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Robust airline scheduling with controllable cruise times and chance constraints

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Robust airline schedules can be considered as flight schedules that are likely to minimize passenger delay. Airlines usually add an additional time—e.g., schedule padding—to scheduled gate-to-gate flight times to make their schedules less susceptible to variability and disruptions. There is a critical trade-off between any kind of buffer time and daily aircraft productivity. Aircraft speed control is a practical alternative to inserting idle times into schedules. In this study, block times are considered in two parts: Cruise times that are controllable and non-cruise times that are subject to uncertainty. Cruise time controllability is used together with idle time insertion to satisfy passenger connection service levels while ensuring minimum costs. To handle the nonlinearity of the cost functions, they are represented via second-order conic inequalities. The uncertainty in non-cruise times is modeled through chance constraints on passenger connection service levels, which are then expressed using second-order conic inequalities. Overall, it is shown, that a 2% increase in fuel costs cuts down 60% of idle time costs. A computational study shows that exact solutions can be obtained by commercial solvers in seconds for a single-hub schedule and in minutes for a four-hub daily schedule of a major U.S. carrier.

Keywords: Robust airline scheduling, second-order cone programming

1. Introduction

During the implementation of airline schedules, numerous disruptions are faced that result in operational delays. The continuous increase in fleet sizes, number of flights, and number of passenger connections result in congestion, which make the effects and propagation of delays very significant. Therefore, airlines need to generate robust flight schedules that can respond to these disruptions during implementation and ensure desired service levels even under uncertainty. We refer to Barnhart and Cohn (2004) for an extensive discussion on flight operations.

As reported in Barnhart *et al*. (2012), the total cost of delays in the United States in 2007 was estimated at \$31.2 billion, \$8.3 billion being direct cost to airlines and \$16.7 billion to passengers. Moreover, approximately \$6 billion of the total direct cost to airlines and passengers was associated with the additional time—e.g., idle time or schedule padding—airlines add to scheduled gate-to-gate flight times to make their schedules less susceptible to disruptions.

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It is important to note that there is a critical trade-off between any kind of buffer time and daily aircraft productivity. As stated in Cook (2007), a waiting aircraft with unused buffer time always includes a sunk cost—e.g., just 5 minutes of unused buffer—at-gate, for a B-767-300 ER, would amount to over €50,000 over a period of 1 year (or €27.40 a minute) on just one leg per day. Consequently, some airlines place more emphasis on aircraft utilization and add almost no slack (or idle time) into their schedules, which makes them more vulnerable to variability and disruptions. In other applications, the block time for each flight is calculated independently without considering the propagation of delays or the impact of variability on the entire network. Since the extra time on the ground is cheaper, the additional slack time is usually included in the aircraft turnaround time at the destination airport.

A growing literature highlights the importance of robustness, increasing service levels, delay reduction, and cost management in airline operations. An extensive review for irregular airline operations can be found in Barnhart (2009) and Clausen *et al*. (2010). Lan *et al*. (2006) considered flight delays in two categories as propagated and nonpropagated delay. An aircraft routing is a sequence of flights flown by a single aircraft, so a delay in one of these flights propagates to the following fight if there is no slack time in between.

They suggested that propagated delay can be reduced by assigning slack optimally to aircraft routings. Delay propagation for airline networks was analyzed and robustness measures were developed in Arıkan *et al*. (2013). Arıkan and Deshpande (2012) analyzed the impact of scheduled block times on on-time performance. Dunbar *et al*. (2012) presented a mathematical model to minimize propagated delay costs while integrating aircraft routing and crew pairing problems. Other researchers have addressed the problem of slack distribution and its effects on schedule performances. Ahmadbeygi *et al*. (2010) and Chiraphadhanakul and Barnhart (2013) worked on re-allocating already scheduled slack to have a more effective slack distribution in the schedule that can better absorb disruptions. Petersen *et al*. (2012) studied an integrated airline recovery problem using a single-day horizon and proposed separate mixed-integer mathematical models for the schedule, aircraft, and crew and passenger recovery problems. They utilized a Benders decomposition scheme together with the column generation approach to achieve coordination between these four mathematical models. Since it could take a significant computation time to solve the overall problem, they also proposed a sequential recovery algorithm.

Sohoni *et al*. (2011) took an alternative approach and modeled block-time distributions using chance constraints. They perturbed the departure times of an initial schedule to achieve improved passenger and network service levels while maximizing operational profits. To solve the model, they developed linear approximations on chance constraints. Marla and Barnhart (2010) employed two approaches to robust airline optimization focusing on the aircraft routing problem: the extreme value-based approach and chance-constrained programming approach. In the extreme value-based approach, the main focus is on minimizing the worst-case propagated delay, whereas the chance-constrained programming approach tries to minimize the probability of passenger misconnections as in Sohoni *et al*. (2011) or the probability of a certain flight being delayed less than a prespecified threshold as in Marla and Barnhart (2010).

In the existing literature on robust airline schedules, the cruise speed of an aircraft is taken as a fixed parameter, although the current industry practice of using a cost-index ratio allows airlines to dynamically adjust the cruise speed as discussed in Cook *et al*. (2009). Therefore, the most common approach is to insert idle times into the block times in order to deal with potential variability or disruptions. One significant advantage of having controllable cruise times is the added flexibility. Flight times can be reduced by increasing the cruise speeds to compensate the time losses in the system with, of course, additional fuel costs. Since there is an upper bound on the compression amount of the cruise times, cruise speed controllability should be considered simultaneously with idle time insertion to ensure the desired service levels for passenger connections at a minimum cost. In an airline schedule recovery problem, Marla *et al*. (2011) used cruise speed control as a means to decrease delay costs. In their time – space network model, they created flight copies with different cruise speeds to generate alternative schedules against a given disruption. We take a different approach and consider the nonlinear relationship between fuel consumption and cruise speed by expressing the cruise speed as a continuous variable instead of approximating it through a set of discrete variables. Furthermore, we use the recent advances in second-order cone programming to solve the resulting nonlinear programming formulation using an exact optimization approach.

In this study, we develop an optimization model that uses both idle time insertion and aircraft speed control to generate a robust schedule of minimum cost that satisfies given passenger connection service levels. We take all passenger connections into account and superimpose the passenger connection network with the flight network to consider the effects of aircraft delays on passenger service-level requirements, which are modeled through chance constraints. In previous studies, chance constraints (Charnes and Cooper, 1959) have been used to model the desired service levels; however, the resulting models were solved by approximation methods. For a general review on mathematical programming formulations with probabilistic constraints, we refer to Luedtke *et al*. (2010). In our study, instead of developing approximations, chance constraints are transformed into second-order conic inequalities and solved exactly in very short times. More information on conic programming can be found in Ben-Tal and Nemirovski (2001) and Günlük and Linderoth (2010). To the best of our knowledge, these methods have not previously been applied to robust airline scheduling.

The contributions and scope of this study can be summarized as follows. We start by taking an initial daily sequence of flights, passenger itineraries, and the specified time windows for the departure times. For each flight, the block time is defined as the time interval from which the plane departs from the gate at the origin location to the time the plane arrives at a gate at the destination point, where cruise time is the longest and most steady part of the block time and it is not significantly affected by variability. Therefore, we consider cruise times to be controllable and non-cruise times to be random variables. Obviously, increasing the cruise speed (albeit at the cost of additional fuel costs) is always more beneficial in terms of aircraft utilization as opposed to inserting idle times into the schedule.

In this study, the uncertainty associated with the random variables is modeled with chance constraints on the service level of passengers on their flight connections. Fast and exact solutions to this large-sized model of probabilistic constraints and nonlinear cost components are obtained by the use of second-order cone programming. We are able to solve the resulting nonlinear model in reasonable computation time using commercial solvers such as IBM ILOG CPLEX. As will be demonstrated in the Computational Study section, using published schedules of a major

U.S. carrier, the makespan of a day-to-day operation of an aircraft is significantly decreased (or, equivalently, the aircraft utilization is increased) when we introduce the controllable cruise times. Moreover, we show that slack times can be drastically reduced, achieving significant cost savings, by using speed controllability while satisfying the existing service levels on passenger connections.

Another important contribution is the incorporation of origin and destination information of a flight when calculating non-cruise time variability of each flight. It is known that airport congestion levels are different at each airport and an aircraft taking off from a non-hub location spends a lot less time for take-off compared with an aircraft that originates from a hub location, with the same concept applying to landing times. Therefore, the variability in non-cruise times in this study is calculated separately for each flight, considering the effect of the origin–destination airport congestions.

In the next section, the mathematical model is explained and a numerical example is provided to explain the mechanics. In Section 3, a conic reformulation of the model is shown in detail. Section 4 is devoted to a comprehensive computational study. We test the proposed model on two different flight schedules retrieved from the "Airline On-Time Performance Data" database of the Bureau of Transportation Statistics (BTS, 2010a). Conclusions and future study opportunities are discussed in Section 5.

2. Proposed model

Given the routings of aircraft, flight sequences, and passenger itinerary information, the model perturbs the departure times of flights in the initial schedule by inserting slack into the schedule and speeding up aircraft as necessary and hence determining proposed departure times and cruise time durations for all flights. While doing so, the objective is to minimize costs, with a given constraint on passenger connection service levels. As a result, the model generates a more robust schedule that can respond better to delays and ensure a given target of passenger connection service level of the overall network.

2.1. *Model definition*

The notation used in the mathemohed model is as follows:

Parameters

J	:	set of all flight legs
T	:	set of aircraft types
B	:	set of airports
t_i	:	the aircraft type of flight $i \in J$, $t_i \in T$
O_i	:	origin of flight $i \in J$
D_i	:	destination of flight $i \in J$
FIL_i	:	number of passengers in flight $i \in J$
f_i^{u}	:	original cruise time duration of flight $i \in J$
TP_{ij}	:	turnaround time needed to connect passengers between flights $i, j \in J$
TA_{ij}	:	turnaround time needed to prepare an aircraft between flights $i, j \in J$
PAS_{ij}	:	normalized passenger connection level between flights $i, j \in J$
C_t	:	fuel burn rate of aircraft type $t \in T$ in tons of fuel per minute
I_t	:	unit idle time cost of aircraft type $t \in T$ in dollars per minute
$[lb_i, ub_i]$	:	time window for departure time of flight $i \in J$
$[f_i^{\mathrm{l}}, f_i^{\mathrm{u}}]$	:	time window for cruise time of flight $i \in J$
P_i	:	set of flights that has a passenger connection with flight $i \in J$
$PAIR$	:	set of pairs of consecutive flights of the same aircraft
e_b	:	airport congestion coefficient for $b \in B$
γ	:	desired minimum service level for passenger connections
cf	:	fuel cost per ton of aircraft fuel

Decision Variables

x_i	:	departure time of flight $i \in J$
s_i	:	idle time after flight $i \in J$
f_i	:	cruise time of flight $i \in J$
γ_{ij}	:	service level for passenger connections between flights $i, j \in J$

f_i^{u} is the ideal duration of the flight, which corresponds to the scheduled duration in the initial plan. This duration in flight operations is decided by airlines using the cost index ratio (Cook *et al.*, 2009). The cost index is a number between zero and 999 (or sometimes zero and 99), where zero corresponds to the cruise speed that minimizes the fuel burn per unit distance and 999 corresponds to the maximum cruise speed that can be achieved by the aircraft. f_i^u in the model is associated with the zero cost index, generally known as the maximum range cruise speed. This will serve as an upper bound on the cruise time decision variable in the model.

