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On the role of commodity futures in portfolio diversification

Hooi Hooi Lean^a , Duc Khuong Nguyen^{b,c,*}, Ahmet Sensoy^d and Gazi Salah Uddin^e

^aSchool of Social Sciences, Universiti Sains Malaysia, Penang, Malaysia ^bIPAG Business School, Paris, France ^cInternational School, Vietnam National University, Hanoi, Vietnam ^dFaculty of Business Administration, Bilkent University, Ankara, Turkey

^eLinköping University, Linköping, Sweden

E-mail: hooilean@usm.my [Lean]; duc.nguyen@ipag.fr [Nguyen]; ahmet.sensoy@bilkent.edu.tr [Sensoy]; gazi.salah.uddin@liu.se [Uddin]

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Abstract

The last two decades have witnessed major financial crises that led investors to seek alternative assets and investment strategies to reduce their portfolio risk. In this article, we provide information on the role of commodity futures in designing portfolios and managing risk based on an appealing operational framework. Using more than 20 years of sample data, we first investigate the conditional mean and volatility dynamics of equity and commodity futures markets within a dynamic conditional correlation model setup. We then form alternative equity-commodity futures portfolios by changing the weights of commodity futures and examine if the diversified commodity-equity portfolios perform superior to the all-equity portfolios and four well-known investment strategies that suit most practitioners. Stochastic dominance approach shows that including commodity futures in diversified portfolios does not always improve the risk-return performance, except for gold in some particular portfolio setups. Accordingly, commodity assets have behaved like financial assets (stocks) and tend to be driven by the same pricing factors in general, which reduces the benefits of diversification.

Keywords: commodity futures; equity markets; portfolio diversification; stochastic dominance

1. Introduction

In the past decade, commodity futures have increasingly been viewed as a different financial asset class for portfolio risk diversification. The main argument for this increasing tendency is the diversifying potential that commodity futures may offer given their high expected returns with low

*Corresponding author.

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correlation with traditional asset classes such as equity and bonds. For instance, the modern portfolio theory predicts no common factors driving equity and commodity markets. In his seminal paper, Lintner (1983) looks at the role of managed commodity futures in institutional portfolios of stocks and bonds and documents that the resulting diversified portfolios exhibit higher risk-return performance. More recently, Gorton and Rouwenhorst (2006) examine some stylized facts of commodity futures and particularly their relationships with equity markets over the period 1959–2004. These authors explicitly uncover two important characteristics suggesting that commodities provide positive diversification benefits:33 i) commodity futures have the same average returns as equities along with a negative correlation between bonds and equities; ii) they generate less volatile returns.

This evidence of portfolio diversification opportunities from the inclusion of commodity futures is also confirmed by subsequent studies (Conovor et al., 2010; Arouri et al., 2011; Daskalaki and Skiadopoulos, 2011; Hammoudeh et al., 2013; Daskalaki et al., 2017; Wen and Nguyen, 2017).¹Daskalaki and Skiadopoulos (2011) show, for example, that adding commodities in a portfolio consisting of traditional asset classes yield significant diversification benefits during the 2005-2008 commodity boom period. Arouri et al. (2011) find that the inclusion of the crude oil asset into a well-diversified portfolio of European and US stocks improves its risk-adjusted performance of sector stock portfolios over time. The financialization of the commodity markets and the increased instability in international equity markets following the onset of the global financial crisis 2008-2009 have also incited the holding of commodity futures in financial portfolios (Hamilton and Wu, 2013).² For example, applying a stochastic dominance efficiency approach, Daskalaki et al. (2017) find that commodities provide diversification benefits both in- and out-of-sample. In their recent study that revisits the diversification, hedging, and safe haven potentials of 21 commodities for stock market movements in 49 countries, Ali et al. (2020) document the roles of some specific commodities (particularly precious and industrial metals) as a hedge and a safe haven, with gold providing the strongest safe haven. According to their literature survey, gold also got a similar distinction in the majority of past studies, followed by some supportive evidence for crude oil and other precious metals as hedge and diversifiers.

The diversifying potential of commodity (futures) markets has, however, been found to be lower than actual expectations over recent years, owing to their increasing financial behavior resulting from financial investors' activity in both commodity and traditional (bond and stock) markets. Silvennoinen and Thorp (2013) document that diversification benefits of commodity futures have reduced since the early 2000s due to the increase in their return correlations with equity returns, particularly during the recent global financial crisis. Büyüksahin and Robe (2011) find evidence of no increase in the commodity-equity comovement until 2008, whereas a positive correlation is found between equity and commodity returns after the fall of 2008. When analyzing the return

¹From a broader perspective, designing optimal portfolios under certain circumstances for risk management has been one of the main problems in Finance since the beginning. For recent theoratical and empirical studies, see Filippi et al. (2020), Saadaoui and Ghadhab (2020), Bilbao-Terol et al. (2021), Saadaoui (2021), and Sun et al. (2022).

²Other studies have investigated the implications of financialization on the behavior of commodity markets as well as the factors driving their returns (e.g., Domanski and Heath 2007; Choi and Hammoudeh, 2010; Dwyer et al., 2011; Irwin and Sanders, 2011; Henderson et al., 2012; Cheng et al., 2012; Baker, 2012; Fattouh et al., 2013). For example, Choi and Hammoudeh (2010) show that the prices of five strategic commodities (crude oil and precious metals) are affected by macroeconomic variables and exhibit herding behavior. Cheng et al. (2012) find evidence of increased systematic risk in commodities futures markets due to their financialization.

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relationships between 25 commodities and the S&P 500 index over the period 2001–2011, Creti et al. (2013) find that their conditional correlations are time-varying and highly volatile. They also document an increase in the strength of these correlations during the 2007–2008 financial crisis. More recently, Nguyen et al. (2020) use three-month futures prices of 11 commodities to investigate the changes in their links with equity markets over the period from 1998 to 2017. They find evidence of positive correlations of shocks to both commodity and equity futures except for gold futures that exhibit a negative and insignificant correlation of residual return shocks. This evidence suggests, on the one hand, the increasing financialization of commodity markets, and, on the other hand, the continuing potential of safe haven status for gold futures. As to energy commodities like crude oil, they provided a good hedge for stock investments only in the 1990s. These results remain robust when the time-varying nature of commodity-equity relationships and real return heteroscedasticity are considered.

Looking at the above-mentioned papers, one can put forward a convincing argument that while the commodities had offered substantial diversification benefits to market participants in the late 1990s and early 2000s, this situation has changed in later periods mainly due to the growing financialization of commodity markets.³ However, interestingly, commodities might be in a definancialization stage over recent years. For instance, Bianchi et al. (2020) show a strengthening in the financialization of energy commodities during the global financial crisis (2008–2009). Similarly, Gagnon et al. (2020) find that while the diversification potential of commodities was limited during their financialization era, the post-financialization period offers new opportunities. The evidence of de-financialization thus prompted new studies to quantify the value of commodities in portfolio construction and allocation, while taking into account possible trends, instabilities, and various portfolio design scenarios.

