Minimum Cost Path Problem
for Plug-in Hybrid Electric Vehicles

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Abstract

We introduce a practically important and theoretically challenging problem: finding the minimum cost path for plug-in hybrid electric vehicles (PHEVs) in a network with refueling and battery switching stations, considering electricity and gasoline as sources of energy with different cost structures and limitations. We show that this problem is NP-complete even though its electric vehicle and conventional vehicle special cases are polynomially solvable. We propose three solution techniques: (1) a mixed integer quadratically constrained program that incorporates non-fuel costs such as vehicle depreciation, battery degradation and stopping, (2) a dynamic programming based heuristic and (3) a shortest path heuristic. We conduct extensive computational experiments using both real world road network data and artificially generated road networks of various sizes and provide significant insights about the effects of driver preferences and the availability of battery switching stations on the PHEV economics. In particular, our findings show that increasing the number of battery switching stations may not be enough to overcome the range anxiety of the drivers.

Keywords: plug-in hybrid electric vehicles, minimum cost path, vehicle routing, energy management, integer programming, dynamic programming

1. Introduction

The interest in electric vehicles (EVs) and their variants such as Plug-in Hybrid Electric Vehicles (PHEVs) is on the rise due to the economic, environmental and security concerns associated with gasoline. A PHEV provides reduction in both transportation costs and greenhouse gas emissions with respect to a comparable conventional vehicle (CV) (Windecker and Ruder 2013). It has an electric motor and an internal combustion engine (ICE) as its power resources. It has the capabilities of an EV such as recharging from a regular power outlet and the convenience of a gasoline powered CV such as long-range trips. On charge sustaining (CS) mode, it travels using gasoline as the only energy resource. On charge depleting (CD) mode, PHEVs can travel exclusively on electricity or blended with both electricity and gasoline (Pistoia 2010, Axsen and Kurani 2010, Axsen et al. 2008, Markel and Wipke 2001). In blended fashion, the PHEV travels primarily using the electric motor, supported by the ICE using gasoline for operations that require extra power. *All-electric CD mode drive* is assumed in recent research including Traut et al. (2011) and He et al. (2013). Similarly, in this article, we focus on PHEVs that operate exclusively using electricity on CD mode. However, the proposed methodology can also be regarded as a close approximation for those PHEVs that operate in blended mode since the primary source of energy is again electricity and ICE is only used as a supplement.

Recent research related to PHEVs focus mainly on the energy management problem (Sioshansi 2012, Wei and Guan 2014), refueling station location problem (Kuby and Lim 2005, MirHassani and Ebrazi 2013) and demand analyses (Glerum et al. 2013, Dagsvik et al. 2002). In this research, we approach PHEVs from the cost perspective. A driver of a vehicle may prefer to minimize total travel distance, total travel time or total travel cost of a trip, and these problems have been extensively studied in the existing literature. In terms of cost, there are

various studies that separately investigate the minimum cost path problem for CVs (MCPP-CV) and for EVs (MCPP-EV) as we review below, and polynomial time algorithms are proposed for both problems. In this study, we formally present the minimum cost path problem for PHEVs (MCPP-PHEV) and efficient solution methodologies. To the best of our knowledge, this study is the first attempt to address the MCPP-PHEV.

Several articles addressed the MCPP-CV in the literature (Lin et al. 2007, Khuller et al. 2007, Lin 2008a,b, 2012, Suzuki 2008, 2009, 2012, Adler et al. 2013). Mixed Integer Programming (MIP) formulations, heuristic techniques and linear-time algorithms with dynamic programming approach are proposed as solution methodologies for both fixed and non-fixed path assumptions. On the EV side, the problem of energy efficient routing of EVs has been addressed in the literature by considering limited cruising range and regenerative breaking capabilities of EVs (Artmeier et al. 2010, Sachenbacher et al. 2011, Eisner et al. 2011) and polynomial time algorithms have been developed. These problems only consider routing in a network without charging facilities. Kobayashi et al. (2011) and Siddiqi et al. (2011) further include battery recharging stations in their models and propose heuristic techniques as solution methodologies. Schneider et al. (2014) also consider time windows beside recharging stations. Note that assuming the electricity as a commodity similar to gasoline, the algorithms mentioned above for MCPP-CV can also be used as solution methodologies for MCPP-EV. In such a case, we also need to assume that the EVs are charged at recharging stations. However, due to long charging times of EV batteries, battery switching stations with short battery switching times are more convenient for EVs. Even though it is presented in a different context, Laporte and Pascoal (2011) present a methodology that can be customized to solve the MCPP-EV problem in a network with battery switching stations. In the existing MCPP-EV studies, battery degradation costs are not

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considered. Furthermore, all the aforementioned studies consider a single energy resource, either gasoline or electricity. Thus, their solution methodologies cannot be directly used for the solution of MCPP-PHEV.

An important problem related to the minimum cost path problems is the shortest weight-constrained path problem (SWCPP) which is known to be NP-complete (Desrosiers et al. 1984, Desrochers and Soumis 1989). In SWCPP, there are typically two independent measures such as cost and time associated with a path (e.g. Desaulniers and Villeneuve 2000, Ahuja et al. 2002). It can efficiently be solved by a shortest path algorithm if one of the measures is disregarded or the two measures are consistent. Even though MCPP-PHEV has only the cost measure, we conclude in Section 2 that it is equivalent to SWCPP and thus is NP-complete. Note that the MCPP-PHEV is a generalization of MCPP-CV and MCPP-EV. Furthermore, shortest path and minimum hop problems are also special cases of the MCPP-PHEV.

The problem defined in this study is a challenging and a fundamental one for long distance travels of a PHEV that possibly require several refueling/battery switching stops. Moreover, it captures the drivers' reluctance for the extra mileage and frequent stops. There are four main contributions:

- We introduce the MCPP-PHEV and present its complexity status.

- We propose a realistic extension to the MCPP-PHEV that incorporates three new dimensions: battery degradation cost, vehicle depreciation cost and stopping cost. Our study is the first that addresses the battery degradation cost in the MCPP context.

- We present a mixed integer quadratically constrained programming (MIQCP) formulation, a dynamic programming based heuristic algorithm, and a shortest path heuristic as solution methodologies.

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- We provide significant insights about the effects of driver preferences and the availability of battery switching stations on the economics of PHEVs.

2. Minimum Cost Path Problem for PHEVs (MCPP-PHEV)

We provide the basic definitions and assumptions necessary for the formalization of MCPP-PHEV. Consider a directed transportation graph $G = (N, A)$ and a PHEV traveling from an origin node $s \in N$ to a destination node $t \in N$. Refueling and/or battery switching stations are located at some of the nodes of the graph and pricing may vary between nodes. Therefore, a PHEV can reduce its travel costs by a proper choice of refueling or battery switching stations.

Proposition 1. *If a PHEV does not refuel or switch battery when traveling from node $i \in N$ to node $j \in N$, then the minimum cost path is the shortest path between nodes i and j.*

The proof of Proposition 1 is straightforward. Next, we introduce a graph transformation which will be useful for the solution methodologies. A similar construction in a complete different application setting is provided by Chen et al. (2010), Smith et al. (2012) and Yıldız and Karaşan (2014).

Definition 1. *Given a weighted graph $G = (N, A)$: let $\widehat{N} = \{s, t\} \cup \{i \in N : i$ has a battery switching and/or refueling station$\}$ and $\widehat{A} = \{(i, j) : i, j \in \widehat{N}$ and j is reachable from i if a PHEV at node i with a full tank of gasoline and fully charged battery can reach node j along a shortest path in $G\}$. Arc $(i, j) \in \widehat{A}$ has a distance equal to the shortest path distance, say $d_{ij}^\star$, from i to j in G. The graph $\widehat{G} = (\widehat{N}, \widehat{A})$ is called the **meta-network** of G.*

Proposition 1 implies that an optimal solution of a MCPP on a given graph can also be obtained by solving the same MCPP instance on its meta-network. Now, consider nodes B, C and D in graph G in Figure 1. Only node C has a refueling

station. The meta-network $\widehat{G}$ is also shown in the same figure. Observe that the arc from s to t is redundant and corresponds to traveling on the path $s \to C \to t$. Since the shortest path from s to t contains a node with a refueling station in the original graph G, arc (s,t) can be omitted.
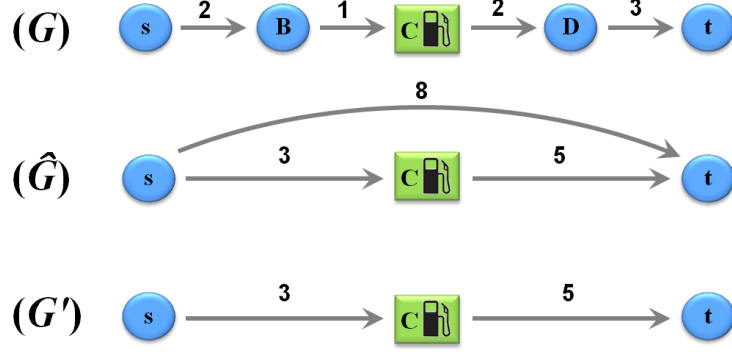

Figure 1: Graph Transformation

Meta-networks can be very dense due to the combined CD and CS mode ranges. The size of the graph is a burden on the solution efficiency, and thus it is useful to omit the redundant arcs in the meta-network. We refer to the graph formed by the omission of redundant arcs as the **reduced meta-network** denoted by G' in Figure 1. In particular, the arcs that are present in the reduced meta-network G' correspond to shortest paths in the original graph G that contain no intermediate nodes with refueling or battery switching stations.