P_i represents the set of flights for which i has an immediate passenger connection at the destination point of i; i.e., set P_i consists of flights that passengers from flight i use to continue their itineraries. Flights having the same destination as to origin of flight i are not allowed in the connection set, as passengers immediately returning back to the origin point is an unrealistic situation.

$[lb_i, ub_i]$ is the optional time window for the departure time of flight i. The model chooses departure times within this interval. This time window allows the planner to capture marketing and resource considerations when setting the departure time of a flight.

The set $PAIR$ holds the pairs of consecutive flights flown by the same aircraft. For flights $(i, j) \in PAIR$, TA_{ij} represents the turnaround time needed by the aircraft between two consecutive flights. The realized turnaround times depend on the congestion levels at the airports. They are also affected by the type of aircraft, since each aircraft needs a different amount of time for this operation.

2.1.1. *Random variable A_i*

An airline can influence the cruise time of an aircraft by adjusting the cruising speed, but it has no influence on the taxi-in and taxi-out phases. Taxi-in and taxi-out times are highly uncertain and can cause significant delays, especially at congested airports. The descend phase of a flight is also subject to uncertainty due to air traffic congestion, weather conditions, and airborne holding. Our purpose is to incorporate cruise time controllability and the uncertainty arising from airport congestion in the same framework. Thus, when modeling the flight time, cruise time is considered as a decision variable and the other stages of the flight are pooled into a random variable. Pooling is a mathematically necessitated assumption which enables deriving closed-form expressions for the random variable constraints.

This pooled random variable, A_i, represents the portion of the block time except the cruise time for each flight $i \in J$. Arıkan and Deshpande (2012) performed an extensive study on airline flight schedules across several U.S. airlines. They tested several distributions and showed that block times fitted a log Laplace distribution, so A_i values are assumed to be log Laplace random variables. We also assume A_i variables are independent random variables for each flight i. Note that the propagation of delays in the network actually might cause a correlation between A_is, especially for flights of the same aircraft. This has not been considered in this study.

We include the effects of congestion on flight time uncertainty by adjusting the distribution parameters α and β. For each airport $b \in B$, e_b represents the congestion coefficient, which is a measure of the level of congestion at that airport. These coefficients are used for calculating the turnaround time of an aircraft and for deciding related parameters of the random variable A_i. More information on the values of congestion coefficients used in this study can be found in Section 4.

Each random time A_i is associated with two parameters, α and β_i. Since the origin and destination airport congestion affects the block time duration separately and independently, for each flight, β_i is calculated by multiplying a base parameter β with a real-valued function of two congestion coefficients corresponding to origin and destination airports of the flight. In other words, the mean and variance of the random variable changes depending on the origin and destination airports. A robust verbal explanation for this is that higher congestion levels will result in a higher parameter β_i, which translates into a higher value for the random variable. β_i can be expressed as

$$\beta_i = \beta \times g(e_{O_i}, e_{D_i}),$$

where O_i and D_i are the origin and destination airports of flight $i \in J$. The A_is are assumed to be symmetric log Laplace random variables, therefore the tail grows one-sided; i.e., depending on the level of variability, the mean of the random variable grows.

The properties for a symmetric log Laplace random variable X with parameters α and $\beta_i > 0$, where e^α is a scale parameter and $1/\beta_i$ is the tail parameter, are given as

$$F_X(x) = \begin{cases} \frac{1}{2} e^{\frac{(\ln(x)-\alpha)}{\beta_i}} & \text{if } \ln(x) < \alpha \\ 1 - \frac{1}{2} e^{\frac{-(\ln(x)-\alpha)}{\beta_i}} & \text{if } \ln(x) \geq \alpha, \end{cases}$$

$$f_X(x) = \begin{cases} \frac{1}{2 \times \beta_i \times x} e^{\frac{(\ln(x)-\alpha)}{\beta_i}} & \text{if } \ln(x) < \alpha \\ \frac{1}{2 \times \beta_i \times x} e^{\frac{-(\ln(x)-\alpha)}{\beta_i}} & \text{if } \ln(x) \geq \alpha, \end{cases}$$

with quantile function

$$F_X^{-1}(p) = \begin{cases} (2p)^{\beta_i} \times e^\alpha & \text{if } p < \frac{1}{2} \\ \frac{e^\alpha}{(2-2p)^{\beta_i}} & \text{if } p \geq \frac{1}{2}. \end{cases} \tag{1}$$

The quantile function of random variable X will be used in the chance constraints in the proposed mathematical model presented in Section 2.3, as well as in the conic reformulation of the model in Section 3.

2.1.2. *Service level*

In this study, we superimpose the aircraft routing network with the passenger connection network to achieve the desired service levels. For flights $i \in J$ and $j \in P_i$, TP_{ij} equals the time needed by passengers to connect between flights where the decision variable γ_{ij} represents the percentage of passenger connections satisfied between i and j. The γ_{ij}s are calculated using chance constraints for the above-described random variable such that the probability of the time between arrival of flight i and departure of flight j being greater than the required connection time TP_{ij} is at least γ_{ij}. The weighted average of these γ_{ij} values using weights PAS_{ij} needs to be greater than or equal to γ, the overall service level of the schedule. PAS_{ij} values are assigned to flight connections in a manner that they represent the relative share of a given connection among all other passenger connections based on the number of passengers connecting. These values are normalized over the whole flight network and are used as weights when calculating the schedule service level.

Calculating the service level of the schedule using a weighted average of individual passenger connection service levels provides more accurate information on actual service levels, since the value of each connection may be different to the airline company. In this study, we weigh the connections based on the number of passengers connecting, but a different weighting scheme such as percentage of higher class customers within all connecting passengers could be used as well. It is also possible to add lower-bound constraints to each connection in the following manner to ensure minimum connection service levels for each flight:

$$\gamma_{ij} \geq \gamma_{ij}{}^{\mathrm{d}} \qquad i \in J, j \in P_i,$$

where $\gamma_{ij}{}^{\mathrm{d}}$ represents the minimum desired connection service level.

2.1.3. *Fuel cost function*

The fuel cost function for the cruise stage of flight $i \in J$ is given as

$$K_{t_i}(f_i) = \frac{C_{t_i} \times cf \times (f_i^{\mathrm{u}})^{m_i}}{f_i^{m_i-1}}. \tag{2}$$

The fuel burn rate of the aircraft in tons per minute is multiplied with the cost per ton of fuel to get how much fuel an aircraft burns in monetary terms in 1 minute. This resulting cost term is used in the nonlinear cost function by multiplying it with the term $(f_i^{\mathrm{u}})^{m_i}/f_i^{m_i-1}$, where f_i^{u} stands for the original planned cruise time of flight i, f_i is the associated decision variable for the new cruise time of flight i, and m_i is the flight-specific cost exponent. In accordance with the airline manufacturers' technical specifications as reported in Airbus (1998) and Boeing (2007), we present a convex and increasing function to express the change in fuel cost as the speed increases. Marla *et al*. (2011) also used a nonlinear fuel cost function in an airline schedule recovery problem to handle a cruise speed control strategy, although they approximated the nonlinear fuel cost function by a set of discrete settings in their approach.

There is a trade-off between fuel costs and idle time costs. Note that it can be cheaper to speed up the aircraft and then insert an idle time, if necessary, to compensate for the variability, since fuel costs are defined by nonlinear functions. Cruise time controllability is a practical alternative to idle time insertion. In this study, we show that we can solve the resulting nonlinear models by using second-order conic programming.

A numerical example is provided in the next subsection, before giving the mathematical model, so that the model mechanics are easily understood.

2.2. *Numerical example*

The schedule used in this numerical example is a small sample that consists of the daily plans of two aircraft. The sample is taken from BTS (2010a) and is given in Table 1. Tail numbers of the aircraft and the assigned flights to these aircraft are given in the first and second columns, respectively. The next two columns provide origin and destination information for flights, the following three columns list planned and announced departure times, flight durations, and arrival times. In the next column, actual departure time information from the BTS database is listed, and finally turnaround times are given in the last column. Note that there is a through flight 336 for the first aircraft, which can be defined as a single flight with one or more intermediate stops and allows passenger connections in these intermediate destination points.

Due to delays, actual departure times are different than planned departure times. There can be various reasons for the delays. First of all, because of variability, the actual duration of the block times can be different than the planned durations. In some cases, delayed arrival of a flight may cause a departure delay for the succeeding flight. If the time between the planned arrival time of a flight and planned departure time of its successor is longer than needed, then there is unnecessary idle time for the aircraft. It is also important to note that delays propagate through the network. For example, if there is insufficient time between two consecutive flights, then even a short delay in a flight will affect the next flight of the same aircraft.

