2011), Babich et al. (2012), and Khanjari et al. (2014). Similar to our approach, these papers assume a finite set of states where each state is represented by a known demand distribution. One of the supply chain parties exactly knows the state, whereas the other party has a probabilistic knowledge. Different than our study, incentive mechanisms are designed through supply contracts in the form of wholesale, buyback, or capacity investment contracts, in these examples.

Delegation of operational decisions to an intermediary (agent) by the business owner (principal) falls into the context of the well-investigated “principal-agent” problem in the economics literature [see Laffont and Martimort (2009) for a comprehensive review of this problem]. In its most general form, a principal delegates a certain task to an agent through a contract, which induces the agent to act in alignment with the principal’s objective. van Ackere (1993) and Schenk-Mathes (1995) analyze this problem from an operations management perspective. In particular, the agent is the more informed salesperson who decides how much effort to exert to generate and flourish the demand, and the principal offers a corresponding incentive scheme (such as a sales target and bonus). Zhang and Zenios (2008) extend the basic model to multiple periods and dynamic information structures. Chen (2000), Chu and Lai (2013), and Dai and Jerath (2013) extend the basic principal-agent model in a “salesforce compensation” context to include inventory replenishment decisions, which we also consider. More recently, the static setting of these studies is extended to dynamic models in a multiperiod setting by Saghafian and Chao (2014) and Schöttner (2017). The main incentive mechanisms considered in this stream are sales-quota-based bonus or sales commission contracts. Unlike our setting, these studies assume that the inventory replenishment decisions are made by the principal, and they exploit the interactions between product availability, the effort spent by the agent, and the incentive mechanisms. The main differences in our grocery retail setting are the existence of exogenous demand (which is not influenced by the agent’s sales effort) and the structure of the information asymmetry.

Similar to our problem, Baldenius and Reichelstein (2005) deal with the delegation of the inventory management decisions to a store manager (agent) by a less-informed principal through goal-congruent measures. In particular, the incentive mechanism considered is an accounting-based performance measure through inventory valuation. Unlike our setting, the emphasis of

this literature on inventory management is on cases in which a product is manufactured and sold in different periods under deterministic demand.

DeHoratius and Raman (2007) and Dai et al. (2018) consider incentive mechanisms for store managers in retail, similar to our setting. Store managers are multitasking agents who allocate effort between increasing sales (marketing) and decreasing inventory shrinkage (operations). DeHoratius and Raman (2007) show through an empirical study that the particular incentive system used for store managers has a substantial effect on overall retail performance. Dai et al. (2018) formally study this incentive design problem using a moral hazard principal-agent framework. In both of these papers, it is assumed that the store managers can only control the shrinkage in a given inventory by working on problems, such as shoplifting, errors in paperwork, and supplier fraud. In contrast, we assume in this paper that the inventory levels are determined by store managers who possess private information about demand and who consider designing incentives so that the store managers make use of this information for the best interest of the retailer.

The incentive mechanisms that we consider involve a fixed penalty for stock-outs. Inventory management under lump-sum penalty costs for shortages has been studied before in the inventory literature (see, e.g., Bell and Noori 1984, Cetinkaya and Parlar 1989, and Benkherouf and Sethi 2010). We contribute to this stream of literature by (1) characterizing the optimal solution when the fixed penalty is based on observing a stock-out occasion at an earlier time instant and (2) extending these models to the case of asymmetric information. Fixed-penalty contracts are also used in coordinating the supply chain. See Sieke et al. (2012) for some examples from different industries and an analysis of a “flat penalty contract,” in which the supplier is penalized if she cannot satisfy a percentage of the orders placed by a manufacturer. The emphasis in this line of research is supply chain coordination. Our emphasis in this paper, however, is the delegation of replenishment decisions.

3. Problem Statement and Incentive Schemes

We consider a single item that is offered to the market through several stores or sales channels. The principal, the owner or the main stakeholder of the item, hires agents (e.g., store or inventory managers) who are in charge of inventory replenishment decisions and sales operations. Stores could be different in size, located at distant regions, or have structurally different demand patterns. The agents have the most complete information at their store for forecasting their future demand. We assume that the item observes exogenous demand, indicating that the sales

effort exerted by the agents is fixed or does not influence the demand. The planning horizon consists of N periods, $n = 1, \dots, N$. Without loss of generality, we assume that each period has length 1. In each period, the item observes stochastic demand on a continuous basis from $t = 0$ to $t = 1$, the start and the end of the period, respectively. We assume that the accumulated demand at time t of period n , $X^n(t)$, follows a Wiener process with drift μ^n and standard deviation σ^n . By definition of the Wiener process, $X^n(0) = 0$, and the joint distribution of $X^n(t_0), X^n(t_1), \dots, X^n(t_{k-1}), X^n(t_k)$ when $t_k > t_{k-1} > \dots > t_1 > t_0 > 0$ satisfies the following condition: The differences $X^n(t_k) - X^n(t_{k-1})$ (total demand observed between t_{k-1} and t_k) are mutually independent, normally distributed random variables with mean $(t_k - t_{k-1})\mu^n$ and variance $(t_k - t_{k-1})(\sigma^n)^2$.

The Wiener process or Brownian motion is occasionally used in the inventory literature (e.g., Rudi et al. 2009). In order to ensure that the probability of negative demand in a given period n is negligibly small, one can assume that the drift μ^n is sufficiently larger than the standard deviation σ^n (e.g., $\mu^n > 3.5\sigma^n$). Because each period has length 1, the demand observed throughout period n , denoted by D^n , follows normal distribution with mean μ^n and standard deviation σ^n .

The demand observed in a given period is dependent on the state of the world in that period. There are J states with state j having the probability λ_j , $j = 1, \dots, J$, where $\sum_{j=1}^J \lambda_j = 1$. State of the world is randomly drawn at the beginning of each period. If the state in period n is j , then $(\mu^n, \sigma^n) = (\mu_j, \sigma_j)$. The principal does not know the exact state in any period, but she is fully informed about the possibilities and the parameters of the Wiener process, that is, λ_j , μ_j , and σ_j . The agent, on the other hand, learns the state of the world at the beginning of each period. Let D_j^n be the random demand in period n if the state of the world in that period is j , that is, D_j^n is a normal random variable with mean μ_j and standard deviation σ_j .

Knowing the state of the world at the beginning of the period and based on initial inventory, the agent places an order to be received immediately. We assume that the ordering epochs and the epochs at which the state of the world is observed overlap. This assumption is also used in a significant number of papers with Markov-modulated demand in a periodic setting (e.g., Iglehart and Karlin 1962, Chen et al. 2017). However, different from these papers, we assume a continuous demand process that changes with the state of the world at the beginning of each period. We also assume that the state of the world and thus the demand process remain the same throughout each period. This is particularly reasonable in our retail setting, where the decisions regarding

promotions (e.g., price discounts, inserts) that influence demand are usually made at the same periodicity with operational decisions (e.g., inventory levels and replenishment), typically weekly.

There is no fixed cost of ordering. At the end of period n , any leftover inventory is carried to the next period at a cost of c_o^n per unit. Any unsatisfied demand is lost, costing the retailer c_u^n per unit. Given an order-up-to-level S , the expected cost of the principal in period n can be written as

$$\begin{aligned} ETC_j^n(S) &= c_u^n E[(S - D_j^n)^+] + c_o^n E[(D_j^n - S)^+] \\ &= c_u^n \int_0^S (S - x) f_j(x) dx + c_o^n \int_S^\infty (x - S) f_j(x) dx, \end{aligned} \quad (1)$$

where f_j is the probability density function (pdf) of D_j^n .

