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Explicit time-delay compensation in teleoperation: An adaptive control approach

K. Abidi[1,*,†], Y. Yildiz[2] and B. E. Korpe[2]

[1] *School of Electrical and Electronic Engineering, Newcastle University, Newcastle Upon Tyne, NE1 7RU, UK*
[2] *Department of Mechanical Engineering, Bilkent University, Cankaya, Ankara 06800, Turkey*

SUMMARY

This paper proposes a control framework that addresses the destabilizing effect of communication time delays and system uncertainties in telerobotics, in the presence of force feedback. Force feedback is necessary to obtain transparency, which is providing the human operator as close a feel as possible of the environment where the slave robot is operating. Achieving stability and providing transparency are conflicting goals. This is the major reason why, currently, a very few, if at all, fully operational force feedback teleoperation devices exist except for research environments. The proposed framework handles system uncertainty with adaptation and communication time delays with explicit delay compensation. The technology that allows this explicit adaptive time-delay compensation is inspired by Massachusetts Institute of Technology (MIT)'s Adaptive Posicast Controller. We provide simulation results that demonstrate stable explicit adaptive delay compensation in a force-reflecting teleoperation set up. Copyright © 2016 John Wiley & Sons, Ltd.

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1. INTRODUCTION

In a teleoperation system, a human operator commands a master robot, and a remote slave robot follows the motions of the master robot. See Figure 1 [1]. If the forces on the slave side are reflected back to the master robot, that system is called a 'force reflecting telerobotics system". Because of the two-way information exchange, these systems are also called 'bilateral telerobotics system'. The main challenge in force reflecting telerobotics is satisfying two competing goals: transparency and stability. Transparency is a term used in telerobotics field, which refers to the extent that the system is capable of giving the feel of the operation environment of the slave robot to the human operator. Force feedback, vision feedback, and acoustic feedback are some of the tools that can be used to achieve this goal. Among these tools, arguably the most problematic one is the force feedback because it has the most detrimental effect on stability. As demonstrated in [2], even a small stimulus can make a time-delayed teleoperation system with force feedback dangerously unstable. A comprehensive comparative study among the most common control schemes proposing solutions to transparency and the stability problem is given in [3]. A study showing closed loop stability conditions in time-delayed teleoperation is presented in [4].

One of the most creative solutions to the stability problem in force feedback teleoperation is using scattering and passivity theories [2, 5, 6]. These methods provide stability independent of the communication time-delay value. The main problem with these approaches, however, is that transparency may be sacrificed to obtain stability. There have been studies to obtain better transparency

*Correspondence to: K. Abidi, School of Electrical and Electronic Engineering, Newcastle University International Singapore, Nanyang Polytechnic, 569830, Singapore.
†E-mail: khalid.abidi@newcastle.ac.uk

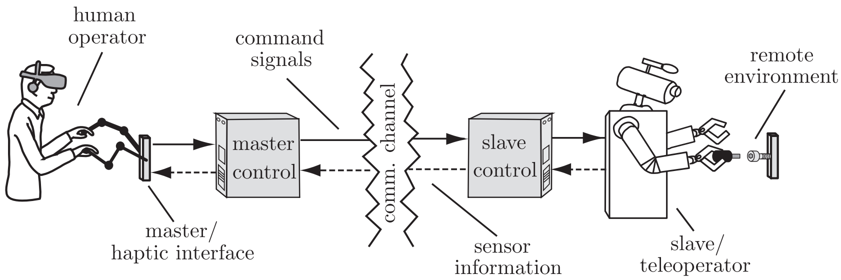

Figure 1. Teleoperation System [1].

using these approaches [7–13]. One of the approaches to provide increased transparency is using predictive control [5, 14]. In this approach, the main idea is predicting slave robot's future behavior using known system dynamics and feeding it back to the master robot. Smith predictors are one common approach used for this purpose [3]. The problem with this approach is that Smith predictors are known to be sensitive to modeling errors in the known system dynamics and errors in the known amount of the time delay. Therefore, modeling uncertainties or uncertainties caused by actuator degradation, parameter changes due to temperature variation and component aging, can cause dangerous instabilities if this method is not used with caution. A different approach proposed in the literature is to use local impedence controllers to stabilize the slave and master robots improving robustness to time delay [15]. This approach is also sensitive to modeling errors and does not preserve transparency. Another approach proposed in the literature to increase transparency and stability in the presence of time delays and uncertainties is employing adaptive control [16]. In this approach, each manipulator has its local adaptive controller to address modeling uncertainties. The controller in [16] is designed in continuous time, and a switching coefficient is used, which is set according to free motion or contact scenarios. This switching may cause erratic behavior if not handled properly, and it requires effort to obtain smooth switching between operation conditions. The proposed approach in this paper does not require switching. In addition, the proposed controller is designed in discrete time, which eliminates inaccuracies emanating from discrete approximations of continuous time controllers in real applications.

There are also studies in the literature that utilizes fuzzy approximation-based controllers [17]*, controllers that employ neural networks* [18] *and also approaches that are designed for systems where master and slave kinematics are equivalent* [19].

