# Achieving Delay Diversity in Asynchronous Underwater Acoustic (UWA) Cooperative Communication Systems

Mojtaba Rahmati and Tolga M. Duman, *Fellow, IEEE*

***Abstract*—In cooperative UWA systems, due to the low speed of sound, a node can experience significant time delays among the signals received from geographically separated nodes. One way to combat the asynchronism issues is to employ orthogonal frequency division multiplexing (OFDM)-based transmissions at the source node by preceding every OFDM block with an extremely long cyclic prefix (CP) which reduces the transmission rates dramatically. One may increase the OFDM block length accordingly to compensate for the rate loss which also degrades the performance due to the significantly time-varying nature of UWA channels. In this paper, we develop a new OFDM-based scheme to combat the asynchronism problem in cooperative UWA systems without adding a long CP (in the order of the long relative delays) at the transmitter. By adding a much more manageable (short) CP at the source, we obtain a delay diversity structure at the destination for effective processing and exploitation of spatial diversity by utilizing a low complexity Viterbi decoder at the destination, e.g., for a binary phase shift keying (BPSK) modulated system, we need a two-state Viterbi decoder. We provide pairwise error probability (PEP) analysis of the system for both time-invariant and block fading channels showing that the system achieves full spatial diversity. We find through extensive simulations that the proposed scheme offers a significantly improved error rate performance for time-varying channels (typical in UWA communications) compared to the existing approaches.**

***Index Terms*—Asynchronous communication, cooperative systems, underwater acoustics, OFDM.**

## I. INTRODUCTION

COOPERATIVE UWA communications which refers to a group of nodes, known as relays, helping the source to deliver its data to a destination is a promising physical layer solution to improve the performance of UWA systems [1], [2], [3]. In a UWA cooperative communication system, the time differences among signals received from geographically separated nodes can be excessive due to the low speed of sound in water. For example, if the relative distance between two nodes with respect to another one is $500$ m, then their transmissions experience a relative delay of $333$ ms. Considering, for instance, that in an OFDM-UWA cooperative communication scheme with $512$ sub-carriers over a total bandwidth of $8$ kHz, the OFDM block duration is only $64$ ms, the excessive delay of $333$ ms becomes problematic. Furthermore, UWA channels are highly time varying due to the large Doppler spreads and Doppler shift effects (or Doppler scaling) [4]. Therefore, a practical non-centralized UWA cooperative communication system is asynchronous with large relative delays among the nodes and sees highly time-varying frequency selective channel conditions.

Our focus in this paper is on asynchronous cooperative UWA communications where only the destination node is aware of the relative delays among the nodes. Existing signaling solutions for asynchronous radio terrestrial cooperative communications rely on quasi-static fading channels with limited delays among signals received from different relays at the destination, e.g., see [5] and references therein, in which every transmitted block is preceded by a time guard not less than the maximum possible delay among the relays. Therefore, we cannot directly apply them for cooperative UWA communications. Our main objective is to develop new OFDM based signaling solutions to combat the asynchronism issues arising from excessively large relative delays without preceding each OFDM block by a large CP (in the order of the maximum possible relative delay).

In systems employing OFDM, e.g., [6], [7], the existing solutions are effective when the maximum length of the relative delays among signals received from various nodes are less than the length of an OFDM block which is not a practical assumption for the case of UWA communications. In [6], a space-frequency coding approach is proposed which is proved to achieve both full spatial and full multipath diversities. In [7], OFDM transmission is implemented at the source node and relays only perform time reversal and complex conjugation. A trivial generalization of existing OFDM-based results to compensate for large relative delays may be to increase the OFDM block lengths. The main drawback in this case is that inter carrier interference (ICI) is increased due to the time variations of the UWA channels. Another trivial solution is to increase the length of the CP. This is not an efficient solution either, since it dramatically decreases the spectral efficiency of the system.

There are several single carrier transmission based solutions reported in the literature as well, e.g., [1], [8], [9], [10], [11], [12]. In [1] a time reversal distributed space time block code (DSTBC) is proposed for UWA cooperative commu-

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M. Rahmati is with the School of Electrical, Computer and Energy Engineering (ECEE) of Arizona State University, Tempe, AZ 85287-5706, USA (e-mail: mojtaba@asu.edu).

T. M. Duman is with the Department of Electrical and Electronics Engineering, Bilkent University, Bilkent, Ankara, 06800, Turkey (e-mail: duman@ee.bilkent.edu.tr). He is on leave from the School of ECEE of Arizona State University.

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nication systems under quasi-static multipath fading channel conditions. In [8], a DSTBC transmission scheme by decode and forward (DF) relaying is proposed which achieves both full spatial and multipath diversities. A distributed space time trellis code with DF relaying is proposed in [9], [10] which under certain conditions can achieve full spatial and multipath diversities. In [11], [12], space time delay tolerant codes are proposed for decode and forward relaying strategies where in [11] a family of fully delay tolerant codes and in [12] a family of bounded delay tolerant codes are developed for asynchronous cooperative systems.

In this paper, we focus on OFDM based cooperative UWA communication systems with full-duplex AF relays where all the nodes employ the same frequency band to communicate with the destination. We assume an asynchronous operation and potentially very large delays among different nodes (known only at the destination). We present a new scheme which can compensate for the effects of the long delays among the signals received from different nodes without adding an excessively long CP. We demonstrate that we can extract delay diversity out of the asynchronism among the cooperating nodes. The main idea is to add an appropriate CP (much shorter than the long relative delays among the relays) to each OFDM block at the transmitter side to combat multipath effects of the channels and obtain a delay diversity structure at the destination.

The paper is organized as follows. In Section II, the system model and the structure of the OFDM signals at the source, relays and destination are presented. The proposed signaling scheme which includes appropriate CP addition at the source and CP removal at the destination is explained in Section III. Furthermore, it is shown that the proposed scheme gives a delay diversity structure at the destination for large relative delays among the relays. In Section IV, we present another transmission scheme which is also useful for delay values less than one OFDM block and provides the same delay diversity structure for longer delay values. In Section V, the PEP analysis of the system under both quasi-static and block fading channel models is provided. In Section VI, the performance of the proposed scheme is evaluated through some numerical examples. Finally, conclusions are given in Section VII.

## II. System and Signal Models

We consider a full-duplex AF relay system with two relays, shown in Fig. 1, in which there is no direct link between source (S) and destination (D), and the relays help the source deliver its data to the destination by using the AF method. No power allocation strategy is employed at the relay nodes and they use fixed power amplification factors. Note that the model can be generalized to a system with an arbitrary number of relays and a direct link between source and the destination, and optimal power allocation can be used in a straightforward manner. We assume that the channels from the source to the relays and the relays to the destination are time-varying multipath channels where $h_i(t,\tau)$ and $g_i(t,\tau)$ represent the source to the $i$-th relay and the $i$-th relay to the destination channel responses at time $t$ to an impulse applied at time $t-\tau$, respectively.

Before presenting the system model of the new relaying scheme, we would like to give an example to demonstrate how

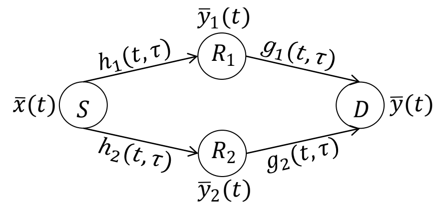


Fig. 1. Relay channel with two relays.

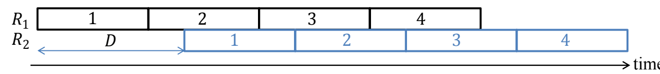


Fig. 2. The structure of the received OFDM blocks from two different relays of the proposed delay diversity scheme for a relative delay of $D$ seconds.

a delay diversity structure is obtained. For illustration, Fig. 2 shows the received OFDM block structure of the proposed delay diversity scheme from two different relays with a relative delay of $D$ seconds. In Fig. 2, $D$ is in a range that each block relayed through the relay $R_1$ is overlapped with its preceding block relayed through the relay $R_2$. E.g., under quasi-static fading scenario, each subcarrier of a received block is a summation of the corresponding subcarriers from two successively transmitted blocks which results in a delay diversity structure [13].

### A. Signaling Scheme

At the transmitter, we employ a conventional OFDM transmission technique with $N$ subcarriers over a total bandwidth of $B$ Hz. We consider successive transmission of $M$ data blocks of length $N$ symbols. In discrete baseband signaling form, the $m$-th ($m \in \{1,\ldots,M\}$) data vector (in time) is denoted by $\boldsymbol{X}^m = [X_0^m,\ldots,X_{N-1}^m]^T$ and the samples of the $m$-th transmitted OFDM block are represented by $\boldsymbol{x}^m = \text{IFFT}(\boldsymbol{X}^m) = [x_0^m,\ldots,x_{N-1}^m]^T$, where $(.)^T$ denotes the transpose operation. Therefore, we have $x_n^m = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} X_k^m e^{j\frac{2\pi k}{N}n}$. After adding a CP of length $N_{CP}$ to $\boldsymbol{x}^m$, the CP-assisted transmission block $\bar{\boldsymbol{x}}^m$ results. By digital to analog (D/A) conversion of $\bar{\boldsymbol{x}}^m$ with sampling period $T_s = \frac{1}{B}$ seconds, we obtain the continuous time signal $\bar{x}^m(t)$ with time duration of $T = (N+N_{CP})T_s$ seconds which can be written as

$$\bar{x}^m(t) = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} X_k^m e^{j\frac{2\pi k}{NT_s}t} R(t), \tag{1}$$

where $x_n^m = \bar{x}^m(nT_s)$, $R(t) = u(t+N_{CP}T_s) - u(t-NT_s)$ and $u(t)$ denotes the unit step function. Furthermore, for the continuous time transmitted signal $\bar{x}(t)$, we can write $\bar{x}(t) = \sum_{m=1}^{M}\bar{x}^m(t-(m-1)T)$.

At the $i$-th relay ($i \in \{1,2\}$), the signal $\bar{y}_i(t)$ is received, hence the part of $\bar{y}_i(t)$ corresponding to the $m$-th transmitted block, i.e., $\bar{y}_i^m(t) = \bar{y}_i(t+(m-1)T)R(t)$, can be written as

$$\begin{aligned}\bar{y}_i^m(t) &= \int_{-\infty}^{\infty}\bar{x}^m(t-\tau)h_i^m(t,\tau)d\tau + z_{1,i}^m(t)\\ &\underbrace{+\sum_{m'\neq m}\int_{-\infty}^{\infty}\bar{x}^{m'}(t-(m'-m)T-\tau)h_i^m(t,\tau)d\tau}_{\text{ISI}},\end{aligned} \tag{2}$$

where $z_{1,i}^m(t) = z_{1,i}(t + (m-1)T)R(t)$, $h_i^m(t,\tau) = h_i(t + (m-1)T, \tau)R(t)$ and $z_{1,i}^m(t)$ are independent complex Gaussian random processes with zero mean and power spectral density (PSD) of $\sigma_{1,i}^2$. By taking only the resolvable paths into account, we can write $h_i(t,\tau) = \sum_{l=1}^{L_{h_i}} h_{i,l}(t)\delta(\tau - \tau_{h_i,l})$, where $L_{h_i}$ denotes the number of resolvable paths from the source to the $i$-th relay, $h_{i,l}(t)$ are independent zero-mean (for different $i$ and $l$) complex Gaussian wide-sense stationary (WSS) processes with a total envelope power of $\sigma_{h_i,l}^2$ (i.e., independent time-varying Rayleigh fading channel tap gains) assuming that $\sum_{l=1}^{L_{h_i}} \sigma_{h_i,l}^2 = 1$, and $\tau_{h_i,l} \geq 0$ denotes the delay of the $l$-th resolvable path from the source to the $i$-th relay. Assuming $\tau_{h_i,L_{h_i}} \leq N_{CP}T_s$, i.e., the length of the CP overhead is greater than the delay spread of the channel (the main job of the CP to guarantee robustness against multipath), and defining $I_{1,i}^m(t) = \sum_{l=1}^{L_{h_i}} h_{i,l}^m(t)\bar{x}^{m-1}(t + T - \tau_{h_i,l})$, we can rewrite (2) as

$$\bar{y}_i^m(t) = \sum_{l=1}^{L_{h_i}} h_{i,l}^m(t)\bar{x}^m(t - \tau_{h_i,l}) + I_{1,i}^m(t) + z_{1,i}^m(t). \tag{3}$$

We assume that the signal passing through the second relay is received $D$ seconds later than the signal passing through the first relay and we also assume $\tau_{h_i,1} = \tau_{g_i,1} = 0$ for $i = \{1,2\}$, i.e., the delay spread of the channel $h_i$ ($g_i$) is $\tau_{h_i,L_{h_i}}$ ($\tau_{g_i,L_{g_i}}$). Therefore, by denoting the amplification factor of the $i$-th relay by $\sqrt{P_i}$, for the received signal at the destination $\bar{y}(t)$, we have

$$\begin{aligned}\bar{y}(t) = &\int_{-\infty}^{\infty} \sqrt{P_1}\bar{y}_1(t-\tau)g_1(t,\tau)d\tau \\ &+ \int_{-\infty}^{\infty} \sqrt{P_2}\bar{y}_2(t-D-\tau)g_2(t,\tau)d\tau + z_2(t),\end{aligned} \tag{4}$$

where $z_2(t)$ is a Gaussian random processes with zero mean and PSD of $\sigma_2^2$. By employing $g_i(t,\tau) = \sum_{l=1}^{L_{g_i}} g_{i,l}(t)\delta(\tau - \tau_{g_i,l})$ in (4), we obtain

$$\begin{aligned}\bar{y}(t) = &\sum_{l=1}^{L_{g_1}} \sqrt{P_1}g_{1,l}(t)\bar{y}_1(t - \tau_{g_1,l}) \\ &+ \sum_{l=1}^{L_{g_2}} \sqrt{P_2}g_{2,l}(t)\bar{y}_2(t - \tau_{g_2,l} - D) + z_2(t).\end{aligned}$$

Defining $z(t) = z_2(t) + \sum_{l=1}^{L_{g_1}} \sqrt{P_1}g_{1,l}(t)z_{1,1}(t - \tau_{g_1,l}) + \sum_{l=1}^{L_{g_2}} \sqrt{P_2}g_{2,l}(t)z_{1,2}(t - \tau_{g_2,l})$ which represents a Guassian random process conditioned on known $g_{i,l}(t)$ for all $i$ and $l$, we can write

$$\begin{aligned}\bar{y}(t) = &\sum_{l=1}^{L_{g_1}}\sqrt{P_1}g_{1,l}(t)\sum_{q=1}^{L_{h_1}} h_{1,q}(t-\tau_{g_1,l})\bar{x}(t-\tau_{g_1,l}-\tau_{h_1,q}) + z(t) \\ &+ \sum_{l=1}^{L_{g_2}}\sqrt{P_2}g_{2,l}(t)\sum_{q=1}^{L_{h_2}} h_{2,q}(t-D-\tau_{g_2,l})\bar{x}(t-D-\tau_{g_2,l}-\tau_{h_2,q}).\end{aligned}$$

Note that without conditioning on $g_{i,l}(t)$, $z(t)$ represents a complex random process with zero mean and PSD of $\sigma^2 = P_1\sigma_{1,1}^2 + P_2\sigma_{1,2}^2 + \sigma_2^2$. Therefore, we define the received signal to noise ratio (SNR) as $\frac{P_1+P_2}{\sigma^2}$. We also define $L = \left\lceil \frac{\max_i(\tau_{h_i,L_{h_i}} + \tau_{g_i,L_{g_i}})}{T_s} \right\rceil$, where $\tau_{h_i,L_{h_i}} + \tau_{g_i,L_{g_i}}$ is the delay spread of the overall channel experienced at the destination through $R_i$.