Next, we define two benchmarks for the principal. The first one is the complete information benchmark, where we assume that the principal has access to the exact state information in each period (i.e., the principal has the same information as the agent). Under this benchmark, the principal's optimal policy is an order-up-to policy, because the lead time is zero (Zipkin 2000, section 9.4.6). Let $\tilde{g}_j^n(I^n)$ be the retailer's minimum expected cost in periods $n, n+1, \dots, N$ if the retailer starts the period n with I^n units of inventory, and the state of the world is j . The Bellman equation for periods $n = 1, \dots, N-1$ and the minimum expected cost for the last period for the complete information benchmark are

$$\begin{aligned} \tilde{g}_j^n(I^n) &= \min_{S \geq I^n} ETC_j(S) + \sum_{k=1}^J \lambda_k E[\tilde{g}_k^{n+1}((S - D_j^n)^+)], \\ \tilde{g}_j^N(I^N) &= \min_{S \geq I^N} ETC_j(S). \end{aligned} \quad (2)$$

Given an initial inventory of I^1 , the minimum expected cost of the complete information benchmark over the planning horizon is then $\tilde{g}^1(I^1) = \sum_{j=1}^J \lambda_j \tilde{g}_j^1(I^1)$.

The principal does not have access to the state information prior to observing the demand, so $\tilde{g}^1(I^1)$ is the best (lowest) cost that can be obtained by the principal. If the principal obtains the cost $\tilde{g}^1(I^1)$ through an incentive scheme, we say that this incentive scheme leads to *perfect alignment*.

The second benchmark is when the principal places orders centrally for each store without knowing the state of the world. This is the so-called "computer-aided ordering" (CAO) in retail, where a software solution creates forecasts and orders at headquarters without consulting with stores. Such a system relies on a data stream that inherits a mixture of J normal random variables, and it is equivalent to assuming that the demand is a random variable \tilde{D}^n with pdf $\tilde{f} = \sum_{j=1}^J \lambda_j f_j$. Order-up-to policy is also optimal for this

benchmark. Let $\tilde{g}^n(I^n)$ be the minimum expected cost in periods $n, n+1, \dots, N$ if the retailer starts period n with I^n units of inventory under CAO. The Bellman equation for periods $n = 1, \dots, N-1$, and the minimum expected cost for the last period can be written as

$$\begin{aligned} \tilde{g}^n(I^n) &= \min_{S \geq I^n} \sum_{j=1}^J \lambda_j (ETC_j(S) + E[\tilde{g}^{n+1}((S - D_j^n)^+)]), \\ \tilde{g}^N(I^N) &= \min_{S \geq I^N} \sum_{j=1}^J \lambda_j ETC_j(S). \end{aligned}$$

Given an initial inventory I^1 , the retailer's minimum expected cost over the horizon under centralized ordering is $\tilde{g}^1(I^1)$. Centralized ordering does not utilize any information about the state of the world, so any incentive scheme that allows the agent to use his private information is expected to generate a cost lower than $\tilde{g}^1(I^1)$.

We propose two incentive schemes, $[M]$ and $[M, t]$, through which the principal delegates ordering decisions to the agent to utilize his private information. In both schemes, the agent learns the state of the world at the beginning of the period and makes a stocking decision prior to observing the demand in that period. In both schemes, the agent's performance is measured using two indicators: excess inventory and occurrence of stock-out. We also note that for both schemes, the agent makes an ordering decision in a given period by considering his performance only in that period, similar to the setting in Saghaian and Chao (2014). This is primarily due to the complexity of the problem that the agent needs to solve if he is to consider a multiperiod problem and, typically, the limited time for him to make an ordering decision. In addition, the principal is free to change the parameters of the incentive scheme from one period to the other and will not necessarily reveal future parameters to the agent ahead of time. Considering multiple periods and the possibility of parameter changes in agent's decisions will require the analysis of a multiperiod game between the principal and the agent. To formally define the incentive schemes, in a given period n , let I_t^n be the on-hand inventory at time t within the period and $1_{\{I_t^n=0\}}$ be an indicator function, which is equal to 1 if $I_t^n = 0$, and 0 otherwise. The next two sections analyze the proposed schemes for a given period n , and, hence, we drop the script n for brevity.

3.1. Scheme $[M]$

In this scheme, the inventory management performance (IMP) score only depends on the inventory level at the end of a period. In particular, the IMP score is calculated based on the following quantity, which we call the total penalty measure under scheme $[M]$:

$$TPM_{[M]} = \hat{c}_o \times I_1 + M \times 1_{\{I_1=0\}},$$

where \hat{c}_0 and M are the parameters to be set by the principal. The best inventory management performance of the store manager is to end the period with just 1 unit of inventory (the best match between demand and supply that can be observed by the principal), leading to $TPM_{[M]} = \hat{c}_0$. When this happens, the IMP score of the store manager should be the maximum score W_{IMP} . Any other $TPM_{[M]}$ value can be linearly mapped to an IMP score between 0 and W_{IMP} . Therefore, maximizing the IMP score for the store manager is equivalent to minimizing $TPM_{[M]}$. Because the demand is uncertain, and assuming that the store manager is risk neutral, given an initial inventory level I and state of the world j in a given period, the agent (store manager) solves the following problem:

$$\begin{aligned} \min_{S \geq I} ETPM_{[M]}(S) &= \hat{c}_0 E[(S - D_j)^+] + MP\{D_j \geq S\} \\ &= \int_{-\infty}^S \hat{c}_0 (S - x) f_j(x) dx + \int_S^{\infty} M f_j(x) dx. \end{aligned} \quad (3)$$

Predicating on the prospective decision of the agent, the principal has the liberty to set the \hat{c}_0 and M values, so that the order-up-to level chosen by the agent (the value that solves (3)) is as close as possible to the optimal order-up-to level in problem (2), the complete information benchmark.

3.2. Scheme $[M, t]$

Scheme $[M, t]$ is similar to scheme $[M]$, with the difference that a stock-out penalty M is accounted for if on-hand inventory at time $t < 1$ is equal to zero, rather than at $t = 1$. The scheme can be called an “early inspection” scheme and builds on the fact that discovering a stock-out earlier in the horizon may lead to a better understanding of the actual amount of unsatisfied demand for the principal. This scheme provides more information to the principal, because a stock-out at given time $t < 1$ also means a stock-out at time 1, but not vice versa. We will show in Section 4 that this scheme strictly outperforms scheme $[M]$ under certain conditions. The total penalty measure that derives the IMP score under the scheme $[M, t]$ is given by

$$TPM_{[Mt]} = \hat{c}_0 \times I_1 + M \times 1_{\{I_1=0\}},$$

where \hat{c}_0 , M , and t are the parameters to be set by the principal. The projection of $TPM_{[Mt]}$ into the IMP score can be done in a similar way as in scheme $[M]$.

To express the agent’s problem formally, we let Y_{tj} be the random variable denoting demand during $[0, t]$ in any given period. Then we have $X(t) - X(0) = Y_{tj} \sim N(t\mu_j, \sqrt{t}\sigma_j)$ by the definition of the Wiener process.

Let h_j denote the pdf of Y_{tj} . The agent’s problem is then given by

$$\begin{aligned} \min_{S \geq I} ETPM_{[Mt]}(S) &= \hat{c}_0 E[(S - D_j)^+] + MP\{Y_{tj} \geq S\} \\ &= \hat{c}_0 \int_{-\infty}^S (S - x) f_j(x) dx + M \int_S^{\infty} h_j(x) dx. \end{aligned} \quad (4)$$

Similar to $[M]$ scheme, the principal aims to set \hat{c}_0 , M , and t so that solutions to problems (4) and (2) are as close as possible.

4. Single Period

In this section, we analyze the optimal characteristics of a single period problem, in which the principal enforces an incentive scheme based on the available on-hand inventory level and the demand over the immediate replenishment cycle, by ignoring the long-term effects of the current decision. This setting fits the management of perishable items and provides an excellent heuristic for the multiple periods problem.

4.1. Scheme $[M]$

4.1.1. Agent’s Problem. If the principal imposes the $[M]$ scheme, the agent solves the minimization problem in (3). We first show in Theorem 1 that the agent’s optimal ordering decision is a function of the reversed hazard rate of the demand distribution (all proofs are provided in the online appendix). Reversed hazard rate is defined by $\frac{f(x)}{F(x)}$ for any random variable with pdf of $f(x)$ and cdf of $F(x)$. Marshall and Olkin (2007) show that this function is decreasing if the random variable has log-concave density. Many densities including uniform and normal are log-concave (Bagnoli and Bergstrom 2005).