The resulting flight network for the sample schedule can be seen in Fig. 1. Continuous lines for flights show the actual arrival and departure times of flights, and the dashed lines show the planned arrival and departure times. Continuous ground lines correspond to turn times of aircraft, and the dashed ground lines represent unnecessary waiting. We have 5 minutes of idle time after flight 2303 and

Table 1. Published schedule

Tail no.	*Flight no.*	*From*	*To*	*Departure time*	*Duration*	*Arrival time*	*Actual Departure*	*TA time*
N531AA	2303	ORD	LGA	7:35	2:05	9:40	7:35	0:39
	2336	LGA	ORD	10:30	2:15	12:45	10:30	0:41
	1053	ORD	DFW	13:15	3:00	16:15	13:33	0:40
	336	DFW	ORD	16:50	3:00	19:50	17:20	0:21
	336	ORD	LGA	20:20	2:05	22:25	20:49	
N4WPAA	2311	ORD	DFW	7:45	2:25	10:10	7:45	0:37
	2348	DFW	ORD	11:30	2:25	12:55	11:30	0:38
	1797	ORD	LGA	14:00	2:20	16:20	14:41	0:36
	1982	LGA	ORD	17:20	2:00	19:20	17:44	0:38
	1339	ORD	SAN	20:20	4:30	0:50	20:29	

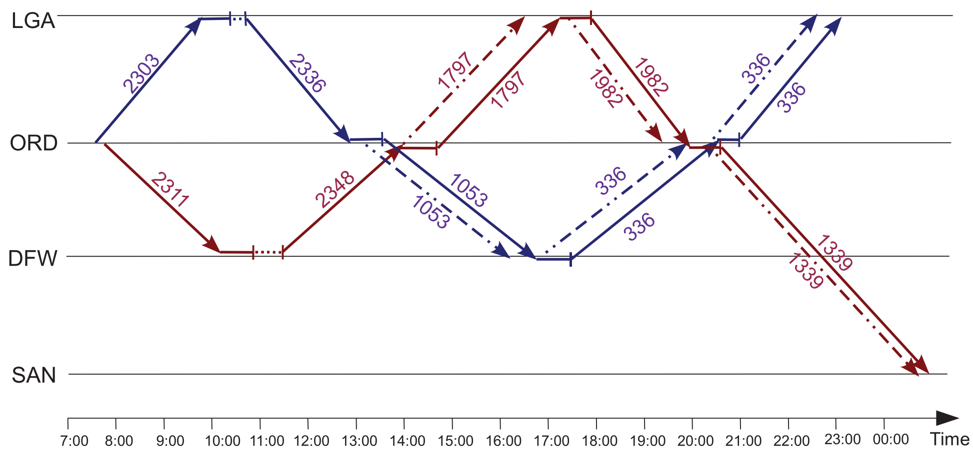

Fig. 1. Network graph for the published schedule.

35 minutes of idle time after flight 2311. Thus, we have unnecessary waiting times for some flights and we have some other flights with delays. These delays can also cause connecting passengers to miss their flights since a certain length of time is required for passengers to connect to their next flight. Passenger connected flight pairs in this schedule are 2336-1053, 336-336, 336-1339, 2348-1797, and 1982-1339. We explain the calculation of average non-cruise times in Section 2.1.1. In this example, we use $\alpha = \ln 20$, $\beta = 0.05$, and the airport congestion coefficients that will be given in Table 4.

Since schedule delays are costly, a better distribution of slack time can reduce idle time costs and avoid flight delays as reported in Ahmadbeygi *et al.* (2010). The departure times for a perturbed schedule with a different distribution of idle times is drawn in Fig. 2, where delay is completely avoided, and the overall passenger service level is not less than of the original schedule. It can be seen that in the new schedule, two idle time slots are inserted after the first leg of flight 336 and flight 2348, and there is no delay in the schedule.

The adjusted departure times for this schedule can be observed in Table 2. In this schedule, idle times are 48 minutes after flight 2348 and 10 minutes after first connecting flight of 336. Note that total idle time is increased but delay costs are totally avoided in this case. If we compare the costs of two schedules without taking delay costs into consideration, the total costs increased by around 5%. However, when delay costs are considered, there is a total cost saving of 32% when the second schedule is used.

In the schedule given in Fig. 2, flight times are taken as fixed parameters. As stated earlier, costs can be improved further without decreasing the service levels by utilizing cruise time controllability. In exchange for extra fuel burn, an aircraft can fly a route faster. For the same service level, we can trade-off extra fuel cost and idle time costs to minimize the total cost. We give an alternative schedule with adjusted flight times and idle times in Fig. 3. The adjusted departure times for this schedule can be observed in Table 2 with the comparison to the case where cruise time control is not allowed. Flights 2303, 2336, 1053, and the first connecting part of flight 336 have decreased cruise times. The

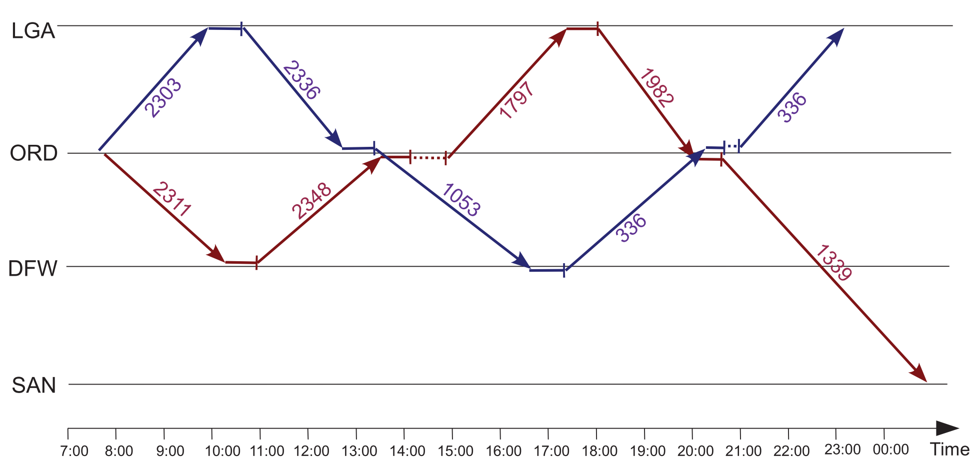

Fig. 2. Network graph with adjusted departure times.

Table 2. Adjusted departure times

	No cruise control allowed			*Cruise control allowed*		
Flight no.	*Departure time*	*Speeding*	*Idle time*	*Departure time*	*Speeding*	*Idle time*
2303	7:35	0:00	0:00	7:35	0:10	0:00
2336	10:26	0:00	0:00	10:16	0:11	0:00
1053	13:28	0:00	0:00	13:08	0:15	0:00
336	17:16	0:00	0:10	16:41	0:15	0:10
336	20:55	0:00	0:00	20:05	0:00	0:00
2311	7:45	0:00	0:00	7:45	0:00	0:00
2348	10:55	0:00	0:48	10:55	0:00	0:00
1797	14:55	0:00	0:00	14:06	0:00	0:00
1982	17:58	0:00	0:00	17:09	0:00	0:00
1339	20:44	0:00	0:00	19:54	0:00	0:00

original durations of the flights are drawn in dashed lines. In this case, the idle time after flight 2348 is not needed anymore since passenger service levels are ensured with speed control. This new schedule has 8% more fuel cost than the initial schedule, but idle time costs have decreased by 74% and there is a 35% improvement in total cost.

The model given in the next section works with these mechanics. The objective is to minimize the total cost including idle time, fuel, and delay costs. By using cruise time control, significant cost savings can be achieved by reducing idle time and delay costs in exchange for an increase in fuel costs.

2.3. *Mathematical model*

Balancing cruise time reduction and idle time insertion is a complex problem and decisions should be made for the whole network due to delay propagations. Therefore, we propose a global optimization tool that can generate flight schedules that are likely to minimize passenger misconnections through a set of chance constraints while minimizing the operational costs (e.g., the sum of idle time and fuel costs) that will be incurred while executing the airline schedule as planned:

$$\min \quad \sum_{i\in J} s_i \times I_{t_i} + \frac{C_{t_i} \times cf \times (f_i^{\mathrm{u}})^m}{f_i^{m-1}} \tag{3}$$

$$\text{s.t.} \quad \Pr[A_i + f_i \le x_j - x_i - TP_{ij}] \ge \gamma_{ij}, i \in J, j \in P_i, \tag{4}$$

$$\sum_{i\in J}\sum_{j\in P_i} PAS_{ij} \times \gamma_{ij} \ge \gamma, \tag{5}$$

$$lb_i \le x_i \le ub_i, \quad i \in J, \tag{6}$$

$$x_j - x_i - TA_{ij} - f_i - E[A_i] - s_i = 0, \ (i, j) \in PAIR, \tag{7}$$

$$f_i^{\mathrm{l}} \le f_i \le f_i^{\mathrm{u}}, \quad i \in J, \tag{8}$$

$$s_i \ge 0, \quad i \in J, \tag{9}$$

$$\gamma_{ij}{}^{\mathrm{d}} \le \gamma_{ij} \le 1, \ i \in J, j \in P_i. \tag{10}$$

The objective function (3) minimizes the sum of idle time and fuel costs. We calculate fuel costs as explained in Section 2.1.3. We utilize the idea of service levels through a set of chance constraints in Equation (4) such that the probability of the time between arrival of flight i and departure of flight j being greater than the required connection time

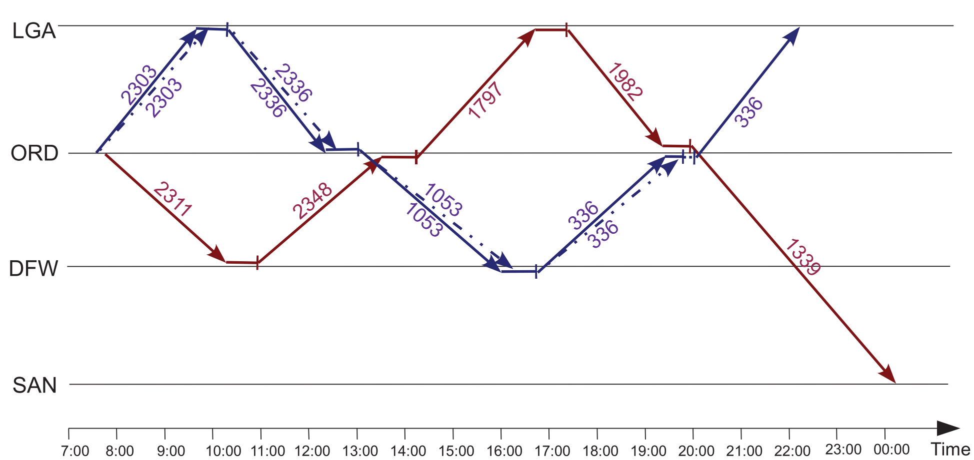

Fig. 3. Network graph with adjusted departure times and speed control.

TP_{ij} is at least γ_{ij} for every passenger connection. In constraint (5), we require the weighted sum of γ_{ij}s to be greater than the desired service level γ. In other words, passenger connections are incorporated using a chance constraint based on the block time variability. Note that we allow the passengers that continue their itineraries on the same aircraft to experience some inconvenience due to aircraft delays (i.e., their service levels do not have to be 100%). In constraint (6), we give the time window constraints for all flights. In constraint (7), we guarantee that the minimum aircraft connection time is available between two consecutive flights of the same aircraft, using the mean value of the random variable, since a flight cannot depart until the requisite aircraft has arrived. In other words, we maintain the aircraft routing network for each flight path. The expected value of a log Laplace variable is derived in Proposition A5 given in Appendix A. In constraint (8), we give the allowed boundaries for aircraft's cruise speed.

In summary, we take all passenger connections into account and superimpose the passenger itinerary network with the aircraft routing network in constraints (4), (6), and (7). As a result, we can satisfy the given minimum passenger connection service levels by either retiming the departures of flights within a given time window or by adjusting cruise speeds of incoming flights, or both, while minimizing their overall cost impact on the whole network.