Our present study also contributes to the related literature on equity-commodity market links by re-examining the role of commodity futures in diversified portfolios of stocks, to the extent that their degree of financialization varies through time. We address this issue in several steps. First, a bivariate DCC-EGARCH is used to estimate the dynamic conditional correlations (DCCs) between commodity futures and equity returns. We then investigate the time-varying features of these correlations (strength, time trends, and structural breaks) as well as optimal portfolio designs and hedging involving the holding of commodity futures. Finally, we adopt the stochastic dominance analysis to compare the performance across a number of constructed commodity-equity portfolios including the one with optimal weights based on DCC-EGARCH estimates and four of the most popular practitioner's investment strategies (naive strategy, Sharpe ratio maximization strategy, momentum strategy, and contrarian strategy).

Overall, our approach allows us to evaluate the benefits of adding commodity futures into stock portfolios through modeling the dynamics of their time-varying comovement with stocks and drawing the stochastic frontier of performance across simulated portfolios. This approach is particularly

³Financialization in commodity markets is mostly attributed to commodity ETFs and commodity mutual funds that allow investors to buy or sell various commodity products directly in financial markets (Arunanondchai et al., 2020; Del Brio et al., 2020). For example, when they are used excessively to hedge against the movements in the equity markets, that is, commodities and equities are bought (or sold) together, the buying (selling) pressure toward the commodities increases the correlation between them and the equities. This breaks the natural correlation mechanism between commodities and equities, and creates so-called "the curse of diversification," making commodities ineffective for diversifying equity portfolios.

relevant to our study period (April 1990 to November 2017) as it easily accommodates the conditional heteroscedasticity and the nonstandard asymmetry found in the volatility process of all financial return series. The DCC-EGARCH model has been proven suitable for modeling asymmetric volatility due to market bearish and bullish periods. On the other hand, the comparison of portfolio performance based on the stochastic dominance analysis is not exposed to parameter biases. Our obtained results for the S&P500 index and 19 commodity futures widely traded in the New York Mercantile Exchange, the Chicago Board of Trade and the ICE Futures exchanges show that it is not always possible to improve the performance of stock portfolios with the inclusion of commodity futures, except for gold portfolio under several specific portfolio designs. Our results are partially in line with those studies concluding that the hedging and/or safe haven properties of various commodities have diminished in recent years, pointing out to the commodity financialization era and its negative impact on portfolio design. However, as gold can still play a safe-haven role under specific setups, it has noticeably different characteristics from the other commodities.

The rest of the paper is designed as follows. Section 2 presents the empirical method. Section 3 describes the data. Section 4 reports and discusses the results. Section 5 concludes the paper.

2. Methodology

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2.1. The DCC-EGARCH model for correlation dynamics

The standard Dynamic Conditional Correlation (DCC) – Generalized Autoregressive Conditional Heteroscedasticity (GARCH) was developed by Engle (2002). This model offers an easy and insightful way to deal with the conditional correlations for a large number of series included in the system. In this paper, we use the bivariate DCC-EGARCH model to accommodate the dynamics of return correlations between each of our commodity futures and the equity market represented by the S&P500 market index. To the extent that our study period contains multiple episodes of financial instability and crises, the DCC-EGARCH models allows us to directly infer the dynamics of cross-market correlations, while capturing the potential of volatility asymmetry (i.e., the conditional volatility reacts more to negative return changes than to positive return changes) that may occur in the time-paths of return series.

The Exponential-GARCH (EGARCH) model specifications capture the asymmetries in the conditional volatility process of equity and commodity futures market returns. Assume that returns are normally distributed with zero mean and conditional variance-covariance matrix H_t , our bivariate DCC-EGARCH model for the return on the S&P500 index and the return of each commodity futures in our sample can be specified as follows:

$$\begin{cases} r_t = \mu_t + \varepsilon_t \\ \varepsilon_t \mid I_{t-1} \to N(0, \Sigma_t), \\ \Sigma_t \equiv D_t \Psi_t D_t \end{cases}$$
(1)

where $\mu_{i,t} = \delta_{i0} + \delta_{i1}r_{i,t-1} \cdot r_t$ is the (2×1) vector of the returns on the S&P500 index and commodity futures contract under consideration. ε_t is a (2×1) vector of zero mean return innovations conditionally on the information available at time (t – 1). Ψ_t is the (2×2) time-varying symmetric

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conditional correlation matrix. D_t is a (2×2) diagonal matrix whose main diagonal elements are the conditional standard deviations of the returns from the univariate EGARCH model for each return series. D_t is given by

$$D_t = \operatorname{diag}(\sigma_{11t...} \sigma_{nnt}). \tag{2}$$

According to the classification of Nelson (1991), the EGARCH(1,1) approach incorporates the asymmetric response to negative and positive shocks affecting returns, and is expressed in following equation:

$$\ln\left(\sigma^{2}_{it}\right) = \omega_{i} + \alpha_{i} \left[\left| \frac{\varepsilon_{it-1}}{\sigma_{it-1}} \right| - \sqrt{2/\Pi} \right] + \beta_{i} \ln\left(\sigma^{2}_{it-1}\right) + \theta_{i1} \frac{\varepsilon_{it-1}}{\sigma_{it-1}}, \tag{3}$$

where θ_1 capture the volatility asymmetries. The negative shocks of the same magnitude then have greater effects on the conditional volatility than positive shocks if θ_1 is less than 0.

The correlation dynamics in the DCC process can be expressed in the following form:

$$\Psi_t = (\text{diag}Q_t)^{-1/2}Q_t(\text{diag}Q_t)^{-1/2},$$
(4)

$$Q_{t} = (1 - \alpha' - \beta') \bar{Q} + \alpha' u_{t-1} u_{t-1}' + \beta' Q_{t-1},$$
(5)

where $u_{it} = \varepsilon_{it} / \sigma_{it} \cdot Q_t$ is an (2×2) conditional variance–covariance matrix of standardized residuals. \bar{Q} is the (2×2) unconditional variance matrix of u_t . α' and β' are nonnegative scalar parameters satisfying $\alpha' + \beta' < 1$

The conditional correlation coefficient ρ_{ij} between two individual return series, *i* and *j*, is then given by

$$\rho_{ij} = \frac{(1 - \alpha' - \beta')\,\bar{q}_{ij} + \alpha' u_{i,t-1} u_{j,t-1} + \beta' q_{ij,t-1}}{\left((1 - \alpha' - \beta)\,\bar{q}_{ii} + \alpha' u_{i,t-1}^2 + \beta' q_{ii,t-1}\right)^{1/2} \left((1 - \alpha' - \beta')\,\bar{q}_{jj} + \alpha' u_{j,t-1}^2 + \beta' q_{jj,t-1}\right)^{1/2}}.$$
(6)

In Equation (6), q_{ij} refers to the element located in the *i*th row and *j*th column of the symmetric positive definite matrix Q_t .

In sum, the bivariate DCC-EGARCH model above is estimated using a two-stage procedure. First, univariate ARMA(m, n)-EGARCH(p, q) model is estimated for each return series under consideration. Second, the standardized residuals obtained from the first stage are used to infer the dynamic conditional correlations. The latter are expected to vary over time given the frequent changes in market conditions.