Theorem 1. Let $r(x) = \frac{f(x)}{F(x)}$ be the reciprocal of the reversed hazard rate function of the demand distribution. The optimal order-up-to level of the agent, S^a , that maximizes his performance under scheme $[M]$ is given by $S^{a*} = r^{-1}\left(\frac{M}{\hat{c}_0}\right)$ if $r^{-1}\left(\frac{M}{\hat{c}_0}\right) \geq I$ and $S^{a*} = I$ otherwise, where I is the starting inventory level.

If the principal uses scheme $[M]$ with parameters \hat{c}_0 and M as the performance measure of the agent, and knows the demand distribution with certainty as well as the starting inventory level, then she can anticipate that the agent will order $r^{-1}\left(\frac{M}{\hat{c}_0}\right) - I$ units before the planning horizon if $r^{-1}\left(\frac{M}{\hat{c}_0}\right) \geq I$ and will not order otherwise.

4.1.2. Principal’s Problem Under Complete Information.

Suppose that there is no information asymmetry between the agent and the principal. This implies that $J = 1$ and $\lambda_1 = 1$. Principal’s objective is given by $ETC_j(S^p)$ defined in (1). This is the well-studied Newsvendor problem, which is minimized at $S^{p*} = F^{-1}(\alpha)$,

where $\alpha = \frac{c_u}{c_u + c_o}$. The principal wants the agent to bring the inventory level up to S^{p*} for any given starting inventory level I . The principal can achieve this by anticipating the optimal action of the principal stated in Theorem 1 and imposing the incentive scheme $[M]$ with the values of the parameters \hat{c}_o and M that satisfy

$$S^{a*} = S^{p*} \Rightarrow \frac{M}{\hat{c}_o} = \frac{F(S^{a*})}{f(S^{a*})} = \frac{F(S^{p*})}{f(S^{p*})} = \frac{F(F^{-1}(\alpha))}{f(F^{-1}(\alpha))} = \frac{\alpha}{f(F^{-1}(\alpha))}. \quad (5)$$

Without loss of generality, \hat{c}_o can simply be set to 1, and the parameter M can be set to $\frac{\alpha}{f(F^{-1}(\alpha))}$.

The function $s(\tau) = \frac{1}{f(F^{-1}(\tau))}$ is called the sparsity function by Tukey (1965) or the quantile density function by Parzen (1979). The sparsity function for normal density with mean μ and standard deviation σ is given

by $s_N(\tau) = \frac{\sqrt{2\pi}\sigma e^{\frac{(\Phi^{-1}(\tau))^2}{2}}}{\Phi^{-1}(\tau)}$, where Φ is the cdf of the standard normal random variable. Consequently, a perfect alignment is possible under scheme $[M]$ and complete information, when the principal sets the parameters to $\hat{c}_o = 1$ and $M = \alpha s_N(\alpha)$. We note that this quantity is independent of the mean demand μ and is only a function of the standard deviation, σ .

4.1.3. Principal's Problem Under Incomplete Information.

Suppose that the exact demand in the upcoming period is D_j with mean μ_j and standard deviation σ_j , and the agent decides to order S_j units. Because the exact demand distribution could come from any of the J states-of-the-world with probability λ_j , the principal's expected cost becomes $\sum_{j=1}^J \lambda_j ETC_j(S_j)$. In the remainder of this paper, we use the standard normal transformation to denote the order-up-to-level of the agent by z_j where $z_j = \frac{S_j - \mu_j}{\sigma_j}$ and to denote the expected cost of the agent by $L_j(z_j)$, where $ETC_j(S_j) \equiv L_j(z_j) = c_o z_j \sigma_j + (c_o + c_u) \sigma_j [\phi(z_j) - z_j(1 - \Phi(z_j))]$. Note that $r(z) = \frac{\Phi(z)}{\phi(z)} \sigma$ under this transformation.

Under the imposed M and \hat{c}_o values, the agent will first find his "ideal" order-up-to level, \bar{z}_j that satisfies $r(\bar{z}_j) = \frac{M}{\hat{c}_o}$, and will set the "actual" order-up-to level, z_j , by using the results of Theorem 1, which is equal to \bar{z}_j or $z_j^I = \frac{I - \mu_j}{\sigma_j}$ depending on the value of the starting inventory level, I . Consequently, by presetting the parameter $\hat{c}_o = 1$ without loss of generality, the principal solves the following optimization problem:

$$PP^M(J|I): \min_{z_j, \bar{z}_j, M \geq 0} \sum_{j=1}^J \lambda_j L(z_j) \quad (6)$$

$$\text{s.t. } \frac{\phi(\bar{z}_j)}{\phi(z_j)} \sigma_j = M \text{ for } j = 1, \dots, J, \quad (7)$$

$$z_j = \begin{cases} \bar{z}_j, & \text{if } \bar{z}_j \geq z_j^I, \\ z_j^I, & \text{otherwise,} \end{cases} \quad (8)$$

to find the value for the parameter M that will incentivize the agent to order in a way that minimizes the principal's expected total cost under $[M]$ scheme.

$PP^M(J|I)$ is a nonlinear optimization problem. It can be transformed into a nonlinear mixed integer programming problem by linearizing the constraint set (8) using auxiliary binary variables, so that it can be solved by an off-the-shelf solver (such as Knitro).

Clearly, the principal's cost under incomplete information is higher than or equal to her cost under complete information, as a single M value may not lead the agent to order $S^{p*} = \Phi^{-1}(\alpha)$ in each state of the world. If this could be achieved, then a perfect alignment would be instated, resulting in $r_1(S^{p*}) = r_2(S^{p*}) = \dots = r_J(S^{p*})$. In such a case, setting $M = r_j(S^{p*})$ for any state j would lead to perfect alignment. The following theorem characterizes a situation where this is attainable.

Corollary 1. *If $\sigma_j = \sigma$, for each state of the world $j = 1, \dots, J$, then setting $M^* = \alpha s(\alpha)$ leads the agent to select S^{p*} in each state for any given starting inventory level I .*

Corollary 1 states that perfect alignment is possible under scheme $[M]$ with a reasonable demand scenario.¹ This scenario is observed when the variability of demand is exogenous to the factors that differentiate alternative states of the world. In such cases, the differentiating factor will be the expected value of demand. In all cases other than this scenario, perfect alignment is not possible, and a nonlinear optimization problem given by (6)–(8) should be solved. Next, we present two special cases ($J \geq 2$, $I = 0$ in Theorem 3 and $J = 2$, $I \geq 0$ in Theorem 4), where this problem can be solved easily in polynomial time.

Proposition 1. *The following system of linear equations:*

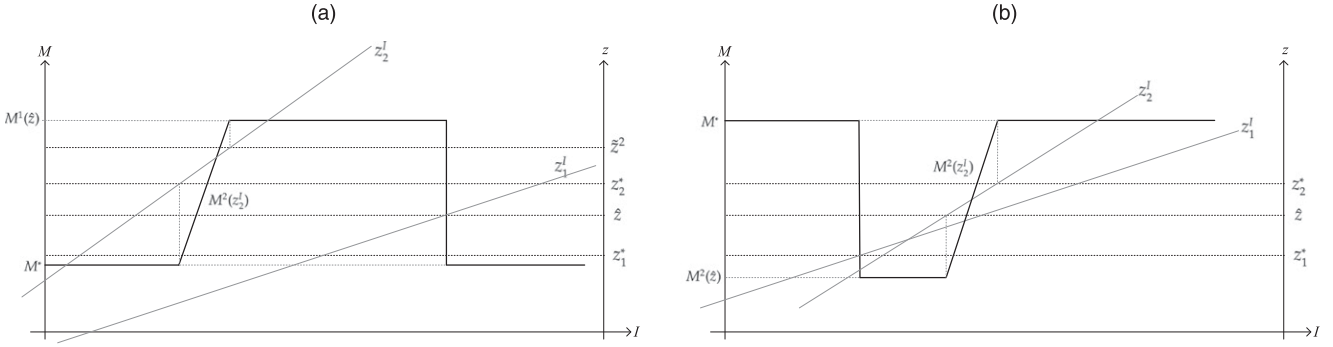
$$\sum_{j=1}^J \frac{\lambda_j (\Phi(z_j) - \alpha)}{1 + z_j \frac{\Phi(z_j)}{\phi(z_j)}} = 0 \text{ and } \sigma_j \Phi(z_j) = M \phi(z_j), \quad j = 1, \dots, J$$