The solution of the model is a challenge. There is nonlinearity in the objective function, and there are probabilistic constraints in the model. In the previous literature, chance constraints are usually handled with approximations, but for non-cruise flight times showing log Laplace distribution, we show that chance constraints can be handled in their exact form. The methodology is to first transform these probabilistic constraints into their closed-form expressions. Later, we will represent them and the nonlinear cost terms using second-order conic inequalities. The next section explains this methodology in detail.

3. Conic reformulation of the model

Using a conic reformulation allows us to solve for the chance constraints exactly to optima, as opposed to using approximations. To achieve the conic reformulation, the nonlinear expressions in the model are rewritten as second-order cone programming constraints.

3.1. *Conic representation of chance constraints*

In the mathematical model constraint (4) formulates chance constraints for passenger connections. Property 1 states that constraint (4) can be expressed using the quantile function of the probability distribution of random variable A_i.

Property 1. For $i \in J$, $j \in P_i$,

$$\Pr[A_i \le x_j - x_i - TP_{ij} - f_i] \ge \gamma_{ij}$$

is equivalent to

$$F^{-1}(\gamma_{ij}) \le x_j - x_i - TP_{ij} - f_i.$$

Random variable A_i has log Laplace distribution as previously discussed. The quantile function for a log Laplace random variable X with parameters α and β_i is given as

$$F_X^{-1}(p) = \begin{cases} (2p)^{\beta_i} \times e^{\alpha}, & \text{if } 0 \le p < \frac{1}{2} \\ \dfrac{e^{\alpha}}{(2-2p)^{\beta_i}}, & \text{if } \frac{1}{2} \le p \le 1. \end{cases}$$

$F_X^{-1}(p)$ is a piecewise function. Then, constraint (4) can be expressed as

$$(2\gamma_{ij})^{\beta_i} \times e^{\alpha} \le x_j - x_i - TP_{ij} - f_i, \quad \text{if } 0 \le \gamma_{ij} < \frac{1}{2}, \quad i \in J, j \in P_i, \tag{11}$$

$$\frac{e^{\alpha}}{(2-2\gamma_{ij})^{\beta_i}} \le x_j - x_i - TP_{ij} - f_i, \quad \text{if } \frac{1}{2} \le \gamma_{ij} \le 1, \quad i \in J, j \in P_i. \tag{12}$$

In Proposition A5 in Appendix A we show that the mean of a log Laplace random variable A_i is finite only if $\beta_i < 1$. Thus, in the rest of the article we will assume that $\beta_i < 1$ for all i. For the case $\beta_i \ge 1$, the conic reformulations of constraints (11) and (12) are discussed in Appendix A.

We will also restrict our analysis to the case where $\gamma_{ij} \ge 1/2$ for each passenger connection. This is a reasonable assumption, since offering a service level below 50% to its passengers on any given flight would not be desirable for an airline. Thus, we can drop constraint (11).

We will first prove the convexity of the expression on the left-hand side of inequality (12) in Proposition 1. Then, we will give the second-order conic representation of Equation (12) in Proposition 2.

Proposition 1. *For $i \in J$, $j \in P_i$,*

$$f(\gamma_{ij}) = \frac{e^{\alpha}}{(2-2\gamma_{ij})^{\beta_i}}$$

is a convex function when $0 \le \gamma_{ij} \le 1$ and if function parameter $\beta_i \in [0, 1]$.

Proof. The second derivative of $f(\gamma_{ij})$ is

$$f''(\gamma_{ij}) = \frac{e^{\alpha}\beta_i(\beta_i+1)}{2^{\beta_i}(1-\gamma_{ij})^{\beta_i+2}}.$$

$f''(\gamma_{ij}) \ge 0$ when $0 \le \gamma_{ij} \le 1$ and $0 \le \beta_i \le 1$. ∎

Proposition 2. *For $i \in J$, $j \in P_i$, if $0 < \beta_i < 1$ and the condition $^1/_2 \le \gamma_{ij} \le 1$ holds then constraint (12),*

$$\frac{e^{\alpha}}{(2-2\gamma_{ij})^{\beta_i}} \le x_j - x_i - TP_{ij} - f_i,$$

can be represented via second-order conic inequalities.

Proof. First, introduce two auxiliary variables $w_{ij} \geq 0$ and $v_{ij} \geq 0$ and make the following conversions:

$$w_{ij} = 2 - 2\gamma_{ij}, \quad (13)$$
$$v_{ij} = x_j - x_i - TP_{ij} - f_i. \quad (14)$$

Then, express

$$\beta_i = \frac{a_i}{b_i},$$

where a_i, b_i are positive integers. Then, inequality (12) becomes

$$e^{\alpha} \leq w_{ij}^{a_i/b_i} v_{ij},$$

which can be equivalently expressed as

$$e^{\alpha b_i} \leq w_{ij}^{a_i} v_{ij}^{b_i}.$$

Next, we choose l_i such that

$$l_i = \lceil \log_2(a_i + b_i) \rceil$$

and define a new auxiliary variable $y_{ij} \geq 0$ and add equation:

$$y_{ij} = e^{\frac{\alpha b_i}{2^{l_i}}} \quad (15)$$

to the model.

Now, constraint (12) can be written as

$$y_{ij}^{2^{l_i}} \leq w_{ij}^{a_i} v_{ij}^{b_i}. \quad (16)$$

An inequality of the form

$$r^{2^l} \leq s_1 s_2 \cdots s_{2^l} \quad (17)$$

with restrictions $s_i \geq 0$ for $i = 1, \ldots, 2^l$ defines the hypograph of the geometric mean of 2^l variables, $s_1, s_2, \ldots, s_{2^l}$. This hypograph is a convex set. It is easy to observe that inequality (16) with restrictions $w_{ij} \geq 0$ and $v_{ij} \geq 0$ also defines a hypograph of the geometric mean of 2^l variables. a_i of those variables equal to w_{ij}, b_i of them equal to v_{ij} and remaining $2^{l_i} - a_i - b_i$ equal to one.

Due to Ben-Tal and Nemirovski (2001), a hypograph of the geometric mean of 2^l variables is known to be second-order conic programming representable. The hypograph can be equivalently represented by $O(2^l)$ variables and $O(2^l)$ hyperbolic inequalities of the form

$$u^2 \leq v_1 v_2, \quad u, v_1, v_2 \geq 0.$$

which can be represented by the following second-order conic inequality:

$$\|(2u, v_1 - v_2)\| \leq v_1 + v_2,$$

which concludes the proof. ■

Example 1: Suppose that $\alpha = 1$, $\beta_i = 2.5$ and thus $a_i = 5$ and $b_i = 2$. Then, following the steps of the proof of Proposition 2 inequality (12) for flight pair i, j:

$$\frac{e}{(2 - 2\gamma_{ij})^{2.5}} \leq x_j - x_i - TP_{ij} - f_i$$

is first represented by the system

$$\begin{aligned} e^2 &\leq w^5 v^2, \\ w &= x_j - x_i - TP_{ij} - f_i, \\ v &= 2 - 2\gamma_{ij}, \\ w &\geq 0, \\ v &\geq 0. \end{aligned}$$

Then, adding auxiliary variable y and constraint

$$y = e^{1/4},$$

the first inequality becomes

$$y^8 \leq w^5 v^2,$$

which can be expressed by the following three inequalities and two new nonnegative auxiliary variables $u_1, u_2 \geq 0$:

$$\begin{aligned} u_1^2 &\leq w \times 1, \\ u_2^2 &\leq u_1 \times v, \\ y^2 &\leq u_2 \times w. \end{aligned}$$

Figure 4 shows the generation of the inequalities.

These constraints can be represented by the following conic quadratic inequalities:

$$\begin{aligned} 4u_1^2 + (w-1)^2 &\leq (w+1)^2, \\ 4u_2^2 + (u_1 - v)^2 &\leq (u_1 + v)^2, \\ 4y^2 + (u_2 - w)^2 &\leq (u_2 + w)^2, \end{aligned}$$

which can easily be input to a conic optimization software such as IBM ILOG CPLEX.

3.2. *Conic representation of the fuel cost function*

The fuel cost function associated with the speeding of an aircraft is a nonlinear function and it can be expressed via second-order conic constraints.

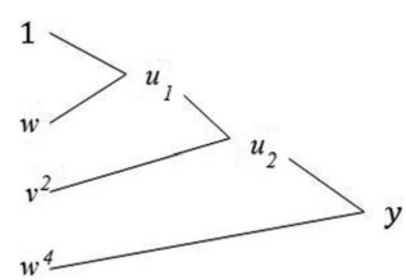

Fig. 4. Illustration of generation of conic quadratic constraints.

Proposition 3. *For $i \in J$, the fuel cost function*

$$K_{t_i}(f_i) = \frac{C_{t_i} \times cf \times (f_i^{\mathrm{u}})^{m_i}}{f_i^{m_i-1}}$$

is second-order cone programming representable.

Proof. $K_{t_i}(f_i)$ appears in the objective function and can be replaced with an auxiliary variable $q_i \geq 0$ and sent to the constraint set. The objective function is now linear and is written as

$$\min \sum_{i \in J} s_i \cdot I_{t_i} + q_i.$$

Then, we add the following constraints to the model for each $i \in J$:

$$\frac{C_{t_i} \times cf \times (f_i^{\mathrm{u}})^{m_i}}{f_i^{m_i-1}} \leq q_i.$$

Define $n_i = \lceil \log_2 m_i \rceil$ and introduce variable $z_i = (C_{t_i} \times cf \times (f_i^{\mathrm{u}})^{m_i})^{1/2^{n_i}}$ to rewrite the inequality as

$$z_i^{2^{n_i}} \leq q_i f_i^{m_i-1},$$

which is second-order come programming representable as discussed in the proof of Proposition 2. ■

3.3. *Reformulated model*

After the above-described changes, the model becomes

$$\min \quad \sum_{i \in J} s_i \times I_{t_i} + q_i, \tag{18}$$

$$\text{s.t.} \quad y_{ij}^{2^{l_i}} \leq w_{ij}^{a_i} v_{ij}^{b_i}, i \in J, j \in P_i, \tag{19}$$

$$w_{ij} = 2 - 2\gamma_{ij}, i \in J, j \in P_i, \tag{20}$$

$$v_{ij} = x_j - x_i - TP_{ij} - f_i, i \in J, j \in P_i, \tag{21}$$

$$y_{ij} = e^{\frac{b_i \alpha}{2^{l_i}}}, i \in J, j \in P_i, \tag{22}$$

$$\sum_{i \in J} \sum_{j \in P_i} PAS_{ij} \times \gamma_{ij} \geq \gamma, \tag{23}$$

$$z_i^{2^{n_i}} \leq q_i f_i^{m_i-1}, i \in J, \tag{24}$$

$$z_i = (C_{t_i} \times cf \times (f_i^{\mathrm{u}})^{m_i})^{1/2^{n_i}}, i \in J, \tag{25}$$

$$x_j - x_i - TA_{ij} - f_i - E[A_i] - s_i = 0, (i, j) \in PAIR, \tag{26}$$

$$lb_i \leq x_i \leq ub_i, i \in J, \tag{27}$$

$$f_i^l \leq f_i \leq f_i^{\mathrm{u}}, i \in J, \tag{28}$$

$$0.5 \leq \gamma_{ij} \leq 1, i \in J, j \in P_i, \tag{29}$$

$$s_i \geq 0, i \in J, \tag{30}$$

$$q_i, z_i \geq 0, i \in J, \tag{31}$$

$$w_{ij}, v_{ij}, y_{ij} \geq 0, i \in J, j \in P_i. \tag{32}$$

In this new formulation, constraints (19) to (22) are due to the reformulation discussed in Proposition 2. Due to Proposition 3 we now have q_i variables in the objective and constraints (24) and (25) in the model. We have all auxiliary variables required by reformulations. Constraints (19) and (24) can be represented via second-order conic inequalities as discussed in Propositions 2 and 3. Note that this resulting model is solvable via commercial solvers in reasonable computation time and can easily be used by airline practitioners. We will demonstrate the performance of the model through an extensive computational study in the next section.