2.2. Performance analysis with stochastic dominance approach

Once the time-varying features of dynamic correlations and optimal portfolio designs from the DCC-EGARCH are analyzed, we use the stochastic dominance technique to compare the performance of various constructed commodity-equity portfolios. This step allows us to empirically

examine whether adding the commodity futures to stock portfolios generates diversification benefits for investors.

There are three forms of stochastic dominance (SD). First, under the first-order SD, utility functions must exhibit nonsatiation, where more is preferred to less. Under the second-order SD, nonsatiation and risk aversion are required. Finally, under the third-order SD, nonsatiation, risk aversion, and decreasing absolute risk aversion are required. The SD analysis is attractive because it is a nonparametric method where no explicit specifications of the agent's utility function or restrictions on the functional form of the probability distribution are required. Instead, the SD method rather relies on general assumptions about investor nonsatiety and risk preferences and considers the entire distribution rather than just the two first moments.

In order to test for the dominance of any pair of the portfolio returns, Davidson and Duclos' (2000) (hereafter DD) nonparametric SD statistics is applied. In computation, we follow the distribution functions possess developed by Lean et al. (2010) and Al-Khazali et al. (2014) and testing the null hypothesis for pre-designed finite numbers proposed procedure by Bishop et al. (1992) and Richmond (1982) and Bai et al. (2011) recommendation to get simulated critical values for DD statistics. The details discussion is available at Levy (1992).⁴

3. Data

We use monthly frequency data of continuous futures contracts, which are derived from individual futures contracts based on the Type 0 and Type 2 roll methods⁵, for three energy commodities (natural gas, heating oil, and crude oil), eight metals (gold, palladium, platinum, silver, copper, tin, zinc, and aluminum) and eight agricultural, food and livestock commodities (corn, wheat, coffee, cotton, soybeans, sugar, feeder cattle, and live cattle). The S&P500 market index is employed to represent the diversified stock portfolio of the U.S. equity markets.

We focus on the period running from April 1990 to November 2017, whereby commodity prices (and thus their links with equity prices) have widely fluctuated given the occurrence of major financial crises, market crashes, and events over the last 25 years (e.g., Asian financial crisis 1997–1998, the dot-com bubble burst in 2001, the global financial crisis 2008–2009, and natural disasters such as the Katrina Hurricane in the North Atlantic ocean and the 2000s droughts in Australia). It is worth noting that the financial turbulence in the U.S. markets following the 2007 subprime crisis has encouraged investors to shift their investments from equity to commodities, which may intensify the equity-commodity market linkages. All monthly data are obtained from Datastream International.

We compute monthly returns by taking the logarithm differential between two consecutive prices. Table 1 provides the summary statistics for the returns on the S&P500 market index and the returns on 19 commodity futures contracts. On average, monthly returns range from 0.1% (natural gas, corn, soybeans, and wheat) to 0.7% (Palladium). The return on the S&P500 reached 0.6%. Returns on cotton futures are very close to zero. The monthly unconditional volatility as measured by the standard deviation is substantial with values ranging from 4.2% (S&P500) to 13.4% (natural contracts) and the returns of the standard deviation is substantial with values ranging from 4.2% (S&P500) to 13.4% (natural contracts) and the standard deviation is substantial with values ranging from 4.2% (S&P500) to 13.4% (natural contracts) and the standard deviation is substantial with values ranging from 4.2% (S&P500) to 13.4% (natural contracts) and the standard deviation is substantial with values ranging from 4.2% (S&P500) to 13.4% (natural contracts) and the standard deviation is substantial with values ranging from 4.2% (S&P500) to 13.4% (natural contracts) and the standard deviation is substantial with values ranging from 4.2% (S&P500) to 13.4% (natural contracts) and the standard deviation is substantial with values ranging from 4.2% (S&P500) to 13.4% (natural contracts) and the standard deviation is substantial with values ranging from 4.2% (S&P500) to 13.4% (natural contracts) and the standard deviation is substantial with values ranging from 4.2% (S&P500) to 13.4% (natural contracts) are used to the standard deviation is substantial with values ranging from 4.2% (S&P500) to 13.4% (natural contracts) are used to the standard deviation is substantial with values ranging from 4.2% (S&P500) to 13.4% (natural contracts) are used to the standard deviation is substantial with values ranging from 4.2% (S&P500) to 13.4% (natural contracts) are used to the standard deviation is substantial with values) aready deviation is substantial with val

⁴See Lean et al. (2010) and Al-Khazali et al. (2014) for more technical details of the stochastic dominance tests.

⁵Thomson Reuters Datastream, 2010. Futures continuous series: Methodology and definitions.

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| | Mean | Std. dev. | Skewness | Kurtosis | J–B | Sharpe ratio | Correlation with S&P500 |
|---------------|-------|-----------|----------|----------|---------------|-----------------|----------------------------|
| S&P500 | 0.006 | 0.042 | -0.969 | 5.409 | 115.880* | 0.142 | 1 |
| Crude oil | 0.004 | 0.096 | -0.402 | 4.039 | 20.908^* | 0.042 | 0.044 |
| Heating oil | 0.005 | 0.091 | -0.204 | 3.296 | 3.085 | 0.055 | 0.137 |
| Natural gas | 0.001 | 0.134 | -0.069 | 3.445 | 2.633^{*} | 0.007 | -0.028 |
| Gold | 0.004 | 0.047 | 0.097 | 6.057 | 113.794* | 0.085 | -0.082 |
| Palladium | 0.007 | 0.104 | -0.389 | 6.351 | 143.498^{*} | 0.067 | 0.185 |
| Copper | 0.005 | 0.078 | -0.222 | 5.078 | 54.759* | 0.064 | 0.336 |
| Platinum | 0.003 | 0.065 | -0.545 | 6.213 | 139.584^{*} | 0.046 | 0.260 |
| Silver | 0.004 | 0.081 | -0.379 | 5.510 | 83.363* | 0.049 | 0.114 |
| Tin | 0.005 | 0.071 | -0.309 | 4.564 | 34.289* | 0.070 | 0.261 |
| Zinc | 0.004 | 0.074 | -0.443 | 5.168 | 66.545^{*} | 0.054 | 0.322 |
| Aluminum | 0.002 | 0.057 | -0.355 | 3.872 | 15.350^{*} | 0.035 | 0.263 |
| Corn | 0.001 | 0.082 | -0.232 | 4.870 | 44.990^{*} | 0.012 | 0.096 |
| Cotton | 0.000 | 0.085 | -0.463 | 5.962 | 116.767^{*} | 0.000 | 0.166 |
| Coffee | 0.002 | 0.105 | 0.941 | 6.509 | 192.307^{*} | 0.019 | 0.094 |
| Soybeans | 0.001 | 0.068 | -0.277 | 4.232 | 22.128^{*} | 0.014 | 0.155 |
| Sugar | 0.001 | 0.093 | -0.029 | 3.481 | 2.852 | 0.010 | -0.023 |
| Wheat | 0.001 | 0.086 | 0.622 | 4.730 | 55.063* | 0.011 | 0.051 |
| Feeder Cattle | 0.002 | 0.044 | -0.532 | 3.938 | 24.376^{*} | 0.045 | 0.113 |
| Live cattle | 0.002 | 0.046 | -0.394 | 3.301 | 8.627^{*} | 0.043 | 0.086 |

 Table 1

 Summary statistics of equity and commodity futures returns

Notes. J-B refers to the empirical statistics of Jarque–Bera test for normality. *, **, and *** indicates the rejection of the null hypotheses at the 1%, 5%, and 10% levels, respectively.

gas). The skewness is negative in most cases, except for some commodity futures (gold, coffee and wheat). Kurtosis coefficients are above three for all the return series, suggesting that the latter have a leptokurtic behavior (fat tails). The Jarque–Bera (JB) test validates that our return series are not distributed normally as it is strongly rejected for all series at the 1% level except heating oil and sugar.