solves $PP^M(J|0)$ for z_j^ . M^* can be calculated by (7).*

Theorem 2. *Suppose that $J = 2$, $\sigma_1 \geq \sigma_2$, and the starting inventory level is I . Let z_1^*, z_2^* , and M^* be the solution to problem $PP^M(2|0)$, which can be obtained by Proposition 1. Let $\hat{z} = \Phi^{-1}(\frac{c_u}{c_u + c_o})$ and $z_j^I = \frac{I - \mu_j}{\sigma_j}$ for $j \in \{1, 2\}$. Then the optimal M value that must be set by the principal when the starting inventory is z_j^I , M_1^* , is characterized by*

$$M_1^* = \begin{cases} M^*, & \text{if } z_1^I \leq z_1^* \text{ and } z_2^I \leq z_2^*, \\ M^1(\hat{z}), & \text{if } z_1^I \leq \hat{z} \text{ and } z_2^* \leq z_2^I \text{ and } \hat{z}^2 \leq z_2^I, \\ M^2(z_2^I), & \text{if } z_1^I \leq \hat{z} \text{ and } z_2^* \leq z_2^I \text{ and } z_2^I \leq \hat{z}^2, \\ M^2(\hat{z}), & \text{if } z_1^* \leq z_1^I \text{ and } z_2^I \leq \hat{z}, \\ M^2(z_2^I), & \text{if } z_1^* \leq z_1^I \text{ and } \hat{z} \leq z_2^I \leq z_2^*, \\ M^*, & \text{if } z_1^* \leq \hat{z} \leq z_1^I \text{ and } z_2^* \leq z_2^I, \end{cases}$$

where $M^1(z) = \frac{\Phi(z)}{\phi(z)} \sigma_j$ and \hat{z}^2 is such that $\frac{\Phi(\hat{z}^2)}{\phi(\hat{z}^2)} \sigma_2 = M^1(\hat{z})$.

Figure 1. Optimal Policy for $J = 2$ and $I \geq 0$ 

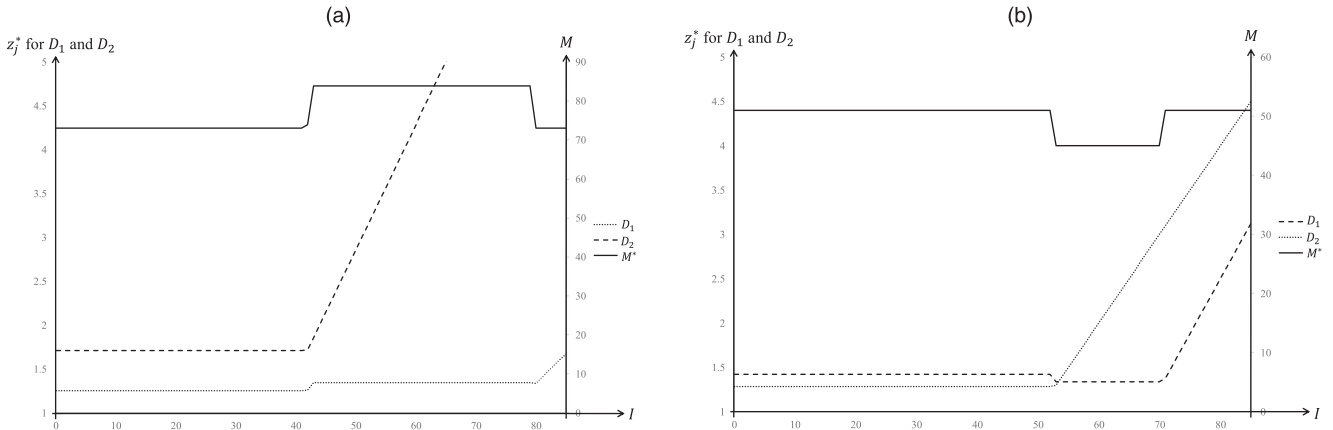
Depending on the problem parameters, the optimal policy can be represented in one of the graphs given in Figure 1. As I increases, if z_1^I exceeds z_1^* after z_2^I exceeds z_2^* , then the policy in Figure 1(a) will be observed. In this case, the principal sets M to M^* , $M^2(z_2^I)$, or $M^1(z)$ until z_1^I exceeds z_1^* , depending on whether z_2^I is less than z_2^* , between z_2^* and \hat{z} , or greater than \hat{z} , respectively. When both z_1^I and z_2^I exceed \hat{z} , then it is optimal to set $M = M^*$. The graph in Figure 1(b) shows the alternative case, where z_1^I exceeds z_1^* before z_2^I exceeds z_2^* . The optimal policy now depends on the relative values of z_2^I with respect to \hat{z} and z_2^* , as shown in the figure.

When the principal optimally sets the incentive parameter M , the agent's resultant replenishment policy is not necessarily an order-up-to policy. Figure 2 depicts two examples. In Figure 2(a), the optimal postreplenishment inventory level for D_1 increases as the starting inventory level increases. In particular, when $0 \leq I \leq 41$, $(M^*, z_1^*, z_2^*) = (73, 1.25, 1.71)$; when $I = 42$, $(M^*, z_1^*, z_2^*) = (73, 1.25, z_2^I = 1.72)$; when

$43 \leq I \leq 79$, $(M^*, z_1^*, z_2^*) = (84, \hat{z} = 1.34, z_2^I)$; and when $I \geq 80$, $(M^*, z_1^*, z_2^*) = (73, z_1^I, z_2^I)$. The optimal postreplenishment inventory level can also decrease as the starting inventory level increases, as depicted in Figure 2(b). This case corresponds to the scenario in Figure 1(b). The deviation from the order-up-to policy is observed when the starting inventory level gets sufficiently high, and not ordering becomes the optimal action for one state. In this case, the M value can be set by considering only the other states, leading to an increase or decrease in postreplenishment inventory levels for them. The agent's replenishment policy may also deviate from an order-up-to policy under more general settings analyzed in the remainder of the text (including $[M, t]$ scheme studied next).

4.2. Scheme $[M, t]$

4.2.1. Agent's Problem. Under scheme $[M, t]$, the agent's objective is to minimize the function $ETPM_{[Mt]}(S^a)$ for $S^a \geq I$ as defined by (4) to find his optimal order-up-to level S^a . By letting $\hat{c}_0 = 1$, $z = \frac{S^a - \mu}{\sigma}$,

Figure 2. Optimal Order-up-to Levels and M^* Values for Two Problem Instances with $J = 2$, $\lambda_1 = 0.5$, $c_o = 1$, $c_u = 10$, and $\hat{z} = 1.34$ 

Note. (a) $(\mu_1, \sigma_1) = (60, 15)$, $(\mu_2, \sigma_2) = (30, 7)$; (b) $(\mu_1, \sigma_1) = (60, 8)$, $(\mu_2, \sigma_2) = (40, 10)$.

$z_t = \frac{S^a - t\mu}{\sqrt{t}\sigma}$, $k = \frac{\mu}{\sigma}$, $a = k(1-t)$ and noting that $z_t = \frac{a+z}{\sqrt{t}}$, we have

$$\begin{aligned} ETPM'_{[Mt]}(S^a) &= F(S^a) - Mg(S^a) \equiv \omega(z) \\ &= \Phi(z) - \frac{M}{\sqrt{t}\sigma} \phi\left(\frac{a+z}{\sqrt{t}}\right), \end{aligned} \quad (9)$$

$$\begin{aligned} ETPM''_{[Mt]}(S^a) &= f(S^a) - Mg'(S^a) = \omega'(z) \equiv \delta(z) \\ &= \phi(z) + \frac{M(a+z)}{t\sqrt{t}\sigma} \phi\left(\frac{a+z}{\sqrt{t}}\right). \end{aligned} \quad (10)$$

Lemma 1. If $M > \frac{e^{-\frac{1}{2}(k^2(1-t))}\sqrt{t}\sigma}{k}$, then $\omega(z)$ has only one local finite minimum. Otherwise, $\omega(-k) \leq \omega(z)$ for all $z \geq -k$.