4. Computational study

In this section, we test the computational performance of the conic reformulation of the problem. We analyze the CPU time performance of the model. We compare the schedule generated by our model against the published schedule by using various performance criteria. The idea is to evaluate the service level of a given schedule and then to solve the model to get a robust schedule for the same flight set and for the given service level. In order to evaluate possible impacts of different problem parameters, we performed a 2^k full-factorial experimental design. The four experimental factors and their levels are given in Table 3. For each factor combination we took five replications.

Fuel cost represents the unit price for jet fuel ($/ton). We use two levels of fuel price $600 and $1200 per ton.

Compression level represents the maximum allowable compression in percentage over planned cruise time for a flight. In the low setting, an aircraft is allowed to speed up to shorten the cruise time by 10%, whereas in the higher setting this value is 15%. For example, in the low setting, a flight with a cruise time duration of 120 minutes is allowed to be expedited by a maximum of 12 minutes.

β is a parameter of the log Laplace distribution as described in Section 2. We use β to adjust the mean and variance of the non-cruise flight time random variable. We adjusted this parameter for each flight using airport congestion coefficients. We used a fixed α value of ln 20 in our experiments.

Finally, connection density represents the percentage of the possible passenger connections between flights. A passenger connection is possible between flight i and a consecutive flight j only if destination of j is a different airport than the origin of i and the scheduled departure time of j

Table 3. Factor values

		Levels	
Factor	*Description*	*Low (0)*	*High (1)*
A	Fuel cost ($/ton)	600	1200
B	Compression level (%)	10	15
C	β	0.01	0.05
D	Connection density (%)	50	100

Table 4. Congestion coefficients

Airport	*Location*	*Coefficient*	*Airport*	*Location*	*Coefficient*
MIA	Miami, FL	1.96	DCA	Washington, DC	1.17
ORD	Chicago, IL	1.88	SAN	San Diego, CA	1.10
LAX	Los Angeles, CA	1.82	STL	St. Louis, MO	1.10
DEN	Denver, CO	1.82	MCI	Kansas City, MO	1.04
DFW	Dallas, TX	1.74	AUS	Austin, TX	1.00
LGA	New York, NY	1.69	RDU	Raleigh/Durham, NC	1.00
BOS	Boston, MA	1.69	MSY	New Orleans, LA	0.96
ATL	Atlanta, GA	1.63	SNA	Santa Ana, CA	0.96
PHX	Phoenix, AZ	1.56	SAT	San Antonio, TX	0.90
LAS	Las Vegas, NV	1.56	RSW	Fort Myers, FL	0.90
SFO	San Francisco, CA	1.44	SJU	San Juan, PR	0.84
MSP	Minneapolis, MN	1.32	PBI	West Palm Beach, FL	0.81
PHL	Philadelphia, PA	1.32	TUS	Tuscan, AZ	0.77
EWR	Newark, NJ	1.25	MCO	Orlando, FL	0.72
FLL	Fort Lauderdale, FL	1.25	EGE	Eagle, CO	0.72
SLC	Salt Lake City, UT	1.17	HDN	Hayden, CO	0.64

is within the range of 30 to 180 minutes of the scheduled arrival time of i. When connection density is set to 100%, all possible passenger connections are realized. When it is low (50%), there exists a passenger connection between two flights with a 50% probability.

Flight-specific β_i values were calculated using the congestion coefficients e_{O_i} and e_{D_i} that were introduced earlier in Section 2. The functional form used in the calculation was

$$\beta_i = \beta \times (e_{O_i})^2 \times (e_{D_i})^2.$$

The functional form employed for this calculation had no effect on the complexity of the model or the computational times; therefore, the planner is free to use the function he sees fit. In Table 4, we give the airport congestion coefficients used in this study. These coefficients take a value between 0.6 and 2, the latter representing the most congested airport. These values were decided based on the number of passengers visiting the airports from T-100 market data BTS (2010b). Specifically, the volume of passengers the airports see were scaled and rounded to be between 0.6 and 2. These upper and lower bound values were determined so that the finiteness of the means of the log Laplace random variables were ensured, the conditions for which are provided in Proposition A5. The values for these coefficients and the form of the function used can be decided jointly by the airline depending on the congestions they observe.

The flight-specific cost component, m_i, was assumed to be the same and equal to three for each flight $i \in J$. The original cruise time duration f_i^{u} was calculated as 20 minutes less than the published block time for each flight $i \in J$.

Passenger connection times were randomly generated from a uniform distribution between 25 and 40 minutes. The number of passengers in a flight was randomly generated from a uniform distribution with a lower bound of 60% and an upper bound of 100% of full seat capacity. There were six different types of aircraft used in our experiments, each with different costs and seat capacities. The values for these parameters are provided in Table 5.

In our experiments, we estimated the turnaround time required for a given aircraft at a given airport by multiplying the base aircraft turnaround time with the square root of the congestion coefficient of the airport. The resulting turnaround times are compatible with the findings of Arıkan *et al.* (2013). For example, for two selected airports MIA and HDN, the turnaround times for different types of aircraft are as given in Table 6. It can be observed that these values are close to the calculated averages in Arıkan *et al.* (2013). Moreover, if the flight is a connecting flight, the turnaround time is taken to be 70% of the calculated

Table 5. Aircraft parameters

Aircraft Type	*1*	*2*	*3*	*4*	*5*	*6*
Idle time cost ($/min)	140	142	136	144	147	150
Fuel burn rate (tons/min)	0.12	0.108	0.064	0.065	0.058	0.083
Base turn time (min)	36	26	40	28	30	32
Number of seats	261	262	158	159	131	190

Table 6. Turnaround times for selected airports

	Turn around time (min)	
Type	*MIA*	*HDN*
1	50.4	28.8
2	36.4	20.8
3	56	32
4	39.2	22.4
5	42	24
6	44.8	25.6

value, since the number of passengers and cargo loading and unloading times are significantly less in case of a connection. Again, airlines can set true values for the necessary turnaround times in their operations.

In order to be consistent with the published schedules, the departure time of the first flight for each aircraft was set to the published value in the original schedule. In this computational study, we did not impose time window constraints on other flights, since they could lead to infeasibilities in the published schedule when there is a high variability in block times or when we are trying to attain higher service levels on passenger connections. Consequently, we can evaluate the impact of delay propagation on service levels and operational costs for all problem instances.

4.1. *Results for single hub data*

We first tested our model on a problem instance used in a recent work by Aktürk *et al*. (2014). The instance is called Single Hub Data as the flight network is formed by considering aircraft that originate their first flight from Chicago O'Hare International Airport (ORD) and revisit ORD at least once during the same day. The schedule was retrieved from the BTS database. The schedule is given in Table 7. It includes 114 flights operated by 31 different aircraft. We added the randomly generated passenger connection network to assess the impact of the proposed robustness on the given flight schedule.

In Table 8, a comparison between different cost components of the optimum solution of the proposed model and the published schedule are given. The costs are calculated by simply using the unit idle time, fuel and delay costs, and the respective total idle time, total delay, and speed reduction of the schedule. The values in the table correspond to reduction in idle time costs, increase in fuel costs, reduction in total costs, and reduction in total costs without including delay costs. The motivation for considering total costs in two cases is that the unit delay cost can be difficult to accurately measure, but it is evident that our model performs better cost-wise even when the unit delay costs are assumed to be zero. The improvement percentages were calculated using the following formula:

$$\textit{Improvement} = 100 \times \frac{\textit{Published schedule costs} - \textit{Proposed model costs}}{\textit{Published schedule costs}},$$

where the percentage of cost increases are calculated using the negative of the same formula.

Before analyzing the results, it is important to mention that fuel costs make up approximately 90% of the total costs for the proposed model results and they make up approximately 75% of the total cost for the published schedule.

Costs for the published schedule were also calculated from scratch by summing up the fuel, idle time, and delay costs. The fuel costs for the published schedule were calculated using the fuel cost function for the model where the published cruise time f_i^{u} was submitted in place of both f_i and f_i^{u}. The idle time cost for each flight was calculated by multiplying the unit idle time cost of the assigned aircraft type with the scheduled idle time for that flight. Similarly, delay costs were calculated by multiplying the minutes of delay of each flight by a unit delay cost per minute. This unit delay cost was taken to be constant at \$200 merely for comparison since it is shown that the model proves improvements even in the case of a \$0 delay cost.

Factor A—i.e., fuel price—has significant effects on the total cost and total fuel cost improvements as expected. The result is that our model achieves better total cost savings when the fuel price is low, and the performance of the model in improving idle time costs is slightly affected by fuel price. The reason for this behavior is that since the idle time cost contribution to the total cost is lower than that of fuel cost, the increase in unit fuel price results in slightly lower idle time cost improvements overall.

Factor B represents the maximum allowed compression level on the cruise time of the flight. In solutions to our model, we observed that compression in flight times did not hit this boundary even in the low setting. Therefore, the change of this compression level does not have a statistically significant affect on the model performance.

Factor C—i.e., the log Laplace random variable base parameter β—shows another interesting result. It is observed that an increase in fuel costs does not change for higher levels of β whereas all other cost improvements are decreased. The reason for this behavior is that a higher β causes a higher variance in block times, which necessitates more idle time insertion into the final schedule and therefore more idle time costs.