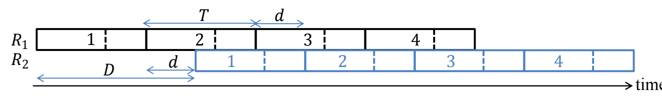


Fig. 3. The structure of the received signal.

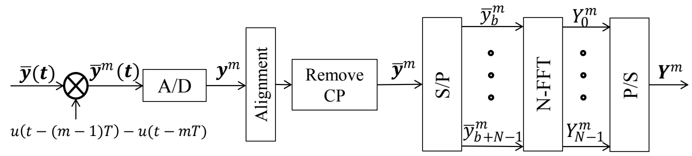


Fig. 4. The structure of the receiver.

## III. Delay Diversity Structure

To achieve a delay diversity structure and overcome ISI at the destination, we need to add an appropriate CP at the source and perform CP removal at the destination.

### A. Appropriate CP Length

In a conventional OFDM system, if we have a window of length $(N+L)T_s$ seconds corresponding to one OFDM block, then by removing the first $L$ samples of the considered window and feeding the remaining $N$ samples to the FFT block, the ISI is completely removed. Therefore, in our scheme, to guarantee robustness of the system against ISI, we need to have an overlap of length $(N+L)T_s$ seconds between two blocks received from two different relays at the destination. Fig. 3 shows the structure of the received signal at the destination for the case that the blocks relayed by $R_2$ are received $D$ seconds later than the blocks relayed by $R_1$, where $T < D < 2T$ and $d = \mathrm{mod}(D,T)$ with $d = \mathrm{mod}(D,T)$ denoting the remainder of division of $D$ by $T$. To obtain the appropriate overlap structure, we need to have $T - d \geq (N+L)T_s$ or $d \geq (N+L)T_s$ or both which results in $T \geq 2(N+L)T_s$, i.e., $N_{CP} \geq N + 2L$.

### B. Received Signal at the Destination

The baseband signaling structure of the receiver is shown in Fig. 4, where $\bar{\boldsymbol{y}}^m = [\bar{y}_0^m, \ldots, \bar{y}_{N+N_{CP}-1}^m]$ denotes the sampled vector of the received signal in the $m$-th signaling interval and $b$ is the starting point of the $m$-th FFT window which is decided by the destination based on the delay value $D$. Since $N_{CP} \geq N + 2L$, by defining $\bar{y}^m(t) = \bar{y}(t + (m-1)T)R(t)$, we can write

$$\begin{aligned}\bar{y}^m(t) = &I_1^m(t) + I_2^m(t) + z^m(t) \\ &+ \sum_{l=1}^{L_{g_1}} \sqrt{P_1}g_{1,l}^m(t)\sum_{q=1}^{L_{h_1}} h_{1,q}^m(t - \tau_{g_1,l})\bar{x}^m(t - \tau_{g_1,l} - \tau_{h_1,q}) \\ &+ \sum_{l=1}^{L_{g_2}} \sqrt{P_2}g_{2,l}^m(t)\Bigg[\sum_{q=1}^{L_{h_2}} h_{2,q}^{m-BD}(t - d_r - \tau_{g_2,l})\times \\ &\times \bar{x}^{m-BD}(t - d_r - \tau_{g_2,l} - \tau_{h_2,q})\Bigg],\end{aligned}$$

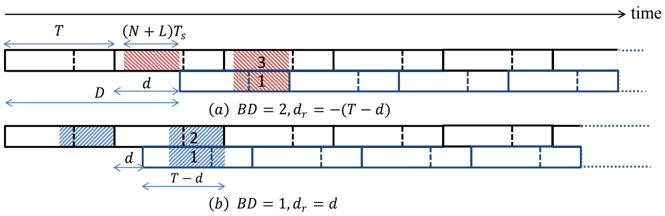


Fig. 5. Example of different situations for $BD$ and $d_r$.

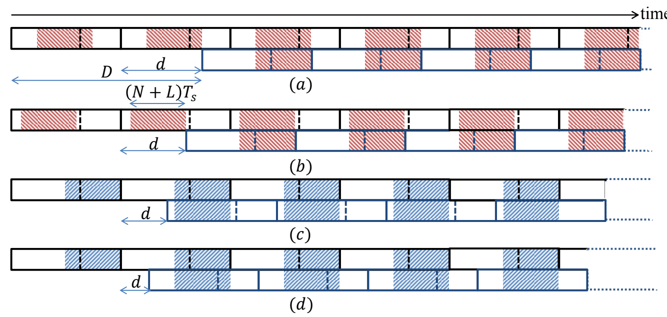


Fig. 6. Different possible FFT windowings for different ranges of $d$ (a) $d \geq (N+2L)T_s$, (b) $(N+L)T_s \leq d < (N+2L)T_s$, (c) $NT_s < d < (N+L)T_s$, and (d) $d \leq NT_s$

where

$$I_1^m(t) = \sum_{l=1}^{L_{g_1}} \sqrt{P_1} g_{1,l}^m(t) \sum_{q=1}^{L_{h_1}} h_{1,q}^m(t-\tau_{g_1,l}) \times \bar{x}^{m-1}(t+T-\tau_{g_1,l}-\tau_{h_1,q})$$

and

$$I_2^{m+BD}(t) = \sqrt{P_2}\sum_{l=1}^{L_{g_2}} g_{2,l}^{m+BD}(t)\sum_{q=1}^{L_{h_2}} h_{2,q}^m(t-d_r-\tau_{g_2,l})\times \left[\bar{x}^{m-1}(t+T-d_r-\tau_{g_2,l}-\tau_{h_2,q})+\bar{x}^{m+1}(t-T-d_r-\tau_{g_2,l}-\tau_{h_2,q})\right]$$

represent the ISI, and $BD$ and $d_r$, as shown in Fig. 5, denote the effective OFDM block delay and effective residual delay observed at the destination, respectively. For $BD$ and $d_r$, we have

$$BD = \begin{cases} \lfloor \frac{D}{T} \rfloor & , \quad d \leq (N+L)T_s \\ \lceil \frac{D}{T} \rceil & , \quad d > (N+L)T_s \end{cases}, \tag{5}$$

and

$$d_r = \begin{cases} d & , \quad d \leq (N+L)T_S \\ d-T & , \quad d > (N+L)T_s \end{cases}, \tag{6}$$

respectively (note that when $m-BD<0$, $\bar{x}^{m-BD}(t)=0$ for all values of $t$). More precisely, $BD$ represents the number of block delays between two received OFDM blocks which have at least an overlap of length $(N+L)T_s$ seconds (necessary to combat the ISI). As discussed in Section III-A, by choosing $N_{CP} \geq N+2L$, achieving the appropriate overlap between the received OFDM blocks is guaranteed. By appropriate CP removal (whose details are explained in Section III-C), $\boldsymbol{y}^m$ is obtained as $\boldsymbol{y}^m = [\bar{y}^m(bT_s), \ldots, \bar{y}^m((b+N-1)T_s)]$. By taking FFT of $\boldsymbol{y}^m$, we have $\boldsymbol{Y}^m = [Y_0^m, \ldots, Y_{N-1}^m] = \mathrm{FFT}(\boldsymbol{y}^m)$ where $Y_k^m$ are given in (7) at the top of the next page, which can be written as

$$Y_k^m = \boldsymbol{GH}_1^m(k)\boldsymbol{X}^m + \boldsymbol{GH}_2^{m-BD}(k)\boldsymbol{X}^{m-BD} + Z_k^m,$$

where $\boldsymbol{GH}_i^m(k) = [GH_i^m[k,0], \cdots, GH_i^m[k,N-1]]$ with $GH_i^m[k,k']$ given in (8) at the top of the next page and $Z_k^m = \frac{1}{\sqrt{N}}\sum_{n=b}^{b+N-1} z^m(nT_s)e^{-\frac{j2\pi n}{N}k}$ conditioned on channel state information are complex Gaussian random variables with zero mean. Hence, by defining $\boldsymbol{X}^m = \boldsymbol{0}_N$ for $m<1$ and $m>M$ and $\boldsymbol{X}^m = [X_0^m, X_1^m, \cdots, X_{N-1}^m]^T$ for $1 \leq m \leq M$, we can write

$$\boldsymbol{Y}^m = \boldsymbol{GH}_1^m \boldsymbol{X}^m + \boldsymbol{GH}_2^{m-BD}\boldsymbol{X}^{m-BD} + \boldsymbol{Z}^m, \tag{9}$$

where $\boldsymbol{GH}_i^m = \left[\boldsymbol{GH}_i^m(0)^T, \ldots, \boldsymbol{GH}_i^m(N-1)^T\right]^T$. In fact, $\boldsymbol{GH}_i^m$ represents the effective $S-R_i-D$ channel seen by the destination in frequency domain which depends on both $S-R_i-D$ channel and the position of the FFT window.

### C. Appropriate CP Removal at the Destination

To take FFT at the destination, we need to choose the FFT window by appropriate CP removal. Since the received OFDM blocks are not synchronized, we align the receiver FFT window with one of the relays. By precise alignment, an overlap of length $(N+L)T_s$ seconds between the OFDM blocks received through $R_1$ and $R_2$ can be achieved which is determined with the value of $d$. Note that an overlap of at least $N+L$ samples is necessary to guarantee robustness of the transmission against ISI. As shown in Fig. 6, for $d \geq (N+L)T_s$, the receiver FFT window is aligned with $R_2$ and for $d < (N+L)T_s$ it is aligned with $R_1$. The only effect of unaligned FFT windowing in time at the destination, as long as appropriate CP removal is done, is phase shift at the frequency domain included in the definition of $\boldsymbol{GH}_i^m$.

### D. Detection by Viterbi Algorithm

For the time-invariant channel scenario the noise samples $Z_k^m$ are independent complex Gaussian random variables for all $m$ and $k$ and i.i.d. for any specific $k$. Therefore, for time-invariant channel conditions, $N$ parallel Viterbi detectors with $M^{BD}$ states (assuming M-PSK modulation) can be employed for ML detection of the transmitted symbols, where the $k$-th Viterbi detector gets $\boldsymbol{Y}_k$ as input to detect the transmitted symbols over the $k$-th subcarrier. On the other hand, for the time-varying channel scenarios, the received noise samples at each OFDM block are dependent complex Gaussian random variables conditioned on known channel state information. Note also that the noise samples corresponding to different FFT windows at the destination are independent but not necessarily identically distributed. However, by approximating the received noise samples $Z_k^m$ as independent complex Gaussian random variables, Viterbi algorithm can be employed as a detector at the destination and extract delay diversity out of the asynchronous system.