The following theorem characterizes the optimal order-up-to level of the agent under different parameter ranges.

Theorem 3. Suppose that the starting inventory level is I with $z^I = \frac{I-\mu}{\sigma}$. The optimal order-up-to level z^* such that $z^* = \frac{S^a - \mu}{\sigma}$ is characterized by the following rules:

1. If $M > \frac{\Phi(-k)}{\phi(-k\sqrt{t})}\sqrt{t}\sigma$ then $z^* : \omega(z^*) = 0$ is unique. Then $z^* = z^I$ if $z^I \geq z^I$ and $z^* = z^I$ otherwise.
2. If $M < \frac{\Phi(-k)}{\phi(-k\sqrt{t})}\sqrt{t}\sigma$ and $M < \frac{e^{-\frac{1}{2}(k^2(1-t))}\sqrt{t}\sigma}{k}$ then $z^* = z^I$.
3. If $\frac{e^{-\frac{1}{2}(k^2(1-t))}\sqrt{t}\sigma}{k} < M < \frac{\Phi(-k)}{\phi(-k\sqrt{t})}\sqrt{t}\sigma$, then there exist at most two values of z^I such that $\omega(z^I) = 0$. Then z^* takes either one of these two z^I values or $z^* = z^I$.

4.2.2. Principal's Problem. In this part, we assume that $M > \frac{\Phi(-k)}{\phi(-k\sqrt{t})}\sqrt{t}\sigma_j$, for all $j = 1, \dots, J$ under which the optimal order-up-to level of the agent is characterized by a unique value that satisfies the first order condition (Theorem 3.1). From (9), the optimal order-up-to level of the agent satisfies the following equality when the state of the world is

$$j: \frac{\Phi(z_j)}{\phi\left(\frac{k_j(1-t)+z_j}{\sqrt{t}}\right)}\sqrt{t}\sigma_j = M,$$

where $k_j = \frac{\mu_j}{\sigma_j}$. Then the principal solves the following problem:

$$\begin{aligned} PP^{Mt}(J|I): \quad & \min_{z_j, \bar{z}_j, M \geq 0, 0 \leq t \leq 1} \sum_{j=1}^J \lambda_j L_j(z_j) \\ \text{s.t.} \quad & q_j(\bar{z}_j, t) = M, \text{ for } j = 1, \dots, J, \\ & z_j = \begin{cases} \bar{z}_j, & \text{if } \bar{z}_j \geq z_j^I, \\ z_j^I, & \text{otherwise,} \end{cases} \end{aligned} \quad (11)$$

where

$$q_j(z_j, t) = \frac{\Phi(z_j)}{\phi\left(\frac{k_j(1-t)+z_j}{\sqrt{t}}\right)}\sqrt{t}\sigma_j.$$

Similar to PP^M , we can linearize the constraint set (11) to solve the problem with a commercial nonlinear integer programming solver.

The principal prefers the agent to calculate $z_j^* = \Phi^{-1}\left(\frac{c_u}{c_o + c_u}\right) = \hat{z}$ and set his order-up-to level as $S_j = \mu_j + \hat{z}\sigma_j$.

This will lead to perfect alignment. The following result shows that this is achievable when $J=2$ under a mild condition.

Proposition 2. When $J=2$ and $\sigma_2 < \sigma_1$, if $k_2 > k_1$, then there exists a τ such that $0 < \tau < 1$, which satisfies

$$e^{-\frac{1}{2}\left[\frac{(k_2(1-\tau)+\hat{z})^2 - (k_1(1-\tau)+\hat{z})^2}{\tau}\right]} = \frac{\sigma_2}{\sigma_1}.$$

The scheme $[M, t]$ with parameters $\hat{c}_0 = 1$, $t = \tau$, and

$$M = \frac{\Phi(\hat{z})}{\phi\left(\frac{k_1(1-\tau)+\hat{z}}{\sqrt{\tau}}\right)}\sqrt{\tau}\sigma_1 = \frac{\Phi(\hat{z})}{\phi\left(\frac{k_2(1-\tau)+\hat{z}}{\sqrt{\tau}}\right)}\sqrt{\tau}\sigma_2$$

incentivizes the agent to set the order-up-to level to an amount that corresponds to \hat{z} , which is equal to the optimal order-up-to quantity for the principal. If this quantity is less than the starting inventory level, both sides choose not to order.

Note that the condition of Proposition 2 is also satisfied when $\mu_1 = \mu_2$ and $\sigma_1 > \sigma_2$. This result has two implications. First, it broadens the conditions in which perfect alignment is possible under incomplete information. Second, the scheme $[M, t]$ outperforms scheme $[M]$ under certain parameter ranges. Next, we generalize the latter result for $J > 2$.

Theorem 4. Suppose that $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_J$. If $k_1 \leq k_2 \leq \dots \leq k_J$, then there exists a scheme $[M, t]$ with $t < 1$, which yields lower expected costs for the principal than scheme $[M]$.

5. Multiple Periods

In this section, we provide two approaches to solve the original multiperiod problem, in which the principal sets the parameters of the selected scheme on a periodic basis by considering the dynamics of the whole planning horizon. We only consider the $[M, t]$ incentive scheme, which is the general version. We first present the optimal algorithm, which requires the principal to declare separate incentive parameters M^n and t^n in each period n , based on the starting inventory level, I^n . Then we present an easy-to-implement heuristic solution approach.

5.1. Optimal Approach

Let $\bar{z}_j(M, t)$ be the “ideal” order-up-to value if the state of the world is j for the agent, under given parameters M and t for any given period. In other words, $\bar{z}_j(M, t) = \bar{z}$ such that

$$\frac{\Phi(\bar{z})}{\phi\left(\frac{k_j(1-t)+\bar{z}}{\sqrt{t}}\right)}\sqrt{t}\sigma_j = M.$$

Let $Z_j(I, M, t)$ be the “actual” order-up-to value for the agent if the state of the world is j , given the starting inventory level I , and parameters M and t . That is,

$$Z_j(I, M, t) = \begin{cases} \bar{z}_j(M, t), & \text{if } \bar{z}_j(M, t) \geq (I - \mu_j)/\sigma_j, \\ (I - \mu_j)/\sigma_j, & \text{otherwise.} \end{cases} \quad (12)$$

Let $g^n(I)$ be the total minimum expected cost of the principal in periods $n, n+1, \dots, N$, when the performance of the agent is assessed by scheme $[M, t]$ throughout the planning horizon. Then the following dynamic programming formulation can be used to find the optimal parameters M^n and t^n for all $n = 1, \dots, N$. For all $I^N \geq 0$,

$$g^N(I^N) = \min_{\{(M^N, t^N): 0 \leq M^N, 0 \leq t^N \leq 1\}} \left\{ \sum_{j=1}^J \lambda_j L_j(z_j) : z_j = Z_j(I^N, M^N, t^N) \right\}.$$

For all $n = 1, \dots, N-1$ and for all $I^n \geq 0$,

$$g^n(I^n) = \min_{\{(M^n, t^n): 0 \leq M^n, 0 \leq t^n \leq 1\}} \left\{ \sum_{j=1}^J \lambda_j (L_j(z_j) + E[g^{n+1}((\mu_j + z_j \sigma_j - D_j^+)^+)] : z_j = Z_j(I^n, M^n, t^n) \right\},$$

where $Z_j(\cdot)$ is as defined in (12). The minimum expected cost for the principal over the planning horizon through the use of incentive scheme $[M, t]$ is then $g^1(I^1)$.