Definition 2. *A vehicle instance (vehicle) is a vector with 6 entries:*
$\langle \overline{P}, \underline{P}, \overline{G}, \underline{G}, \varepsilon, \rho \rangle$ *where* $\overline{P}$ *and* $\underline{P}$ *are the battery maximum and minimum energy capacities, respectively (kWh),* $\overline{G}$ *and* $\underline{G}$ *are the maximum and minimum tank capacities, respectively (gallons),* ε *is the average electricity usage (kWh/mile) and* ρ *is the average gasoline usage (gallon/mile).*

Definition 3. *A network instance (network) is a 7-tuple:*
$\langle N, A, s^e, s^g, c^e, c^g, d \rangle$ *where* N, A *are the sets of nodes and arcs,* $s^e : N \to \{0,1\}$

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and $s^g : N \rightarrow \{0, 1\}$ are functions indicating whether a battery switching or refueling station is located at a node, respectively, $c^e : N \rightarrow \mathbb{R}^+$ is the electricity price function (¢/kWh), $c^g : N \rightarrow \mathbb{R}^+$ is the gasoline price function (¢/gallon) and $d : A \rightarrow \mathbb{R}^+$ is the length function (miles).

Definition 4. *The Minimum Cost Path Problem for PHEV (MCPP-PHEV) is defined as finding a path for a vehicle V from a departure node s to a destination node t in a network, and deciding on how much to refuel and where to switch battery on the path. More formally, the decision version of the problem is:*

INSTANCE: $\langle V, X, s, t, P_s, G_s, P_t, G_t \rangle$ where V is a vehicle instance, X is a network instance, nodes s and t are departure and destination nodes, P_s and G_s are the initial electricity and gasoline storages at node s, P_t and G_t are the minimum final electricity and gasoline storage requirements at node t, respectively, and a positive number C.

QUESTION: Is there a path from s to t in network X that can be traveled by vehicle V with initial electricity and gasoline levels of P_s and G_s and final electricity and gasoline levels of at least P_t and G_t for a cost less than or equal to C?

The solution of the MCPP-PHEV is a triplet $\langle x, e^+, g^+ \rangle$ where x is the incidence vector of the optimal path, e^+ and g^+ are vectors of size $|N|$ representing the electricity and gasoline purchases that are transferred to PHEV at each node, respectively.

2.1. NP-Completeness

Consider the shortest weight-constrained path problem (SWCPP) for directed graphs which is known to be NP-Complete (Garey and Johnson 1979):

INSTANCE: A directed graph $G = (N, A)$ with length $l_{ij} \in \mathbb{Z}^+$ and weight $w_{ij} \in \mathbb{Z}^+$ for each $(i, j) \in A$, specified nodes $s, t \in N$ and positive integers K and

W.

QUESTION: Is there a path in G from s to t with total length K or less and total weight W or less?

First, note that multiplying both W and w_{ij} $\forall (i,j) \in A$ by a positive constant ϕ does not change the solution in SWCPP, and the question in the original instance has a YES answer if and only if the modified instance has a YES answer.

Theorem 1. *The MCPP-PHEV is NP-complete.*

Proof. Proof Observe that the MCPP-PHEV is in NP: given a solution and a value C, one can verify in polynomial time if the solution is feasible and the associated cost is at most C. Given an instance $\langle G, l, w, s, t, K, W \rangle$ to SWCPP, let $l^{min} = \min_{(i,j) \in A} l_{ij}$, $l^{max} = \max_{(i,j) \in A} l_{ij}$, $w^{max} = \max_{(i,j) \in A} w_{ij}$, $\phi = \dfrac{l^{min}}{2 \times l^{max} \times w^{max}} > 0$, $\hat{W} = \phi \times W$ and $\hat{w}_{ij} = \phi \times w_{ij}$ $\forall (i,j) \in A$. Now, consider an equivalent SWCPP instance $\langle G, l, \hat{w}, s, t, K, \hat{W} \rangle$.
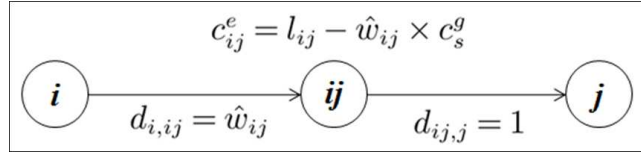

Figure 2: Graph Transformation

We now transform this SWCPP instance into an MCPP-PHEV instance by the following polynomial time transformation: we add a node, say node ij, on each arc $(i,j) \in A$ as shown in Figure 2. Let N' be the set of newly added nodes, A_1 be the set of arcs from node i to node ij $\forall (i,j) \in A$ with distance equal to $\hat{w}_{ij}$ and A_2 be the set of arcs from node ij to node j $\forall (i,j) \in A$ with distance equal to 1 mile. The graph is then transformed into $G' = (N \cup N', A_1 \cup A_2)$. In the transformed graph, no gasoline or battery switching station is located at node $i \in N/\{s\}$. We locate only a refueling station at the source node and the cost of

gasoline at this node is $c_s^g = l^{max}$. We also locate a battery switching station, but no refueling station, at every node $ij \in N'$ and the cost of electricity at node ij is $c_{ij}^e = l_{ij} - \hat{w}_{ij} \times c_s^g = l_{ij} - \phi \times w_{ij} \times l^{max}$. Replacing ϕ, we get $c_s^g > c_{ij}^e > 0$ for all nodes $ij \in N'$ so that traveling on electricity is always preferable to traveling on gasoline. Let X be this transformed network. Let V be the vehicle $\langle 1, 0, \hat{W}, 0, 1, 1 \rangle$. That is, PHEV V has 1 mile of CD mode range and $\hat{W}$ miles of CS mode range.

Consider the MCPP-PHEV instance $\langle V, X, s, t, 0, 0, 0, 0 \rangle$, i.e. a PHEV V travels from node s to node t in network X with zero initial and final gasoline and electricity levels. Let K be the associated cost input. In Figure 2, V at node i with minimum electricity level needs to spend $\hat{w}_{ij}$ units of gasoline in order to arrive at node ij. Since electricity is preferable to gasoline, it switches its battery at node ij with a fully charged battery and travels to node j on the CD mode. At node j, its battery depletes and it starts running on CS mode again. The cost of electricity at node ij and the distance between nodes ij and j are such that the total cost of traversing this arc is $l_{ij} - \hat{w}_{ij} \times c_s^g$ cents. Observe that the vehicle needs to buy the required level of gasoline at the source node at a cost of $\hat{w}_{ij} \times c_s^g$ in order to travel from node i to node j.

Now, it is easy to observe that V has a path from node s to t with cost at most K if and only if the SWCPP has a path from s to t with length at most K and weight at most $\hat{W}$. □

2.2. Extensions

In order to model real world more closely, non-fuel costs such as vehicle depreciation or stopping costs need to be taken into account (Suzuki 2008). To this end, we extend the MCPP-PHEV from three aspects and refer to this problem as the Extended MCPP-PHEV (E-MCPP-PHEV). The first extension is **vehicle depreciation cost**. A PHEV incurs electricity and gasoline costs while traveling. Furthermore, it loses its value with increasing mileage. Therefore, it incurs a vehi-

cle depreciation cost for every mile traveled. Unless depreciation cost is included in the objective function, an optimal path might get much longer than the shortest path which cannot be tolerated even for the most cost averse driver. Therefore, we indirectly avoid long trip distances by including the depreciation cost in the model. In a sense, the depreciation cost can be considered as the cost of tolerating longer distances, and high depreciation costs would force the E-MCPP-PHEV solutions to follow the shortest path.