We also computed the realized Sharpe ratios (risk-adjusted performance) by dividing the average return by the corresponding standard deviation. The results show that the Sharpe ratio values range from 0.000 (Cotton) to 0.142 (S&P500), with equity, crude oil, heating oil, gold, palladium, platinum, and silver representing the best investments. As expected, the unconditional correlation between the U.S. equity market and commodity futures returns is positive in most cases (except for natural gas, gold, and sugar). This finding potentially reflects the increased financialization of commodity markets and the increased interest of investors on commodity diversification over recent years. Nevertheless, the relatively low correlation and negative coefficients suggest that room for diversification benefits from investing in both equity and commodities exists. The higher correlation between equity market and copper and zinc futures returns can be explained by the important role played by these industrial nonferrous metals in the U.S. economy, particularly in the construction and electric equipment sectors.

Figure 1 displays the dynamics of equity and commodity returns through time. The observed patterns show that they react to some common shocks related to major international economic and

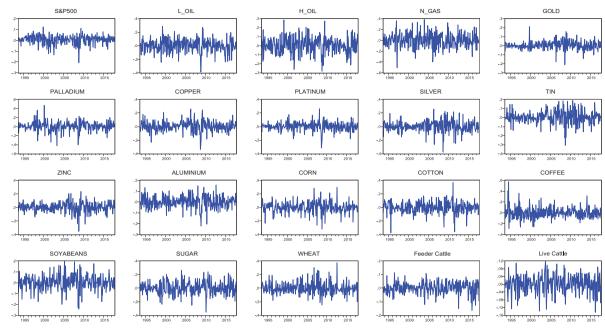


Fig. 1. Time-paths of equity and commodity futures returns.

Notes. We use monthly data of continuous futures contracts, which are derived from individual futures contracts based on the Type 0 and Type 2 roll methods for three energy commodities (crude oil, heating oil, and natural gas), eight metals (copper, platinum, gold, silver, palladium, tin, zinc, and aluminum), and eight agricultural, food, and livestock commodities (corn, wheat, coffee, cotton, soybeans, sugar, feeder cattle, and live cattle). The S&P500 market index is employed to represent the diversified stock portfolio of the U.S. equity markets. Data sample period: April 1990 to November 2017.

financial events such as the Asian financial crisis of 1997–1998 and the global financial crisis of 2008–2009. The return movements of the commodity futures are generally larger than the ones of the diversified stock portfolio.

4. Empirical results

4.1. Results from the DCC-EGARCH

Table 2 reports the estimation result of the bivariate DCC-EGARCH model for the U.S. equity market and each of the 19 commodity futures markets under consideration. Recall that the conditional mean of returns follows an autoregressive process, while the conditional variance of returns is modeled by an EGARCH model that explicitly accommodates the asymmetric volatility property.

The results related to the mean equations show that the autoregressive (AR) coefficients are only statistically significant for gold, corn, cotton, and coffee. This evidence suggests that commodity futures returns are not predictable in most cases when using their one-period lagged values—an

| | | | Heating | Natural | | | | | | | | | | | | | | | Feeder | |
|----------------|---------------|----------------------|-------------|----------|--|-----------|-----------|----------------------------------|------------------------------------|----------------------|--------------|------------|----------------------|---------------|--------------|----------------------|-----------|-----------|-------------|-------------------------|
| | S&P500 | S&P500 Crude oil oil | lio | gas | Gold | Palladiu | m Copper | Palladium Copper Platinum Silver | m Silver | Tin | Zinc | Aluminu | Aluminum Corn | Cotton Coffee | Coffee | Soybeans Sugar | Sugar | Wheat | cattle | Live cattle |
| Const(m) | 0.007*** | 0.004 | 0.006 | 0.003 | 0.004* | 0.006 | 0.005 | 0.003 | 0.001 | 1 0.001 | 0.001 | 0.002 | 0.003 | 0.003 | -0.000 | 0.003 | 0.002 | 0.004 | 0.001 | 0.001 |
| AR(1) | -0.029 | 0.042 | -0.059 | -0.027 | -0.109^{*} | -0.052 | 0.068 | -0.031 | -0.121 | 1 -0.038 | -0.014 | -0.073 | 0.109^{**} | * -0.113** | | 0.024 | 0.042 | -0.045 | 0.133 | 0.071 |
| Cst(V) | 2.566^{a} | -4.735*** | 0.002** | -4.071 | -4.071^{***} -2.200^{***a} 0.000 | **a 0.000 | -5.170*** | *** 2.215 ^a | 5 ^a 0.578 | | | 1 -5.775* | -5.775***a -5.039*** | ** -5.010*** | | * -5.420* | -4.758*** | ** -4.870 | 2.086^{a} | -6.213 ^{***} a |
| $ARCH(\alpha)$ | 0.021 | | 0.056 | 1.005 | 0.912** | | | 0.220** |)** 0.013 | | ** 0.086 | *** 2.231 | 1.045** | * 0.805 | 0.348*** | * 0.120*** | * 0.979 | 1.270 | 0.021 | 0.980 |
| $GARCH(\beta)$ | 0.683^{***} | -0.581*** | 0.615*** | * -0.177 | -0.011 | 0.900*** | ** 0.035 | | s*** 0.937* | | *** 0.890*** | *** -0.034 | -0.418^{**} | * -0.117 | 0.392*** | * 0.810 [*] | -0.328* | -0.280*** | 0.809*** | -0.581*** |
| EGARCH | 0.264 | 0.082 | 0.216^{*} | -0.177 | 0.035** | * -0.060 | 0.314** | ** -0.208 | 1.652 | | 0.000 | -0.019 | 0.246^{**} | * -0.017 | -0.433 | * -0.021 | -0.010 | 0.043 | 0.118 | 0.116 |
| (l_{θ}) | | | | | | | | | | | | | | | | | | | | |
| Log (L) | 530.75 | 279.02 | 296.59 | 179.45 | 492.81 | 255.54 | 341.93 | 404.94 | | 379.99 | 292.51 | 427.72 | 326.12 | 317.99 | 64 | | 280.74 | 286.84 | 507.68 | 493.63 |
| h Stock | 1 | -0.032 | 0.157 | -0.059 | -0.020 | 0.263 | | 0.303*** 0.202** | 2 ^{**} 0.121 [*] | 1 [*] 0.020 | 0.294 | | 0.252*** 0.051 | 0.124* | 0.146^{**} | 0.138^{**} | -0.054 | 0.074 | 0.057 | 0.091 |
| Q(10) | NO | NO | NO | NO | NO | NO | NO | NO | NO | NO | NO | NO | NO | NO | YES | NO | NO | NO | NO | YES |
| 10) | NO | NO | NO | YES | NO | NO | NO | NO | NO | NO | NO | NO | NO | NO | NO | NO | NO | NO | NO | NO |
| ARCH(10) | NO | NO | NO | NO | NO | NO | NO | NO | NO | NO | NO | NO | NO | NO | NO | NO | NO | NO | NO | NO |