5.2. A Myopic Policy

The optimal solution requires the principal to set potentially different and inventory-level-dependent M^n and t^n values in every period of the planning horizon. In practice, this might be confusing for the agent, so an easy-to-implement heuristic approach is to mandate the optimal M and t values of the single period problem as the fixed policy parameters throughout the planning horizon, independent of the period and the starting inventory level. In particular, the principal solves $PP^{Mt}(J|0)$ to find the corresponding myopic optimal M^{myo} and t^{myo} values and imposes them as the performance measure parameters consistently throughout the horizon.

6. Numerical Study

In this section, we present the results of a numerical study conducted to gain insights about the suggested inspection scheme, their practical relevance, and the performance of the proposed heuristic approach suggested for multiple periods. In all numerical analysis, we assume that the initial inventory is zero. Let $ETC_{PI} = \tilde{g}^1(0)$ denote the minimum expected total cost of the principal, under complete demand information. Let $ETC_{Mt} = g^1(0)$ denote the total minimum expected cost of the principal when she imposes the $[M, t]$ incentive scheme, with the convention that ETC_{M1} corresponds to the optimal cost when $t = 1$

and, hence, the $[M]$ scheme. Similarly, let $ETC_{CAO} = \tilde{g}^1(0)$ be the minimum total cost incurred if the principal adopts the CAO system, and let ETC_{MMyo} and ETC_{MtMyo} be the minimum total costs incurred with the myopic policy under $[M]$ and $[M, t]$ schemes, respectively. We define the performance metrics that we use in our numerical analysis as

$$\begin{aligned} Inc_M &= \frac{ETC_{M1} - ETC_{PI}}{ETC_{PI}}, Inc_{Mt} = \frac{ETC_{Mt} - ETC_{PI}}{ETC_{PI}}, \\ Inc_{MMyo} &= \frac{ETC_{MMyo} - ETC_M}{ETC_M}, \\ Inc_{MtMyo} &= \frac{ETC_{MtMyo} - ETC_{Mt}}{ETC_{Mt}}, \\ Sav_{CAO} &= \frac{ETC_{CAO} - ETC_{Mt}}{ETC_{CAO}}. \end{aligned}$$

Lower values of Inc_M and Inc_{Mt} correspond to better alignment between the principal and the agent. Sav_{CAO} measures the superiority of the proposed incentive schemes to the common practice of disregarding the local demand information. Finally, Inc_{MMyo} and Inc_{MtMyo} measures the quality of the myopic policies proposed for ease of implementation in multiperiod settings.

6.1. Performance of Incentive Schemes: Single Period

In this section, we investigate the value of the proposed schemes in aligning incentives of both parties in a single period setting. We experiment with two test beds: the first one contains demand streams with the same coefficient of variations, and the second one consists of randomized instances.

In the first test bed, the number of possible demand distributions, J , is set to 3 with drift parameters $\mu_1 = 20, \mu_2 = 30$, and $\mu_3 = 40$ and standard deviations $\sigma_j = CoV \cdot \mu_j \ \forall j = 1, 2, 3$ with $CoV \in \{0.1, 0.15, 0.2, 0.25\}$, where CoV is the coefficient of variation. The overage cost, c_o , is set to 1, and the underage cost, c_u , takes one of the following values: $\{2, 5, 10, 20\}$.

In order to study the effect of information asymmetry, we use a measure that is the ratio of the variance faced by the principal (variance of \tilde{D}) to the expected value of the variance faced by the store manager. Namely, this measure is defined as

$$IAL = \frac{\text{Var}[\tilde{D}]}{E[\sigma^2]} = \frac{\sum_{j=1}^J \lambda_j \sigma_j^2 + \sum_{j=1}^J \lambda_j \mu_j^2 - (\sum_{j=1}^J \lambda_j \mu_j)^2}{\sum_{j=1}^J \lambda_j \sigma_j^2}.$$

When $J = 3$, $\mu_j = \sigma_j / CoV$ and $\lambda_1 = \lambda_3 = \frac{1-\lambda_2}{2}$, the expression above simplifies to

$$IAL = 1 + (1/CoV)^2 - (1/CoV)^2 \cdot \left[\frac{[\sigma_1 + \sigma_3 - \lambda_2(\sigma_1 - 2\sigma_2 + \sigma_3)]^2}{2[\sigma_1^2 + \sigma_3^2 - \lambda_2(\sigma_1^2 - 2\sigma_2^2 + \sigma_3^2)]} \right].$$

When $\sigma_2 = \frac{1}{2}(\sigma_1 + \sigma_3)$, as in our case, the λ_2 term in the denominator cancels out, and IAL becomes strictly decreasing in λ_2 . Therefore, λ_2 can be used as a measure of information asymmetry. We use four values for λ_2 : $\{1/3, 0.5, 0.75, 0.90\}$ in this first test bed. In total, we generate $4 \times 4 \times 4 = 64$ different problem instances by enumerating all possible combinations of the different values of the parameters c_u (overage cost), CoV (demand variation), and λ_2 (level of information asymmetry).

The average results are shown in Table 1, separately for each value of CoV, c_u , and λ_2 . Out of the 64 problem instances solved, the minimum, maximum, and average values of Inc_{Mt} are 0.01%, 0.12%, and 0.05%, respectively. The same statistics are 0.17%, 3.28%, and 1.12% for Inc_M and 8.29%, 68.02%, and 37.97% for Sav_{CAO} . These numbers reveal the overall strength of the scheme $[M, t]$ for aligning incentives of the agent and the principal. $[M]$ scheme is also effective, but on average performs about 1% worse than $[M, t]$ scheme, which may be significant in certain settings. The performance of CAO relative to $[M, t]$ and $[M]$ is very low; not utilizing the store manager's private information and setting the inventory levels centrally lead to undesirable results. We also observe that the average t^* values are not close to 1 in $[M, t]$ scheme (inspection is much earlier than the end-of-horizon). The optimal inspection time can be as early as 0.39 (when $c_u = 2$ and $CoV = 0.25$).

As the demand variability (CoV) increases, it is better to review the inventory status at an earlier time in the $[M, t]$ scheme. This is true because, when there is more variability, earlier stock-out information is more valuable. As CoV increases, we also see that the $[M, t]$ scheme gets better at aligning incentives. The demand variability does not affect the performance of the $[M]$ scheme. The losses due to the CAO system, on the other hand, are smaller, as the demand variability increases. As the underage cost increases, the principal will set a higher penalty for the stock-out, and the agent will have a larger order quantity. In this case, it is more likely that the retailer will not face any stock-out throughout the horizon. Therefore, the relative benefit of earlier stock-out information goes down, and it is better to review the inventory status at a later time, leading to higher t^* in the $[M, t]$ scheme. For the same reason, the performance of the $[M, t]$

scheme gets worse, and the performance of the $[M]$ scheme gets better as c_u increases. Finally, as the information asymmetry decreases (as λ_2 increases), all schemes, including the CAO system, perform better.

We created a second test bed in order to investigate the impact of the number of potential demand processes by letting J take one of the values in $\{2, 3, 5\}$. For each J , we generated 50 random problem instances by varying μ_j and CoV parameters by picking a random value from the following ranges: $\mu_j \in [20, 100]$, $\forall j = 1, \dots, J$; $CoV \in [0.1, 0.25]$ and set $\sigma_j = CoV \cdot \mu_j$.

Table 2 depicts statistics for t^* , Inc_{Mt} , Inc_M , and Sav_{CAO} values for different J values when $N = 1$ and $I = 0$. Alignment through the use of $[M, t]$ scheme is excellent, as Inc_{Mt} is below 1% on average, and the highest value it takes is about 3.2%. $[M, t]$ scheme also provides significant improvement over $[M]$ scheme; on average, the improvement is around 1.5%. Once again, not utilizing the store manager's private information leads to significant losses; Sav_{CAO} is 35.02%, 43.45%, and 52.41% for $J = 2$, $J = 3$, and $J = 5$, respectively. The loss relative to $[M, t]$ scheme can be as high as 81.88%.

6.2. Performance of Incentive Schemes: Multiple Periods

We study a 10-period version of the setting in Table 2 in order to investigate the performance of $[M]$ and $[M, t]$ schemes and their myopic counterparts. We take the cost parameter (c_u^n) to be the same for all ten periods. This assumes that the shortage and inventory holding costs are time-invariant and that the last period is no different from other periods; that is, any leftover inventory at the end of the horizon can be salvaged at unit cost. The results are shown in Table 3.