The complexity of the Viterbi algorithm for the time varying case is prohibitive due to the ICI effects. Therefore, we implement a suboptimal detector in which we ignore the ICI effects and assume that $Z_k^m$ are i.i.d. for any given subcarrier $k$, and employ the same structure as in the time-invariant case. Hence, $\boldsymbol{Y}'_k = [Y_k'^1, \ldots, Y_k'^M]$ is given to the $k$-th Viterbi decoder where $[Y_0'^m, \ldots, Y_{N-1}'^m] = diag(\boldsymbol{Y}_m)$ and $diag(\boldsymbol{A})$, with $\boldsymbol{A}$ being a square matrix, denotes a vector of diagonal elements of $\boldsymbol{A}$.

$$
\begin{aligned}
Y_k^m =& \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} y_n^m e^{-j\frac{2\pi n}{N}k} = \frac{1}{\sqrt{N}} \sum_{n=b}^{b+N-1} \bar{y}^m(nT_s) e^{-j\frac{2\pi n}{N}k} \\
=& \frac{1}{\sqrt{N}} \sum_{n=b}^{b+N-1} \Bigg[ \sum_{l=1}^{L_{g_1}} \sqrt{P_1} g_{1,l}^m(nT_s) \sum_{q=1}^{L_{h_1}} h_{1,q}^m(nT_s-\tau_{g_1,l}) \bar{x}^m(nT_s-\tau_{h_1,q}-\tau_{g_1,l}) + z^m(nT_s) \\
&+ \sum_{l=1}^{L_{g_2}} \sqrt{P_2} g_{2,l}^m(nT_s) \sum_{q=1}^{L_{h_2}} h_{2,q}^{m-BD}(nT_s-d_r-\tau_{g_2,l}) \bar{x}^{m-BD}(nT_s-d_r-\tau_{h_2,q}-\tau_{g_2,l}) \Bigg] e^{-j\frac{2\pi n}{N}k}. \qquad (7)
\end{aligned}
$$

$$
\begin{aligned}
GH_1^m[k,k'] &= \frac{\sqrt{P_1}}{N} \sum_{n=b}^{b+N-1} \sum_{l=1}^{L_{g_1}} g_{1,l}^m(nT_s) \sum_{q=1}^{L_{h_1}} h_{1,q}^m(nT_s-\tau_{g_1,l}) e^{j\frac{2\pi n}{N}(k'-k)} e^{-j\frac{2\pi k'}{NT_s}(\tau_{g_1,l}+\tau_{h_1,q})}, \\
GH_2^{m-BD}[k,k'] &= \frac{\sqrt{P_2}}{N} \sum_{n=b}^{b+N-1} \sum_{l=1}^{L_{g_2}} g_{2,l}^m(nT_s) \sum_{q=1}^{L_{h_2}} h_{2,q}^{m-BD}(nT_s-d_r-\tau_{g_2,l}) e^{j\frac{2\pi}{NT_s}\left[n(k'-k)T_s-k'(d_r+\tau_{g_2,l}+\tau_{h_2,q})\right]}, \qquad (8)
\end{aligned}
$$

## IV. A Modified Amplify and Forward Relaying Scheme

In Section III, we presented a new scheme which achieves the delay diversity structure for $BD > 0$; however, for $BD = 0$, the scheme does not provide spatial diversity. To address this limitation, we present a slightly modified version of the proposed scheme in this section which achieves the delay diversity structure for large values of the relative delay $D$, i.e., $BD \geq 1$, and also provides diversity for small values of $D$, i.e., $BD = 0$.

We still employ full duplex amplify and forward relay nodes. Similar to the scheme described in Section III, the second relay simply amplifies and forwards its received signal. The only modification is at the first relay in which instead of forwarding the received signal unchanged, a complex conjugated version of the received signal is amplified and forwarded to the destination. At the receiver, if the signal from the second relay is received $D$ seconds later than the signal from the first relay, by following the same steps as in Section III-B, we can write

$$
\begin{aligned}
Y_k^m &= \boldsymbol{GH}_2^{m-BD}(k)\boldsymbol{X}^{m-BD} + Z_k^m + \sqrt{\frac{P_1}{N}} \sum_{n=b}^{b+N-1} e^{-j\frac{2\pi n}{N}k} \times \\
&\left[ \sum_{l=1}^{L_{g_1}} g_{1,l}^m(nT_s) \sum_{q=1}^{L_{h_1}} h_{1,q}^m(nT_s-\tau_{g_1,l})^* \bar{x}^m(nT_s-\tau_{h_1,q}-\tau_{g_1,l})^* \right] \\
&= \overline{\boldsymbol{GH}}_1^m(k)\tilde{\boldsymbol{X}}^m + \boldsymbol{GH}_2^{m-BD}(k)\boldsymbol{X}^{m-BD} + Z_k^m, \qquad (10)
\end{aligned}
$$

where $\overline{\boldsymbol{GH}}_i^m(k) = [\overline{GH}_i^m[k,0], \cdots, \overline{GH}_i^m[k,N-1]]$ with

$$
\begin{aligned}
\overline{GH}_1^m[k,k'] &= \frac{\sqrt{P_1}}{N} \sum_{n=b}^{b+N-1} \sum_{l=1}^{L_{g_1}} g_{1,l}^m(nT_s) \times \\
&\times \left[ \sum_{q=1}^{L_{h_1}} h_{1,q}^{m\,*}(nT_s-\tau_{g_1,l}) e^{j\frac{2\pi n}{N}(k'-k)} e^{-j\frac{2\pi k'}{NT_s}(\tau_{g_1,l}+\tau_{h_1,q})} \right],
\end{aligned}
$$

$\boldsymbol{GH}_2^{m-BD}(k)$ is as given in (7), $\tilde{\boldsymbol{X}}^m = [X_0^*, X_{N-1}^*, \ldots, X_1^*]$, and $Z_k^m$ has the same statistical properties as in (7). Obviously, since $h_{1,q}^{m\,*}(nT_s-\tau_{g_1,l})$ and $h_{1,q}^m(nT_s-\tau_{g_1,l})$ have the same probability density function, then $\overline{GH}_1^m[k,k']$ and $GH_1^m[k,k']$ have the same density function as well.

For the block fading scenario, where $\overline{GH}_1^m(k,k') = 0$ and $GH_2^m(k,k') = 0$ for $k \neq k'$ and all $m$, we arrive at

$$
Y_k^m = \overline{GH}_{1,k}^m X_{N-k}^{m\,*} + GH_{2,k}^{m-BD} X_k^{m-BD} + Z_k^m \qquad (11)
$$

for $k \neq 0$, and $Y_0^m = \overline{GH}_{1,0}^m X_0^{m*} + GH_{2,0}^{m-BD} X_0^{m-BD} + Z_0^m$ for $k = 0$.

For $BD = 0$, if we focus on $Y_k^m$ and $Y_{N-k-1}^m$ ($k \neq 0, \frac{N}{2}$), then we have

$$
\begin{bmatrix} Y_k^m \\ Y_{N-k}^{m\,*} \end{bmatrix} = \boldsymbol{C}_k^m \begin{bmatrix} X_k^m \\ X_{N-k}^{m\,*} \end{bmatrix} + \begin{bmatrix} Z_k^m \\ Z_{N-k}^{m\,*} \end{bmatrix} \qquad (12)
$$

with

$$
\boldsymbol{C}_k^m = \begin{bmatrix} GH_{2,k}^m & \overline{GH}_{1,k}^m \\ \overline{GH}_{1,N-k}^{m\,*} & GH_{2,N-k}^{m\,*} \end{bmatrix}.
$$

Therefore, based on the optimal maximum likelihood (ML) detection criteria (assuming $[Z_k^m, Z_{N-k}^{m\,*}]$ as white Gaussian noise), for the optimal detector, we obtain

$$
\begin{aligned}
[\hat{X}_k^m, \hat{X}_{N-k}^m] =& \underset{X_k^m, X_{N-k}^m}{\arg\max} \Bigg[ \mathrm{Re}\left\{ [Y_k^{m*}, Y_{N-k}^m] \boldsymbol{C}_k^m \begin{bmatrix} X_k^m \\ X_{N-k}^{m\,*} \end{bmatrix} \right\} \\
&- \frac{1}{2} [X_k^{m*}, X_{N-k}^m] \boldsymbol{C}_k^{mH} \boldsymbol{C}_k^m \begin{bmatrix} X_k^m \\ X_{N-k}^{m\,*} \end{bmatrix} \Bigg],
\end{aligned}
$$

which offers spatial diversity. Note that for $k = 0$ and $k = \frac{N}{2}$, no diversity is provided; however, in detection of the remaining sub carriers spatial diversity is extracted out of the proposed system. The worst case is to not occupy the sub-carriers $k = 0$ and $k = \frac{N}{2}$ for data transmission which results in a very small loss in rates, e.g., in an OFDM transmission with $N = 1024$ subcarriers, the system experiences a rate loss of less than $0.2\%$.

On the other hand, for $BD > 0$, the received signal preserves the delay diversity structure presented in Section II. Obviously, for $k \in \{0, \frac{N}{2}\}$, $Y_0^m$ depends only on $X_k^m$ and $X_k^{m-BD}$ and the delay diversity structure is similar to the previous scheme. For $k \neq 0$ and $\frac{N}{2}$, by focusing on $X_k^m$ and $X_{N-k}^m$ and considering the block-fading case, we have

$$\begin{bmatrix} Y_k^1 \\ Y_{N-k}^{1+BD*} \\ Y_k^{1+2BD} \\ \vdots \end{bmatrix} = \begin{bmatrix} \overline{GH}_{1,k}^1 X_{N-k}^{1}{}^{*} & \\ \overline{GH}_{1,k}^{1+BD*} X_k^{1+BD} & + \; GH_{2,N-k}^{1}{}^{*} X_{N-k}^{1}{}^{*} \\ \overline{GH}_{1,k}^{1+2BD} X_{N-k}^{1+2BD*} & + \; GH_{2,k}^{1+BD} X_k^{1+BD} \\ & \vdots \end{bmatrix} + \begin{bmatrix} Z_k^1 \\ Z_{N-k}^{1+BD*} \\ Z_k^{1+2BD} \\ \vdots \end{bmatrix}.$$

Therefore, we expect to achieve the same performance as the scheme introduced in Section II. Moreover, by this new scheme we are also able to extract diversity out of the system for $BD = 0$.

## V. Pairwise Error Probability Analysis

Design of the space-time codes is out of the scope of this work; however, we present the PEP performance analysis of the system under quasi-static and block fading frequency selective channel conditions which can be useful in a diversity order analysis of the proposed scheme and possible space-time code designs. In the following, we present the PEP analysis for the quasi-static and block fading frequency selective channels, respectively.

### *A. Quasi-Static Frequency-Selective Channels*

In this section, we consider the PEP performance analysis for ML detection presented in Section III-D. We provide the result under the condition that the channels from the source to the relays have significantly higher SNRs than the channels from the relays to the destination, i.e., $\frac{1}{\sigma_{1,i}^2} \gg \frac{P_i}{\sigma_2^2}$. We assume that the channels are quasi-static Rayleigh fading, i.e., the channel gains in time domain are random variables but fixed for the transmission of $M$ consecutive OFDM blocks. We denote $h_{i,l}^m(nT_s) = h_{i,l}$ and $g_{i,l}^m(nT_s) = g_{i,l}$ for $n = \{0, \ldots, N-1\}$ and $m = \{1, \ldots, M\}$, where $h_{i,l}$ and $g_{i,l}$ are zero mean circularly symmetric complex Gaussian random variables with variances of $\sigma_{h_i,l}^2$ and $\sigma_{g_i,l}^2$, respectively, with $\sum_{l=1}^{L_{h_i}} \sigma_{h_i,l}^2 = 1$ and $\sum_{l=1}^{L_{g_i}} \sigma_{g_i,l}^2 = 1$. For the PEP of the under consideration scenario, we obtain (see Appendix A),

$$P\left(\boldsymbol{X}_k \to \boldsymbol{X}_k'\right) \leq \frac{8\sigma_2^4}{P_1 P_2 \left(s_k^4 - f_k^4\right)} \log\left(1 + \frac{P_1}{4\sigma_2^2}\sqrt{s_k^4 - f_k^4}\right) \log\left(1 + \frac{P_2}{4\sigma_2^2}\sqrt{s_k^4 - f_k^4}\right), \tag{13}$$

where $f_k^2 = \left|\sum_{m=BD+1}^{M} (X_k^m - X_k'^m)(X_k^{m-BD} - X_k'^{m-BD})^*\right|$ and $s_k^2 = \sum_{m=1}^{M} |X_k^m - X_k'^m|^2$. We observe from (13) that the system achieves the diversity order of 2. For instance, for $P_1 = P_2$, we have $SNR = \frac{2P_1}{\sigma_2^2}$ and $P\left(\boldsymbol{X}_k \to \boldsymbol{X}_k'\right) \leq \frac{32 SNR^{-2}}{(s_k^4 - f_k^4)} \log\left(1 + \frac{SNR}{8}\sqrt{s_k^4 - f_k^4}\right)^2$.

### *B. Block Fading Frequency-Selective Channels*

In this section, we analyze the PEP performance of the proposed scheme under block fading frequency selective channels. Similar to the analysis for the quasi-static fading scenario, we assume that the channels from the source to the relays have significantly higher SNRs than the channels from the relays to the destination. We first give the considered block fading channel model. We then provide a discussion on the discrete noise samples at the destination under the block fading channels and at the end provide the PEP analysis for which similar to the quasi-static channel conditions, we assume that no coding is employed over the subcarriers and focus on the spatial diversity analysis of the system.

*1) Block Fading Frequency-Selective Channel Model:* Here, we follow the same channel model and procedure used in [14] in which the PEP performance analysis of space-time coded OFDM multi-input multi-output (MIMO) system over correlated block fading channels has been considered. The main difference between the system model in [14] and the one in this paper is in the effective channel model and the noise experienced at the destination. In fact, we need to make some simplifying approximations to be able to derive a closed form upper bound on the PEP of the system. By approximating the received noise samples $Z_k^m$ as complex Gaussian random variables (see Section V-B2), we provide a PEP analysis under the block fading channel scenario in which channel coefficients are fixed during each block transmission and change block by block based on the following Fourier expansion relation [15] (for ease of presentation we assume that $L_{g_i} = L_{h_i} = L$, $\tau_{h_i,l} = \tau_{g_i,l} = \tau_l$ and all the channels experiencing the same Doppler frequency shift $f_d$)

$$h_{i,l}^m(t) = h_{i,l}^m \approx \sum_{n=-\frac{L_t-1}{2}}^{\frac{L_t-1}{2}} \alpha_{i,l}[n] e^{j\frac{2\pi n(mT)}{MT}}, \tag{14}$$

and

$$g_{i,l}^m(t) = g_{i,l}^m \approx \sum_{n=-\frac{L_t-1}{2}}^{\frac{L_t-1}{2}} \beta_{i,l}[n] e^{j\frac{2\pi n(mT)}{MT}}, \tag{15}$$

in which $\alpha_{i,l}[n]$ and $\beta_{i,l}[n]$ are independent circularly symmetric complex Gaussian random variables with zero mean and variance of $\frac{\sigma_{h_i,l}^2}{L_t}$ and $\frac{\sigma_{h_i,l}^2}{L_t}$, respectively, with $L_t = \lceil 2f_dMT + 1\rceil$. $h_{i,l}^m$ and $g_{i,l}^m$ can also be represented as $h_{i,l}^m = \boldsymbol{\alpha}_i(l)^T \boldsymbol{w}_t(m)$ and $g_{i,l}^m = \boldsymbol{\beta}_i(l)^T \boldsymbol{w}_t(m)$, where $\boldsymbol{w}_t(m) = [e^{-j2\pi Mf_dT}, \ldots, 1, \ldots, e^{j2\pi Mf_dT}]^T$, $\boldsymbol{\alpha}_i(l) = \left[\alpha_{i,l}[-\frac{L_t-1}{2}], \ldots, \alpha_{i,l}[\frac{L_t-1}{2}]\right]^T$, and $\boldsymbol{\beta}_i(l) = \left[\beta_{i,l}[-\frac{L_t-1}{2}], \ldots, \beta_{i,l}[\frac{L_t-1}{2}]\right]^T$.