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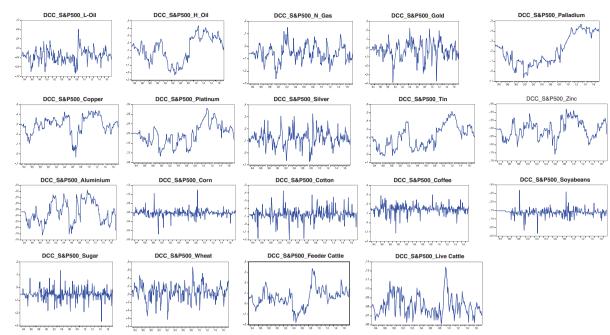


Fig. 2. Dynamic conditional correlations between equity and commodity future returns.

evidence of weak-form market efficiency for commodity futures markets. The small size and negative sign of three out of four AR coefficients may be explained by the decreasing trend of these commodity futures prices over recent years.

With respect to the estimated parameters of the conditional variance equations, we see that the sum of the ARCH and GARCH coefficient is less than unity in most cases. The majority of the GARCH coefficients is significant, except for natural gas, gold, copper, aluminum, and cotton. The coefficients associated with palladium and silver have a value above 0.9, thus indicating the high volatility persistence over time. The result of the asymmetric adjustment of the volatility model shows that the positive shock and negative news may have different effects on the volatility. The volatility-asymmetric parameter (θ_1) is negative and significant only for the case coffee futures. Accordingly, bad news have stronger effects on its conditional volatility than good news. For other commodities including natural gas, gold, copper, and corn, the sign of leverage coefficient is positively significant, which thus implies that positive shocks have more impacts on conditional volatility than negative shocks.

The result reported in Table 2 show the diagnostic tests applied to standardized residuals and standardized squared residuals indicate that our selected EGARCH type of model are correctly specified because the hypothesis of no autocorrelation is rejected in most cases, while ARCH effects are no longer present for all cases.

The dynamics of conditional correlations between the U.S. equity market and commodity futures, provided by the bivariate DCC-EGARCH models, is plotted in Fig. 2. We observe considerable variations over time and the dynamic patterns are not alike across commodity futures under consideration. The average time-varying correlations between equity-commodity returns, reported

in Table 2, are very similar to the unconditional correlations provided in Table 1. Theyare generally inferior to 30.3% and only significant at conventional levels for 7 out of 19 commodity futures (copper, platinum, silver, aluminum, cotton, coffee, and soybeans). This finding suggests that there is room for portfolio diversification between equity and commodity market. However, one common picture reflected in equity-commodity market linkages (Fig. 2) is their large fluctuations around 2008–2009 global financial crisis. Most commodities across the energy, metal, and agriculturalfood-livestock sectors experienced a synchronized boom and bust cycles during this period, which make commodity returns more volatile. According to the metals group, gold is different from other commodities. The correlation is negative and consistent with the safe-haven role of gold (Baur and Mcdermott, 2010). The agriculture commodities also have an interesting pattern of comovement. The major scenario of the correlation is that volatility varies over time, being quite stable before the 2007–2008 crisis and becoming excessive during the great financial turmoil. The findings are consistent with the results reported in Creti et al. (2013).

For a better understanding of the equity-commodity market comovement through time, we examine the time trend of the DCC obtained from the DCC-EGARCH model by estimating a linear time-trend model accounting for potential structural changes in commodity market returns. We allow for the structural breaks in either the constant or the time trend of the model, and Quandt–Andrews breakpoint test is used to identify the unknown breakpoint. Note that the breakpoint test is conducted using two model specifications: breakpoint in the time-trend only and breakpoint in the constant term only. More specifically, for each commodity, we estimate the following time-trend models where the *break*_t variable takes the value of 0 for all observations prior to the identified breakpoint and the value of 1 for all subsequent observations:

$$DCC_t = \alpha_0 + \beta_{0*}t + \beta_1t * break_t + e_t \tag{7}$$

$$DCC_t = \alpha_0 + \alpha_{1*}break_t + \beta_{0*}t + e_t,$$
(8)

where α and β capture, respectively, the long-term level of comovement between the equity and commodity futures markets, and its rates of changes with respect to the time.

The results of the Quandt–Andrews breakpoint test are reported in Table 3. We see that breakpoints are commodity-specific and mostly occurred around major crisis and geopolitical events (1996–1997, 2003–2004, 2008–2009). They are all significant at conventional levels, except for crude oil, heating oil, natural gas, platinum, tin, zinc corn, soybean, and sugar when we tested for breakpoints in the time trend only. For feeder cattle and live cattle, the result is insignificant when we tested for breakpoints in the constant only.

Table 4 reports the results of the linear time-trend models for commodities experiencing significant breakpoints, that is, Equations (7) and (8). Panel A shows that the β_0 parameters are positive and significant for all commodity futures, except for natural gas, gold, silver, corn, coffee, soyabeans, and sugar. On the other hand, the β_1 parameters are insignificant for all cases, except for heating oil and natural gas futures at the 10% level and silver at the 5% level. Altogether, these results suggest the increased equity-commodity market comovement for most cases over time until the breakpoints, but no particular trend (increasing or decreasing) is observed. On the other hand, we find in Panel B that the overall (long-term) level of equity-commodity comovement captured by α_0 parameters is positive (except for crude oil, heating oil, natural gas, gold, and sugar) and

| Table 3 | |
|------------------------------|--|
| Zivot and Andrews break test | |

| Commodity | Constant only | Trend only | Commodity | Constant only | Trend only |
|--|--|--|--|---|---|
| Crude oil Heating oil Natural gas Gold Palladium Copper Platinum | 2008M08** 2008M07** 2006M01** 2011M10** 2001M02*** 2006M06** 2008M06** | 2004M09 2004M09 2000M04 2007M11** 2002M07** 2005M09* 2005M12 | Aluminum Corn Cotton Coffee Soybeans Sugar Wheat | 2008M08** 2008M07** 2011M04*** 2002M03*** 2008M07** 2004M01** 2008M04** | (-)* 2010M10 (-)* 1997M11* 2007M10 2009M08 2007M07* |
| Silver Tin Zinc | 2011M05*** 2011M05** 2007M01*** | 2006M05 ⁺ 2007M01 2014M02 | Feeder cattle Live cattle | 2012M03 2014M03 | (-)* (-)* |