In this work, we propose a telerobotics framework that may lead the way towards making fully operational, stable bilateral teleoperation a possibility without sacrificing transparency. We build upon the earlier successful research results, presented earlier, by eliminating the need for precise system models and eliminating the sacrifice of transparency by developing an adaptive controller in tandem with an explicit delay-compensating controller. MIT's Adaptive Posicast Controller (APC) [20], which is partly developed and improved by author Yildiz, is at the heart of this work. The main contribution of this paper is merging explicit delay compensation and adaptation in the teleoperation framework, in a mathematically rigorous way. There are key distinctions of this work compared with earlier studies. Firstly, unlike most passivity-based approaches, transparency will not be sacrificed for stability, and there will be no need for precise plant models. Secondly, unlike earlier adaptive approaches, there will be no need for persistently exciting (rich) input excitations for parameter identification. Thirdly, the time delay in the system will be explicitly compensated instead of building a control system that is robust to delays. These distinctions provide stability and increased transparency at the same time. *Although the proposed approach is inspired from the APC, the resultant control framework is different: the proposed control framework consists of three different controllers, which help compensate for time delays and uncertainties and due to the peculiar structure of the telerobotics problem, the resultant control algorithms and adaptation laws are different than the APC.*

The main approach, explicit adaptive delay compensation, will be achieved by employing a discrete adaptive controller locally and explicit time-delay compensating controller inspired by

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the discrete-time version of APC [21]. APC is an adaptive controller based on the ideas of Smith Predictor [22–26], finite spectrum assignment [27], and adaptation [28, 29]. The key features of the APC, different from the existing adaptive time-delay controllers, are that APC does not have any restrictions on plant pole multiplicities, it is simple to implement and computationally less expansive and most importantly, APC is experimentally proven to be very effective by the author Yildiz and his collaborators for automotive control problems [30–34]. To see a list of delay-compensating controllers and an investigation of predictive laws for delay perturbations, see [35]. In addition, the book [36] provides recent research results on adaptive control of time-delay systems together with investigations on other delay related issues.

The organization of this paper is as follows: the problem formulation is presented in Section 2. Section 3 presents the fixed controller design. Section 4 introduces the adaptive controller design. Stability of the overall system is presented in Section 5. Simulation results are shown in Section 6 followed by the conclusions in Section 7.

2. PROBLEM FORMULATION

Consider the Euler-Lagrange equations [13, 37] of an n_m-link master and n_s-link slave teleoperation system with a description given as

$$M_m(q_m)\ddot{q}_m + C_m(q_m, \dot{q}_m)\dot{q}_m + g_m(q_m) = \tau_m(t) + J_m^T(q_m)f_h(t) \tag{1}$$

$$M_s(q_s)\ddot{q}_s + C_s(q_s, \dot{q}_s)\dot{q}_s + g_s(q_s) = \tau_s(t) - J_s^T(q_s)f_e(t) \tag{2}$$

where $q_m \in \Re^{n_m \times 1}$, $q_s \in \Re^{n_s \times 1}$ are the joint displacement vectors, $\dot{q}_m \in \Re^{n_m \times 1}, \dot{q}_s \in \Re^{n_s \times 1}$ are the joint velocity vectors, $\tau_m(t) \in \Re^{n_m \times 1}$, $\tau_s(t) \in \Re^{n_s \times 1}$ are the joint torque vectors, $M_m(q_m) \in \Re^{n_m \times n_m}$, $M_s(q_s) \in \Re^{n_s \times n_s}$ are the inertia matrices, $C_m(q_m, \dot{q}_m), C_s(q_s, \dot{q}_s)$ are the Centripetal and Coriolis torques matrices, $g_m(q_m) \in \Re^{n_m \times 1}$, $g_s(q_s) \in \Re^{n_s \times 1}$ are the gravitational torque vectors, $J_m(q_m) \in \Re^{l \times n_m}$, $J_s(q_s) \in \Re^{l \times n_s}$ are the Jacobian matrices, $f_h \in \Re^{l \times 1}$ is the operator hand force vector and $f_e \in \Re^{l \times 1}$ is the contact force vector on the slave robot. In this work, the master and slave parameters are assumed to be uncertain. The Euler–Lagrange equations (1) and (2) have the following useful property due to their structure [9].

Property 1
The Lagrangian dynamics are linearly parameterizable [38], which gives the form

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = Y(q, \dot{q}, \ddot{q})\,\theta = \tau(t)$$

where θ is a constant p-dimensional vector of parameters and $Y(q, \dot{q}, \ddot{q}) \in \Re^{n \times p}$ is the matrix of known functions of the joint displacements and their first and second derivatives.

In this implementation, the operator hand force is modeled as

$$f_h(t) = \alpha_0 - K_h x_m(t) - B_h \dot{x}_m(t) \tag{3}$$

where α_0 represents a constant non-passive force exerted by the operator resisted by the passive component $-K_h x_m(t) - B_h \dot{x}_m(t)$, and $x_m(t), \dot{x}_m(t)$ are the diplacement and velocity vectors of the master robot end-effector [9]. The contact force on the slave robot is modeled as a passive force of the form

$$f_e(t) = K_e x_s(t) + B_e \dot{x}_s(t) \tag{4}$$

where $x_s(t), \dot{x}_s(t)$ are the diplacement and velocity vectors of the slave robot end-effector. Note that the matrices K_h, K_e, B_h, and B_e represent uncertain stiffness and damping of the operator and