Since all the channels are block fading, for $GH_1^m[k,k']$ and $GH_2^m[k,k']$, we have

$$GH_1^m[k,k'] = \sqrt{P_1}\left[\sum_{l=1}^{L_{g_1}} \sum_{q=1}^{L_{h_1}} g_{1,l}^m h_{1,q}^m e^{-\frac{2\pi jk'(\tau_{h_1,q} + \tau_{g_1,l})}{NT_s}}\right]\delta(k-k'),$$

$$GH_2^m[k,k'] = \sqrt{P_2} e^{-2\pi j\frac{k'}{NT_s}d_r}\left[\sum_{l=1}^{L_{g_2}} g_{2,l}^m e^{-2\pi j\frac{k'\tau_{g_2,l}}{NT_s}} \sum_{q=1}^{L_{h_2}} h_{2,q}^m e^{-2\pi j\frac{k'\tau_{h_2,q}}{NT_s}}\right]\delta(k-k').$$

$GH_i^m[k,k]$ can also be written as $GH_2^m[k,k] = G_{2,k}^m H_{2,k}^m e^{-2\pi j\frac{k'}{NT_s}d_r}$ and $GH_1^m[k,k] = G_{1,k}^m H_{1,k}^m$ with $H_{i,k}^m = \sum_{l=1}^{L} h_{i,l} e^{-j\frac{2\pi k\tau_l}{NT_s}} = \boldsymbol{h}_i^T(m)\boldsymbol{w}_f(k)$, $G_{i,k}^m = \sum_{l=1}^{L} g_{i,l} e^{-j\frac{2\pi k\tau_l}{NT_s}} = \boldsymbol{g}_i^T(m)\boldsymbol{w}_f(k)$ where $\boldsymbol{h}_i(m) = [h_{i,1}^m, \cdots, h_{i,L}^m]^T$, $\boldsymbol{g}_i(m) = [g_{i,1}^m, \cdots, g_{i,L}^m]^T$

and $\boldsymbol{w}_f(k) = [e^{-j\frac{2\pi k\tau_1}{NT_s}}, \ldots, e^{-j\frac{2\pi k\tau_L}{NT_s}}]^T$. On the other hand, by defining $\boldsymbol{W}_t(m) = diag\{\boldsymbol{w}_t(m), \ldots, \boldsymbol{w}_t(m)\}_{LL_t\times L}$, $\boldsymbol{v}_i = [\boldsymbol{\alpha}_i^T(1), \ldots, \boldsymbol{\alpha}_i^T(L)]^T$, $\boldsymbol{q}_1 = [\boldsymbol{\beta}_1^T(1), \ldots, \boldsymbol{\beta}_1^T(L)]^T$ and $\boldsymbol{q}_2 = [\boldsymbol{\beta}_2^T(1), \ldots, \boldsymbol{\beta}_2^T(L)]^T e^{-2\pi j\frac{k'}{NT_s}d_r}$ we obtain $H_{i,k}^m = \boldsymbol{v}_i^T \boldsymbol{W}_t(m)\boldsymbol{w}_f(k)$ and $G_{i,k}^m = \boldsymbol{q}_i^T \boldsymbol{W}_t(m)\boldsymbol{w}_f(k)$.

*2) Discussion on the Statistics of the Noise Samples:* For the block fading scenario, we have $h_{i,l}^m(t) = h_{i,l}^m$ and $g_{i,l}^m(t) = g_{i,l}^m$, hence we can write

$$Z_k^m = \frac{1}{\sqrt{N}} \sum_{n=b}^{b+N-1} \Bigg[ z_2^m(nT_s) + \sum_{l=1}^{L_{g_1}} \sqrt{P_1} g_{1,l}^m z_{1,1}^m(nT_s - \tau_{g_1,l}) + \sum_{l=1}^{L_{g_2}} \sqrt{P_2} g_{2,l}^m z_{1,2}(nT_s - \tau_{g_2,l}) \Bigg] e^{-\frac{j2\pi n}{N}k}.$$

Since the noise samples from the OFDM block durations $m$ and $m'$ ($m \neq m'$) are independent then obviously $E\{Z_k^m Z_k^{m'}\} = 0$. Furthermore, for $E\{|Z_k^m|^2\}$, we can write

$$\begin{aligned} E\{|Z_k^m|^2\} = \frac{1}{N} E\Bigg\{ &\sum_{n=b}^{b+N-1} \sum_{v=b}^{b+N-1} \Bigg[ z_2^m(nT_s) \\ &+ \sum_{l=1}^{L_{g_1}} \sqrt{P_1} g_{1,l}^m z_{1,1}^m(nT_s - \tau_{g_1,l}) + \sum_{l=1}^{L_{g_2}} \sqrt{P_2} g_{2,l}^m z_{1,2}^m(nT_s - \tau_{g_2,l}) \Bigg] \\ &\times \Bigg[ z_2^{m*}(vT_s) + \sum_{i=1}^{L_{g_1}} \sqrt{P_1} {g^m}_{1,i}^* {z_{1,1}^m}^*(vT_s - \tau_{g_1,i}) \\ &+ \sum_{i=1}^{L_{g_2}} \sqrt{P_2} {g^m}_{2,i}^* {z_{1,2}^m}^*(vT_s - \tau_{g_2,i}) \Bigg] e^{-\frac{j2\pi(n-v)}{N}k} \Bigg\}. \end{aligned}$$

By using the facts that $z_{j,i}(t)$ are zero mean independent Gaussian random processes (as a result $E\{z_{1,1}(t)z_{1,2}(t')\} = 0$ for all $t$ and $t'$) and $E\{z_{j,i}(t)z_{j,i}(t')\} = \sigma_{j,i}^2\delta(t - t')$, we obtain

$$\begin{aligned} E\{|Z_k^m|^2\} = &\frac{1}{N} \sum_{n=b}^{b+N-1} \sum_{v=b}^{b+N-1} \Bigg[ \sigma_2^2 \delta(n - v) \\ &+ P_1 \sum_{l=1}^{L_{g_1}} \sum_{i=1}^{L_{g_1}} g_{1,l}^m {g_{1,i}^m}^* \sigma_{1,1}^2 \delta(nT_s - \tau_{g_1,l} - vT_s + \tau_{g_1,i}) \\ &+ P_2 \sum_{l=1}^{L_{g_2}} \sum_{i=1}^{L_{g_2}} g_{2,l}^m {g_{2,i}^m}^* \sigma_{1,2}^2 \delta(nT_s - \tau_{g_2,l} - vT_s + \tau_{g_2,i}) \Bigg] e^{-\frac{j2\pi(n-v)}{N}k} \end{aligned}$$

which leads to

$$\begin{aligned} E\{|Z_k^m|^2\} = \sigma_2^2 &+ P_1\sigma_{1,1}^2 \sum_{l=1}^{L_{g_1}} |g_{1,l}^m|^2 + P_2\sigma_{1,2}^2 \sum_{l=1}^{L_{g_2}} |g_{2,l}^m|^2 \\ &+ 2\sigma_{1,1}^2 Re\Bigg[ \sum_{f_1, \tau_{g_1,l} - \tau_{g_1,i} = f_1 T_s} g_{1,l}^m {g_{1,i}^m}^* e^{-\frac{j2\pi k}{N} f_1} \Bigg] \\ &+ 2\sigma_{1,2}^2 Re\Bigg[ \sum_{f_2, \tau_{g_2,l} - \tau_{g_2,i} = f_2 T_s} g_{2,l}^m {g_{2,i}^m}^* e^{-\frac{j2\pi k}{N} f_2} \Bigg], \end{aligned}$$

where $f_1$ and $f_2$ only take positive integer values. Since $Z_k^m$ are independent for a specific $k$ but not identically distributed, the optimal ML detection can be obtained by employing Viterbi detector over the normalized received signals according to $E\{|Z_k^m|^2\}$. However, in the following, we provide the PEP analysis by approximating the received noise samples $Z_k^m$ as i.i.d. complex Gaussian random variables with zero mean and variance of $\sigma^2$ to match with the sub-optimal detector we considered for the general time-varying channel case in Section III-D.

*3) PEP Analysis:* Conditioned on known channel state information at the receiver, for the considered Viterbi detector[1], we have

$$\boldsymbol{X}'_k = \arg\min_{\boldsymbol{X}_k} \sum_{m=1}^{M+BD} \left| Y_k^m - GH_{1,k}^m X_k^m - GH_{2,k}^m X_k^{m-BD} \right|^2,$$

where $X_k^m = 0$ for $m < 1$ or $m > M$, and $GH_{1,k}^m = G_{1,k}^m H_{1,k}^m$ and $GH_{2,k}^m = G_{2,k}^m H_{2,k}^m e^{-2\pi j\frac{k'}{NT_s}d_r}$. Therefore, under the assumption that $\frac{1}{\sigma_{1,i}^2}$ are sufficiently larger than $\frac{P_i}{\sigma_2^2}$, we obtain (see Appendix B)

$$\begin{aligned} P(\boldsymbol{X}_k \to \boldsymbol{X}'_k) &\leq \frac{1}{2} E\left\{ e^{-\frac{1}{4\sigma_2^2}\sum_{c=1}^{r_k} \lambda_{k,c} \left| \boldsymbol{U}_{k,c}^H \boldsymbol{q}(k) \right|^2} \right\} \\ &\leq \frac{1}{2} \prod_{c=1}^{r_k} PEP_{k,c}, \end{aligned} \tag{16}$$

where

$$\begin{aligned} PEP_{k,c} &= E_V\left\{ \frac{2\sigma_2^2}{2\sigma_2^2 + \lambda_{k,c}V} \right\} \\ &= \sum_{p\in S_0} \frac{\pi_p}{\sigma_{\chi,k,p}^2 \sigma_{q,p}^2} \frac{2\sigma_2^2}{\lambda_{k,c}} e^{\frac{2\sigma_2^2}{\lambda_{k,c}\sigma_{\chi,k,p}^2\sigma_{q,p}^2}} E_1\left( \frac{2\sigma_2^2}{\lambda_{k,c}\sigma_{\chi,k,p}^2\sigma_{q,p}^2} \right) \\ &+ \sum_{j=1}^{J} \sum_{p\in S_j} \frac{(2\sigma_2^2)^{N_j}(-1)^{N_j-1}}{(N_j-1)!(\lambda_{k,c}\sigma_{\chi,k,p}^2\sigma_{q,p}^2)^{N_j}} \times \\ &\times \Bigg[ e^{\frac{2\sigma_2^2}{\lambda_{k,c}\sigma_{\chi,k,p}^2\sigma_{q,p}^2}} E_1\left( \frac{2\sigma_2^2}{\lambda_{k,c}\sigma_{\chi,k,p}^2\sigma_{q,p}^2} \right) \\ &+ \sum_{k=1}^{N_j-1} (k-1)! \left( \frac{-\lambda_{k,c}\sigma_{\chi,k,p}^2\sigma_{q,p}^2}{2\sigma_2^2} \right)^k \Bigg]. \end{aligned} \tag{17}$$

## VI. Simulation Results

To provide numerical examples we assume that the total occupied bandwidth is 8 kHz (over the frequency band from 12 kHz to 20 kHz). We define $f_d$ as the Doppler frequency shift observed at the destination in Hz, and $\sigma_{h_i} = [\sigma_{h_i,0}, \ldots, \sigma_{h_i,L_{h_i}}]$. We assume $P_i = 1$, $\tau_{h_i,l} = \tau_{g_i,l} = lT_s = 125l$ $\mu$s, and $\sigma_{i,1}^2 = 2\sigma_2^2$ ($i \in \{1, 2\}$).

In Figs. 7 and 8, we compare the performance of the proposed scheme with the performance of the scheme proposed in [7] for different values of $f_dT_s$ under two different scenarios where quadrature phase-shift keying (QPSK) modulated symbols are transmitted over $N$ subcarriers. The parameters of the two scenarios are reported in Table I in which to make a fair comparison, both schemes are set to the same data transmission rate. We generate time varying Rayleigh fading channel tap gains following the Jakes' model [16]. We chose [7] for comparison since it also considers an OFDM

[1]For the considered block fading channels, the optimal ML detection is obtained by normalizing the received signals over each subcarrier with variance of its corresponding noise; however, we present the result for the case that there is no normalization to match with the Viterbi detection of the general time-varying case.