Another cost component of a vehicle trip is the ***stopping cost***. This cost component can be a measure of the tolerance for stops on the route. That is, for high enough stopping costs, the optimal solution would be the one with the least number of stops. Note that by including the stopping cost, we avoid excessive number of stops on the optimal path which is not tolerable even for the most cost averse driver.
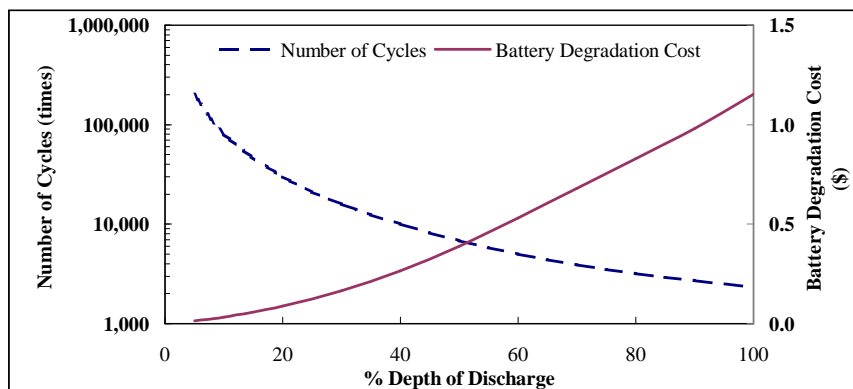

Figure 3: Cycle Life of PHEV Batteries as a Function of DoD

At a battery switching station, a PHEV owner is charged for switching his/her battery. The PHEV arrives at a battery switching station with a fully depleted battery, or some remaining charge. Therefore, the PHEV is charged for the net charge difference between arrival and departure. Furthermore, there is the battery

degradation component of the cost. Similar to vehicle depreciation, the battery deteriorates through usage and the PHEV incurs a **battery degradation cost** for each battery charge/discharge cycle. In this context, we assume that a PHEV is billed by the switching station for the net charge difference and the corresponding battery degradation cost. To the best of our knowledge, Sioshansi and Denholm (2010) are the first to include battery degradation cost in their energy management model. The battery of a PHEV has a limited lifespan, and its life shortens at each cycle. The number of cycles is a nonlinear function of depth of discharge (DoD) as reported by Electric Power Research Institute (2005) and Millner (2010). A sample cycle life function is presented in Figure 3 by dashed lines. The more the battery is discharged, the less the number of cycles is. For instance, consider a battery worth $2650 being discharged to 40% DoD throughout its lifetime. The expected number of cycles at this DoD is approximately 10000. Therefore each discharging costs the PHEV owner 26.5 ¢ ($2650 × 1/10000). A sample degradation cost function for a $2650 battery is presented in Figure 3. In our study, we assume that a cycle is completed each time a battery is switched at a station and a PHEV owner incurs a battery degradation cost depending on the DoD level upon arrival to a battery switching station. We determine this cost by evaluating a quadratic function of DoD.

Within this context, the cost components of a PHEV trip are the gasoline cost, the electricity cost, the battery degradation cost, the vehicle depreciation cost and the stopping cost. For simplicity, in representing an E-MCPP-PHEV instance, we use the MCPP-PHEV instance representation and assume that all cost components are embedded in the corresponding network instance.

3. Solution Techniques

In this section, we provide a mathematical formulation for the E-MCPP-PHEV. Then we present a dynamic programming based heuristic, a shortest path heuristic,

and their extended versions.

3.1. E-MCPP-PHEV Mathematical Model

The parameters and variables to be used in the formulation of the E-MCPP-PHEV are presented below:

- *Parameters*

N, A	:	Sets of nodes and arcs
s, t	:	Source and destination nodes
s_i^e, s_i^g	:	1 if there is an electricity or refueling station, respectively, at node i, and 0 otherwise
$\overline{P}, \underline{P}$	:	Battery maximum and minimum energy capacities, respectively (kWh)
$\overline{G}, \underline{G}$	:	Maximum and minimum tank capacities, respectively (gallons)
P_s, P_t	:	Initial and final energy stored in battery of the PHEV (kWh), respectively
G_s, G_t	:	Initial and final gasoline stored in tank of the PHEV (gallons), respectively
ε	:	Average electricity usage of the PHEV (kWh/mile)
ρ	:	Average gasoline usage of the PHEV (gallon/mile)
d_{ij}	:	Length of arc (i, j) (miles)
c_i^e	:	Price of electricity at node i (¢/kWh)
c_i^g	:	Price of gasoline at node i (¢/gallon)
c^{st}	:	Stopping cost (¢)
c^{dep}	:	Depreciation cost of traveling for a mile (¢/miles)

- *Variables*

e_i^α, e_i^β	:	Charge level at node i at arrival and departure, respectively (kWh)
e_i^+	:	Net electric energy change at node i (kWh)
g_i^α, g_i^β	:	Gasoline level at node i at arrival and departure, respectively (gallons)
g_i^+	:	Gasoline transferred to the PHEV at node i (gallons)
x_{ij}	:	1 if arc (i, j) is on the minimum cost path, 0 otherwise
v_i	:	1 if the PHEV switches battery at node i, and 0 otherwise
r_i	:	1 if the PHEV refuels and/or switches battery at node i, and 0 otherwise
δ_i	:	Depth of Discharge (DoD) at node i at arrival
$c^{bat}(\delta_i)$	:	Degradation cost of the PHEV battery at node i
		We assume the expected battery replacement cost as a quadratic function of DoD δ, i.e.,
		$c^{bat}(\delta) = a \times \delta^2 + b \times \delta$ where a and b are coefficients for a given battery type
d_{ij}^{cd}, d_{ij}^{cs}	:	Travel distance in charge-depleting (CD) and charge-sustaining (CS) mode while traveling on arc (i, j), respectively (miles)

The formulation is as follows:

$$\text{minimize} \sum_{i \in N} c_i^e \times e_i^+ + \sum_{i \in N} c_i^g \times g_i^+ + \sum_{i \in N} c^{bat}(\delta_i) + \sum_{(i,j) \in A} d_{ij} \times c^{dep} \times x_{ij} + \sum_{i \in N} c^{st} \times r_i$$

(1)

subject to

$$\sum_{j:(i,j) \in A} x_{ij} - \sum_{j:(i,j) \in A} x_{ji} = 1 \qquad\qquad i = s \qquad (2)$$

$$\sum_{j:(i,j) \in A} x_{ij} - \sum_{j:(i,j) \in A} x_{ji} = 0 \qquad\qquad \forall i \in N/\{s,t\} \qquad (3)$$

$$\sum_{j:(i,j) \in A} x_{ij} - \sum_{j:(i,j) \in A} x_{ji} = -1 \qquad\qquad i = t \qquad (4)$$

$$e_i^\beta = e_i^\alpha + s_i^e \times e_i^+ \qquad\qquad \forall i \in N \qquad (5)$$

$$M \times (x_{ij} - 1) \leq e_j^\alpha - e_i^\beta + \varepsilon \times d_{ij}^{cd} \leq M \times (1 - x_{ij}) \qquad \forall (i,j) \in A \qquad (6)$$

$$\underline{P} \leq e_i^\alpha \leq \overline{P} \qquad\qquad \forall i \in N \qquad (7)$$

$$\underline{P} \leq e_i^\beta \leq \overline{P} \qquad\qquad \forall i \in N \qquad (8)$$

$$e_i^+ \leq v_i \times \overline{P} \qquad\qquad \forall i \in N \qquad (9)$$

$$e_i^\beta \geq v_i \times \overline{P} \qquad\qquad \forall i \in N \qquad (10)$$

$$v_i \leq r_i \qquad\qquad \forall i \in N \qquad (11)$$

$$e_s^\alpha = P_s \qquad\qquad (12)$$

$$e_t^\alpha \geq P_t \qquad\qquad (13)$$

$$\delta_i = \frac{e_i^+}{\overline{P}} \qquad\qquad \forall i \in N \qquad (14)$$

$$c^{bat}(\delta_i) \geq a \times (\delta_i)^2 + b \times \delta_i - M \times (1 - v_i) \qquad\qquad \forall i \in N \qquad (15)$$

$$g_i^\beta = g_i^\alpha + s_i^g \times g_i^+ \qquad\qquad \forall i \in N \qquad (16)$$

$$M \times (x_{ij} - 1) \le g_j^\alpha - g_i^\beta + \rho \times d_{ij}^{cs} \le M \times (1 - x_{ij}) \qquad \forall (i,j) \in A \qquad (17)$$

$$\underline{G} \le g_i^\alpha \le \overline{G} \qquad\qquad \forall i \in N \qquad (18)$$

$$\underline{G} \le g_i^\beta \le \overline{G} \qquad\qquad \forall i \in N \qquad (19)$$

$$g_i^+ \le r_i \times \overline{G} \qquad\qquad \forall i \in N \qquad (20)$$

$$g_s^\alpha = G_s \qquad\qquad (21)$$

$$g_t^\alpha \ge G_t \qquad\qquad (22)$$

$$d_{ij}^{cs} + d_{ij}^{cd} = d_{ij} \qquad\qquad \forall (i,j) \in A \qquad (23)$$

$$x_{ij}, v_k, r_k \in \{0,1\}; \ d_{ij}^{cd}, d_{ij}^{cs}, e_k^\alpha, e_k^\beta, e_k^+, g_k^\alpha, g_k^\beta, g_k^+, \delta_k^\alpha, c_k^{bat} \ge 0$$

$$\forall k \in N, \forall (i,j) \in A \qquad (24)$$

The objective function minimizes the cost of traveling. The cost components are the cost of obtaining electricity and gasoline, the battery degradation cost, the depreciation cost and the stopping cost. Constraints (2)-(4) enforce the solution to be a path from s to t. Constraints (5) are the electricity balance equations for nodes. The level of electricity upon leaving node i equals the entering electricity level plus the electricity obtained at node i. Similarly, Constraints (6) are the electricity balance equations for those arcs that are on the path. For the non-path arcs, the constraints are relaxed. Constraints (7)-(8) set the upper and lower bounds for the electricity level when entering or leaving a node. Constraints (9) assign binary v_i variable a value of 1 if battery is switched at node i. Because a switched battery is necessarily full, Constraints (10) force the charge level upon leaving the node to be full if the battery is switched. Constraints (11) require that r_i is set to 1 if v_i equals 1 and therefore a stopping cost is incurred in the objective function if the PHEV stops to switch its battery. Constraints (12)-(13) set the electricity level at nodes s and t, respectively. Constraints (14) assign proper

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depth of discharge values and Constraints (15) calculate the battery degradation for each node if battery is switched. Constraints (16)-(22) are the counterparts of constraints (5)-(13) for the gasoline case. Constraints (23) make sure that the sum of the distances on CS and CD modes is equal to the arc length if the arc is on the path. Constraints (24) are the domain requirements.