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the environment. It is possible to rewrite the force models (3) and (4) in the parameterized form as follows

$$f_h(t) = \alpha_0 - K_h x_m(t) - B_h \dot{x}_m(t) = \alpha_0 - \Theta_h \chi_m \tag{5}$$

and

$$f_e(t) = K_e x_s(t) + B_e \dot{x}_s(t) = \Theta_e \chi_s \tag{6}$$

where $\Theta_h \in \Re^{l \times 2l}, \Theta_e \in \Re^{l \times 2l}$ are constant matrices of the uncertain parameters and $\chi_{m,k} = \left[x_{m,k}^T, \dot{x}_{m,k}^T\right]^T \in \Re^{2l}$, $\chi_{s,k} = \left[x_{s,k}^T, \dot{x}_{s,k}^T\right]^T \in \Re^{2l}$. Combining (1), (2) and *Property 1* the system reduces to the form

$$Y_m\left(q_m, \dot{q}_m, \ddot{q}_m\right)\theta_m = \tau_m(t) + J_m^T(q_m) f_h(t) \tag{7}$$

$$Y_s\left(q_s, \dot{q}_s, \ddot{q}_s\right)\theta_s = \tau_s(t) - J_s^T(q_s) f_e(t) \tag{8}$$

Consider that there exists fictitious control inputs at the end effectors $f_m(t)$ *and* $f_s(t)$. The objective is to design the control inputs $\tau_m(t)$ and $\tau_s(t)$ in discrete-time such that $x_s(t) \to x_m(t)$ when the slave robot is in free motion and $f_s(t) \to R f_h(t)$, for some scaling factor R, when the slave robot end-effector is in contact with a surface. These objectives must be satisfied when there is communication time delay between the master robot and the slave robot. The time delay can be specified as forward communication time delay and backward communication time delay. The forward communication time delay can be represented in number of time-steps, namely, as d_1 where $(d_1 - 1)T \leqslant t_1 \leqslant d_1 T$ with t_1 being the actual time delay and T being the sampling period. Similary, the backward communication time delay can be represented as d_2 time-steps.

3. FIXED CONTROLLER DESIGN

In order to design the controller, the problem will be divided into two parts: (i) local adaptive controllers that cancel the nonlinear dynamics $Y\left(q, \dot{q}, \ddot{q}\right)\theta$ and impose the impedence $M\ddot{x} + B\dot{x} = f$, where f is a fictitious control, at the end-effectors and (ii) design the fictitious control f such that $\lim_{t\to\infty} \|x_s(t) - x_m(t)\| \to 0$ when the slave robot is in free motion and $\lim_{t\to\infty} \|f_s(t) - R f_h(t)\| \to 0$ when the slave robot end-effector is in contact with a surface. During contact with a surface, $\lim_{t\to\infty} \|f_s(t) - R f_h(t)\| \to 0$ would imply that $\lim_{t\to\infty} \|f_e(t) - R f_m(t)\| \to 0$; thus, the contact force $f_e(t)$ will be reflected back to the operator in the form of $f_m(t)$.

3.1. Local controller design

Local controller design involves no interaction between the master robot and slave robot; therefore, there will exist no time delay in any of the signals. Consider the system (7) and (8). Because this will be a discrete-time implementation, any time dependent function $\rho(t)$ will be replaced with ρ_k where k is the index of the sampling instant, also, for convenience, let $Y_{m,k} \equiv Y_m\left(q_m, \dot{q}_m, \ddot{q}_m\right)$, $Y_{s,k} \equiv Y_s\left(q_s, \dot{q}_s, \ddot{q}_s\right)$, $J_{m,k} \equiv J_m(q_m)$ and $J_{s,k} \equiv J_s(q_s)$. *To facilitate the controller design, the parameters of the robots and the contact forces are assumed to be known. (In the adaptive controller design, which is presented in the next section, this assumption is eliminated.) Thus, the control law is selected as*

$$\tau_{m,k} = Y_{m,k}\theta_m - J_{m,k}^T\left(M\ddot{x}_{m,k} + B\dot{x}_{m,k} + f_{m,k}\right) \tag{9}$$

$$\tau_{s,k} = Y_{s,k}\theta_s - J_{s,k}^T\left(RM\ddot{x}_{s,k} + RB\dot{x}_{s,k} - f_{s,k}\right) \tag{10}$$

where R is a diagonal positive-definite constant matrix used for scaling the environmental contact forces. The mass matrix M and damping matrix B are selected to reflect a desired impedence of

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the slave robot. Substituting the control laws (9) and (10) into (7) and (8) to obtain the closed-loop system of the master robot as

$$-J_{m,k}^T\left(M\ddot{x}_{m,k} + B\dot{x}_{m,k} + f_{m,k} - f_{h,k}\right) = 0 \tag{11}$$

and slave robot as

$$-J_{s,k}^T\left(RM\ddot{x}_{s,k} + RB\dot{x}_{s,k} - f_{s,k} + f_{e,k}\right) = 0, \tag{12}$$

ensuring that the desired impedences are imposed at the end-effector of the master and slave robots.

Remark 1
Unlike position and velocity, the acceleration terms in (9) and (10) may not be easily available. However, advances in sensor technology such as that which is shown in [39, 40], and [41] have made it possible for the accurate measurement of accelerations and forces, and there have been controllers proposed in the literature for stable teleoperation that assumes such measurements are available [12, 42, 43]. However, depending on the application, the need for filtering may introduce robustness issues. In this paper, it is assumed that this is not the case. A modified version of the proposed controller that does not require these measurements is the topic of future research.