TABLE I
PARAMETERS OF TWO DIFFERENT SCENARIOS USED TO COMPARE THE PROPOSED SCHEME WITH THE SCHEME IN [7].

| Scenario | Scheme | $N$ | $M$ | $D$ | $\sigma_{h_i} = \sigma_{g_i}$ $(j, i \in \{1, 2\})$ | $N_{CP}$ | $T$ (ms) | Data Rate (kb/s) |
|---|---|---|---|---|---|---|---|---|
| $S_1$ | Proposed Scheme | 512 | 100 | $1039T_s$ | $[1,\ 0.8,\ 0.6]/\sqrt{2}$ | 522 | 129.25 | 7.9226 |
| | Scheme from [7] | 1024 | 2 | $1039T_s$ | $[1,\ 0.8,\ 0.6]/\sqrt{2}$ | 1044 | 258.5 | 7.9226 |
| $S_2$ | Proposed Scheme | 256 | 100 | $527T_s$ | $[0.8,\ 0,\ 0.6]$ | 266 | 65.25 | 7.8467 |
| | Scheme from [7] | 512 | 2 | $527T_s$ | $[0.8,\ 0,\ 0.6]$ | 532 | 130.5 | 7.8467 |

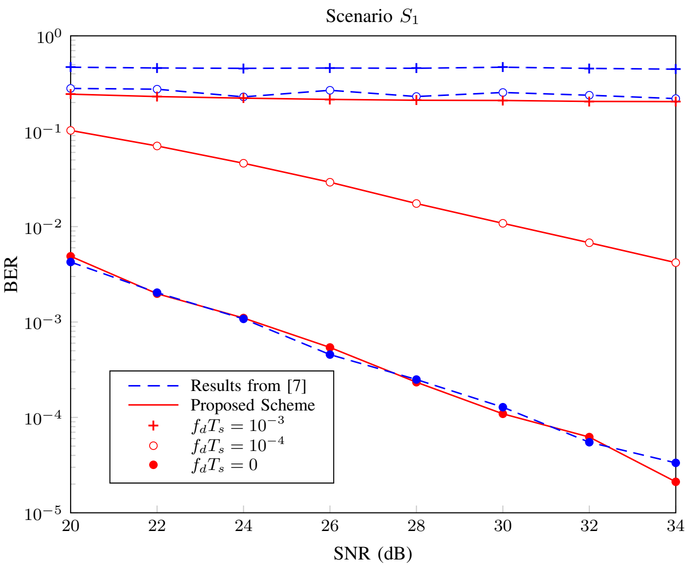


Fig. 7. Comparison between the performance of the proposed scheme with the scheme proposed in [7] under the scenario $S_1$.

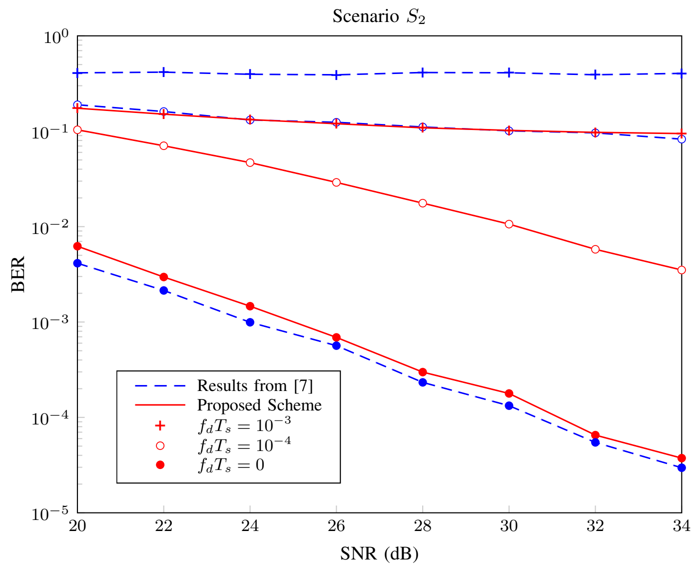


Fig. 8. Comparison between the performance of the proposed scheme with the scheme proposed in [7] under the scenario $S_2$.

based cooperative transmission with full-duplex AF relays. However, in [7], the relays perform time reversal and symbol complex conjugation as well. In Figs. 7 and 8, we observe that for the time-invariant channel case ($f_dT_s = 0$), the performance of both schemes are identical. However, for time varying scenarios, the proposed scheme outperforms the scheme proposed in [7]. The reason is that, for the range of the relative delays considered, to attain the same data rate for both schemes, the scheme proposed in [7] transmits longer OFDM blocks (larger $N$) and as a result more ICI is experienced over the received subcarriers. Obviously, by increasing $f_d$, i.e., faster fading conditions, more ICI are experienced over the subcarriers and the performance becomes worse. We observe that in the SNR range considered, the bit error rate (BER) of the fast fading scenario ($f_dT_s = 10^{-3}$) reaches an error floor. We expect that for higher SNR values, the slow fading scenario ($f_dT_s = 10^{-4}$) converges to an error floor as well. In both cases, we employ a suboptimal Viterbi decoder to detect the transmitted signal, which assumes that the noise samples over the same subcarrier of different blocks are i.i.d. and ignores the ICI effects to reduce complexity.

In Fig. 9, we compare the PEP performance of the proposed scheme with the derived upper bounds for the quasi-static frequency selective channel. We consider transmission of $M = 10$ OFDM blocks with $N = 64$ binary phase-shift keying (BPSK) modulated subcarriers over multipath channels with $\sigma_{h_i} = \sigma_{g_i} = \frac{[1, 0.8, 0.6]}{\sqrt{2}}$ where there is a relative delay of 145 $T_s$ seconds between the signals received from the two relay nodes, i.e., $BD = 1$. Furthermore, we assume that $P_1 = P_2 = 1$, $\sigma^2_{1,1} = \sigma^2_{1,2} = \frac{\sigma_2^2}{10}$. Therefore, the $SNR$ of the system at the receiver is $\frac{5}{3\sigma_2^2}$. We consider

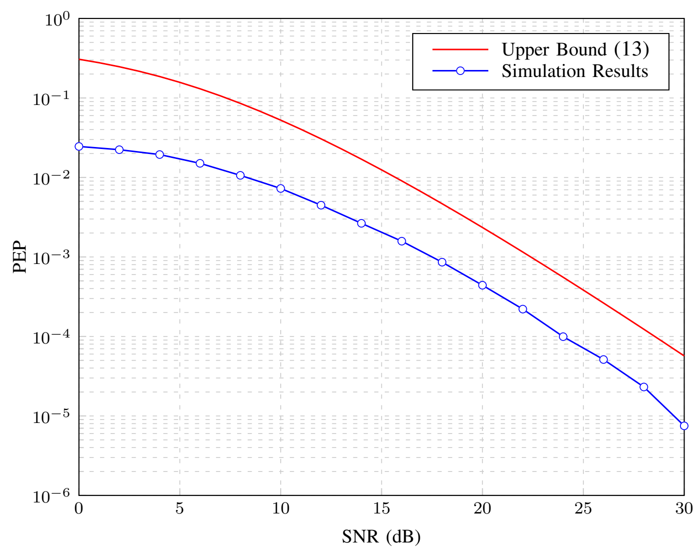


Fig. 9. Comparison between the upper bound (13) and actual PEP for $\boldsymbol{X}_k = \mathbf{1}_M$ and $\boldsymbol{X}'_k = [-1, \mathbf{1}^T_{M-2}, -1]^T$ under quasi-static frequency selective channels.

$\boldsymbol{X}_k = \mathbf{1}_M$, where $\mathbf{1}_M$ represents an all one vector, and $\boldsymbol{X}'_k = [\mathbf{1}^T_{\frac{M}{2}-1}, -1, \mathbf{1}^T_{\frac{M}{2}}]^T$. Note that the considered case is the worst case scenario, i.e., gives the maximum PEP among all the possible pairwise error events ($s^4_k - f^4_k$ is minimized). We observe in Fig. 9 that by increasing SNR we achieve a tighter upper bound on PEP. Furthermore, as we expect for high SNR values, the diversity order of the system is 2 which is due to the delay diversity structure of the system.

In Fig. 10, we compare the derived upper bound on $P(\boldsymbol{X}_k \to \boldsymbol{X}'_k)$, for $\boldsymbol{X}_k = \mathbf{1}_{10}$ and several different $\boldsymbol{X}'_k$ as

| | Case I | Case II | Case III | Case IV |
|---|---|---|---|---|
| $\boldsymbol{X}'_k$ | $[-1, \mathbf{1}_9^T]^T$ | $[-1, \mathbf{1}_8^T, -1]^T$ | $[-\mathbf{1}_5^T, \mathbf{1}_5^T]$ | $[-1, 1, -1, 1, -1, 1, -1, 1, -1, 1]^T$ |
| $D(\boldsymbol{X}_k, \boldsymbol{X}'_k)$ | 2 | 4 | 6 | 10 |

TABLE II
DIFFERENT CASES CONSIDERED IN FIG. 10 WITH $\boldsymbol{X}_k = \mathbf{1}_{10}$.

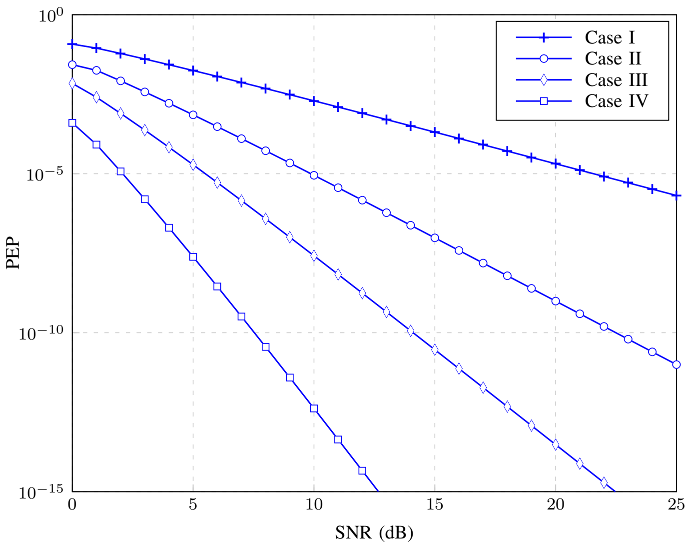


Fig. 10. Comparison between the upper bound (34) for $\boldsymbol{X}_k = \mathbf{1}_{10}$ and $\boldsymbol{X}'_k$ as given in Table II.

given in Table II under the block fading channel conditions considered in Section V-B1 with $f_d T_s = 0.01$. We consider the same transmission specs as considered in the study given in Fig. 9. As we expected, larger $D(\boldsymbol{X}_k, \boldsymbol{X}'_k)$ (as defined in (28)) results in a higher diversity order and a better performance which shows the importance of designing appropriate codes to extract the maximum possible diversity out of the system.

## VII. CONCLUSIONS

We developed a new OFDM transmission scheme for UWA cooperative communication systems suffering from asynchronism among the relays by considering possibly large relative delays among the relays (typical in UWA systems) and time-varying frequency selective channels among the cooperating nodes. The main advantage of the proposed scheme is in managing the asynchronism issues arising from excessively large delays among the relays without adding time guards (or CP in OFDM-based transmissions) in the order of the maximum possible delay, which increases the spectral efficiency of the system and improves the performance in time-varying channel conditions compared with the existing solutions in the literature. In fact, we showed that independent of the maximum possible delay between the relays, by adding an appropriate CP at the transmitter and appropriate CP removal at the receiver, a delay diversity structure can be obtained at the receiver, where a full-duplex AF scheme is utilized at the relays. Through numerical examples, we evaluated the performance of the proposed scheme for time-varying multipath channels with Rayleigh fading channel taps, modeling UWA channels. We compared our results with those of the existing schemes and found that while for time invariant channels, the performance is similar, for time varying cases (typical in UWA communications) the proposed scheme is significantly superior.

## APPENDIX A
## PEP ANALYSIS FOR QUASI-STATIC FREQUENCY-SELECTIVE CHANNELS

Under the quasi-static channel conditions, for the channel gains in frequency domain, we have

$$GH_1^m[k, k'] = \sqrt{P_1}\left[\sum_{l=1}^{L_{g_1}}\sum_{q=1}^{L_{h_1}} g_{1,l} h_{1,q} e^{-\frac{2\pi j k'(\tau_{h_1,q}+\tau_{g_1,l})}{NT_s}}\right]\delta(k-k'),$$

and

$$GH_2[k, k'] = \sqrt{P_2}\left[\sum_{l=1}^{L_{g_2}} g_{2,l} e^{-2\pi j \frac{k'\tau_{g_2,l}}{NT_s}}\right] \times$$
$$\times \left[\sum_{q=1}^{L_{h_2}} h_{2,q} e^{-2\pi j \frac{k'\tau_{h_2,q}}{NT_s}}\right] e^{-2\pi j \frac{k'}{NT_s} d_r}\delta(k - k'),$$

where for a fixed $k$, $G_{i,k} = \sum_{l=1}^{L_{g_i}} g_{i,l} e^{-2\pi j \frac{k\tau_{g_i,l}}{NT_s}}$ and $H_{i,k} = \sum_{q=1}^{L_{h_i}} h_{i,q} e^{-2\pi j \frac{k\tau_{h_i,q}}{NT_s}}$ are independent complex Gaussian random variables with zero mean and unit variance. Hence, for the received signal on the $k$-th subcarrier, we have

$$Y_k^m = GH_1[k, k]X_k^m + GH_2[k, k]X_k^{m-BD} + Z_k^m, \quad (18)$$

where conditioned on $g_{i,l}$ for all $l \in \{1, \ldots, L_{g_i}\}$ and $i \in \{1, 2\}$, $Z_k^m$ are i.i.d. complex Gaussian random variables with zero mean and variance of $\sigma_2^2 + P_1|G_{1,k}|^2\sigma_{1,1}^2 + P_2|G_{2,k}|^2\sigma_{1,2}^2$. The above relation for $BD > 0$ is a delay diversity structure which can be used to extract spatial diversity out of the relay system shown by the PEP analysis. If we define $\boldsymbol{Y}_k = [Y_k^1, \ldots, Y_k^{M+BD}]$, $\boldsymbol{Z}_k = [Z_k^1, \ldots, Z_k^{M+BD}]$, $\boldsymbol{GH}(k) = [GH_1[k, k], GH_2[k, k]]$ and

$$\boldsymbol{X}_k = \begin{bmatrix} X_k^1 & \cdots & X_k^{1+BD} & \cdots & X_k^M & \cdots & 0 \\ 0 & \cdots & X_k^1 & \cdots & X_k^{M-BD} & \cdots & X_k^M \end{bmatrix},$$