Alignment through the use of $[M, t]$ scheme is still excellent, as Inc_{Mt} is below 1% on average, with a worst performance at about 2.7%. $[M]$ scheme also has a reasonable performance as Inc_M is around 2%, and the worst performance is around 5.4%. The savings in comparison with CAO benchmark in multiperiod setting is significant, similar to what is observed in the single period case. The results also show that the myopic counterparts of these schemes perform excellent, resulting in negligible increases in expected cost.

Table 1. Average t^* , Inc_{Mt} , Inc_M , and Sav_{CAO} Values

CoV	t^*	Inc_{Mt}	Inc_M	Sav_{CAO}	c_u	t^*	Inc_{Mt}	Inc_M	Sav_{CAO}	λ_2	t^*	Inc_{Mt}	Inc_M	Sav_{CAO}
0.1	0.73	0.06%	1.12%	52.93%	2	0.52	0.04%	1.93%	35.47%	1/3	0.59	0.09%	1.91%	50.65%
0.15	0.63	0.06%	1.12%	40.39%	5	0.58	0.05%	1.15%	37.44%	0.5	0.59	0.07%	1.48%	46.41%
0.2	0.55	0.05%	1.12%	32.13%	10	0.63	0.06%	0.81%	38.89%	0.75	0.60	0.04%	0.78%	34.82%
0.25	0.49	0.04%	1.12%	26.44%	20	0.67	0.06%	0.60%	40.09%	0.9	0.61	0.02%	0.32%	20.01%

Table 2. Results for Random Problem Instances, Single Period, $c_u = 10$, $\lambda_j = 1/J$, $I^1 = 0$

J	t^*			Inc_{Mt}			Inc_M			Sav_{CAO}		
	Min	Ave	Max	Min	Ave	Max	Min	Ave	Max	Min	Ave	Max
2	0.62	0.85	1.00	0.00%	0.14%	2.34%	0.00%	1.47%	4.03%	0.89%	35.02%	69.91%
3	0.61	0.88	1.00	0.00%	0.54%	3.20%	0.21%	2.04%	5.61%	0.89%	43.45%	78.75%
5	0.70	0.89	0.97	0.03%	0.80%	2.81%	0.15%	2.53%	5.69%	10.25%	52.41%	81.88%

6.3. Application Using the Discount Retailer's Demand Data

We are provided with a data set by the European discount grocery chain that we explained in Section 1. The data set has daily demand data for 50 item-store pairs for 55 weeks prior to the fourth week of 2017 and fixed replenishment frequencies of these items at these stores. We specifically requested the company to include stores that have been in operation for a considerable time and items that have stable and nonseasonal demand patterns. We assume that the state-of-the-world changes once a week, because most of the sales promotional activities, such as price discounts, feature advertising (in the form of run of press ads, free-standing newspaper inserts, or pre-printed fliers delivered directly to houses), or in-store displays are planned and carried out weekly. The effect of these activities on store traffic and the demands for individual items in a given store can be best assessed by the local staff at that store.

As discussed before, each item at a store is replenished based on a weekly schedule, the majority being replenished once at the beginning of each week. We removed the item-store pairs that are replenished more frequently than once a week. As a result, our planning periods are calendar weeks; at the beginning of each, the state of the world changes, and the replenishment orders are received. We removed item-store pairs for which we have no sales consecutively for more than a week, indicating a long-term stock-out. For each item-store pair, we also removed weeks for which there were no sales for more than one day, to avoid censoring due to short-term stock-outs.

For each of the remaining item-store pairs, using the total weekly demand, we seek to group weeks into different clusters, each cluster corresponding to a different normal variate. For this purpose, we used

model-based clustering, which clusters data based on normal mixture modeling (Fraley and Raftery 2002). The clustering algorithm developed in R for this purpose (Fraley et al. 2012) left us with 11 item-store pairs with more than 1 cluster: 10 with 2 clusters and 1 with 3 clusters. We then carried out routine tests for the fitness of the Wiener process using the daily demand data. These tests are used to verify that daily demand fits the normal distribution, is stationary (using augmented Dickey-Fuller test), and does not exhibit autoregression. In the end, we are left with five item-store pairs. Four of these item-store pairs had two Gaussian density clusters, and one had three clusters. There were four distinct items in three stores. These items were bottled water, UHT processed milk, pasta, and margarine. Table 4 lists the mixture probabilities as well as the mean and standard deviation of the best fit clusters provided by the algorithm.

Figure 3 shows the daily and weekly sales of the first item-store pair in Table 4. The item is a 1/2 liter of UHT processed milk. There was no significant seasonality or trend pattern (within week or within year) in the daily sales. The model-based clustering algorithm identified two clusters for the weekly sales. Six of the weeks belonged to the second cluster (higher sales), while the remaining weeks belonged to the first cluster. There was no clear indication (such as national holidays or other calendar effects) that could be used by the retail chain headquarters to explain why there was a higher demand in these six weeks. Our communication with the retail chain, however, suggests that the store representatives may possess some information to identify the weeks in which their stores will face higher than usual demand.

The results of our analysis with these five item-store pairs are presented in Table 5. We use a $c_u^n = c_u$ value of 50 and 100. The latter value is a more realistic scenario

Table 3. Results for Random Problem Instances, Multiple Periods ($N = 10$), $c_u^n = 10$, $\lambda_j = 1/J$, $I^1 = 0$

J	Inc_{Mt}			Inc_{MtMyo}			Inc_M			Inc_{MMyo}			Sav_{CAO}		
	Min	Ave	Max	Min	Ave	Max	Min	Ave	Max	Min	Ave	Max	Min	Ave	Max
2	0.00%	0.13%	2.21%	0.00%	0.02%	0.24%	0.00%	1.38%	3.59%	0.00%	0.01%	0.20%	1.04%	34.96%	69.23%
3	0.00%	0.46%	1.86%	0.00%	0.03%	0.33%	0.21%	1.89%	5.40%	0.00%	0.02%	0.49%	0.94%	43.35%	78.24%
5	0.02%	0.73%	2.71%	0.00%	0.03%	0.35%	0.14%	2.35%	5.18%	0.00%	0.01%	0.26%	10.30%	52.23%	81.89%

Table 4. Parameters of the Four Items Selected from Retailer's Data Set

Item	λ_1	μ_1	σ_1	k_1	λ_2	μ_2	σ_2	k_2	λ_3	μ_3	σ_3	k_3
1	0.89	92.9	19.8	4.69	0.11	161.3	19.8	8.15	—	—	—	—
2	0.78	81.6	16.4	4.98	0.22	187.5	129.1	1.45	—	—	—	—
3	0.9	33.9	9.7	3.49	0.1	71.0	32.9	2.16	—	—	—	—
4	0.56	16.7	4.3	3.88	0.44	29.4	10.6	2.77	—	—	—	—
5	0.78	130.4	23.0	5.67	0.12	41.1	15.2	2.70	0.1	305.3	148.2	2.10

for the chain, corresponding to roughly 20% annual inventory carrying cost per unit and 35% shortage cost per unit.

Perfect alignment is achieved for the first four selected item-store pairs for a single period problem, $N = 1$; their demand parameters listed in Table 4 satisfy the conditions of Proposition 2. The fifth selected item has three demand clusters, and perfect alignment is not possible when $N = 1$. When the planning horizon has $N = 10$ periods, $[M, t]$ scheme yields near perfect alignment (Table 5). $[M, t]$ scheme is about 2.5% away from the ideal case of complete information, but this result is much better than relying on the existing CAO system, for which the cost

can be reduced by 74% by adopting the $[M, t]$ scheme. The performance of the myopic policy for the $[M, t]$ scheme seems also reasonable for use in practice.