A *directed path* is an alternating sequence of nodes $(n_0, n_1, n_2, ..., n_k)$ with $(n_i, n_{i+1}) \in A$, $\forall i = 0, \ldots, k-1$. A directed path is a *non-simple path* if it repeats nodes and *simple path* otherwise. Non-simple paths can occur in transportation networks and as solution to the E-MCPP-PHEV. The presented MIQCP formulation constructs a simple path in the input network $G = (N, A)$. By choosing G as the meta-network or as the reduced meta-network of the input transportation network, a wide group of non-simple paths as potential solutions can be efficiently handled by this formulation. All non-simple path occurrences, including extremely rare ones, can be taken into account by duplicating the nodes in G at the expense of computational inefficiency. In the Appendix, we present possible occurrences of non-simple paths in the optimal solutions and ways to handle those cases.

Observe that one can easily extract the following information from the outputs of the model: the path to travel from node s to node t, how many miles to travel on CD and CS modes on each arc, where to stop to refuel or switch battery, and how much to refuel at each refueling stop.

Lemma 1. $\left(\sum_{i \in N} e_i^+ + P_s - P_t \right)/\varepsilon + \left(\sum_{i \in N} g_i^+ + G_s - G_t \right)/\rho \geq \sum_{(i,j) \in A} x_{ij} \times d_{ij}$ *is a valid inequality to (2)-(24).*

The inequality simply states that we need to have enough electricity and gasoline to travel the trip distance. Computational studies in Section 4 show that the above cut is very effective in improving the relaxation bound.

3.2. Dynamic Programming Based Heuristic

In this subsection, we introduce a dynamic programming based heuristic algorithm referred to as DH. We first define a set of states associated with electricity and gasoline levels at nodes. Then, we present Bellman's equations (Bellman 1956) that should be satisfied by minimum cost path lengths in order to facilitate the dynamic programming solution methodology. Lastly, we present a graph transformation by which the solution of these equations can be accomplished efficiently by solving a shortest path problem on the transformed graph.

Definition 5. *A state is a triplet $\langle i, \sigma, \lambda \rangle$ which represents the arrival at a node $i \in N$ with $\sigma \in [\underline{P}, \overline{P}]$ kWh electricity charge and $\lambda \in [\underline{G}, \overline{G}]$ gallons of gasoline. We will use the notation $\omega_i^{\sigma, \lambda}$ to refer to a state and replace this notation with ω or ω_i when the context does not require specific values of i, σ and λ to be discerned.*

Given an E-MCPP-PHEV instance $\langle V, X, s, t, P_s, G_s, P_t, G_t \rangle$, a solution $\langle x, e^+, g^+ \rangle$ contains a path from s to t which can be extracted from the vector x. With the specific energy (e^+) and gasoline (g^+) purchases at the nodes, the distances to be covered in CD and CS modes on this path can easily be extracted. Together with P_s and G_s, the vectors x, e^+ and g^+ induce the levels of state-of-charge and gasoline at arrival to the nodes on the solution path. So, for every solution of the E-MCPP-PHEV, there is a unique sequence of states that represents this solution. Note that in general, the E-MCPP-PHEV has an uncountable number of feasible solutions. Since each of these solutions maps uniquely to a sequence of *states*, the state space is also uncountable. However this uncountable state space can be approximated with a finite one which is the main idea behind the DH.

Let $\xi, \tau \in \mathbb{N}$ be the discretization parameters for the state space. Consider two sets $\Sigma = \{\sigma_0, \sigma_1, \dots, \sigma_\xi\}$ and $\Lambda = \{\lambda_0, \lambda_1, \dots, \lambda_\tau\}$ where $\sigma_0 = \underline{P}, \lambda_0 = \underline{G}$, $\sigma_k = \sigma_0 + k \times \frac{\overline{P} - \underline{P}}{\xi}$ $\quad \forall k \in \{1, 2, \dots, \xi\}$ and $\lambda_l = \lambda_0 + l \times \frac{\overline{G} - \underline{G}}{\tau}$ $\quad \forall l \in \{1, 2, \dots, \tau\}$.

Every σ_i represents the interval of electricity levels $[\sigma_i, \sigma_{i+1}]$ $\forall i \in \{0, 1, \ldots, \xi - 1\}$ and σ_ξ represent the fully charged battery. The representation for each λ is similar.

For a given E-MCPP-PHEV instance $\langle V, X, s, t, P_s, G_s, P_t, G_t \rangle$, ξ and τ values, we define the *discrete state space* Ω as:

$$\Omega = \{(\omega_i^{\sigma, \lambda} | i \in N - \{s, t\}, \sigma \in \Sigma, \lambda \in \Lambda\} \cup \{\omega_s^{P_s, G_s}, \omega_t^{P_t, G_t}\} \qquad (25)$$

Observe that the cardinality of the *discrete state space* Ω is bounded by $n \times (\xi + 1) \times (\tau + 1)$ where n is the number of nodes in X, and is finite. Algorithm DH uses Ω and incurs an approximation error on representing the amount of electricity charge and gasoline left with the PHEV arriving at a node. Obviously this approximation error can be reduced arbitrarily by choosing ξ and τ large enough.

Definition 6. $\pi : \Omega \to \mathbb{R}$ *is called the value function and* $\pi(\omega_i^{\sigma, \lambda})$ *is defined to be the optimal solution value of the E-MCPP-PHEV instance* $\langle V, X, s, i, P_s, G_s, \sigma, \lambda \rangle$.

The minimum cost transition function $f : \Omega \times \Omega \to \mathbb{R}^+$ takes two states $\omega_i^{\sigma, \lambda}, \omega_j^{\bar{\sigma}, \bar{\lambda}}$ as its arguments and returns the minimum cost of the transition from node i starting with σ kWh charge and λ gallons of gasoline to node j ending with at least $\bar{\sigma}$ kWh charge and $\bar{\lambda}$ gallons of gasoline. When calculating $f(\omega_i^{\sigma, \lambda}, \omega_j^{\bar{\sigma}, \bar{\lambda}})$, we only consider how much to refuel and whether or not to switch battery at node i. Four cases as detailed below should be considered. A feasibility condition is stated for each case. The cost value is as presented if the feasibility condition is met, and is not finite otherwise. Let $d^\star$ represent the shortest path lengths.

- Case 1: No battery switching and no refueling.

  Feasibility Condition: The existing electricity charge and gasoline are enough to travel from node i to node j while satisfying the end-state conditions, i.e.,

$$\sigma \geq \bar{\sigma}, \lambda \geq \bar{\lambda} \text{ and } \frac{(\sigma - \bar{\sigma})}{\varepsilon} + \frac{(\lambda - \bar{\lambda})}{\rho} \geq d^\star_{ij}.$$

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Total Cost: The only cost component to be incurred is the depreciation cost. Thus, $f_1(\omega_i^{\sigma,\lambda}, \omega_j^{\bar{\sigma},\bar{\lambda}}) = c^{dep} \times d_{ij}^\star$.

- Case 2: Refueling but no battery switching.

Feasibility Condition: The existing electricity charge and full tank of gasoline are enough to travel from node i to node j while satisfying the end-state conditions, i.e.,

$$s_i^g = 1, \sigma \geq \bar{\sigma} \text{ and } \frac{(\sigma - \bar{\sigma})}{\varepsilon} + \frac{(\overline{G} - \bar{\lambda})}{\rho} \geq d_{ij}^\star.$$

Total Cost: The minimum cost transition requires to use $(\sigma - \bar{\sigma})$ electricity charge first. Thus $d_{ij}^{cd} = \min\{d_{ij}^\star, \frac{(\sigma - \bar{\sigma})}{\varepsilon}\}$ and $d_{ij}^{cs} = d_{ij}^\star - d_{ij}^{cd}$. On the other hand, we need to purchase enough gasoline at node i to cover the travel distance and retain $\bar{\lambda}$ gallons of gasoline at node j, i.e., $g_i^+ = (d_{ij}^{cs} \times \rho + \bar{\lambda} - \lambda)^+$ gallons of gasoline should be purchased at node i. Note that, by the feasibility condition, we make sure that the purchased gasoline is between the limits, i.e. $0 \leq g_i^+ \leq \overline{G} - \lambda$. Since the battery is not switched, only the gasoline cost, vehicle depreciation cost and stopping cost are included in the total cost function which is $f_2(\omega_i^{\sigma,\lambda}, \omega_j^{\bar{\sigma},\bar{\lambda}}) = c_i^g \times g_i^+ + c^{dep} \times d_{ij}^\star + c^{st}$.