we can write $\boldsymbol{Y}_k = \boldsymbol{GH}(k)\boldsymbol{X}_k + \boldsymbol{Z}_k$. Note that our focus is on extracting spatial diversity out of the asynchronous cooperative system which is attained in the form of the delay diversity. In fact, we assume no explicit channel coding is employed across different subcarriers and as a result no multipath diversity is attained; however, it does not mean that the system does not achieve multipath diversity. Now, let us focus on a given subcarrier, e.g., $k$-th subcarrier, where conditioned on given $g_{i,l}$ for $l \in \{1, \ldots, L_{g_i}\}$ and $i \in \{1, 2\}$, $\boldsymbol{Z}_k$ is a complex white Gaussian vector with zero mean and autocorrelation matrix of $\left(\sigma_2^2 + P_1|G_{1,k}|^2\sigma_{1,1}^2 + P_2|G_{2,k}|^2\sigma_{1,2}^2\right)\boldsymbol{I}_{M+BD}$ with $\boldsymbol{I}_M$ denoting the $M$ by $M$ identity matrix. Therefore, the Viterbi algorithm proposed in Section III-D can be used as an ML detection scheme on symbols transmitted over the $k$-th subcarrier. Furthermore, by employing ML detection at the destination for the conditional PEP over the $k$-th subcarrier, $P\left(\boldsymbol{X}_k \to \boldsymbol{X}'_k|\boldsymbol{GH}(k)\right)$ which shows the probability of deciding in favor of $\boldsymbol{X}'_k$ at the receiver while $\boldsymbol{X}_k$ is the actual transmitted symbol conditioned on the channel realizations, we have

$$P\big(\boldsymbol{X}_k \to \boldsymbol{X}'_k|\boldsymbol{GH}(k)\big) \leq \frac{1}{2} e^{-\frac{d_k^2}{4\left(\sigma_2^2+P_1|G_{1,k}|^2\sigma_{1,1}^2+P_2|G_{2,k}|^2\sigma_{1,2}^2\right)}}, \tag{19}$$

where in deriving the last inequality the Chernoff bound is employed [17, p. 58], $d_k = ||\boldsymbol{GH}(k)(\boldsymbol{X}_k - \boldsymbol{X}'_k)||$ and $||\boldsymbol{e}||$ denotes the Euclidean length of vector $\boldsymbol{e}$. Therefore, we have

$$P\left(\boldsymbol{X}_k \to \boldsymbol{X}'_k\right) \leq \frac{1}{2} E_{d_k}\{e^{-\frac{d_k^2}{4\sigma^2}}\}, \tag{20}$$

where $E_{d_k}\{.\}$ denotes the expected value with respect to the random variable $d_k$. Under the assumption that $\frac{1}{\sigma_{1,i}^2} \gg \frac{P_i}{\sigma_2^2}$, we obtain $\frac{d_k^2}{\sigma^2} \simeq \frac{d_k^2}{\sigma_2^2}$. Furthermore, due to the definition of the Euclidean distance, by defining $\alpha_k^2 = \frac{P_1}{4\sigma_2^2}\sum_{m=1}^{M}|X_k^m - X_k'^m|^2$, $\beta_k^2 = \frac{P_2}{4\sigma_2^2}\sum_{m=1}^{M}|X_k^m - X_k'^m|^2$, and

$$\gamma_k^2 e^{-j\phi_{\gamma,k}} = \frac{\sqrt{P_1P_2}}{4\sigma_2^2}\sum_{m=BD+1}^{M}(X_k^m - X_k'^m)(X_k^{m-BD} - X_k'^{m-BD})^*,$$

we can write

$$\begin{aligned}\frac{d_k^2}{4\sigma_2^2} &= \frac{\boldsymbol{GH}(k)(\boldsymbol{X}_k - \boldsymbol{X}'_k)(\boldsymbol{X}_k - \boldsymbol{X}'_k)^H\boldsymbol{GH}^H(k)}{4\sigma_2^2} \\ &= \alpha_k^2\frac{|GH_1[k,k]|^2}{P_1} + \beta_k^2\frac{|GH_2[k,k]|^2}{P_2} \\ &+ 2Re\left\{\frac{|GH_1[k,k]|}{\sqrt{P_1}}e^{-j\phi_{1,k}}\frac{|GH_2[k,k]|}{\sqrt{P_2}}e^{j\phi_{2,k}}\gamma_k^2 e^{-j\phi_{\gamma,k}}\right\}.\end{aligned}$$

We also have $|GH_i[k,k]| = P_i|H_{i,k}||G_{i,k}|$, where $|H_{i,k}| \sim Rayleigh(\frac{\sqrt{2}}{2})$ and $|G_{i,k}| \sim Rayleigh(\frac{\sqrt{2}}{2})$, i.e., $|H_{i,k}|$ and $|G_{i,k}|$ are Rayleigh distributed random variables, and $\phi_{i,k}$ are uniformly distributed random variables over $[0, 2\pi]$. If we define $a_i = |H_{i,k}|$ and $b_i = |G_{i,k}|$, then $\frac{d_k^2}{4\sigma_2^2} = \alpha_k^2 a_1^2 b_1^2 + \beta_k^2 a_2^2 b_2^2 + 2a_1a_2b_1b_2\gamma_k^2\cos(\phi)$, where $\phi = \angle e^{-j(\phi_{1,k}-\phi_{2,k}+\phi_{\gamma,k})}$ is uniformly distributed over $[0, 2\pi]$. Therefore, we have

$$\begin{aligned}E_{d_k}\left\{e^{-\frac{d_k^2}{4\sigma_2^2}}\right\} &= E_{a_i,b_i,\phi}\left\{e^{-(\alpha_k^2a_1^2b_1^2+\beta_k^2a_2^2b_2^2+2a_1a_2b_1b_2\gamma_k^2\cos(\phi))}\right\} \\ &= E_{a_1,a_2,b_1,b_2}\left\{\frac{1}{2\pi}\int_0^{2\pi}e^{-(\alpha_k^2a_1^2b_1^2+\beta_k^2a_2^2b_2^2+2a_1a_2b_1b_2\gamma_k^2\cos(\phi))}d\phi\right\} \\ &=^a E_{a_1,a_2,b_1,b_2}\left\{e^{-(\alpha_k^2a_1^2b_1^2+\beta_k^2a_2^2b_2^2)}I_0(2a_1a_2b_1b_2\gamma_k^2)\right\} \\ &= E_{a_2,b_1,b_2}\left\{e^{-\beta_k^2a_2^2b_2^2}\int_0^{\infty}e^{-\alpha_k^2a_1^2b_1^2}I_0(2a_1a_2b_1b_2\gamma_k^2)2a_1e^{-a_1^2}da_1\right\}\end{aligned}$$

where the equality $a$ holds due to the definition of the modified Bessel function of the first kind $I_0(.)$. Following the result from [18, p. 294. Eq. 2. 15. 1. 2.] we can write

$$\begin{aligned}E_{d_k}\left\{e^{-\frac{d_k^2}{4\sigma_2^2}}\right\} &= E_{a_2,b_1,b_2}\left\{\frac{e^{-\beta_k^2a_2^2b_2^2}}{\alpha_k^2b_1^2+1}e^{\frac{a_2^2b_1^2b_2^2\gamma_k^4}{\alpha_k^2b_1^2+1}}\right\} \\ &= E_{b_1,b_2}\left\{\int_0^{\infty}\frac{e^{-\beta_k^2a_2^2b_2^2}}{\alpha_k^2b_1^2+1}e^{\frac{a_2^2b_1^2b_2^2\gamma_k^4}{\alpha_k^2b_1^2+1}}2a_2e^{-a_2^2}da_2\right\} \\ &= E_{b_1,b_2}\left\{\frac{1}{1+\alpha_k^2b_1^2+b_1^2b_2^2\alpha_k^2\beta_k^2+\beta_k^2b_2^2-b_1^2b_2^2\gamma_k^4}\right\},\end{aligned} \tag{21}$$

Furthermore it follows from the inequality $\alpha_k^2\beta_k^2 \geq \gamma_k^4$ that $\frac{1}{1+\alpha_k^2b_1^2+b_1^2b_2^2\alpha_k^2\beta_k^2+\beta_k^2b_2^2-b_1^2b_2^2\gamma_k^4} \leq \frac{1}{\left(\sqrt{\frac{\alpha_k^2\beta_k^2-\gamma_k^4}{\alpha_k^2\beta_k^2}}\alpha_k^2b_1^2+1\right)\left(\sqrt{\frac{\alpha_k^2\beta_k^2-\gamma_k^4}{\alpha_k^2\beta_k^2}}\beta_k^2b_2^2+1\right)}$ which leads to (since $\boldsymbol{b}_1$ and $\boldsymbol{b}_2$ are independent)

$$\begin{aligned}E_{d_k}\left\{e^{-\frac{d_k^2}{4\sigma_2^2}}\right\} &\leq E_{b_1}\left\{\frac{1}{\sqrt{\frac{\alpha_k^2\beta_k^2-\gamma_k^4}{\alpha_k^2\beta_k^2}}\alpha_k^2b_1^2+1}\right\}E_{b_2}\left\{\frac{1}{\sqrt{\frac{\alpha_k^2\beta_k^2-\gamma_k^4}{\alpha_k^2\beta_k^2}}\beta_k^2b_2^2+1}\right\} \\ &= \frac{1}{\alpha_k^2\beta_k^2-\gamma_k^4}e^{\frac{\beta_k}{\alpha_k\sqrt{\alpha_k^2\beta_k^2-\gamma_k^4}}}E_1\left(\frac{\beta_k}{\alpha_k\sqrt{\alpha_k^2\beta_k^2-\gamma_k^4}}\right)\times \\ &\times e^{\frac{\alpha_k}{\beta_k\sqrt{\alpha_k^2\beta_k^2-\gamma_k^4}}}E_1\left(\frac{\alpha_k}{\beta_k\sqrt{\alpha_k^2\beta_k^2-\gamma_k^4}}\right) \\ &\leq \frac{1}{\alpha_k^2\beta_k^2-\gamma_k^4}\log\left(1+\alpha_k^2\sqrt{\frac{\alpha_k^2\beta_k^2-\gamma_k^4}{\alpha_k^2\beta_k^2}}\right)\log\left(1+\beta_k^2\sqrt{\frac{\alpha_k^2\beta_k^2-\gamma_k^4}{\alpha_k^2\beta_k^2}}\right),\end{aligned} \tag{22}$$

where $E_1(a)$ denotes the exponential integral function which is given as $E_1(a) = \int_a^{\infty}\frac{e^{-x}}{x}dx$ and the last inequality follows sine $e^aE_1(a) \leq \log(1+\frac{1}{a})$. Invoking the result of (22) in (20), and defining $s_k^2 = \sum_{m=1}^{M}|X_k^m - X_k'^m|^2$ and $f_k^2 = \left|\sum_{m=BD+1}^{M}(X_k^m - X_k'^m)(X_k^{m-BD} - X_k'^{m-BD})^*\right|$ yields

$$\begin{aligned}&P\left(\boldsymbol{X}_k \to \boldsymbol{X}'_k\right) \\ &\leq \frac{8\sigma_2^4}{P_1P_2\left(s_k^4-f_k^4\right)}\log\left(1+\frac{P_1}{4\sigma_2^2}\sqrt{s_k^4-f_k^4}\right)\log\left(1+\frac{P_2}{4\sigma_2^2}\sqrt{s_k^4-f_k^4}\right),\end{aligned} \tag{23}$$

under the assumption that the channels from the source to the relays have higher SNR ratios than the channels from the relays to the destination.

## Appendix B
## PEP Analysis for Block Fading Frequency-Selective Channel

For the Viterbi detector, we have

$$\boldsymbol{X}'_k = \arg\min_{\boldsymbol{X}_k}\sum_{m=1}^{M+BD}\left|Y_k^m - GH_{1,k}^mX_k^m - GH_{2,k}^mX_k^{m-BD}\right|^2,$$

where $X_k^m = 0$ for $m < 1$ or $m > M$, and $GH_{1,k}^m = G_{1,k}^mH_{1,k}^m$ and $GH_{2,k}^m = G_{2,k}^mH_{2,k}^me^{-2\pi j\frac{k'}{NT_s}d_r}$. Therefore, similar to (19) under the assumption that $\frac{1}{\sigma_{1,i}^2}$ are sufficiently larger than $\frac{P_i}{\sigma_2^2}$, we can write

$$P(\boldsymbol{X}_k \to \boldsymbol{X}'_k|\boldsymbol{H}_i, \boldsymbol{G}_i,\ i \in \{1,2\}) \leq \frac{1}{2}e^{-\frac{d^2\left(\boldsymbol{X}_k,\boldsymbol{X}'_k\right)}{4\sigma_2^2}},$$

where $d^2(\boldsymbol{X}_k, \boldsymbol{X}'_k) = \sum_{m=1}^{M+BD} |G_{1,k}^m H_{1,k}^m d_k^m + G_{2,k}^m H_{2,k}^m d_k^{m-BD}|^2$ and by defining $\boldsymbol{W}_{t,f}(m,k) = diag\{\boldsymbol{W}_t(m)\boldsymbol{w}_f(k), \boldsymbol{W}_t(m)\boldsymbol{w}_f(k)\}$, $d_k^m = X_k^m - X_k'^m$ and $\boldsymbol{d}_k(m) = [d_k^m, d_k^{m-BD}]^T$, we can write

$$d^2(\boldsymbol{X}_k, \boldsymbol{X}'_k) = \sum_{m=1}^{M+BD} \boldsymbol{w}_f^T(k)\boldsymbol{W}_t^T(m)[\boldsymbol{v}_1\boldsymbol{q}_1^T, \boldsymbol{v}_2\boldsymbol{q}_2^T]\boldsymbol{W}_{t,f}(m,k)\boldsymbol{d}_k(m) \times \boldsymbol{d}_k^H(m)W_{t,f}^H(m,k)[\boldsymbol{v}_1\boldsymbol{q}_1^T, \boldsymbol{v}_2\boldsymbol{q}_2^T]^H \boldsymbol{W}_t^*(m)\boldsymbol{w}_f^*(k).$$