3.2. Fictitious controller design

Fictitious controller design involves interaction between the master robot and slave robot and, therefore, will be handeled keeping in mind the forward and backward communication time delay. Now, consider the dynamics at the end-effectors given by

$$M\ddot{x}_{m,k} + B\dot{x}_{m,k} = -f_{m,k} + f_{h,k} \tag{13}$$

$$RM\ddot{x}_{s,k} + RB\dot{x}_{s,k} = f_{s,k} - f_{e,k} \tag{14}$$

To proceed with the selection of the fictitious control inputs $f_{m,k}$ and $f_{s,k}$ the system (13) and (14) is written in the sampled-data form

$$\chi_{m,k+1} = \Phi\chi_{m,k} - \Gamma f_{m,k} + \Gamma f_{h,k}, \tag{15}$$

$$\chi_{s,k+1} = \Phi\chi_{s,k} + \Gamma R^{-1} f_{s,k} - \Gamma R^{-1} f_{e,k} \tag{16}$$

where Φ, Γ are the sampled-data state and input matrices computed from

$$\begin{aligned}\Phi &= \exp\left(\begin{bmatrix}\mathbf{0} & \mathbf{1}\\ \mathbf{0} & -M^{-1}B\end{bmatrix} T\right),\\ \Gamma &= \int_0^T \exp\left(\begin{bmatrix}\mathbf{0} & \mathbf{1}\\ \mathbf{0} & -M^{-1}B\end{bmatrix}\sigma\right)\begin{bmatrix}\mathbf{0}\\ M^{-1}\end{bmatrix} d\sigma\end{aligned}$$

where T is the sampling time. In (16), the fictitious control $f_{s,k}$ is selected as a Proportional-Derivative (PD) controller and since there may exist communication time delays between the master and slave robots the PD-controller is given as

$$\begin{aligned}f_{s,k} &= K_s(x_{m,k-d_1} - x_{s,k}) + B_s(\dot{x}_{m,k-d_1} - \dot{x}_{s,k})\\ &= \Theta_s\left(\chi_{m,k-d_1} - \chi_{s,k}\right)\end{aligned} \tag{17}$$

where K_s and B_s are the PD-controller gains, which act as stiffness and damping, and $\Theta_s \equiv diag(K_s, B_s)$. Because the parameters of the system (15) and (16) are known, the gains of the

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controller (17) can be selected so that $\lim_{t\to\infty} \|\chi_{m,k-d_1} - \chi_{s,k}\| = 0$ when the slave robot is in free motion, according to certain control specifications imposed by the task.

Technically, transparency is defined as follows:

$$f_{h,k} = f_{e,k}$$
$$x_{m,k} = x_{s,k}$$

According to this definition, in a transparent system, the slave tracks the master and at the same time the operator feels the external force acting on the slave robot. This is desired in many applications, but there may be situations where a slightly different structure is preferred. For example, during a free motion, that is, when the slave robot is moving freely without any contact to its environment, the operator should not feel anything according to the aforementioned transparency definition. However, there may be situations where this may result in dangerous behavior: if the operator feels no resistance, he/she can move the master robot in a way that can saturate the slave robot actuators and cause the slave robot move in an unpredictable way. In addition, feeling nothing may not be desired by the operator. He/she may require a feel of inertia in his/her hands to 'understand better' the tool (slave robot) he/she is using to use it in a more precise and controlled manner. Similarly, during contact with the environment, the operator may want the feel of the tool he/she using together with the effect of the external environmental force acting on it. For example, a surgeon may desire to feel the inertia of the cutting apparatus he/she is using together with the effect of the tissue on the apparatus. We develop the proposed telerobotics framework based on these considerations, so that the slave follows the master and the operator feels the virtual control force that is applied to the modified slave robot dynamics, that is, $x_{s,k} = x_{m,k}$ *and* $f_{s,k} = f_{h,k}$. We modify the slave and master robot dynamics by local adaptive controllers in such a way that both the master and the slave virtual robot dynamics are the same. This way, the hand force applied (thus felt) by the operator on the (virtual) master robot becomes equal to the force applied on the (virtual) slave robot. Therefore, the framework gives the operator a sense of being virtually present at the remote environment and using his/her tool to manipulate the environment. The dynamics of the fictitious slave input force $f_{s,k}$ can be found by substituting (15) and (16) into a single time-step shifted (17) as

$$\begin{aligned} f_{s,k+1} &= \Theta_s\left(\chi_{m,k-d_1+1} - \chi_{s,k+1}\right) \\ &= \Theta_s\Phi\chi_{m,k-d_1} - \Theta_s\Phi\chi_{s,k} - \Theta_s\Gamma R^{-1} f_{s,k} \\ &\quad + \Theta_s\Gamma f_{h,k-d_1} + \Theta_s\Gamma R^{-1} f_{e,k} - \Theta_s\Gamma f_{m,k-d_1}. \end{aligned} \tag{18}$$

Substitution of (6) and (17) in (18) results in the final form of the fictitious slave input force dynamics

$$\begin{aligned} f_{s,k+1} &= \Theta_s(\Phi - \Gamma R^{-1}\Theta_s)(\chi_{m,k-d_1} - \chi_{s,k}) + \Theta_s\Gamma f_{h,k-d_1} \\ &\quad + \Theta_s\Gamma R^{-1}\Theta_e\chi_{s,k} - \Theta_s\Gamma\delta_{s,k} - \Theta_s\Gamma f_{m,k-d_1}. \end{aligned} \tag{19}$$

Let $\Theta_\phi \equiv \Theta_s(\Phi - \Gamma R^{-1}\Theta_s)$ and $\Theta_\gamma \equiv \Theta_s\Gamma$ then (19) can be rewritten as

$$\begin{aligned} f_{s,k+1} &= \Theta_\phi(\chi_{m,k-d_1} - \chi_{s,k}) + \Theta_\gamma f_{h,k-d_1} + \Theta_\gamma R^{-1} f_{e,k} \\ &\quad - \Theta_\gamma R^{-1}(\delta_{s,k} + \delta_{e,k}) + \Theta_\gamma\delta_{m,k-d_1} - \Theta_\gamma f_{m,k-d_1}. \end{aligned} \tag{20}$$

Remark 2

During contact with a surface, the velocity and acceleration of the slave robot would be approximately zero and, therefore, $f_{s,k} \approx f_{e,k}$. Thus, $\lim_{t\to\infty} \|f_{s,k} - Rf_{h,k}\| \to 0$ would imply that $\lim_{t\to\infty} \|f_{e,k} - Rf_{h,k}\| \to 0$.