Notes. *, **, and *** indicate significance at the10%, 5%, and 5% levels, respectively. (–)*: Singular Matrix

| Table 4 |
|---|
| Time trends and breaks of the commodity-equity dynamic conditional correlations |

| | Panel A: B | reakpoints in time | e trends | Panel B: Brea | ıkpoints in c | onstant terms | |
|---------------|----------------|------------------------------|-------------------------|---------------|----------------|-------------------------|------------------------------|
| | α_0 | eta_0 | β_1 | | α_0 | α_1 | β_0 |
| Crude oil | -0.037*** | $3.36 \times 10^{-5++}$ | -0.000 | Crude oil | -0.037*** | -0.062** | 3.77×10^{-5} |
| Heating oil | -0.009 | 0.001^{+++} | -0.002^{*} | Heating oil | -0.010 | -0.141 | 0.001^{***} |
| Natural gas | -0.056^{***} | 1.07×10^{-6} | -0.001^{*} | Natural gas | -0.057^{***} | -0.071 | 4.81×10^{-6} |
| Gold | -0.029^{***} | 1.69×10^{-5} | 0.001 | Gold | -0.029^{***} | -0.254^{***} | 2.76×10^{-5} |
| Palladium | 0.023^{**} | 0.001^{***} | -0.001 | Palladium | 0.024^{**} | -0.179^{*} | 0.001^{***} |
| Copper | 0.262^{***} | 0.000^{***} | 0.001 | Copper | 0.263*** | -0.257^{***} | 0.000^{***} |
| Platinum | _ | _ | _ | Platinum | 0.160^{***} | -1.15×10^{-5} | 0.000^{***} |
| Silver | 0.117^{***} | 3.48×10^{-5} | -0.001^{**} | Silver | 0.116^{***} | 0.018 | 3.36×10^{-5} |
| Tin | 0.065^{***} | 0.001^{***} | -0.000 | Tin | 0.065^{***} | 0.157 | 0.001^{***} |
| Zinc | 0.271^{***} | 0.000^{***} | -0.000 | Zinc | 0.272^{***} | -0.043 | 0.000^{***} |
| Aluminum | _ | _ | _ | Aluminum | 0.240^{***} | -0.018 | $6.75 \times 10^{-5***}$ |
| Corn | 0.058^{***} | -1.65×10^{-7} | 4.45×10^{-5} | Corn | 0.058^{***} | -0.020^{**} | 4.45×10^{-7} |
| Cotton | _ | _ | _ | Cotton | 0.115*** | 0.018 | 6.72×10^{-5} |
| Coffee | 0.145^{***} | -6.04×10^{-6} | 2.64×10^{-5} | Coffee | 0.145*** | 0.038 | -5.31×10^{-6} |
| Soybeans | 0.139*** | -4.80×10^{-6} | 2.20×10^{-5} | Soybeans | 0.139*** | -0.039^{*} | -4.80×10^{-6} |
| Sugar | -0.052^{***} | -1.14×10^{-5} | 0.000 | Sugar | -0.052^{***} | -0.072^{*} | -1.14×10^{-5} |
| Wheat | 0.125*** | $2.93 \times 10^{-12^{***}}$ | -5.89×10^{-12} | Wheat | 0.125*** | -3.11×10^{-10} | $2.94 \times 10^{-12^{***}}$ |
| Feeder cattle | - | _ | _ | Feeder cattle | 0.014 | 0.098 | 0.000^{***} |
| Live cattle | - | _ | _ | Live cattle | - | _ | _ |

Notes. The table reports the estimated parameters. *, **, and *** indicate significance at the10%, 5%, and 1% levels, respectively. Missing results: Singular Matrix

significant for most cases (except for heating oil and feeder cattle futures). Another interesting result is that while there is an increasing trend of comovement (β_0) in most cases, no pattern of significant shifts in the overall level of comovement following the identified breakpoints is found as the α_1 parameters are all insignificant, except for crude oil, gold, copper, corn, soyabeans, and sugar.

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In sum, the above results indicate that potential diversification benefits can be achieved by adding commodity futures to stock portfolios because the dynamic correlation levels, despite increasing, are at most moderate and there are no significant shifts in conditional correlations after structural breaks are accounted for.

4.2. Economic value of commodity futures

This subsection addresses the question of where the inclusion of commodity futures into portfolios of stocks help improve their risk-adjusted performance. For this purpose, we first construct several commodity-equity portfolios (equity and each of the 19 commodity futures in our sample data) with different holding weights of commodity futures: PF1 (100% equity portfolio represented by S&P500 index), PF2 (with optimal weights of equity and commodity futures determined by DCC-EAGRCH model), PF3 (25% commodity, 75% equity), PF4 (50% commodity, 50% equity), and PF5 (75% commodity, 25% commodity). We then use the stochastic dominance technique to compare the performance of the portfolio PF1 with that of the others (PF2, PF3, PF4, and PF5). Note that we follow the approach of Arouri et al. (2011) to compute the optimal holding weight of commodity futures (W_t^c) for the portfolio PF2 at time t as follows:

$$W_t^c = \frac{h_t^e - h_t^{ec}}{h_t^c - 2h_t^{ec} + h_t^e},\tag{9}$$

subject to

$$W_t^{ec} = \begin{cases} 0, & if \ W_t^{ec} < 0 \\ W_t^{ec}, & if \ 0 \le W_t^{ec} \le 1 \\ 1, & if \ W_t^{ec} > 1 \end{cases}$$

where h_t^c is the conditional volatility of the commodity futures under consideration, h_t^c the conditional volatility of the equity market, and h_t^{ec} the conditional covariance between commodity futures reutrns and equity returns, which are obtained from the DCC-EGARCH model. By construction, the optimal weight of the stock market index in the commodity-equity portfolio is equal to $(1 - W_t^c)$. The optimal weights satisfy the condition that they minimize the overall risk of the commoditystock portfolio without reducing its expected returns.

Using SD criteria, we compare the performance of the PF1 (100% equity) with that of each individual commodity and of the above-mentioned equity-commodity portfolios: PF2, PF3, PF4, and PF5. The SD results for PF1 and individual commodities are reported in Table 5. These results show that the 100% equity portfolio stochastically dominates all commodity futures at second and third orders except gold and the two live stocks that have no SD relation. When we compare the PF1 with the portfolios PF2 that are constructed on the basis of the optimal weights from the DCC-EGARCH model (PF2), the results in Table 6 indicate that the PF1 stochastically dominates 13 optimal equity-commodity portfolios at second and third orders. On the other hand, gold and the two live stocks) that do not display any SD relation.

| Commodity | FSD | SSD | TSD |
|---------------|-----|-------|-------|
| Crude oil | ND | F > G | F > G |
| Heating oil | ND | F > G | F > G |
| Natural gas | ND | F > G | F > G |
| Copper | ND | F > G | F > G |
| Platinum | ND | F > G | F > G |
| Silver | ND | F > G | F > G |
| Palladium | ND | F > G | F > G |
| Gold | ND | ND | ND |
| Aluminum | ND | F > G | F > G |
| Zinc | ND | F > G | F > G |
| Tin | ND | F > G | F > G |
| Corn | ND | F > G | F > G |
| Coffee | ND | F > G | F > G |
| Cotton | ND | F > G | F > G |
| Wheat | ND | F > G | F > G |
| Sugar | ND | F > G | F > G |
| Soya | ND | F > G | F > G |
| Feeder cattle | ND | ND | ND |
| Live cattle | ND | ND | ND |

Notes. FSD, SSD, and TSD denote first-, second-, and third-order stochastic dominance, respectively. PF1 is the portfolio that consists of 100% equity.