Factor D—i.e., the connection density of the network—has a similar effect as Factor C. More passenger connections result in a higher need for idle times, and therefore total cost improvements decrease, since idle time cost savings is a strong advantage of our model.

Overall, it is important to observe that a 2% increase in fuel costs allows for a 60% improvement in total idle time costs. This is because fuel cost is a huge part of the total airline operational costs, and cruise time controllability results in great savings from unnecessary idle times.

As previously mentioned, five replications for each factor combination were perfomed to see whether random values had any effect on objective values. The comparisons for cost improvements for different replications are given in Table 9, which indicates that the randomization does not have a statistically significant effect on the overall results.

Another measure of interest is the service level of the schedules. Results show that the only significant factor affecting service level values is β. The overall average of the service level is 94%. A higher setting of β causes the average service level to drop to 92.7%, whereas a lower setting of β results in service levels of 95.3% on average. We see that the

Table 7. Single hub data

Tail no.	*Flight no.*	*Departure*	*Arrival*	*Departure time*	*Flight time*	*Arrival time*
N530AA	398	ORD	LGA	6:15	2:14	8:29
	319	LGA	ORD	9:25	2:50	12:15
	2329	ORD	DFW	13:35	2:35	16:10
	2364	DFW	ORD	17:00	2:30	19:30
N459AA	394	ORD	LGA	6:50	2:15	9:05
	321	LGA	ORD	10:00	2:50	12:50
	366	ORD	LGA	13:55	2:20	16:15
	347	LGA	ORD	17:15	2:50	20:05
N531AA	2303	ORD	DFW	6:45	2:35	9:20
	2336	DFW	ORD	10:10	2:20	12:30
	1053	ORD	AUS	13:25	2:50	16:15
	336	AUS	ORD	17:00	2:45	19:45
	336	ORD	LGA	20:40	2:05	22:45
N4XGAA	2079	ORD	SAN	8:45	4:30	13:15
	1438	SAN	ORD	14:00	4:10	18:10
	346	ORD	LGA	19:50	2:15	22:05
N598AA	1341	ORD	SFO	7:50	4:55	12:45
	348	SFO	ORD	13:30	4:25	17:55
	1521	ORD	TUS	19:15	3:55	23:10
N439AA	2455	ORD	PHX	7:10	4:00	11:10
	358	PHX	ORD	11:55	3:30	15:25
	358	ORD	LGA	16:25	2:25	18:50
	371	LGA	ORD	20:00	2:35	22:35
N475AA	407	ORD	STL	6:20	1:10	7:30
	755	STL	ORD	8:35	1:15	9:50
	755	ORD	SAT	10:45	3:00	13:45
	408	SAT	ORD	14:30	2:40	17:10
	408	ORD	PHL	18:05	2:05	20:10
N3EEAA	876	ORD	BOS	6:35	2:10	8:45
	413	BOS	ORD	9:35	3:05	12:40
	413	ORD	SNA	13:45	4:35	18:20
	1262	SNA	ORD	19:10	3:50	23:00
N4YDAA	451	ORD	SFO	9:45	4:55	14:40
	554	SFO	ORD	15:45	4:25	20:10
N3ERAA	496	ORD	DCA	6:45	1:40	8:25
	1715	DCA	ORD	9:15	2:10	11:25
	1715	ORD	LAS	12:25	4:05	16:30
	1708	LAS	ORD	17:20	3:40	21:00
N5CLAA	1425	ORD	SNA	8:25	4:40	13:05
	556	SNA	ORD	14:00	4:00	18:00
	1940	ORD	MIA	19:25	3:00	22:25
N535AA	2460	ORD	RSW	6:45	2:45	9:30
	564	RSW	ORD	10:20	3:05	13:25
	1446	ORD	EWR	14:55	2:45	17:40
	1411	EWR	ORD	18:45	2:45	21:30
N3DMAA	568	ORD	FLL	7:25	2:55	10:20
	711	FLL	ORD	11:10	3:15	14:25
	2021	ORD	SJU	15:25	4:35	20:00
N544AA	2463	ORD	MCI	6:25	1:30	7:55
	754	MCI	ORD	8:40	1:30	10:10
	2321	ORD	DFW	11:15	2:35	13:50
	2356	DFW	ORD	14:40	2:20	17:00
	2487	ORD	DEN	17:50	2:45	20:35
N3EBAA	1565	ORD	MSP	6:40	1:30	8:10
	779	MSP	ORD	9:00	1:25	10:25
	779	ORD	SAN	11:35	4:20	15:55
	1358	SAN	ORD	16:45	3:55	20:40
	1358	ORD	BOS	21:50	2:05	23:55
N3ETAA	1704	ORD	EWR	6:35	2:05	8:40
	1883	EWR	ORD	9:30	2:40	12:10
	810	ORD	DCA	13:10	1:45	14:55
	2013	DCA	ORD	15:45	2:15	18:00
	2013	ORD	LAS	19:00	4:10	23:10
N3DYAA	1063	ORD	LAX	8:50	4:35	13:25
	874	LAX	ORD	14:30	4:15	18:45
	874	ORD	BOS	19:45	2:15	22:00
N3DRAA	1021	ORD	LAS	8:30	4:05	12:35
	1544	LAS	ORD	13:25	3:35	17:00
	1544	ORD	DCA	18:00	1:45	19:45
N5DXAA	1048	ORD	MIA	7:35	3:10	10:45
	1763	MIA	ORD	11:55	3:20	15:15
	1899	ORD	MIA	16:20	3:05	19:25
N454AA	2441	ORD	ATL	6:30	2:00	8:30
	1986	ATL	ORD	9:15	2:15	11:30
	1872	ORD	MCO	12:25	2:40	15:05
	1131	MCO	ORD	15:50	3:05	18:55
N4YMAA	1137	ORD	MSY	8:20	2:25	10:45
	1768	MSY	ORD	11:30	2:30	14:00
	1768	ORD	PHL	15:05	2:05	17:10
	1697	PHL	ORD	18:00	2:35	20:35
N467AA	1823	ORD	PBI	9:20	2:55	12:15
	2067	PBI	ORD	13:00	3:20	16:20
	2067	ORD	STL	17:15	1:10	18:25
	1186	STL	ORD	19:10	1:20	20:30
N536AA	2305	ORD	DFW	7:45	2:40	10:25
	2344	DFW	ORD	11:35	2:20	13:55
	1201	ORD	STL	14:50	1:05	15:55
	1815	STL	ORD	17:00	1:20	18:20
	1815	ORD	SLC	19:15	3:40	22:55
N420AA	1686	ORD	RDU	6:50	1:50	8:40
	2435	RDU	ORD	9:25	2:15	11:40
	2435	ORD	PHX	12:35	3:55	16:30
	1206	PHX	ORD	17:15	3:25	20:40
N546AA	1462	ORD	EWR	8:00	2:20	10:20
	1387	EWR	ORD	11:25	2:40	14:05
	1397	ORD	MCO	15:00	2:40	17:40
	1221	MCO	ORD	18:25	2:55	21:20
N4WPAA	2311	ORD	DFW	9:05	2:35	11:40
	2348	DFW	ORD	12:35	2:20	14:55
	1797	ORD	STL	15:50	1:10	17:00
	1982	STL	ORD	18:00	1:20	19:20
	1339	ORD	SAN	20:15	4:30	24:45
N5EBAA	2375	ORD	EGE	8:10	2:55	11:05
	2378	EGE	ORD	12:25	2:45	15:10
	1677	ORD	SNA	18:40	4:30	23:10
N3DUAA	2099	ORD	LAX	7:00	4:30	11:30
	1972	LAX	ORD	12:40	4:05	16:45
	1972	ORD	RDU	17:45	1:55	19:40
N3ELAA	2057	ORD	SJU	8:30	4:50	13:20
	2078	SJU	ORD	14:25	5:35	20:00
N3DTAA	2363	ORD	HDN	9:50	2:50	12:40
	2318	HDN	ORD	13:40	2:50	16:30
N412AA	2345	ORD	DFW	17:15	2:35	19:50
	2374	DFW	ORD	20:40	2:10	22:50

Table 8. Comparison of factor effects

		Idle time cost improvement (%)			*Fuel cost increase (%)*			*Total cost improvement (%)*			*Total cost improvement (%) without delay costs*		
		Min.	*Avg.*	*Max.*	*Min.*	*Avg.*	*Max.*	*Min.*	*Avg.*	*Max.*	*Min.*	*Avg.*	*Max.*
A	0	56.8	66.4	76.2	2.2	2.8	3.7	14.1	17.6	21.1	13.8	17.4	21.1
	1	52.1	60.6	67.7	0.9	1.3	1.7	7.4	9.4	11.6	7.2	9.3	11.6
B	0	52.1	63.4	75.2	0.9	2.0	3.4	7.4	13.5	21.1	7.2	13.3	21.1
	1	52.1	63.7	76.2	0.9	2.1	3.7	7.4	13.5	21.1	7.2	13.4	21.1
C	0	57.8	65.7	76.2	0.9	2.1	3.7	9.4	14.8	21.1	9.4	14.7	21.1
	1	52.1	61.4	73.9	0.9	2.0	3.6	7.4	12.2	18.5	7.2	12.0	18.2
D	0	61.0	68.6	76.2	0.9	2.1	3.7	8.9	14.7	21.1	8.7	14.5	21.1
	1	52.1	58.5	64.0	1.3	2.0	3.1	7.4	12.3	17.7	7.2	12.2	17.6

published schedule actually has really good passenger connection service levels, and our model can achieve the same service levels with significantly lower operational costs. In fact, it will be shown later that our model achieves higher service levels if total cost is allowed to be as much as the published schedule costs.

4.1.1. *Scenario analysis*

The extensive computational study results raise several questions on model behavior and performance. In this part, some insights into model dynamics are provided by considering different scenarios.

What if time compression is not allowed? Since cruise time controllability is an important contribution of the proposed study, we would like to check the performance of the model when speeding is not allowed, and the only tool is schedule padding, as is commonly done in the current literature. Since replications do not affect results, costs were calculated for a single set of replications, with 15% compression allowed in one case and zero compression on the other case. The cost values were compared with the published schedule costs for both cases, the proposed model with time compression allowed and not allowed. The results in Table 10 show that even without cruise time controllability, a better utilization of idle times by the model results in important idle cost and total cost improvements, parallel to the findings in the literature. Moreover, the benefits of cruise time controllability can be observed from the improvement rates.