- Case 3: Battery switching but no refueling.

Feasibility Condition: A full battery charge and existing level of gasoline are jointly enough to travel from node i to node j while satisfying the end-state conditions, i.e.,

$$s_i^e = 1, \lambda \geq \bar{\lambda} \text{ and } \frac{(\overline{P} - \bar{\sigma})}{\varepsilon} + \frac{(\lambda - \bar{\lambda})}{\rho} \geq d_{ij}^\star.$$

Total Cost: We have $e_i^+ = \overline{P} - \sigma$. We first use this electricity charge to

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travel from i to j. Thus, $d_{ij}^{cd} = \min\{d_{ij}^{\star}, \frac{(\overline{P}-\bar{\sigma})}{\varepsilon}\}$ and $d_{ij}^{cs} = d_{ij}^{\star} - d_{ij}^{cd}$. We do not purchase gasoline in this case. The electricity cost, battery degradation cost, vehicle depreciation cost and stopping cost are included in the total cost. Thus the total cost is, $f_3(\omega_i^{\sigma,\lambda}, \omega_j^{\bar{\sigma},\bar{\lambda}}) = c_i^e \times e_i^+ + c^{bat}(\frac{\overline{P}-\sigma}{\overline{P}}) + c^{dep} \times d_{ij}^{\star} + c^{st}$.

- Case 4: Both battery switching and refueling.

  Feasibility Condition: A full battery charge and a full tank of gasoline are enough to travel from node i to node j, while satisfying the end-state conditions, i.e.,
  $$s_i^e = 1, s_i^g = 1 \text{ and } \frac{(\overline{P}-\bar{\sigma})}{\varepsilon} + \frac{(\overline{G}-\bar{\lambda})}{\rho} \geq d_{ij}^{\star}.$$

  Total Cost: In this case, we switch battery and refuel. Similar to Case 3, we necessarily have $e_i^+ = \overline{P} - \sigma$. We first use this electricity charge to travel from i to j. Thus, $d_{ij}^{cd} = \min\{d_{ij}^{\star}, \frac{(\overline{P}-\bar{\sigma})}{\varepsilon}\}$ and $d_{ij}^{cs} = d_{ij}^{\star} - d_{ij}^{cd}$. Similar to Case 2, we need to purchase $g_i^+ = (d_{ij}^{cs} \times \rho + \bar{\lambda} - \lambda)^+$ gallons of gasoline at node i. Note that, by the feasibility condition, we make sure that the purchased gasoline is between the limits, i.e. $0 \leq g_i^+ \leq \overline{G} - \lambda$. All cost components are included in the total cost and thus, $f_4(\omega_i^{\sigma,\lambda}, \omega_j^{\bar{\sigma},\bar{\lambda}}) = c_i^e \times e_i^+ + c_i^g \times g_i^+ + c^{bat}(\frac{\overline{P}-\sigma}{\overline{P}}) + c^{dep} \times d_{ij}^{\star} + c^{st}$.

Considering all possible cases, the minimum cost transition function is defined as:

$$f(\omega, \bar{\omega}) = \min_{i \in \{1,2,3,4\}} \{f_i(\omega, \bar{\omega})\} \tag{26}$$

The following Bellman's equations are based on the principle of optimality:

$$\pi(\omega_s^{P_s, G_s}) = 0 \tag{27}$$

$$\pi(\omega) = \min_{\bar{\omega} \in \Omega} \{\pi(\bar{\omega}) + f(\bar{\omega}, \omega)\} \qquad \forall \omega \in \Omega \tag{28}$$

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Definition 7. $\tilde{G} = (\Omega, \tilde{A})$ *is called the DH-Graph where the node set is the discrete state space Ω. The arc set $\tilde{A}$ includes an arc between states ω_i and $\omega_j \in \Omega$ with a cost of $f(\omega_i, \omega_j)$ if this cost is finite.*

Once the DH-Graph is obtained, solving the Bellman's equations, which is the core of the DH algorithm, reduces to solving the shortest path problem on $\tilde{G}$ from *state* $\omega_s^{P_s, G_s}$ to the *state* $\omega_t^{P_t, G_t}$. Observe that arcs on the shortest path contain the information where the PHEV stops for refueling/recharging and how much electricity charge/gasoline to purchase at those stops. So obtaining the shortest path in $\tilde{G}$ is sufficient to obtain a solution for the E-MCPP-PHEV instance.

$\tilde{G}$ contains $|\Omega|$ nodes and the cardinality of the arc set $\tilde{A}$ is bounded by $|\Omega|^2$. Constant time calculation of the transition function f results in $O(|\Omega|^2)$ run time bound for the generation of the DH-Graph. Using Dijkstra's algorithm to find the shortest path in $\tilde{G}$, the overall run time complexity of DH becomes $O(|\Omega|^2)$.

3.3. Extended Discrete State Space Heuristic (DHE)

Due to discretization of the levels of gasoline and electricity, DH might not always give the optimal solution in terms of refueling and battery switching policies even if the optimal path is correctly identified. To that end, we provide extended version of DH (DHE) in which we take into account the path that is given by the algorithm, but not the refueling and battery switching policies. Instead, we consider the subgraph that consists of only the path nodes and the path arcs. Then, we solve the model presented in Subsection 3.1 on this subgraph. Since the subgraph size is much smaller than the original graph, the solution times of the model formulation reduce drastically and we attain improved refuel and battery switch strategies.

3.4. Extended Shortest Path Heuristic (SPE)

Minimizing the operating cost on the shortest path is a commonly used solution technique to solve the minimum cost path problems in the literature. Since well known efficient algorithms are available for finding shortest paths, such heuristics are also pervasive in industrial and commercial applications as well. In this context, we propose the extended shortest path heuristic (SPE) in which MIQCP model is solved considering the shortest path as the input graph.

4. Computational Study

To test the performances of the proposed solution methodologies and drive insights about the solutions, we conducted extensive numerical experiments using problem instances that represent various network structures and user behaviors. IBM ILOG CPLEX Optimization Studio 12.4 was used on a 4x16C AMD Opteron with 96 GB RAM computer for the computational study. We present the data and the results related to computational performances and several measures in the following subsections. It is important to note that with several preliminary experimentations, we have observed that working with reduced meta-networks is satisfactory in capturing the non-simple paths that might arise in our instances and opted to using reduced meta-networks throughout our computational experiments.

4.1. Data

A 2013 Chevrolet Volt PHEV has the following specifications: 16.5 kWh battery capacity, 9.3 gallon tank capacity, 0.352 kWh per mile and 0.027 gallons per mile (United States Department of Energy 2013) usages. We assume a 20% minimum battery level. Furthermore, we assume that the battery cannot be charged over 85% to avoid overcharging degradation. Hence, we assume a hard bound of 14 kWh on capacity rather than 16.5 kWh. The battery cost of PHEV is assumed to be \$2650 and the cost function with respect to depth of discharge is

$c^{dep}(\delta) = 79.517 \times \delta^2 + 37.854 \times \delta$, as presented in Figure 3. We also assume that the minimum tank capacity is zero and the depreciation cost is 1 ¢/mile. In order to analyze the effects of the stopping cost on the total travel costs, we consider stopping costs of 0, 50, 100, 200 and 500 ¢.

For the network instances, we consider square mesh shaped networks of node sizes 6x6, 7x7, 8x8, 9x9 and 10x10. We generate 10 instances of each size. Every node in a given network is connected with an arc to the next node on the right, left, top and bottom, if there is one. The source and destination nodes are the top left and bottom right nodes of the graph, respectively. The arc distances are random values uniformly distributed between 20 and 40 miles. A refueling station is located at every node and the gasoline prices are uniformly generated in $3.5 and $4.1 range. We assume that battery switching stations are located randomly at 0%, 25%, 50%, 75% and 100% of the total nodes and the electricity prices at battery switching stations change uniformly between 10 ¢ and 12 ¢. In total, we have 250 mesh shaped networks and 5 different stopping cost values, i.e. 1250 runs. For each set of parameters, we report the averages corresponding to 10 network instances.