In order to achieve $\|f_{s,k+1} - Rf_{h,k-d_1}\| \to 0$, note that in (20), the control input, $f_{m,k}$, is delayed by d_1 time-steps, and, therefore, the control law design will require future states as shown in the succeeding discussion

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$$f_{m,k} = \left(I + \Theta_\gamma^{-1} R\right) f_{h,k} + \Theta_\gamma^{-1}\Theta_\phi \chi_{m,k} - \Theta_\gamma^{-1}\Theta_\phi \chi_{s,k+d_1} + R^{-1} f_{e,k+d_1}. \tag{21}$$

Because the future value of $\chi_{s,k}$ and $f_{e,k}$ are not available, these will be estimated from (16). Substitution of (17) in (16), it is obtained that

$$\chi_{s,k+1} = \left(\Phi - \Gamma\left(\Theta_s + R^{-1}\Theta_e\right)\right)\chi_{s,k} + \Gamma\Theta_s\chi_{m,k-d_1}. \tag{22}$$

Writing (22) repeatedly, it is obtained that

$$\chi_{s,k+d} = \Phi_e^d \chi_{s,k} + \sum_{i=0}^{d-1} \Phi_e^{d-1-i}\Gamma\Theta_s\chi_{m,k-d_1+i} \tag{23}$$

where $\Phi_e \equiv \left(\Phi - \Gamma\left(\Theta_s + R^{-1}\Theta_e\right)\right.$ and $d = d_1 + d_2$. Here, d_2 is the backward communication delay in time-steps. Note that because there exists a backward communication delay between the slave and master robots, the future estimate of $\chi_{s,k}$ can be computed only using the availabe measurement $\chi_{s,k-d_2}$. Therefore, the future estimate $\chi_{s,k+d_1}$ is given as

$$\chi_{s,k+d_1} = \Phi_s^d \chi_{s,k-d_2} + \sum_{i=0}^{d-1} \Phi_s^{d-1-i}\Gamma\Theta_s\chi_{m,k-d+i}. \tag{24}$$

All the terms on the right-hand-side of (24) are available, and, therefore, the control law (21) becomes

$$f_{m,k} = \left(I + \Theta_\gamma^{-1} R\right) f_{h,k} + \Theta_\gamma^{-1}\Theta_\phi \chi_{m,k} - \Theta_\gamma^{-1}\Theta_\phi\left[\Phi_e^d \chi_{s,k-d_2} + \sum_{i=0}^{d-1} \Phi_e^{d-1-i}\Gamma\Theta_s\chi_{m,k-d+i}\right]. \tag{25}$$

Controller (25) is in causal form and should reflect the force on the slave robot accurately.

4. ADAPTIVE CONTROLLER DESIGN

In this section, the adaptive controller design is introduced as well as the necessary modifications to the fictitious controller (25) to ensure asymptotic stability.

4.1. *Local adaptive controller*

When the robot parameters θ_m and θ_s are uncertain, then the control laws (9) and (10) can be modified to the form

$$\tau_{m,k} = Y_{m,k}\hat{\theta}_{m,k} - J_{m,k}^T\left(M\ddot{x}_{m,k} + B\dot{x}_{m,k} + f_{m,k} - f_{h,k}\right) \tag{26}$$

$$\tau_{s,k} = Y_{s,k}\hat{\theta}_{s,k} - J_{s,k}^T\left(RM\ddot{x}_{s,k} + RB\dot{x}_{s,k} - f_{s,k} + f_{e,k}\right) \tag{27}$$

where $\hat{\theta}_{m,k}$ and $\hat{\theta}_{s,k}$ are the estimates of θ_m and θ_s, repectively.

Substituting the control laws (26) and (27) into (7) and (8) to obtain the closed-loop system of the master robot as

$$Y_{m,k}\tilde{\theta}_{m,k} = -J_{m,k}^T\left(M\ddot{x}_{m,k} + B\dot{x}_{m,k} + f_{m,k} - f_{h,k}\right) \tag{28}$$

and slave robot as

$$Y_{s,k}\tilde{\theta}_{s,k-d_1} = -J_{s,k}^T\left(RM\ddot{x}_{s,k} + RB\dot{x}_{s,k} - f_{s,k} + f_{e,k}\right) \tag{29}$$

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where $\tilde{\theta}_{m,k} = \theta_m - \hat{\theta}_{m,k}$ and $\tilde{\theta}_{s,k} = \theta_s - \hat{\theta}_{s,k}$ are the parameter estimation errors. From (28) and (29) the adaptation laws are formulated as

$$\hat{\theta}_{m,k+1} = \hat{\theta}_{m,k} - P_{m,k+1} Y_{m,k}^T J_{m,k}^T z_{m,k} \tag{30}$$

$$\hat{\theta}_{s,k+1} = \hat{\theta}_{s,k} - P_{s,k+1} Y_{s,k}^T J_{s,k}^T z_{s,k} \tag{31}$$

where $z_{m,k} \equiv M\ddot{x}_{m,k} + B\dot{x}_{m,k} + f_{m,k} - f_{h,k}$, $z_{s,k} \equiv RM\ddot{x}_{s,k} + RB\dot{x}_{s,k} - f_{s,k} + f_{e,k}$ and d_1 is the forward time delay in time-steps. The matrices $P_{m,k}$, $P_{s,k}$ are symmetric positive definite matrices computed as

$$P_{m,k+1} = P_{m,k} - P_{m,k} Y_{m,k}^T \left(I + Y_{m,k} P_{m,k} Y_{m,k}^T\right)^{-1} Y_{m,k} P_{m,k} \tag{32}$$

$$P_{s,k+1} = P_{s,k-d_1} - P_{s,k-d_1} Y_{s,k}^T \left(I + Y_{s,k} P_{s,k-d_1} Y_{s,k}^T\right)^{-1} Y_{s,k} P_{s,k-d_1}. \tag{33}$$