What if variability was higher? Our expectation is that benefits of the model will be less significant and service levels will be much lower when there is higher variability. For this analysis, Factor C was taken to be 0.07, which is the highest possible value so that each $\beta_i < 1$ for each flight $i \in J$ (Proposition A5). All other factors were taken as their low level values. Computation for a single parameter set resulted in a service level of 88%, which is quite low compared with average service levels that were achieved previously. Delay costs of the published schedule increased significantly since a higher variance caused the block times of flights to increase. Also in the same case, when the total cost of the model was taken to be equal to the original schedule total cost, the proposed model resulted in a service level of 98%.

4.1.2. *Aircraft utilization*

Computational results proved some additional benefits of combining schedule padding with speed controllability apart from the objective function values. The results showed that for the available 31 aircraft paths in the data, the model improved the makespan for almost all paths, and there was a time-wise improvement in the average makespan (or equivalently aircraft utilization) in all factor combinations. The average makespan improvement was taken for each different factor and replication combination. The mean of this improvement over all cases was

Table 9. Cost comparison for different replications

	Idle time cost reduction (%)			*Increase in fuel cost (%)*			*Total cost reduction (%)*		
Rep	*Min.*	*Avg.*	*Max.*	*Min.*	*Avg.*	*Max.*	*Min.*	*Avg.*	*Max.*
1	56.2	63.8	76.2	0.9	2.1	3.7	7.8	13.6	20.9
2	54.8	64.8	75.6	0.9	2.0	3.1	7.8	13.9	21.1
3	54.4	62.8	75.8	1.0	2.0	3.5	7.6	13.4	20.9
4	52.1	62.8	74.9	1.1	2.2	3.6	7.4	13.3	20.6
5	56.4	63.4	74.4	1.1	2.1	3.3	7.8	13.4	20.7

Table 10. Cost improvement (%) with and without flight time compression

A	C	D	*With compression* Fuel	*Idle time*	*Total*	*No compression* Fuel	*Idle time*	*Total*
0	0	0	3.2	74.4	20.7	—	58.9	18.1
0	0	1	2.7	63.2	17.6	—	44.0	13.6
0	1	0	3.3	68.6	16.9	—	53.7	14.8
0	1	1	2.3	58.8	14.9	—	42.3	11.8
1	0	0	1.1	66.9	11.3	—	58.9	10.7
1	0	1	1.5	59.2	9.6	—	44.0	8.0
1	1	0	1.4	61.3	8.9	—	52.5	8.4
1	1	1	1.7	56.4	7.8	—	39.2	6.5

41.5 minutes for 1 day of operation, with a minimum of 28 minutes and a maximum of 59 minutes achieved in one case. The average number of paths for which the makespan improved was 30.5 out of 31, with a minimum of 28 paths and a maximum of 31 paths. Reducing the idle time not only affected the costs but created additional utilization opportunities such as adding an additional flight to a given flight sequence. Note that makespan reduction is just mentioned as a secondary benefit of speed control and is not meaningful cost-wise without the utilization of this additional generated time.

4.2. *Results for four-hub data*

To generate this schedule, data were taken from the BTS database and filtered to include aircraft that originate their first flight from four different hub locations and return to them at least once during the same day. This way, we could consider a schedule for four-hub locations. The airports that were considered as hubs was Dallas–Fort Worth International Airport (DFW), Chicago O'Hare International Airport (ORD), Miami International Airport (MIA), and New York John F. Kennedy International Airport (JFK). This schedule had 469 flights operated by 141 different aircraft.

In Table 11, a comparison among different cost components between model objectives and published schedule is given. The comparison is only done for two factors, A and C, which are the fuel cost and β, respectively. Factor B—i.e. speed compression—is taken as 15% for all runs since we saw that the compression level does not affect the results as the compressions on flights did not hit the boundaries. Similarly, Factor D, the connection density, was taken as 50% throughout, since there are many possible connections in a four-hub problem and 100% passenger connection may not be realistic. Although we showed that replications did not affect results in the single-hub study, we still conducted three replications for each factor combination. We also did not calculate total cost improvement without delay costs separately in this case since it was shown earlier that the model has cost improvements even when delay costs are taken as zero.

It can be seen that results are very promising in the four-hub case as well. Idle time cost savings are approximately 60%, whereas fuel cost increase is approximately 2%. This is very similar to the results in the single-hub case. The fact that model performance is not affected by the size of the data is a good attribute. It shows that cost savings from idle time even out nicely throughout the schedule and the improvement rates are not negatively affected by an increasing data size.

The results show that the idle time cost improvement is decreased by increasing Factor A; i.e., unit fuel costs. This is because speeding becomes more expensive when unit fuel costs are higher, and the model depends more on idle time to achieve robustness. Also, less speeding means a smaller increase in total fuel costs, which can be observed in the results. The total cost improvement is also negatively affected by increasing the fuel costs. Around 80% of total costs are from fuel costs, so doubling the unit fuel cost resulted in the total cost improvements being almost half of the previous values.

Results for Factor C—i.e., the parameter β—are similar to the outcomes in the single-hub schedule as well. The fuel cost increase is not affected by increasing β, whereas idle time and total cost improvements are decreased. As stated previously, this is because an increased variability results in more idle time inserted in to the schedule. The decrease in idle time cost improvements also reflects to the total cost improvements.

Changing levels of β also affects the service level of the schedules. For the case of a low β, the average service level of the schedules is 96%, whereas it is 93% for the case a of high β setting. It is reasonable that higher variability results in lower service levels.

It is also interesting to observe how additional utilization from decreased makespan levels change for the four-hub schedule. Out of 141 flight paths, there were time savings on 135 of the paths on average. The average of this improvement over all cases is 33 minutes, with a minimum of 25 minutes and a maximum of 45 minutes.

Table 11. Comparison of factor effects

		Idle time cost improvement (%)			*Fuel cost increase (%)*			*Total cost improvement (%)*		
		Min.	*Avg.*	*Max.*	*Min.*	*Avg.*	*Max.*	*Min.*	*Avg.*	*Max.*
A	0	59.0	61.1	62.8	2.7	2.8	3.1	14.6	15.6	16.4
	1	52.2	54.6	56.4	1.1	1.2	1.3	7.7	8.2	8.8
C	0	55.9	59.3	62.8	1.2	2.0	3.1	8.6	12.5	16.4
	1	52.2	56.4	60.7	1.1	2.0	2.9	7.7	11.3	14.9

4.3. *Computation time analysis*

All computations were conducted on an Intel Core i5 2410M computer with a 2.30 Ghz processor and 4.00 GB RAM. The problem was modeled in Java language using IBM ILOG CPLEX Optimizer. The model was solved by CPLEX 12.1. In the following subsections, time analysis for the single hub and four-hub schedules are provided.

4.3.1. *Single hub study*

Computation times are very reasonable for all factor settings. Overall average, minimum, and maximum values of computation times in CPU seconds can be seen in Table 12. When we analyze the results we see that Factors A and B do not have a statistically significant effect on computation times. Factor A is the unit fuel cost and it is simply a coefficient term in the model, so changing it does not change the computation times. For Factor B, the maximum allowed compression does not achieve boundaries as stated earlier; thus, it does not affect the computation times. The results are different for Factors C and D, however. Increasing the variability and the number of passenger connections (or network density)—i.e., Factors C and D respectively—increased the problem complexity and overall computation times as expected.

Overall, the average time for all runs is 6.6 CPU seconds. This is a very good result for a problem of that size, having 31 paths and 114 flights. As can be seen, the second-order conic formulation of the chance constraints results in exact and fast solutions.

Table 12. CPU time analysis for the single-hub schedule

		CPU time (sec.)		
Factor	*Level*	*Min.*	*Avg.*	*Max.*
A	0	2.4	6.2	12.1
	1	2.3	6.1	12.1
B	0	2.3	6.2	12.1
	1	2.4	6.1	11.7
C	0	2.3	3.9	6.4
	1	4.7	8.4	12.1
D	0	2.3	4.0	6.9
	1	4.7	8.3	12.1

4.3.2. *Four-hub study*

Computational results prove to be very good time-wise for the four-hub case as well. The size of the problem was 141 paths and 469 flights. Overall average, minimum, and maximum values of computation times in CPU seconds can be seen in Table 13. The average time for all runs was 47.5 CPU seconds in this case. It can be observed that changing unit fuel costs does not affect computation times. However, changing β significantly affects times; e.g., almost doubling them. This is reasonable as β affects variability and increases problem complexity, whereas unit fuel costs are merely coefficients in the model and do not add complexity to the problem.

4.4. *Simulation study*

The computational study above compares the costs of the published schedule and the costs resulting from the optimization model. Since our model is a robust scheduling model that performs under uncertainty, we also performed a simulation study as a better indicator of model performance. We used the single-hub schedule introduced above and imposed the same published times, passenger and aircraft turnaround times, and the same passenger connection network in the simulation. The experimental factor setting that we used in the simulation corresponds to low fuel cost, low compression level, high beta factor, and high connection density.

We performed 10 replications, and random variables associated with non-cruise times were generated from a log Laplace distribution with the same parameters α and β_i for each flight, as in the optimization model. For the single-hub schedule, there were 114 flights as reported in

Table 13. CPU time analysis for the four-hub schedule

		CPU time (sec.)		
Factor	*Level*	*Min.*	*Avg.*	*Max.*
A	0	30.0	48.1	65.6
	1	31.9	47.4	62.6
C	0	30.0	32.4	34.2
	1	59.7	63.0	65.6

Table 14. Summary of simulation results on a single-hub schedule

	Proposed schedule			*Published schedule*		
	Min.	*Avg.*	*Max.*	*Min.*	*Avg.*	*Max.*
Total delay (in minutes)	167.7	297.5	460.8	386.6	492.9	625.2
Maximum delay (in minutes)	12.3	31.7	64	24.7	39.2	72.7
Number of delays > 0 minutes	50	60.4	72	84	91.1	98
Number of delays > 5 minutes	9	16.8	26	24	30.1	39
Number of delays > 15 minutes	0	4.3	10	3	7	10
Missed connections (%)	5.8	7.6	10.1	6.6	7.9	9.2

Table 7, with 1467 connecting passengers in 301 connecting flights. Different performance measures (e.g., the total delay for all flights, the maximum delay, the number of delayed flights over 114 flights, and the percentage of missed passenger connections) for the published schedule and proposed schedule are reported in Table 14 along with the minimum, average, and maximum values over 10 randomly generated runs. The overall service level for this particular setting was 92.2%, which is consistent with the simulation analysis. The results from the simulation study also verified that the proposed schedule could achieve a significantly better delay performance compared with the published schedule, while satisfying the same passenger service levels for the connecting passengers. In summary, the proposed schedule can handle the variability in non-cruise times better than the published schedule as can be seen in several flight delay–related measures summarized in Table 14.