Furthermore, in order to test the performances of the solution techniques in large datasets, we consider a real-world California road network (Li et al. 2005). After processing this network, we have 339 nodes and 1234 arcs as depicted in Figure 4. It is assumed that there is a refueling station in every node, and the nodes on the highway also have battery switching stations. The other settings related to pricing are similar to those of mesh shaped networks. The minimum cost path and refueling/battery switching policies are obtained for each origin-destination pair between 10 randomly selected nodes as depicted in Figure 4.
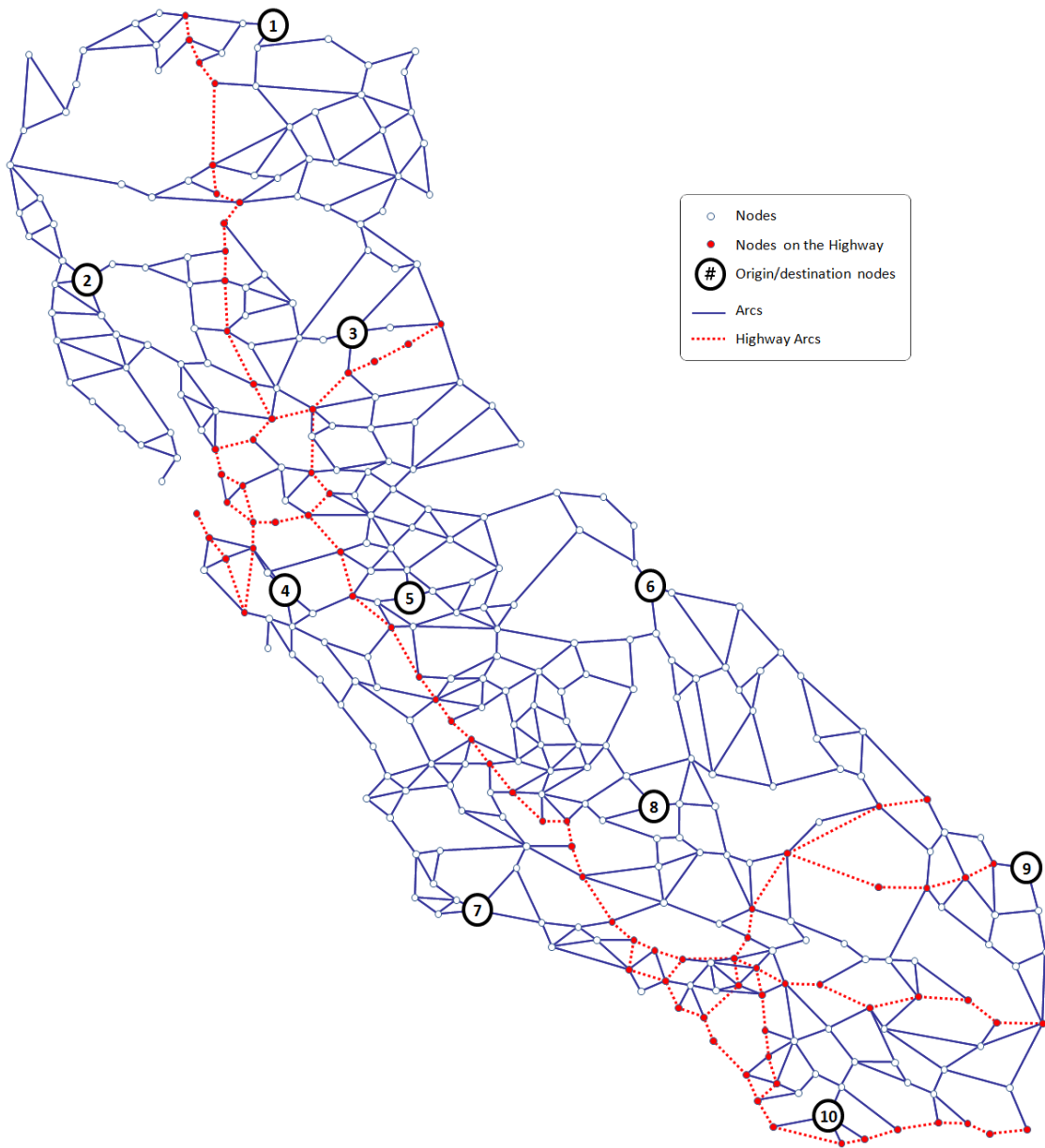
Figure 4: California Network with 339 Nodes and 1234 Arcs

4.2. Performances of the Solution Techniques

We present the basic computational performance measures of the solution methodologies in Table 1. DH is solved with two different levels $\xi = \tau = 4$ and $\xi = \tau = 1$, which we refer to as DH4 and DH1, respectively. The percentage of the optimal solutions for DH4 (DH1) range in 46.4-67.9% (46.4-67.9%) for all instances, which is improved by the extended versions of the algorithms to around 88.4-97.0% (82.4-92.9%). An optimal path is found by DH4 (DH1) in around 86.4-97.6% (81.6-92.9%) of all the instances. Since a high percentage of the optimal solutions (ranging between 64.4-88.7%) coincide with the shortest paths, the SPE heuristic also performs well in minimum cost path problems. However, DHE1 performs equal or better than SPE in the network instances of this study.

We observe that the solution times for the MIQCP starts getting prohibitive as the node number increases. Beyond 100 nodes, there exist problem instances with more than 30 minutes solution times. On the other hand, observe that the average solution time of the DH1 is less than 0.56 seconds on all network sizes. In fact, the average runtime of DH1 for problem instances with 900 nodes is only 40.3 seconds which makes it the suitable solution technique for devices with limited computational capacity. However, since other solution techniques did not scale up to such dimensions, these results are not presented here.

One important fact to note is that the valid inequality presented in Subsection 3.1 greatly contributes to the solution times of the MIQCP. The average gap of the LP relaxation solution from the optimal solution with and without the cut is 29.63% and 90.46%, respectively. We also observe that optimal paths of DH4 (DH1) coincide with the shortest paths on the average 63.2-89.3% (61.6-91.7%) of instances. On the average, the deviation from the shortest path changes in the range of 0.254-0.518% (0.091-0.526%).

Table 1: Computational Results

Node Number	Solution Technique	Opt. sol. found (%)	Avg opt. gap (%)	Opt. path found (%)	Is shortest path? (%)	Avg deviation from the shortest path (%)	Solution Time
36	MIQCP	100.0	0.000	100.0	78.4	0.357	0.552
	DH4	54.0	0.761	95.2	81.6	0.322	0.517
	DH1	54.0	1.762	91.2	79.6	0.376	0.003
	DHE4	95.2	0.011	95.2	81.6	0.322	0.938
	DHE1	91.2	0.060	91.2	79.6	0.376	0.396
	SPE	78.4	0.418	78.4	100.0	0.000	0.261
49	MIQCP	100.0	0.000	100.0	84.4	0.400	1.484
	DH4	52.0	0.712	96.4	82.8	0.421	1.171
	DH1	52.0	1.847	87.6	81.2	0.415	0.003
	DHE4	96.4	0.011	96.4	82.8	0.421	1.681
	DHE1	87.6	0.144	87.6	81.2	0.415	0.539
	SPE	84.4	0.398	84.4	100.0	0.000	0.307
64	MIQCP	100.0	0.000	100.0	70.4	0.334	9.248
	DH4	49.6	0.784	91.2	69.2	0.355	2.235
	DH1	49.6	1.908	83.6	65.6	0.358	0.005
	DHE4	91.2	0.039	91.2	69.2	0.355	2.899
	DHE1	83.2	0.134	83.6	65.6	0.358	0.660
	SPE	70.4	0.548	70.4	100.0	0.000	0.344
81	MIQCP	100.0	0.000	100.0	75.6	0.321	54.779
	DH4	50.8	0.788	90.4	75.2	0.295	4.003
	DH1	50.4	1.925	86.4	74.4	0.343	0.009
	DHE4	90.0	0.052	90.4	75.2	0.295	4.913
	DHE1	86.0	0.132	86.4	74.4	0.343	0.883
	SPE	75.6	0.396	75.6	100.0	0.000	0.370
100	MIQCP	100.0	0.000	100.0	64.4	0.482	294.201
	DH4	46.4	0.836	86.4	63.2	0.518	6.476
	DH1	46.4	2.075	81.6	61.6	0.526	0.015
	DHE4	88.4	0.029	86.4	63.2	0.518	7.823
	DHE1	82.4	0.126	81.6	61.6	0.526	1.322
	SPE	64.4	0.642	64.4	100.0	0.000	0.430
CA[a]	MIQCP	100.0	0.000	100.0	88.7	0.268	501.699
	DH4	67.9	0.556	97.6	89.3	0.254	253.443
	DH1	67.9	1.379	92.9	91.7	0.091	0.560
	DHE4	97.0	0.009	97.6	89.3	0.254	256.294
	DHE1	92.9	0.099	92.9	91.7	0.091	3.338
	SPE	88.7	0.257	88.7	100.0	0.000	0.715

[a] 74.7% of the MIQCP runs were solved to optimality within 30 minutes. The results are given for only those cases that are solved to optimality by the MIQCP.