F > G: the PF1 dominates the respective commodity.

G > F: the respective commodity dominates the PF1.

ND means no stochastic dominance.

Table 6

Table 5

Stochastic dominance results of Davidson and Duclos test: PF1 versus PF2

| Portfolio PF2 | FSD | SSD | TSD |
|---------------|-----|-------|-------|
| Crude oil | ND | F > G | F > G |
| Heating oil | ND | F > G | F > G |
| Natural gas | ND | F > G | F > G |
| Copper | ND | F > G | F > G |
| Platinum | ND | ND | ND |
| Silver | ND | F > G | F > G |
| Palladium | ND | F > G | F > G |
| Gold | ND | G > F | G > F |
| Aluminum | ND | ND | ND |
| Zinc | ND | F > G | F > G |
| Tin | ND | F > G | F > G |
| Corn | ND | G > F | G > F |
| Coffee | ND | F > G | F > G |
| Cotton | ND | F > G | F > G |
| Wheat | ND | F > G | F > G |
| Sugar | ND | F > G | F > G |
| Soya | ND | F > G | F > G |
| Feeder cattle | ND | ND | ND |
| Live cattle | ND | ND | ND |

Notes. FSD, SSD, and TSD denote first-, second-, and third-order stochastic dominance, respectively. PF1 is the portfolio that consists of 100% equity. PF2 is the portfolio that is based on the optimal weights from the DCC-EGARCH model.

F > G: the PF1 dominates the respective PF2.

G > F: the respective PF2 dominates the PF1.

ND means no stochastic dominance.

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| Portfolio PF3 | FSD | SSD | TSD |
|---------------|-----|-------|-------|
| Crude oil | ND | ND | ND |
| Heating oil | ND | ND | ND |
| Natural gas | ND | ND | ND |
| Copper | ND | ND | ND |
| Platinum | ND | ND | ND |
| Silver | ND | ND | G > F |
| Palladium | ND | ND | ND |
| Gold | ND | G > F | G > F |
| Aluminum | ND | G > F | G > F |
| Zinc | ND | ND | ND |
| Tin | ND | ND | ND |
| Corn | ND | ND | ND |
| Coffee | ND | G > F | G > F |
| Cotton | ND | ND | ND |
| Wheat | ND | ND | ND |
| Sugar | ND | ND | ND |
| Soya | ND | G > F | G > F |
| Feeder cattle | ND | G > F | G > F |
| Live cattle | ND | G > F | G > F |

Table 7 Stochastic dominance results of Davidson and Duclos test: PF1 versus PF3

Notes. FSD, SSD, and TSD denote first-, second-, and third-order stochastic dominance, respectively. PF1 is the portfolio that consists of 100% equity. PF3 is the portfolio that consists of 25% commodity and 75% equity.

F > G: the PF1 dominates the respective PF3.

G > F: the respective PF3 dominates the PF1.

ND means no stochastic dominance.

Table 7 reports the SD results for portfolios that are formed by 25% commodity futures and 75% of equity (PF3). The 100% equity portfolio does not stochastically dominates any portfolios, while the gold, aluminum, coffee, and soya-equity portfolios, on the other hand, dominates the 100% equity portfolio at the second and third orders respectively.

Regarding the portfolios that are formed by half of commodity futures and equity (PF4), the SD results are presented in Table 8. We find evidence of 10 pairs of portfolios that do not have any SD relations. Similar to the three SD relations as found for PF3 where the diversified portfolios are stochastically dominates the 100% equity portfolio. On the other hand, six pairs are stochastically dominated by the 100% equity portfolio.

Turning out to PF5, which is composed of 75% commodity futures and 25% equity, Table 9 shows that 14 pairs have significant SD relations where the 100% equity portfolio stochastically dominates all portfolios made of commodity futures and equity, except for the portfolios containing 75% of platinum, gold, aluminum, and the two live stocks where we do not find any SD relations.

To support our findings, we present, graphically in Figs. 3–5, the theoretical DD statistics in black and dark blue and cumulative distribution function (CDF) in red and blue together with empirical order-*j* DD statistics (T1, T2, and T3) of stochastic dominance. Figure 3 shows the case of S&P500 versus individual crude oil futures, where T2 and T3 are significantly negative. The

| Portfolio PF4 | FSD | SSD | TSD |
|---------------|-----|-------|-------|
| Crude oil | ND | F > G | F > G |
| Heating oil | ND | F > G | F > G |
| Natural gas | ND | F > G | F > G |
| Copper | ND | ND | ND |
| Platinum | ND | ND | ND |
| Silver | ND | ND | ND |
| Palladium | ND | F > G | F > G |
| Gold | ND | G > F | G > F |
| Aluminum | ND | ND | ND |
| Zinc | ND | ND | ND |
| Tin | ND | ND | ND |
| Corn | ND | ND | ND |
| Coffee | ND | F > G | F > G |
| Cotton | ND | F > G | F > G |
| Wheat | ND | ND | ND |
| Sugar | ND | ND | ND |
| Soya | ND | ND | ND |
| Feeder cattle | ND | G > F | G > F |
| Live cattle | ND | G > F | G > F |

Stochastic dominance results of Davidson and Duclos test: PF1 versus PF4

Notes. FSD, SSD, and TSD denote first-, second-, and third-order stochastic dominance, respectively. PF1 is the portfolio that consists of 100% equity. PF4 is the portfolio that consists of 50% commodity and 50% equity.

F > G: the PF1 dominates the respective PF4.

G > F: the respective PF4 dominates the PF1.

ND means no stochastic dominance.

Table 9

Table 8

Stochastic dominance results of Davidson and Duclos test: PF1 versus PF5

| Portfolio PF5 | FSD | SSD | TSD |
|---------------|-----|-------|-------|
| Crude oil | ND | F > G | F > G |
| Heating oil | ND | F > G | F > G |
| Natural gas | ND | F > G | F > G |
| Copper | ND | F > G | F > G |
| Platinum | ND | ND | ND |
| Silver | ND | F > G | F > G |
| Palladium | ND | F > G | F > G |
| Gold | ND | ND | ND |
| Aluminum | ND | ND | ND |
| Zinc | ND | F > G | F > G |
| Tin | ND | F > G | F > G |
| Corn | ND | F > G | F > G |
| Coffee | ND | F > G | F > G |
| Cotton | ND | F > G | F > G |
| Wheat | ND | F > G | F > G |
| Sugar | ND | F > G | F > G |
| Soya | ND | F > G | F > G |
| Feeder cattle | ND | ND | ND |
| Live cattle | ND | ND | ND |

Notes. FSD, SSD, and TSD denote first-, second-, and third-order stochastic dominance, respectively. PF1 is the portfolio that consists of 100% equity. PF5 is the portfolio that consists of 75% commodity and 25% equity.

F > G: the PF1 dominates the respective PF4.

G > F: the respective PF4 dominates the PF1.