5. Conclusions and future work

We developed a global optimization tool that can satisfy given passenger connection service levels and avoid flight delays while minimizing the sum of fuel and idle time costs on a combined network of aircraft and passenger itineraries. One of the main results of this study is that significant cost reductions can be achieved from using cruise time controllability in addition to solely utilizing schedule padding, without deteriorating passenger service levels. As far as we know, this is the first time this method of balancing scheduled idle time with cruise time controllability is used together to generate robust airline schedules. We show that, approximately, with a 2% increase in fuel costs, we could achieve a 60% decrease in idle time costs. Since cruise speed decisions significantly affect the overall network and need to be decided globally, the tool we developed will prove many benefits for airlines.

In this study, block times were considered in two separate parts as cruise and non-cruise times. Cruise times are subject to controllability where variability exists in non-cruise times. The variability is modeled with chance constraints on passenger connection service levels. By using a conic formulation, we can exactly solve even large problems as a four-hub daily schedule under a minute of time, where solutions for a single-hub schedule can be obtained in seconds. Moreover, the congestion information on airports was used to arrange the aircraft turnaround times and the variability on each flight. Maintaining a desired level of passenger connection service levels is one of the objectives of this study. However, passenger connection service levels were allowed to be different for each flight as long as their weighted average was above the desired level. This gave a better chance to assess these connections based on to their priority.

The benefits of this research can be extended with further studies. Following the research in Arıkan and Deshpande (2012), the random variable in this study is assumed to be a log Laplace random variable. Our work could be extended to see the properties and performance of the model under a different random distribution.

Another important area for development could be incorporating routing and fleet assignment decisions in to the decision process. Under this new setting, we could more accurately assess possible benefits of the makespan reductions due to cruise time controllability.

In this study, block time random variables were assumed to be independent but, actually, the propagation of delays and the use of shared resources such as airports can mean a correlation between these variables, especially for the flights of the same aircraft. The study can be extended to incorporate this correlation between the block time random variables and benefits of the methodology can be analyzed under this new setting. We expect the benefits to be no less, as an assumed correlation will magnify the effects of delays, but the computation times might grow, due to the added complexity.

The airline scheduling process is a large-sized complex process with many different subproblems. It is important to integrate as many of these subproblems to obtain better solutions that address the whole network with all interacting parts. For example, crew costs are an important part of the operating costs in the airline industry. A problem integrating crew scheduling problem with ours that employs idle time insertion and cruise time controllability is a promising problem for future studies. This will be particularly interesting for the cases when the crew network differs significantly from the aircraft network.

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Appendix A: Formulation allowing low service levels

When $\beta \geq 1$, the expected value of a log Laplace is infinity. In this case, one can formulate the problem by using the geometric mean of log Laplace random variable in constraint (7). We will show that when $\beta \geq 1$ the problem can still be reformulated using second-order conic inequalities. We will have no restriction on γ_{ij}, so the results are valid for all possible values of γ; i.e., $0 \leq \gamma_{ij} \leq 1$.

We first explore the properties of the quantile function given in Equation (1). In Proposition A1, we show that the second piece of quantile function $F_X^{-1}(\gamma)$ is always greater than or equal to the first piece.

Proposition A1. *Denote the first and the second pieces of $F_X^{-1}(\gamma)$ by $f_1(\gamma)$ and $f_2(\gamma)$, respectively, so,*

$$f_1(\gamma) = 2^{\beta_i} \times e^{\alpha} \times \gamma^{\beta_i},$$
$$f_2(\gamma) = \frac{e^{\alpha}}{2^{\beta_i} \times (1-\gamma)^{\beta_i}}.$$

Then, the inequality $f_2(\gamma) \geq f_1(\gamma)$ always holds for $0 \leq \gamma \leq 1$. Furthermore, $f_2(\gamma) = f_1(\gamma)$ only holds when $\gamma = 1/2$.

Proof. Consider

$$f_2(\gamma) - f_1(\gamma) = e^{\alpha}\left(\frac{1 - 4^{\beta_i} \times \gamma^{\beta_i} \times (1-\gamma)^{\beta_i}}{2^{\beta_i} \times (1-\gamma)^{\beta_i}}\right).$$

Notice that, $e^{\alpha} > 0$, $2^{\beta_i} \times (1-\gamma)^{\beta_i} > 0$. Moreover, $1 - 4^{\beta_i} \times \gamma^{\beta_i} \times (1-\gamma)^{\beta_i} \geq 0$ always holds since $\gamma^{\beta_i} \times (1-\gamma)^{\beta_i} \leq 1/4^{\beta_i}$ is always true. Next, observe that $f_2(\gamma) - f_1(\gamma) = 0$ when $\gamma = 1/2$. ■

Proposition A2 states that $F_X^{-1}(\gamma)$ is a convex function.

Proposition A2. *$F^{-1}(\gamma)$ is a convex function when $\beta_i \geq 1$.*

Proof. Both $f_1(\gamma)$ and $f_2(\gamma)$ are convex for $0 \leq \gamma \leq 1$. The result follows. ■

Proposition A3. *The chance constraint* (4)*; that is,*

$$F^{-1}(\gamma_{ij}) \leq (x_j - x_i - TP_{ij} - f_i),$$

in the problem formulation can be replaced with the following constraints:

$$2^{\beta_i} \times e^{\alpha} \times \gamma_{ij}^{\beta_i} \leq x_j - x_i - TP_{ij} - f_i, \quad \text{(A1)}$$

$$\frac{e^{\alpha}}{2^{\beta_i} \times (1-\gamma_{ij})^{\beta_i}} \times z_{ij} \leq x_j - x_i - TP_{ij} - f_i, \quad \text{(A2)}$$

where z_{ij} is a 0-1 decision variable defined as

$$z_{ij} = \begin{cases} 0, & \text{if } \gamma_{ij} < \frac{1}{2} \\ 1, & \text{if } \gamma_{ij} \geq \frac{1}{2} \end{cases}.$$

Proof. Proposition A1 shows that $f_2(\gamma) \geq f_1(\gamma)$ always holds. Thus, when $\gamma_{ij} \geq 1/2$, $z_{ij} = 1$, $f_2(\gamma_{ij})$ bounds the right-hand side $x_j - x_i - TP_{ij} - f_i$; i.e., constraint (A2) is active, whereas when $\gamma_{ij} < 1/2$ and $z_{ij} = 0$, $f_1(\gamma_{ij})$ bounds the right-hand side $x_j - x_i - TP_{ij} - f_i$; i.e. constraint (A1) is active. ■

Proposition A4. *When $\beta \geq 1$, constraints* (A1)–(A2) *are both representable by second-order conic inequalities.*

Proof. First, consider Equation (A1) and

$$2^{\beta_i} \times e^{\alpha} \times \gamma_{ij}^{\beta_i} \leq x_j - x_i - TP_{ij} - f_i.$$

Since $\beta_i \geq 1$, by making the conversion

$$v_{ij} = x_j - x_i - TP_{ij} - f_i,$$

as before, we obtain

$$2^{\beta_i} \times e^{\alpha} \times \gamma_{ij}^{\beta_i} \leq v_{ij},$$

which is obviously in the form of Equation (17) and hence can be represented via conic inequalities.

Next, consider Equation (A2),

$$\frac{e^{\alpha}}{2^{\beta_i} \times (1-\gamma_{ij})^{\beta_i}} \times z_{ij} \leq x_j - x_i - TP_{ij} - f_i,$$

which is obtained by multiplying the left-hand side of constraint (12) by z_{ij}. Constraint (12) is second-order cone programming representable by Proposition 2.

Add auxiliary variables v_{ij} and w_{ij} such that

$$v_{ij} = x_j - x_i - TP_{ij} - f_i, \quad \text{(A3)}$$

$$w_{ij} = 2 - 2\gamma_{ij}, \quad \text{(A4)}$$

and we get

$$e^{\alpha} z_{ij} \leq w_{ij}^{\beta_i} v_{ij}.$$

Suppose that $\beta_i = a_i/b_i$, where a_i, b_i are integers. Since $z_{ij} \in \{0, 1\}$, we can equivalently write

$$e^{\alpha} z_{ij}^{b_i} \leq w_{ij}^{a_i} v_{ij}^{b_i}.$$

To put this inequality in the form of Equation (17), we can increase the exponent of z_{ij} and still get an equivalent inequality since $z_{ij} \in \{0, 1\}$:

$$e^{\alpha} z_{ij}^{2^k} \leq w_{ij}^{a_i} v_{ij}^{b_i},$$

where

$$k = \lceil \log_2(a_i + b_i) \rceil.$$

The proof follows. ■

Proposition A5. *The expected value of log Laplace variable X with parameters α and β_i is finite only for $\beta_i < 1$ and has value $\frac{e^{\alpha}}{(1-\beta_i)\times(1+\beta_i)}$.*

Proof. Define δ such that $\alpha = \ln(\delta)$. Then, we can rewrite $f_X(x)$ as below:

$$f_X(x) = \begin{cases} \dfrac{1}{2 \times \beta_i \times \delta} \left(\dfrac{x}{\delta}\right)^{(1/\beta_i)-1}, & \text{if } x < \delta \\ \dfrac{1}{2 \times \beta_i \times \delta} \left(\dfrac{\delta}{x}\right)^{(1/\beta_i)+1}, & \text{if } x \geq \delta. \end{cases}$$

Using the distribution function, we can calculate expected value of X by

$$E[X] = \int_{-\infty}^{\infty} x \times f_X(x) \times \mathrm{d}x = \int_0^{\delta} \frac{1}{2\beta_i} \left(\frac{x}{\delta}\right)^{1/\beta_i} \times \mathrm{d}x + \int_{\delta}^{\infty} \frac{1}{2\beta_i} \left(\frac{\delta}{x}\right)^{1/\beta_i} \cdot \mathrm{d}x.$$

Define

$$g_1(x) = \int_0^{\delta} \frac{1}{2\beta_i} \left(\frac{x}{\delta}\right)^{1/\beta_i} \cdot \mathrm{d}x$$

$$g_2(x) = \int_{\delta}^{\infty} \frac{1}{2\beta_i} \left(\frac{\delta}{x}\right)^{1/\beta_i} \cdot \mathrm{d}x.$$

Then

$$g_1(x) = \frac{\delta}{2(\beta_i + 1)},$$

whereas

$$g_2(x) = \begin{cases} \dfrac{-\delta}{2(\beta_i - 1)}, & \text{if } \beta_i < 1 \\ \text{undefined}, & \text{if } \beta_i = 1 \\ \infty, & \text{if } \beta_i > 1. \end{cases}$$

Consequently, for α and $0 < \beta_i < 1$ we get

$$E[X] = \frac{e^{\alpha}}{(1-\beta_i)\cdot(1+\beta_i).}$$

■

Biographies

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