On the other hand, we have

$$\begin{aligned}
&\boldsymbol{w}_f^T(k)\boldsymbol{W}_t^T(m)[\boldsymbol{v}_1\boldsymbol{q}_1^T, \boldsymbol{v}_2\boldsymbol{q}_2^T] = \\
&\boldsymbol{w}_f^T(k)\boldsymbol{W}_t^T(m)\Big[[\boldsymbol{\alpha}_1^T(1), \ldots, \boldsymbol{\alpha}_1^T(L)]^T\boldsymbol{q}_1^T, [\boldsymbol{\alpha}_2^T(1), \ldots, \boldsymbol{\alpha}_2^T(L)]^T\boldsymbol{q}_2^T\Big] \\
&= \boldsymbol{w}_f^T(k)\begin{bmatrix} \boldsymbol{w}_t^T(m)\boldsymbol{\alpha}_1(1)\boldsymbol{q}_1^T & , & \boldsymbol{w}_t^T(m)\boldsymbol{\alpha}_2(1)\boldsymbol{q}_2^T \\ \vdots & \vdots & \vdots \\ \boldsymbol{w}_t^T(m)\boldsymbol{\alpha}_1(L)\boldsymbol{q}_1^T & , & \boldsymbol{w}_t^T(m)\boldsymbol{\alpha}_2(L)\boldsymbol{q}_2^T \end{bmatrix} \\
&= \left[\boldsymbol{q}_1^T \sum_{l=1}^{L} e^{-j\frac{2\pi k\tau_l}{NT_s}}\boldsymbol{\alpha}_1^T(l)\boldsymbol{w}_t(m), \boldsymbol{q}_2^T \sum_{l=1}^{L} e^{-j\frac{2\pi k\tau_l}{NT_s}}\boldsymbol{\alpha}_2^T(l)\boldsymbol{w}_t(m)\right].
\end{aligned}$$

By defining

$$\boldsymbol{A}_i(k) = diag\left\{\sum_{l=1}^{L} e^{-j\frac{2\pi k\tau_l}{NT_s}}\boldsymbol{\alpha}_i^T(l), \ldots, \sum_{l=1}^{L} e^{-j\frac{2\pi k\tau_l}{NT_s}}\boldsymbol{\alpha}_i^T(l)\right\}_{LL_t \times LL_t^2},$$

and $\boldsymbol{W}_{\alpha,t}(m) = diag\{\boldsymbol{w}_t(m), \ldots, \boldsymbol{w}_t(m)\}_{LL_t^2 \times LL_t}$, we obtain

$$\begin{aligned}
&\boldsymbol{w}_f^T(k)\boldsymbol{W}_t^T(m)[\boldsymbol{v}_1\boldsymbol{q}_1^T, \boldsymbol{v}_2\boldsymbol{q}_2^T] \\
&= \left[\boldsymbol{q}_1^T\boldsymbol{A}_1(k)\boldsymbol{W}_{\alpha,t}(m), \boldsymbol{q}_2^T\boldsymbol{A}_2(k)\boldsymbol{W}_{\alpha,t}(m)\right].
\end{aligned} \tag{24}$$

Note that $\boldsymbol{A}_i(k)\boldsymbol{W}_{\alpha,t}(m) = \sum_{l=1}^{L} e^{-j\frac{2\pi k\tau_l}{NT_s}}\boldsymbol{\alpha}_{1,1}^T(l)\boldsymbol{w}_t(m)\boldsymbol{I}_{L_tL}$. Furthermore, by defining $\boldsymbol{q}(k) = \left[\boldsymbol{q}_1^T\boldsymbol{A}_1(k), \boldsymbol{q}_2^T\boldsymbol{A}_2(k)\right]^H$, and $\boldsymbol{W}_{A,t}(m) = diag\{\boldsymbol{W}_{\alpha,t}(m), \boldsymbol{W}_{\alpha,t}(m)\}$, we can write

$$\boldsymbol{w}_f^T(k)\boldsymbol{W}_t^T(m)[\boldsymbol{v}_1\boldsymbol{q}_1^T, \boldsymbol{v}_2\boldsymbol{q}_2^T] = \boldsymbol{q}(k)^H\boldsymbol{W}_{A,t}(m). \tag{25}$$

Therefore, defining

$$\boldsymbol{D}_A(\boldsymbol{X}_k, \boldsymbol{X}'_k) = \sum_{m=1}^{M+BD} \boldsymbol{W}_{A,t}(m)\boldsymbol{W}_{t,f}(m,k)\boldsymbol{d}_k(m)\boldsymbol{d}_k^H(m)\boldsymbol{W}_{t,f}^H(m,k)\boldsymbol{W}_{A,t}^H(m),$$

yields to

$$d^2(\boldsymbol{X}_k, \boldsymbol{X}'_k) = \boldsymbol{q}(k)^H\boldsymbol{D}_A(\boldsymbol{X}_k, \boldsymbol{X}'_k)\boldsymbol{q}(k). \tag{26}$$

Since $\boldsymbol{D}_A(\boldsymbol{X}_k, \boldsymbol{X}'_k)$ is a positive semi-definite matrix, we can write

$$\boldsymbol{D}_A(\boldsymbol{X}_k, \boldsymbol{X}'_k) = \boldsymbol{U}_k\boldsymbol{\Lambda}_k\boldsymbol{U}_k^H, \tag{27}$$

where $\boldsymbol{U}_k$ is a unitary matrix and $\boldsymbol{\Lambda} = diag\{\lambda_{k,1}, \ldots, \lambda_{k,r}, 0, \ldots, 0\}$ with $\lambda_{k,i}$ being the positive eigenvalues of $\boldsymbol{D}_A(\boldsymbol{X}_k, \boldsymbol{X}'_k)$. We define $D(\boldsymbol{X}_k, \boldsymbol{X}'_k)$ as the number of values of $m$ where either $X_k^m \neq X_k'^m$ or $X_k^{m-BD} \neq X_k'^{m-BD}$, i.e.,

$$D(\boldsymbol{X}_k, \boldsymbol{X}'_k) = M + BD - \sum_{m=1}^{M+BD} \delta(X_k^m - X_k'^m)\delta(X_k^{m-BD} - X_k'^{m-BD}). \tag{28}$$

Since $\boldsymbol{d}_k(m)\boldsymbol{d}_k^H(m)$ is a rank one matrix whenever $X_k^m \neq X_k'^m$ and/or $X_k^{m-BD} \neq X_k'^{m-BD}$ and a zero matrix otherwise, we can write

$$r_k = rank(D_A(\boldsymbol{X}_k, \boldsymbol{X}'_k)) \leq \min(D(\boldsymbol{X}_k, \boldsymbol{X}'_k), 2LL_t^2),$$

and as a result $r = \min_{\boldsymbol{X}_k, \boldsymbol{X}'_k} r_k \leq \min(D_{eff}(k), 2LL_t^2)$ with $D_{eff}(k) = \min_{\boldsymbol{X}_k, \boldsymbol{X}'_k} D(\boldsymbol{X}_k, \boldsymbol{X}'_k)$. Conditioned on known channel coefficients, it follows from (27) that

$$P(\boldsymbol{X}_k \to \boldsymbol{X}'_k | \boldsymbol{H}_i[k], \boldsymbol{G}_i[k]) \leq \frac{1}{2}e^{-\frac{1}{4\sigma_2^2}\sum_{c=1}^{r_k}\lambda_{k,c}\left|\boldsymbol{U}_{k,c}^H\boldsymbol{q}(k)\right|^2},$$

where $\boldsymbol{U}_{k,c}$ denotes the $c$-th column of $\boldsymbol{U}_k$. Furthermore, if we define $\boldsymbol{\mu}_i(k) = [\mu_{k,i,1}, \ldots, \mu_{k,i,L_t}] = \sum_{l=1}^{L} e^{-j\frac{2\pi k\tau_l}{NT_s}}\boldsymbol{\alpha}_i(l)$, since $\alpha_{i,l}[n]$ are i.i.d. complex Guassian random variables with zero mean, $\mu_{k,i,p}$ are also i.i.d. complex Gaussian random variables with zero mean and variance of $\sigma_{\mu,i}^2 = \sum_{l=1}^{L}\frac{\sigma_{h_i,l}^2}{L_t}$. Furthermore, by denoting $\boldsymbol{q}_1$ as $\boldsymbol{q}_1 = [q_1, \ldots, q_{LL_t}]^H$ and $\boldsymbol{q}_2$ as $\boldsymbol{q}_2 = [q_{LL_t+1}, \ldots, q_{2LL_t}]^H$, we can write

$$\begin{aligned}
\boldsymbol{q}(k) = [&q_1\mu_{k,1,1}, \ldots, q_1\mu_{k,1,L_t}, \ldots, q_{LL_t}\mu_{k,1,1}, \ldots, \\
&\ldots, q_{LL_t}\mu_{k,1,L_t}, q_{LL_t+1}\mu_{k,2,1}, \ldots, q_{LL_t+1}\mu_{k,2,L_t}, \\
&\ldots, q_{2LL_t}\mu_{k,2,1}, \ldots, q_{2LL_t}\mu_{k,2,L_t}].
\end{aligned} \tag{29}$$

Therefore, $\boldsymbol{U}_{k,c}^H\boldsymbol{q}(k)$ can be written as

$$\begin{aligned}
\boldsymbol{U}_{k,c}^H\boldsymbol{q}(k) &= \sum_{p=1}^{LL_t}\sum_{t=1}^{L_t} U_{k,c,(p-1)L_t+t}^* q_p\mu_{k,1,t} \\
&+ \sum_{p=1}^{LL_t}\sum_{t=1}^{L_t} U_{k,c,LL_t^2+(p-1)L_t+t}^* q_{LL_t+p}\mu_{k,2,t} \\
&= \sum_{i=1}^{2}\sum_{p=1}^{LL_t} q_{(i-1)LL_t+p}\chi_{k,(i-1)LL_t+p},
\end{aligned} \tag{30}$$

where $\chi_{k,(i-1)LL_t+p} = \sum_{t=1}^{L_t} U_{k,c,(i-1)LL_t^2+(p-1)L_t+t}^*\mu_{k,i,t}$ are complex Gaussian random variables with zero mean and variance of $\sigma_{\chi,k,(i-1)LL_t+p}^2 = \sum_{t=1}^{L_t}|U_{k,c,(i-1)LL_t^2+(p-1)L_t+t}|^2\sigma_{\mu,i}^2$. To calculate the PEP of the system, we make two simplifying assumptions:

- $\boldsymbol{U}_{k,i}^H\boldsymbol{q}(k)$ and $\boldsymbol{U}_{k,j}^H\boldsymbol{q}(k)$ are independent for different values of $i$ and $j$ ($i \neq j$). (Note that for $\boldsymbol{q}_k$ being a complex Gaussian random vector with zero mean, $\boldsymbol{U}_{k,c}^H\boldsymbol{q}_k$ and $\boldsymbol{U}_{k,j}^H\boldsymbol{q}_k$ are independent complex Gaussian random variables.)
- $\chi_{k,p}$ are i.i.d. complex Gaussian random variables with zero mean and variance of $\sigma_{\chi,k,p}^2$.

For a fixed $c$, by defining $\theta_p = \arccos\left(\frac{\sum_{l=p+1}^{2LL_t}|q_l\chi_{k,l}|\cos(\phi_l)}{R_p}\right)$ and $R_p = \left|\sum_{l=p+1}^{2LL_t} q_l\chi_{k,l}\right|$ in which $\phi_l$ is the angle between the complex valued random vectors $q_l$ and $\chi_{k,l}$ and uniformly distributed over $[0, 2\pi]$, we can write

$$\begin{aligned}|\boldsymbol{U}_{k,c}^H\boldsymbol{q}(k)|^2 &= \left|\sum_{p=1}^{2LL_t} q_p\chi_{k,p}\right|^2\\ &= \sum_{p=1}^{2LL_t} |q_p\chi_{k,p}|^2 + 2Re\left\{\sum_{p=1}^{2LL_t}\sum_{l=p+1}^{2LL_t} q_p\chi_{k,p}q_l^*\chi_l^*\right\}\\ &= \sum_{p=1}^{2LL_t} |q_p\chi_{k,p}|^2 + 2\sum_{p=1}^{2LL_t} |q_p\chi_{k,p}|R_p\cos(\phi_p-\theta_p).\end{aligned}$$

Due to the fact that $\phi_p$ are uniformly distributed over $[0, 2\pi]$, $|\chi_p|$ are Rayleigh distributed and by following the same procedure as in (21), we can write

$$\begin{aligned}&E_{\phi_1,|\chi_{k,1}|}\left\{e^{-\frac{\lambda_{k,c}R_0^2}{4\sigma_2^2}}\right\}\\ &=E_{|\chi_{k,1}|}\left\{e^{-\frac{\lambda_{k,c}\sum_{p=1}^{2LL_t}|q_p\chi_{k,p}|^2}{4\sigma_2^2}}e^{-\frac{\lambda_{k,c}\sum_{p=2}^{2LL_t}|q_p\chi_{k,p}|R_p\cos(\phi_p-\theta_p)}{2\sigma_2^2}}\right.\\ &\qquad\left.\times I_0\left(\frac{\lambda_{k,c}|q_1\chi_{k,1}|R_1}{2\sigma_2^2}\right)\right\}\\ &= \frac{2\sigma_2^2 e^{-\frac{\lambda_{k,c}R_1^2}{4\sigma_2^2}}}{\lambda_{k,c}\sigma_{\chi,k,1}^2|q_1|^2+2\sigma_2^2}e^{\frac{\lambda_{k,c}^2\sigma_{\chi,k,1}^2|q_1|^2R_1^2}{4\sigma_2^2(\lambda_{k,c}\sigma_{\chi,k,1}^2|q_1|^2+2\sigma_2^2)}}\\ &= \frac{2\sigma_2^2}{\lambda_{k,c}\sigma_{\chi,k,1}^2|q_1|^2+2\sigma_2^2}e^{-\frac{\lambda_{k,c}R_1^2}{2\lambda_{k,c}\sigma_{\chi,k,1}^2|q_1|^2+4\sigma_2^2}}. \qquad (31)\end{aligned}$$