4.3. Insights

The cost reduction of a PHEV trip with respect to a CV is due to the CD mode driving technology. How much benefit can be attained is directly proportional with the CD mode driving mileage which is dependent on the number of battery switching stations in the network and the driver's tolerance for stopping. In our numerical experiments, we investigate the effects of these two main parameters: the percentage of nodes with battery switching stations (which we refer to as the penetration level) and the stopping costs (higher stopping costs imply less tolerance for stopping). In the following graphs, we present the optimal results obtained by the MIQCP formulation for 100 nodes network instances. The results for 36, 49, 64 and 81 nodes network instances follow very similar trends to those that we present in these graphs and hence are not presented.
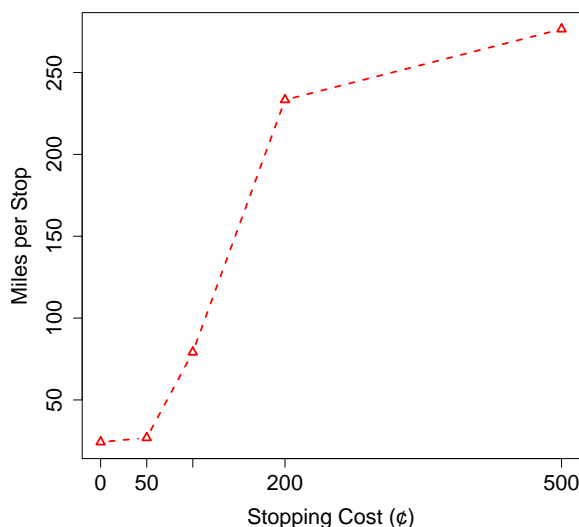
Figure 5: Average Miles per Stop for Different Stopping Costs in a Network With 100 Nodes and 100% Switching Station Penetration Level

Figure 5 depicts the average miles per stop for different stopping costs. In order to depict the sole effect of the stopping cost on the average miles per stop,

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100% penetration is chosen. In other words, a PHEV can stop at every node in the network in order to refuel or switch its battery. Observe that lower stopping costs result in frequent stops. This graph can be used for quantifying one's own stopping cost. Knowing the tolerance for average miles between stops, one can easily obtain his/her dollar value for stopping cost. On the other hand, the graph can also be used to determine how many stops one can tolerate in a trip and the opportunity cost associated with the time spent in these stops.
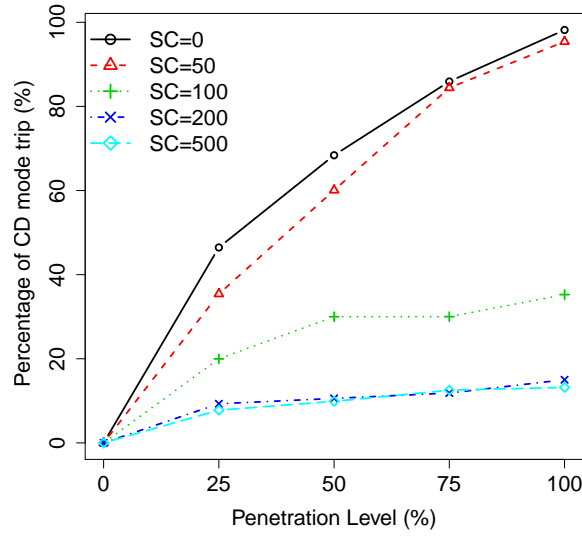
Figure 6: CD Mode Trip Percentage Change for Different Stopping Costs (SC) and Penetration Levels

Figure 6 shows the percentage of the distance covered in CD mode. At zero penetration level, there does not exist any battery switching station in the network and the CD mode mileage is therefore zero. With increasing penetration level, the CD mode mileage increases accordingly. For zero stopping cost, the CD mode trip percentage increases to almost 100% for 100% penetration level. On the other hand, for the stopping costs of more than 200 ¢, the CD mode trip percentage does not go above 10%. This is due to the fact that even though there exists battery

switching opportunities on the path, the driver cannot tolerate for frequent stops and therefore continues on the CS mode rather than CD mode. This implies that for those drivers with less tolerance for stopping, increasing the number of battery switching stations does not necessarily imply more CD mode drive. Increasing the battery capacity is more important than increasing the number of switching stations. On the other hand, if the drivers are more tolerant for stopping, increasing the number of switching stations is equivalent to increasing the battery capacity in terms of CD mode drive percentage. Observe that this result is crucial for both infrastructure investors and governments. We believe that decision makers need to consider the drivers' tolerance for stopping which is neglected in the existing literature and more research must be directed towards determining the utility functions of PHEV drivers' willingness for making frequent stops.
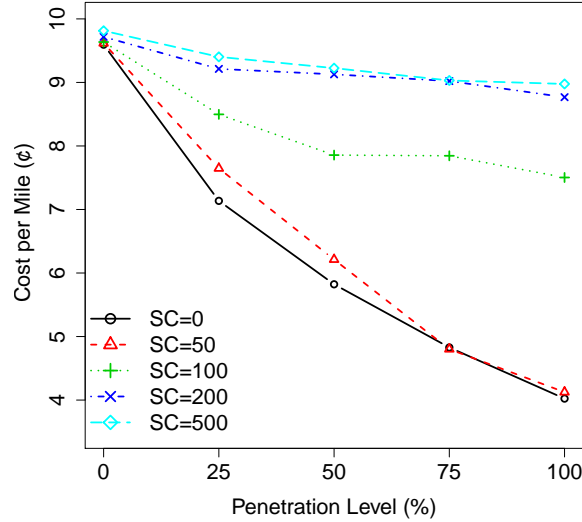

Figure 7: The Effect of Battery Switching Station Penetration Level on the Cost Per Mile for Different Stopping Costs (SC)

The cost per mile graph is depicted in Figure 7 for different stopping costs and penetration levels. When solving the MIQCP model, the objective function

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included the stopping cost, but the cost in the graph is composed of only the following components: electricity cost, gasoline cost, depreciation cost and battery degradation cost. This way, we are able to compare the costs for different stopping cost configurations. Observe that Figure 7 proposes similar results to previous findings. Consider zero stopping cost. As the penetration level increases, the cost per mile decreases to 4 ¢ for 100% penetration level. This result is due to more CD mode trip which can also be observed in Figure 6. The decrease is not as high for 100 ¢ stopping cost case. Note that the cost is almost not affected by penetration level increase for higher stopping costs. These results are also parallel to those in Figure 6.
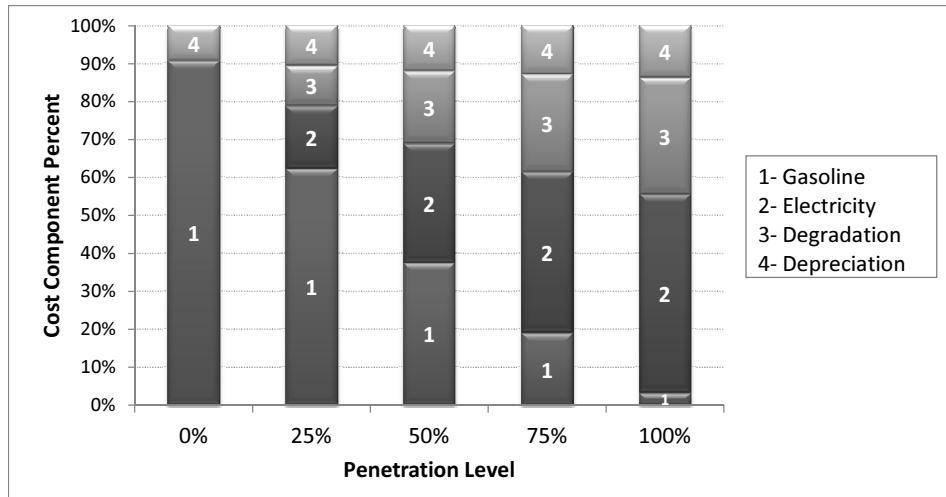

Figure 8: The Effect of Battery Switching Station Penetration Level on the Cost Components for 0¢ Stopping Cost

Lastly, we investigate the change of cost components with increasing penetration level. Figures 8 and 9 depict the percentage of cost components with increasing penetration level for 0 ¢ and 500 ¢ stopping cost values, respectively. The effect of penetration level is significant for no stopping cost and the gasoline usage sig-
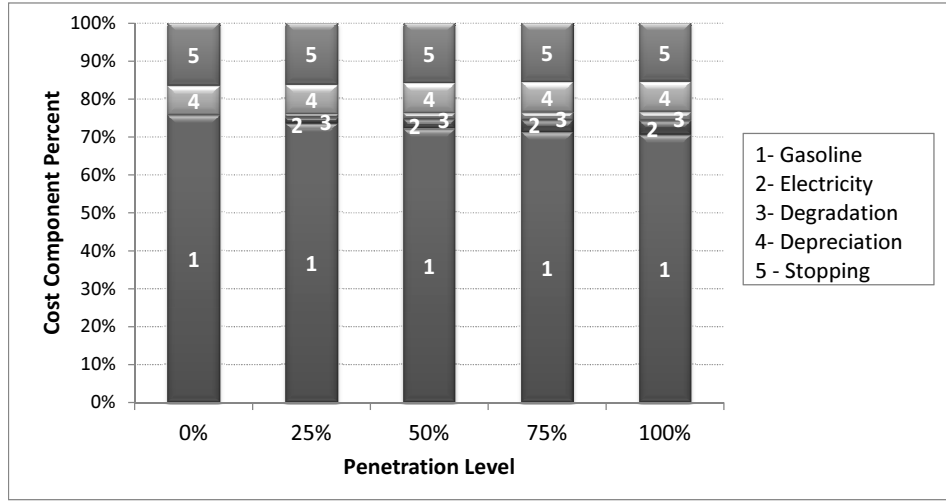
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Figure 9: The Effect of Battery Switching Station Penetration Level on the Cost Components for 500¢ Stopping Cost

nificantly diminishes for 100% penetration level. On the other hand, gasoline is the main source of energy for every penetration level for high stopping costs as depicted in Figure 9 and the PHEV is mainly driven in CS mode.