Other than the five item-store pairs reported here, we identified several item-store pairs for which a single demand distribution (without any mixtures) or a mixture of negative binomial (NB) distributions is a good fit. For items with a single demand cluster, the analysis of Section 4.1.2 prescribes how to attain a perfect alignment with the proposed incentive schemes. In order to investigate the effectiveness of the proposed schemes under a mixture of NB demand, we conducted a preliminary numerical analysis (which is not reported here). This analysis revealed that the

Figure 3. Daily and Weekly Sales for Item-Store 1 with Two Demand Clusters

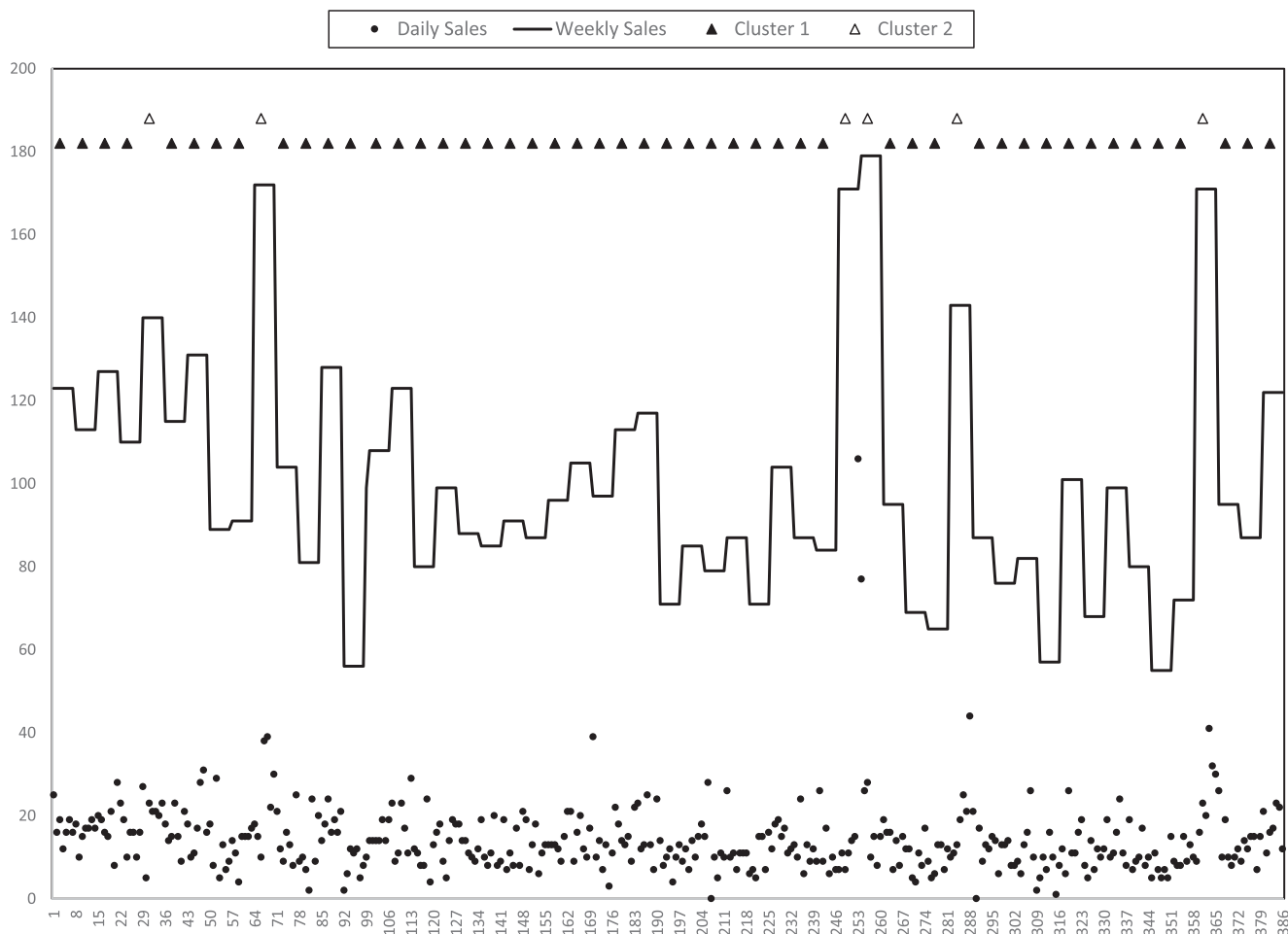


Table 5. Results of the $[M, t]$ Scheme When $N = 10$ and $I^1 = 0$

Item	$c_u = 50$			$c_u = 100$		
	Inc_{Mt}	Inc_{MtMyo}	Sav_{CAO}	Inc_{Mt}	Inc_{MtMyo}	Sav_{CAO}
1	0.00%	0.00%	46.49%	0.00%	0.00%	45.30%
2	0.11%	4.22%	58.91%	0.08%	4.28%	58.47%
3	0.02%	0.87%	61.45%	0.01%	1.05%	62.69%
4	0.01%	0.19%	40.44%	0.01%	0.29%	40.01%
5	2.49%	0.62%	73.09%	0.18%	3.00%	74.02%

proposed schemes are still highly effective in aligning incentives under mixtures of NB demand. All these results show that this chain can have significant savings, if the headquarters delegates replenishment decisions to more informed store representatives and uses the incentive schemes we propose.

7. Conclusion

We consider a problem faced by a principal who delegates the replenishment decisions of a periodically ordered item to an agent. The principal has incomplete demand information, cannot observe lost sales, and needs an incentive scheme so that the agent orders a quantity that minimizes the principal's overage and underage costs. The demand process is assumed to be a Wiener process; the agent knows which particular Wiener process will take place prior to ordering, whereas the principal only knows the set of possible Wiener processes and their probabilities. We propose a scheme where the performance score of the agent depends on how much inventory remains at the end of period and on whether there is any stock-out at a prespecified instant prior to or at the end of the period. We show that when the Wiener processes share the same variance, the principal can perfectly align the agent's incentives by inspecting the stock-out at the end of the period and by setting a proper penalty for a potential stock-out to be deducted from the agent's performance score. Under some mild conditions and when there are only two possible processes, perfect alignment is still possible but interestingly requires the inspection of stock-out before the period ends. In general, such early inspection schemes may lead to strictly better results than only relying on stock-out information at the end of the period. Our numerical results on synthetic and real data show that the scheme we suggest leads to near-perfect alignment and to significant savings over centralized ordering, based on incomplete demand information.

One avenue for further research can be to develop new schemes under the assumptions of this paper. For example, in some settings, it may be possible to detect the exact time the store runs out of stock and design an incentive scheme based on the duration of the

stock-out in a period. An exploratory numerical study we conducted shows that the performance of such a scheme is not essentially different from $[M, t]$ scheme, in terms of incentive alignment. In addition, one may argue that such a scheme may be harder to implement in practice. In addition, this paper focuses on a single-item problem leading to separate incentive parameters for each item in consideration. In practice, working with separate incentive parameters for many items can be difficult. In a preliminary study, we tested a heuristic in which we solve the problem separately for each item and use the median values of the incentive parameters for all items. The performance of this heuristic is impressive; however, determining the common $[M, t]$ pair that minimizes the principal's underage and overage costs over all items can be posed as an interesting and challenging optimization problem.

This paper develops a novel early inspection policy and demonstrates its effectiveness in incentive alignment, when the underlying demand distribution is one of a finite number of normal distributions. Our preliminary investigation shows that this policy also works well when we use negative binomial distributions. A potential future study is to investigate other demand uncertainty structures under which effective alignment of incentives is also possible.

Acknowledgments

The authors thank the department editor (Professor Brian Tomlin), an anonymous associate editor, and two anonymous referees for their valuable comments that greatly improved the paper. The authors also thank Yaşar Altunoğlu, who helped them in the early stages of this work by exploring similar incentive problems and mechanisms in the banking industry.

Endnote

¹ Note that perfect alignment may be obtained for demand processes other than Wiener. For example, the sparsity function for uniform distribution between a and b is given by $s(\tau) = b - a$. Therefore, if the total demand is distributed with one of a number of uniform distributions with same interval length, the principal also can incentivize the agent to order its optimal quantity.

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