ND means no stochastic dominance.

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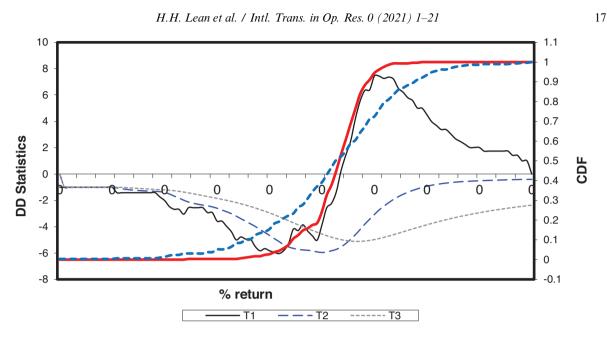


Fig. 3. DD statistics and their cumulative distributions for PF1 versus crude oil futures. Notes. This table shows the DD statistics (black and dark blue), and CDF (red and blue). T1, T2, and T3 denote the first-, second-, and third-order stochastic dominance, respectively.

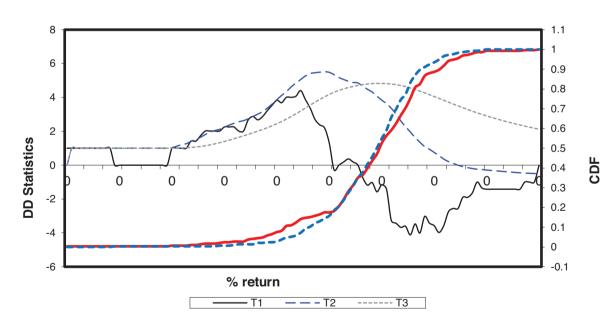


Fig. 4. DD statistics and their cumulative distributions for PF1 versus gold-equity portfolio (PF3). *Notes.* This table shows the DD statistics (black and dark blue), and CDF (red and blue). T1, T2, and T3 denote the first-, second-, and third-order stochastic dominance, respectively.



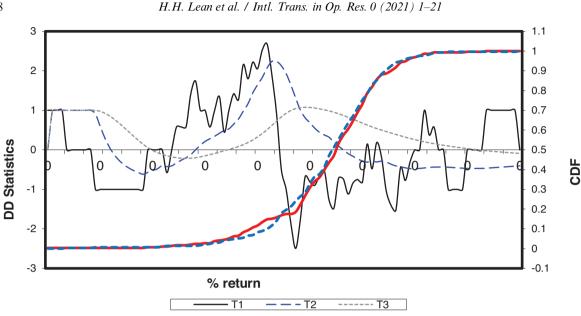


Fig. 5. DD statistics and their cumulative distributions for PF1 versus crude oil-equity portfolio (PF3). Notes. This table shows the DD statistics (black and dark blue), and CDF (red and blue). T1, T2, and T3 denote the first-, second-, and third-order stochastic dominance, respectively.

significant negative T2 and T3 means that the S&P500 dominates the crude oil futures at the second and third orders, respectively. On the other hand, Fig. 4 depicts the case of S&P500 versus portfolio of gold futures (PF3) where T2 and T3 are significantly positive inferring a reverse situation in this pair. We can see a mix of significant positive and negative T2 and T3 in Fig. 5 showing that no SSD and TSD in the PF3 between S&P500 versus crude oil futures. All these figures also support the finding that there is no FSD between any two pairs of returns as their CDFs cross and a mix of significant positive T1.

After the comparison between all equity portfolio (PF1) versus selected mixed portfolios, we further construct alternative portfolios by using some of the most popular practitioner strategies. Then, we again compare our PF1 against these new portfolios. The newly added portfolios are designed in the following way: (i) First strategy is the famous naive portfolio. In this one, we equally weight all sample assets and track the portfolio's monthly return performance; (ii) Second strategy is based on maximized Sharpe ratio (Sharpe Max). We take the first two years of monthly observations to construct the asset return and covariance matrices. Then we find the portfolio weights that give us the highest Sharpe ratio. Then we use these weights to construct the next month's portfolio and track its return. The procedure is repeated on a rolling two years length window; (iii) Third portfolio is based on momentum strategy. Here, we first find the best and worst performing assets in the last 24 months. Using this information, we long (short) the best (worst) performing asset in the next month. Then we pass to the next month and the procedure is repeated; (iv) Finally, we use a contrarian strategy to construct our third portfolio. The idea is similar to the previous portfolio construction. However, the difference is once we find the best and worst performing assets in the last 24 months, we long (short) the worst (best) performing assets in the next month hoping that

Table 10

Stochastic dominance results of Davidson and Duclos test: PF1 versus each strategy

| Strategy | FSD | SSD | TSD |
|------------------|-----|-------|-------|
| Naïve | ND | ND | ND |
| Sharpe Ratio Max | ND | ND | ND |
| Momentum | ND | F > G | F > G |
| Contrarian | ND | F > G | F > G |

Notes. FSD, SSD, and TSD denote first-, second-, and third-order stochastic dominance, respectively. PF1 is the portfolio that consists of 100% equity.

F > G: the PF1 dominates the respective strategy.

G > F: the respective strategy dominates the PF1.

ND means no stochastic dominance.

there will be a return reversal. By shifting one month at a time, the procedure is repeated for the whole sample.

The SD test results for the four above-mentioned strategies are presented in Table 10. We discover that Naïve and Sharpe Max do not possess any SD relations. On the other hand, PF1 stochastically dominates the Momentum and Contrarian portfolios. More precisely, these findings suggest that there is no preference between the equity portfolio and the Naïve or Sharpe Max portfolio, while the equity portfolio is preferred to the Momentum or Contrarian portfolios. Hence, our previous finding that the risk-return performance of portfolios with the inclusion of commodity futures is not always improved still holds when commonly used investment strategies are considered.

5. Conclusion

We provided insights about the role of commodity futures in portfolio optimization and risk management using the implementation of a bivariate DCC-EGARCH model that allows not only for time-varying comovements but also asymmetric volatility reactions to news. The dynamic conditional correlations between the studied S&P500 and the commodity returns are on average weakly positive and significant in 6 out of 15 cases. This implies that there is a room for portfolio diversification between equity and commodity market. In addition, our time-trend analysis shows an increasing equity-commodity market comovement through time, but no significant shifts in both the level of comovement and its rates of change subsequent to the breakpoints identified by the Quandt–Andrews test.

We also investigated whether the diversified portfolios composed of equity and commodity futures (with the application of various allocation rates to commodity futures) as well as portfolios of four most popular investment strategies in real financial world perform better than the all-equity portfolio. Stochastic dominance approach shows that including commodity futures in diversified portfolios does not always improve the risk-return performance, with the exception of gold in some particular portfolio setups, especially when 25% and 50% of the budget are allocated to gold, respectively. This finding supports the fact that gold has different characteristic from the other commodities and is playing safe-haven role.

Our proposed framework in this article has the main advantage of being simple and appealing for investors' portfolio analysis and asset allocation decision-making. Future research can extend it to a more complex modeling in order to incorporate, among others, regime-switching behavior, tail dependence, and chaotic movements of asset returns.

Acknowledgments

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