In the literature, several studies including Wang and Lin (2009) and Romm (2006) argue that the main barrier for the growth of PHEVs on the road is the scarcity of a battery switching station in the road network. However, our results show that increasing the penetration level of the battery switching station infrastructure might not be enough for promoting PHEVs and the tolerance for stopping need to be taken into account as well. For drivers with less tolerance for stopping, increasing the battery capacity of a PHEV is more important than increasing the number of battery switching stations. This result might affect each of the stake holders, namely potential PHEV users, infrastructure investors and governments.

More detailed analyses on the impacts of battery characteristics, driver preferences and road network features on travel costs of a PHEV for long-distance trips

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is carried out by Arslan et al. (2014) using the presented problem and the solution methodology.

5. Conclusion

In this article, we introduce a practically important and theoretically challenging problem: finding the minimum cost path for plug-in hybrid electric vehicles. The theoretical challenge arises due to two modes of drive (CS and CD). In fact, we show that this problem is NP-complete even though there are polynomial time algorithms to solve its electric and gasoline special cases. Fluctuations in fuel/electricity costs, battery degradation issues and scarcity of battery switching stations add further and realistic challenges to our problem. Our computational studies show that the proposed MIQCP formulation can solve problems with realistic sizes. A dynamic programming based heuristic and a shortest path heuristic methodologies further extend the sizes of the solvable problems drastically and produce near optimal solutions. The methodologies that we present in this article are not only applicable for PHEVs, but also for all types of hybrid vehicles that run on two types of energy resources. Furthermore, our solution methodologies encompass fast-charging option of PHEVs as well.

Our study reveals one strategic insight about the alternative energy vehicles: In the literature, most of the studies related to alternative energy vehicles - EV and PHEV in particular - discuss the problem of availability of refueling and battery switching stations as a barrier to proliferation of those vehicles. However, the limited range of a non-fossil-fuel-energy drive not only brings the problem of finding battery switching stations on the route, but also results in frequent battery switching stops which may not be preferable for most of the drivers. Our study shows that this neglected problem can also be a significant barrier. Governments that put subsidies to promote the development and proliferation of alternative

energy vehicles and industries that make decisions about directing their R&D efforts and infrastructure investments need to take drivers' tolerance for stopping into consideration as well.

In this appendix, we demonstrate examples of non-simple paths that might appear as the optimal solution of the E-MCPP-PHEV problem and present methods to handle these non-simple paths by the mathematical model presented in Section 3.1.

First, note that all of the non-simple paths can be handled by duplicating every node in the graph G (as many times as the drivers are willing to revisit the same node in the same trip or as the number of nodes in the worst case). But this implies a much larger graph size and brings along computational burden. Thus, we first present ways to handle those cases by modifying the input graph for the MIQCP model before resorting to the costly node duplication.

Problem Instance

- We consider the vehicle Instance $V = \langle \overline{P} = 1, \underline{P} = 0, \overline{G} = 9, \underline{G} = 0, \varepsilon = 1, \rho = 1 \rangle$. Thus the gasoline range of a PHEV is 9 miles and the electricity range is 1 mile.

- To illustrate different cases, we use a different network instance for each of the three examples (Figure 10). In all three networks, nodes A, B and C are points on the highway.

- The problem instance is given as $\langle V, X, A, C, 1, 9, 0, 0 \rangle$ where X is the input network instance. For the sake of simplicity, we also assume that battery degradation, vehicle depreciation and stopping costs are all zero.

Case-1: Detour from the highway to refuel

We can think of nodes A, B and C as points on the highway, and D is a refueling station just one mile away from the highway. Node B is deleted in the
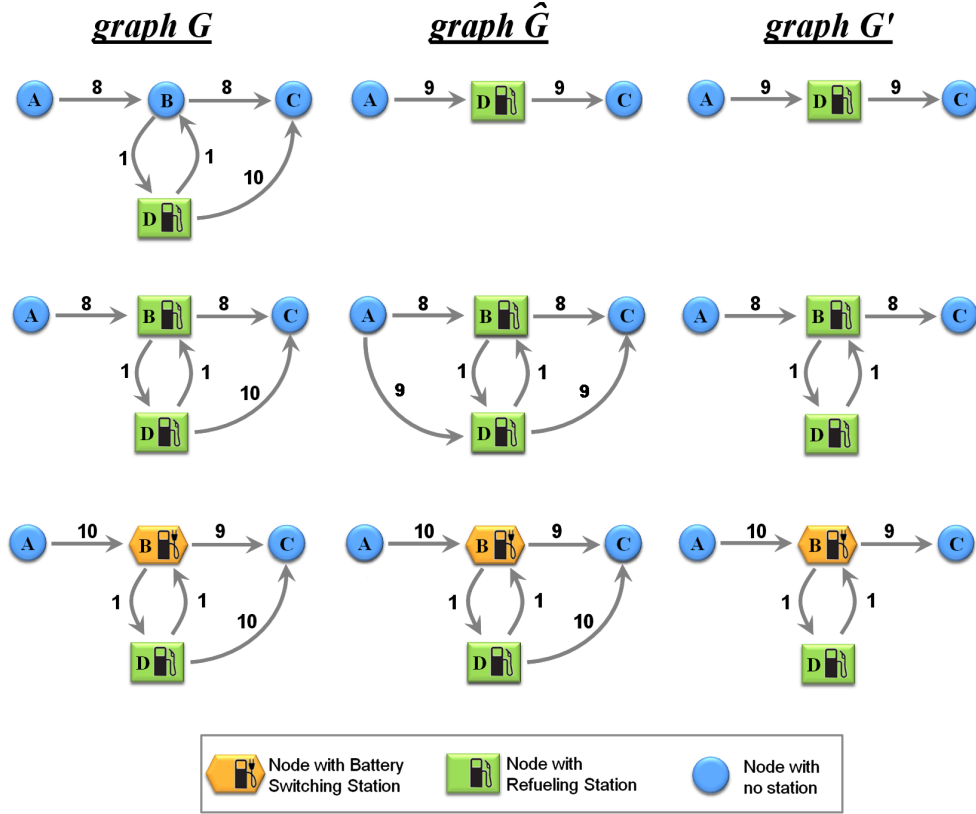
Figure 10: Non-Simple Path Examples

meta-network or reduced meta-network since it does not have a station. The optimal non-simple path from A to C in G is $A \rightarrow B \rightarrow D \rightarrow B \rightarrow C$ and can be attained by using the reduced meta-network G' as input to the MIQCP formulation.

Case-2: Detour from the highway to refuel in a cheaper station

The middle figure illustrates a detour from the highway. But this time, there is a refueling station on the highway (possibly with a more expensive gasoline price) at which the PHEV can detour and go to node D in order to refuel. In this case, MIQCP formulation can handle the optimal non-simple path $A \rightarrow B \rightarrow D \rightarrow B \rightarrow C$ by using meta-network $\hat{G}$ as the input graph. Indeed, simple path

$A \rightarrow D \rightarrow C$ in $\hat{G}$ will correspond to this solution.

Case-3: Refuel twice in the same station

Now, consider the bottom figure. This time, node B has a battery switching station and node D has a refueling station. Observe that there is only one feasible solution for this problem: $A \rightarrow B \rightarrow D \rightarrow B \rightarrow C$. The PHEV switches battery at node B, travels to node D to refuel. Then it necessarily returns back to node B and switches its battery again in order to be able to reach to node C. In this example, the optimal path is a non-simple path in all three graph types and thus, MIQCP formulation can only handle such non-simple paths by a node duplication in this case.

Note that this particular instance can be generalized so that more than two visits to the same node, and hence more than one duplication of the node set, is necessary. Note also that this is a rather rare occurrence. The emergence of such non-simple paths is not only due to price differences, but also to range limitations as well. In the example, node B is reachable from node A, but node D is not. Considering the combined gasoline and electric range of existing PHEVs, this example is not very representative of the real network instances under our scope.

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