The covariance matrix P has some useful properties, [44]

Property 2
$P_{k+1}^{-1} = P_k^{-1} + Y_k^T Y_k$

Property 3
$Y_k P_{k+1} Y_k^T = \left(I + Y_k P_k Y_k^T\right)^{-1} Y_k P_k Y_k^T$

The adaptation laws (30) and (31) are implemented to guarantee that $\lim_{t\to\infty} \|z_{m,k}\| \to 0$ and $\lim_{t\to\infty} \|z_{s,k}\| \to 0$ or, in other words, the desired impedence is imposed at the end-effectors of both the master and slave robots. The asymptotic stability of the closed-loop system (28) and (29) with the adaptation laws (30) and (31) will be shown in Section 5.

Next, consider the external force model (6), if Θ_e is uncertain then

$$\hat{f}_{e,k} = \hat{K}_{e,k} x_{s,k} + \hat{B}_{e,k} \dot{x}_{s,k} = \hat{\Theta}_{e,k} \chi_{s,k} \tag{34}$$

where $\hat{\Theta}_{e,k}$ is the estimate of Θ_e. Since $f_{e,k}$ is measured, it is possible to write

$$f_{e,k} - \hat{f}_{e,k} = \Theta_e \chi_{s,k} - \hat{\Theta}_{e,k} \chi_{s,k} = \tilde{\Theta}_{e,k} \chi_{s,k}. \tag{35}$$

An adaptation law can be formulated for $\hat{\Theta}_{e,k}$ as follows

$$\hat{\Theta}_{e,k+d_2} = \hat{\Theta}_{e,k} - P_{e,k+d_2} \chi_{s,k}^T \left(f_{e,k} - \hat{f}_{e,k}\right) \tag{36}$$

$$P_{e,k+d_2} = P_{e,k} - \frac{P_{e,k} \chi_{s,k} \chi_{s,k}^T P_{e,k}}{1 + \chi_{s,k}^T P_{e,k} \chi_{s,k}}. \tag{37}$$

4.2. *Modified fictitious controller*

Consider the dynamics at the end-effectors given by

$$M\ddot{x}_{m,k} + B\dot{x}_{m,k} = -f_{m,k} + f_{h,k} + \delta_{m,k} \tag{38}$$

$$RM\ddot{x}_{s,k} + RB\dot{x}_{s,k} = f_{s,k} - \hat{f}_{e,k} + \delta_{s,k} + \delta_{e,k} \tag{39}$$

where $\delta_{m,k}$, $\delta_{s,k}$ and $\delta_{e,k}$ are the errors incurred from the adaptive laws (30), (31), and (36). In sampled-data form, the system (38) and (39) are written as

$$\chi_{m,k+1} = \Phi\chi_{m,k} - \Gamma f_{m,k} + \Gamma f_{h,k} + \Gamma\delta_{m,k}, \tag{40}$$

$$\chi_{s,k+1} = \Phi\chi_{s,k} + \Gamma R^{-1} f_{s,k} - \Gamma R^{-1}\hat{\Theta}_{e,k}\chi_{s,k} + \Gamma R^{-1}\left(\delta_{s,k} + \delta_{e,k}\right) \tag{41}$$

The slave input force dynamics (19) is modified to

$$\begin{aligned} f_{s,k+1} = {} & \Theta_\phi(\chi_{m,k-d_1} - \chi_{s,k}) + \Theta_s\Gamma f_{h,k-d_1} + \Theta_s\Gamma R^{-1}\hat{f}_{e,k} \\ & - \Theta_s\Gamma R^{-1}\delta_{s,k} + \Theta_s\Gamma\delta_{m,k-d_1} - \Theta_s\Gamma f_{m,k-d_1}. \end{aligned} \tag{42}$$

In order to achieve $\lim_{t\to\infty}\|f_{s,k+1} - Rf_{h,k-d_1}\| \to 0$, the control law (21) remains the same. Consider $\chi_{s,k}$, the future estimate is computed the same way as in (23). Substituting (17) and (34) in (16) it is obtained that

$$\chi_{s,k+d} = \Phi_s^d\chi_{s,k} + \sum_{i=0}^{d-1}\Phi_s^{d-1-i}\left(\Gamma\Theta_s\chi_{m,k-d_1+i} - \Gamma R^{-1}\hat{f}_{e,k+i} + \Gamma R^{-1}\delta_{e,k+i}\right) \tag{43}$$

where $\Phi_s \equiv (\Phi - \Gamma\Theta_s)$ and $\delta_{e,k}$ is the error $\tilde{f}_{e,k}$. Writing (43) repeatedly it is obtained that