Therefore, by taking the expected value with respect to $\phi_1, |\chi_{k,1}|, \ldots, \phi_{2LL_t}, |\chi_{k,2LL_t}|$, we arrive at

$$\begin{aligned}&E_{\phi_1,|\chi_{k,1}|,\ldots,\phi_{2LL_t},|\chi_{k,2LL_t}|}\left\{e^{-\frac{\lambda_{k,c}R_0^2}{4\sigma_2^2}}\right\}\\ &= \frac{2\sigma_2^2}{2\sigma_2^2+\lambda_{k,c}\sigma_{\chi,k,1}^2|q_1|^2}\frac{2\sigma_2^2+\lambda_{k,c}\sigma_{\chi,k,1}^2|q_1|^2}{2\sigma_2^2+\lambda_{k,c}(\sigma_{\chi,k,1}^2|q_1|^2+\sigma_{\chi,k,2}^2|q_2|^2)}\\ &\quad\cdots\frac{2\sigma_2^2+\lambda_{k,c}\sum_{p=1}^{2LL_t-1}\sigma_{\chi,k,p}^2|q_p|^2}{2\sigma_2^2+\lambda_{k,c}\sum_{p=1}^{2LL_t}\sigma_{\chi,k,p}^2|q_p|^2}\\ &= \frac{2\sigma_2^2}{2\sigma_2^2+\lambda_{k,c}\sum_{p=1}^{2LL_t}\sigma_{\chi,k,p}^2|q_p|^2} \qquad (32)\end{aligned}$$

To obtain the above expected value, we first define $V = \sum_{p=1}^{2LL_t}\sigma_{\chi,k,p}^2|q_p|^2$ in which $|q_p| \sim Rayleigh(\frac{\sigma_{q,p}}{\sqrt{2}})$ and $|q_p|$ are independent for all $p$, then obtain the expected value over the new defined random variable. Let us define $S_0 = \left\{p\middle|\sigma_{\chi,k,p}^2\sigma_{q,p}^2 \neq \sigma_{\chi,k,l}^2\sigma_{q,l}^2 \forall l \neq p\right\}$, i.e., there are $|S_0|$ different values of $\sigma_{\chi,k,p}^2\sigma_{q,p}^2$ such that $\sigma_{\chi,k,p}^2\sigma_{q,p}^2 \neq \sigma_{\chi,k,l}^2\sigma_{q,l}^2$ and $\forall l \neq p$. Furthermore, assume that there are $J$ distinct values $a_j^2$ for $j \in \{1, \ldots, J\}$ for which there are $p$ and $l$ ($p \neq l$ and $p, l \in \{1, \ldots, 2LL_t\}$) such that $\sigma_{\chi,k,p}^2\sigma_{q,p}^2 = \sigma_{\chi,k,l}^2\sigma_{q,l}^2 = a_j^2$. We also define $S_j = \left\{p\middle|\sigma_{\chi,k,p}^2\sigma_{q,p}^2 = a_j^2\right\}$ for $j \in \{1, \ldots, J\}$ and $N_j = |S_j|$. It follows from [17, p. 876] and the definition of the Gamma distribution that

$$\begin{aligned}p_V(v) &= \sum_{p\in S_0}\frac{\pi_p}{\sigma_{\chi,k,p}^2\sigma_{q,p}^2}e^{-\frac{v}{\sigma_{\chi,k,p}^2\sigma_{q,p}^2}}\\ &\quad+\sum_{j=1}^{J}\sum_{p\in S_j}\frac{v^{N_j-1}}{(N_j-1)!(\sigma_{\chi,k,p}^2\sigma_{q,p}^2)^{N_j}}e^{-\frac{v}{\sigma_{\chi,k,p}^2\sigma_{q,p}^2}},\end{aligned}$$

where $\pi_p = \prod_{l\in S_0, l\neq p}\frac{\sigma_{\chi,k,p}^2\sigma_{q,p}^2}{\sigma_{\chi,k,p}^2\sigma_{q,p}^2-\sigma_{\chi,k,l}^2\sigma_{q,l}^2}$. Therefore, we can write

$$\begin{aligned}&E_{|q_1|,\ldots,|q_{2LL_t}|}\left\{\frac{2\sigma_2^2}{2\sigma_2^2+\lambda_{k,c}\sum_{p=1}^{2LL_t}\sigma_{\chi,k,p}^2|q_p|^2}\right\}\\ &= E_V\left\{\frac{2\sigma_2^2}{2\sigma_2^2+\lambda_{k,c}V}\right\}\\ &= \sum_{p\in S_0}\frac{\pi_p}{2\sigma_{\chi,k,p}^2\sigma_{q,p}^2}\int_0^\infty\frac{2\sigma_2^2}{2\sigma_2^2+\lambda_{k,c}v}e^{-\frac{v}{2\sigma_{\chi,k,p}^2\sigma_{q,p}^2}}\\ &+\sum_{j=1}^{J}\sum_{p\in S_j}\frac{1}{(N_j-1)!(\sigma_{\chi,k,p}^2\sigma_{q,p}^2)^{N_j}}\int_0^\infty\frac{2\sigma_2^2v^{N_j-1}}{2\sigma_2^2+\lambda_{k,c}v}e^{-\frac{v}{\sigma_{\chi,k,p}^2\sigma_{q,p}^2}}.\end{aligned}$$

By using the integral calculated in [19, p. 325. Eq. 2. 3. 6. 14.] and due to the definition of the exponential integral function $E_1(a) = \int_a^\infty \frac{e^{-z}}{z}dz$, we obtain

$$\begin{aligned}PEP_{k,c} &= E_V\left\{\frac{2\sigma_2^2}{2\sigma_2^2+\lambda_{k,c}V}\right\}\\ &= \sum_{p\in S_0}\frac{\pi_p}{\sigma_{\chi,k,p}^2\sigma_{q,p}^2}\frac{2\sigma_2^2}{\lambda_{k,c}}e^{\frac{2\sigma_2^2}{\lambda_{k,c}\sigma_{\chi,k,p}^2\sigma_{q,p}^2}}E_1\left(\frac{2\sigma_2^2}{\lambda_{k,c}\sigma_{\chi,k,p}^2\sigma_{q,p}^2}\right)\\ &\quad+\sum_{j=1}^{J}\sum_{p\in S_j}\frac{(2\sigma_2^2)^{N_j}(-1)^{N_j-1}}{(N_j-1)!(\lambda_{k,c}\sigma_{\chi,k,p}^2\sigma_{q,p}^2)^{N_j}}\times\\ &\times\left[e^{\frac{2\sigma_2^2}{\lambda_{k,c}\sigma_{\chi,k,p}^2\sigma_{q,p}^2}}E_1\left(\frac{2\sigma_2^2}{\lambda_{k,c}\sigma_{\chi,k,p}^2\sigma_{q,p}^2}\right)\right.\\ &\left.+\sum_{k=1}^{N_j-1}(k-1)!\left(\frac{-\lambda_{k,c}\sigma_{\chi,k,p}^2\sigma_{q,p}^2}{2\sigma_2^2}\right)^k\right] \qquad (33)\end{aligned}$$

Finally, for the PEP of the system under block fading channel conditions, we obtain

$$\begin{aligned}P(\boldsymbol{X}_k\to\boldsymbol{X}_k') &\le \frac{1}{2}E\left\{e^{-\frac{1}{4\sigma_2^2}\sum_{c=1}^{r_k}\lambda_{k,c}\left|\boldsymbol{U}_{k,c}^H\boldsymbol{q}(k)\right|^2}\right\}\\ &\le \frac{1}{2}\prod_{c=1}^{r_k}PEP_{k,c}, \qquad (34)\end{aligned}$$

where $PEP_{k,c}$ are given in (33).

## REFERENCES


[1] M. Vajapeyam, S. Vedantam, U. Mitra, J. C. Preisig, and M. Stojanovic, "Distributed space-time cooperative schemes for underwater acoustic communications," *IEEE J. Oceanic Eng.*, vol. 33, no. 4, pp. 489–501, Oct. 2008.

[2] Z. Han, Y. L. Sun, and H. Shi, "Cooperative transmission for underwater acoustic communications," in *Proc. 2008 IEEE Int. Conf. Commun.*, pp. 2028–2032.

[3] K. Tu, T. M. Duman, J. G. Proakis, and M. Stojanovic, "Cooperative MIMO-OFDM communications: receiver design for Doppler-distorted underwater acoustic channels," in *Proc. 2010 Asilomar Conf. Signals, Syst., Comput.*, pp. 1335–1339.

[4] M. Stojanovic, "Underwater acoustic communications," *Wiley Encyclopaedia of Electrical and Electronics Engineering*, vol. 22, pp. 688–698, 1999.

[5] H. M. Wang and X.-G. Xia, "Asynchronous cooperative communication systems: a survey on signal designs," *SCIENCE CHINA Inf. Sciences*, vol. 54, no. 8, pp. 1547–1531, 2011.

[6] Y. Li, W. Zhang, and X.-G. Xia, "Distributive high-rate space-frequency codes achieving full cooperative and multipath diversities for asynchronous cooperative communications," *IEEE Trans. Veh. Technol.*, vol. 58, no. 1, pp. 207–217, 2009.

[7] Z. Li, X.-G. Xia, and M. H. Lee, “A simple orthogonal space-time coding scheme for asynchronous cooperative systems for frequency selective fading channels,” *IEEE Trans. Commun.*, vol. 58, no. 8, pp. 2219–2224, 2010.

[8] X. Li, “Space-time coded multi-transmission among distributed transmitters without perfect synchronization,” *IEEE Signal Process. Lett.*, vol. 11, no. 12, pp. 948–951, Dec. 2004.

[9] Z. Zhong, S. Zhu, and A. Nallanathan, “Distributed space-time trellis code for asynchronous cooperative communications under frequency-selective channels,” *IEEE Trans. Wireless Commun.*, vol. 8, no. 2, pp. 796–805, Feb. 2009.

[10] ——, “Delay tolerant distributed linear convolutional space-time code with minimum memory length under frequency-selective channels,” *IEEE Trans. Wireless Commun.*, vol. 8, no. 8, pp. 3944–3949, 2009.

[11] M. O. Damen and A. R. Hammons, “Delay-tolerant distributed-TAST codes for cooperative diversity,” *IEEE Trans. Inf. Theory*, vol. 53, no. 10, pp. 3755–3773, 2007.

[12] M. Nahas, A. Saadani, and G. Rekaya, “Bounded delay-tolerant space time block codes for asynchronous cooperative networks,” *IEEE Trans. Wireless Commun.*, vol. 10, no. 10, pp. 3288–3297, 2011.

[13] D. Gore, S. Sandhu, and A. Paulraj, “Delay diversity codes for frequency selective channels,” in *Proc. 2002 IEEE Int. Conf. Commun.*, vol. 3, 2002, pp. 1949–1953.

[14] B. Lu, X. Wang, and K. Narayanan, “LDPC-based space-time coded OFDM systems over correlated fading channels: performance analysis and receiver design,” *IEEE Trans. Commun.*, 2002.

[15] S. G. Wilson, *Digital Modulation and Coding*. Prentice-Hall, 1996.

[16] W. C. Jakes, *Microwave Mobile Communication*. Wiley, 1974.

[17] J. G. Proakis and M. Salehi, *Digital Communications*, 5th ed. McGraw-Hill, 2007.

[18] B. Y. A. Prudnikov, A. P. and O. I. Marichev, *Integrals and Series*. Gordon and Breach Science Publishers, 1986, vol. 2.

[19] ——, *Integrals and Series*. Gordon and Breach Science Publishers, 1986, vol. 1.


**Mojtaba Rahmati** (S'11) received his B. S. degree in electrical engineering in 2007 from University of Tehran, Iran, his M. S. degree in telecommunication systems in 2009 from Sharif University of Technology, Tehran, Iran and his Ph.D. in electrical engineering in 2013 from Arizona State University, Tempe, AZ. He is currently a research engineer at Mobile Solutions Lab, Samsung Information Systems America. His research interests include wireless and mobile communication, digital communications, digital signal processing and information theory.

**Tolga M. Duman** Tolga M. Duman (S'95–M'98–SM'03–F'11) is a Professor of Electrical and Electronics Engineering Department of Bilkent University in Turkey, and is on leave from the School of ECEE at Arizona State University. He received the B.S. degree from Bilkent University in Turkey in 1993, M.S. and Ph.D. degrees from Northeastern University, Boston, in 1995 and 1998, respectively, all in electrical engineering. Prior to joining Bilkent University in September 2012, he has been with the Electrical Engineering Department of Arizona State University first as an Assistant Professor (1998-2004), then as an Associate Professor (2004-2008), and starting August 2008 as a Professor. Dr. Duman's current research interests are in systems, with particular focus on communication and signal processing, including wireless and mobile communications, coding/modulation, coding for wireless communications, data storage systems and underwater acoustic communications.

Dr. Duman is a Fellow of IEEE, a recipient of the National Science Foundation CAREER Award and IEEE Third Millennium medal. He served as an editor for IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS (2003-08), IEEE TRANSACTIONS ON COMMUNICATIONS (2007-2012) and IEEE COMMUNICATIONS SURVEYS AND TUTORIALS (2002-07). He is currently the coding and communication theory area editor for IEEE TRANSACTIONS ON COMMUNICATIONS (2011-present) and an editor for *Elsevier Physical Communications Journal* (2010-present).