$$\begin{aligned} \chi_{s,k+d} = {} & \left(\prod_{j=0}^{d-1}\Phi_{e,k-d_1+j}\right)\chi_{s,k} + \sum_{i=0}^{d-1}\left(\prod_{j=i+1}^{d-1}\Phi_{e,k-d_1+j}\right)\Theta_\gamma\chi_{m,k-d_1+i} \\ & + \sum_{i=0}^{d-1}\left(\prod_{j=i+1}^{d-1}\hat{\Phi}_{e,k-d_1+j}\right)\Gamma(\delta_{e,k+i} + \delta_{s,k+i}) \end{aligned} \tag{44}$$

where $\hat{\Phi}_{e,k} \equiv \left[\Phi - \Gamma\left(\Theta_s + R\hat{\Theta}_{e,k}\right)\right]$ and $d = d_1 + d_2$. Similar to (24), the future estimate of $\chi_{s,k}$ can be computed only using the availabe measurement $\chi_{s,k-d_2}$. Note that from (23), the future of the transient errors $\delta_{s,k}$ is needed to compute the future value of $\chi_{s,k+d_1}$. To circumvent, a future estimate of $\chi_{s,k+d_1}$ is given as

$$\hat{\chi}_{s,k+d_1} = \left(\prod_{j=0}^{d-1}\Phi_{e,k-d+j}\right)\chi_{s,k-d_2} + \sum_{i=0}^{d-1}\left(\prod_{j=i+1}^{d-1}\Phi_{e,k-d+j}\right)\Theta_\gamma\chi_{m,k-d+i}. \tag{45}$$

All the terms on the right-hand-side of (45) are available, and, therefore, the control law (25) becomes

$$f_{m,k} = \left(I + \Theta_\gamma^{-1}\right) f_{h,k} + \Theta_\gamma^{-1}\Theta_\phi\chi_{m,k} - \left(\Theta_\gamma^{-1}\Theta_\phi - R\hat{\Theta}_{e,k}\right)\hat{\chi}_{s,k+d_1}. \tag{46}$$

Thus, the controller (46) is computed in causal form. The stability of the overall system with the adaptive controller (26), (27) as well as the fictitious controllers (17) and (46) are summarized in the next section.

Remark 3
Dropping the transient error term $\delta_{s,k}$ does not undermine the stability of the system and, as it will be shown in the next section, adds a transient error term to the force tracking error $\|f_{s,k} - \hat{f}_{h,k-d_1}\|$ that converges to zero aymptotically.

5. STABILITY ANALYSIS

Consider the system (42), substitution of the controller (46) will result in the error dynamics

$$\begin{aligned} e_{s,k+1} &= f_{s,k+1} - f_{h,k-d_1} \\ &= \left(\Theta_\phi - \Theta_\gamma R\hat{\Theta}_{e,k}\right)\left(\hat{\chi}_{s,k} - \chi_{s,k}\right) - \Theta_\gamma\delta_{s,k} + \Theta_\gamma\delta_{m,k-d_1} \end{aligned} \tag{47}$$

where the difference $\hat{\chi}_{s,k} - \chi_{s,k}$ can be found from (23) and (24) to be given as

$$\hat{\chi}_{s,k} - \chi_{s,k} = -\sum_{i=0}^{d-1} \left(\prod_{j=i+1}^{d-1} \hat{\Phi}_{e,k-d+j} \right) \Gamma(\delta_{s,k-d+i} + \delta_{e,k-d+i}). \tag{48}$$

From (47) and (48), it is seen that the force tracking error $e_{s,k}$ is dependent on the convergence of the adaptive controller errors $\delta_{m,k}$, $\delta_{s,k}$, and $\delta_{e,k}$. The convergence of the adaptive controller errors $\delta_{m,k}$ and $\delta_{s,k}$ is summarized in the following theorem.

Theorem 1
Consider a discrete-time Lyapunov–Krasovskii functional (LKF) given by

$$V_k = \tilde{\theta}_{m,k}^T P_{m,k}^{-1} \tilde{\theta}_{m,k} + \sum_{i=k-d_1}^{k} \tilde{\theta}_{s,i}^T P_{s,i}^{-1} \tilde{\theta}_{s,i} \tag{49}$$

The parameter adaptation errors $\tilde{\theta}_{m,k}$ and $\tilde{\theta}_{s,k}$ are bounded, and the desired impedence tracking errors $z_{m,k}$ and $z_{s,k}$ are asymptotically stable.

Proof
In LKF (49), the difference between two time steps can be obtained as

$$\begin{aligned}\Delta V_k &= V_{k+1} - V_k \\ &= \tilde{\theta}_{m,k+1}^T P_{m,k+1}^{-1} \tilde{\theta}_{m,k+1} - \tilde{\theta}_{m,k}^T P_{m,k}^{-1} \tilde{\theta}_{m,k} + \sum_{i=k-d_1+1}^{k+1} \tilde{\theta}_{s,i}^T P_{s,i}^{-1} \tilde{\theta}_{s,i} - \sum_{i=k-d_1}^{k} \tilde{\theta}_{s,i}^T P_{s,i}^{-1} \tilde{\theta}_{s,i} \\ &= \tilde{\theta}_{m,k+1}^T P_{m,k+1}^{-1} \tilde{\theta}_{m,k+1} - \tilde{\theta}_{m,k}^T P_{m,k}^{-1} \tilde{\theta}_{m,k} + \tilde{\theta}_{s,k+1}^T P_{s,k+1}^{-1} \tilde{\theta}_{s,k+1} - \tilde{\theta}_{s,k-d_1}^T P_{s,k-d_1}^{-1} \tilde{\theta}_{s,k-d_1}.\end{aligned} \tag{50}$$

From (32) and (33), the parameter adaptation errors are found as

$$\tilde{\theta}_{m,k+1} = \tilde{\theta}_{m,k} + P_{m,k+1} Y_{m,k}^T J_{m,k}^T z_{m,k} \tag{51}$$

$$\tilde{\theta}_{s,k+1} = \tilde{\theta}_{s,k-d_1} + P_{s,k+1} Y_{s,k}^T J_{s,k}^T z_{s,k}. \tag{52}$$

Substitution of (51) and (52) in (50) and simplifying it is obtained that

$$\begin{aligned}\Delta V_k = &\left(\tilde{\theta}_{m,k} + P_{m,k+1} Y_{m,k}^T J_{m,k}^T z_{m,k} \right)^T P_{m,k+1}^{-1} \left(\tilde{\theta}_{m,k} + P_{m,k+1} Y_{m,k}^T J_{m,k}^T z_{m,k} \right) \\ &- \tilde{\theta}_{m,k}^T P_{m,k}^{-1} \tilde{\theta}_{m,k} + \left(\tilde{\theta}_{s,k} + P_{s,k+1} Y_{s,k}^T J_{s,k}^T z_{s,k} \right)^T P_{s,k+1}^{-1} \left(\tilde{\theta}_{s,k} + P_{s,k+1} Y_{s,k}^T J_{s,k}^T z_{s,k} \right) \\ &- \tilde{\theta}_{s,k-d_1}^T P_{s,k-d_1}^{-1} \tilde{\theta}_{s,k-d_1} \\ = &\, \tilde{\theta}_{m,k}^T \left(P_{m,k+1}^{-1} - P_{m,k}^{-1} \right) \tilde{\theta}_{m,k} + 2 z_{m,k}^T J_{m,k} Y_{m,k} \tilde{\theta}_{m,k} \\ &+ z_{m,k}^T J_{m,k} Y_{m,k} P_{m,k+1} Y_{m,k}^T J_{m,k}^T z_{m,k} + \tilde{\theta}_{s,k}^T \left(P_{s,k+1}^{-1} - P_{s,k-d_1}^{-1} \right) \tilde{\theta}_{s,k} \\ &+ 2 z_{s,k}^T J_{s,k} Y_{s,k} \tilde{\theta}_{s,k-d_1} + z_{s,k}^T J_{s,k} Y_{s,k} P_{s,k+1} Y_{s,k}^T J_{s,k}^T z_{s,k}.\end{aligned} \tag{53}$$

Substituting *Property 2*, *Property 3,* and the closed-loop dynamics (28), (29) in (53) it is obtained that

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$$\begin{aligned}\Delta V_k &= -z_{m,k}^T J_{m,k} J_{m,k}^T z_{m,k} + z_{m,k}^T J_{m,k} \left(I + Y_{m,k} P_{m,k} Y_{m,k}^T\right)^{-1} Y_{m,k} P_{m,k} Y_{m,k}^T J_{m,k}^T z_{m,k} \\ &\quad - z_{s,k}^T J_{s,k} J_{s,k}^T z_{s,k} + z_{s,k}^T J_{s,k} \left(I + Y_{s,k} P_{s,k-d_1} Y_{s,k}^T\right)^{-1} Y_{s,k} P_{s,k-d_1} Y_{s,k}^T J_{s,k}^T z_{s,k} \\ &= -z_{m,k}^T J_{m,k} \left(I + Y_{m,k} P_{m,k} Y_{m,k}^T\right)^{-1} J_{m,k}^T z_{m,k} \\ &\quad - z_{s,k}^T J_{s,k} \left(I + Y_{s,k} P_{s,k-d_1} Y_{s,k}^T\right)^{-1} J_{s,k}^T z_{s,k} < 0.\end{aligned} \tag{54}$$

Thus ΔV_k is nonincreasing, implying that $\tilde{\theta}_{m,k}$ and $\tilde{\theta}_{s,k}$ are bounded. Further, applying (54) repeatedly for any $k \in [k_0, \infty)$, it is obtained that

$$V_k = V_{k_0} + \sum_{i=k_0}^{k} \Delta V_i \tag{55}$$

when $k \to \infty$, according to (54)

$$\begin{aligned}\lim_{k\to\infty} V_k = V_{k_0} - \lim_{k\to\infty} \sum_{i=k_0}^{k} &\left[z_{m,i}^T J_{m,i} \left(I + Y_{m,i} P_{m,i} Y_{m,i}^T\right)^{-1} J_{m,i}^T z_{m,i} \right. \\ &\left. + z_{s,i}^T J_{s,i} \left(I + Y_{s,i} P_{s,i-d_1} Y_{s,i}^T\right)^{-1} J_{s,i-d_1}^T z_{s,i} \right].\end{aligned} \tag{56}$$

Consider that V_k is nonnegative, V_{k_0} is finite, thus according to the convergence theorem of the sum of series, it is obtained that

$$\begin{aligned}\lim_{k\to\infty} &\left[z_{m,k}^T J_{m,k} \left(I + Y_{m,k} P_{m,k} Y_{m,k}^T\right)^{-1} J_{m,k}^T z_{m,k} \right. \\ &\left. - z_{s,k}^T J_{s,k} \left(I + Y_{s,k} P_{s,k-d_1} Y_{s,k}^T\right)^{-1} J_{s,k}^T z_{s,k} \right] \to 0.\end{aligned} \tag{57}$$

Further, the boundedness of $\tilde{\theta}_{m,k}$ and $\tilde{\theta}_{s,k}$ imply that $Y_{m,k}$ and $Y_{s,k}$ are sector bounded as $\|Y_{m,k}\| \leqslant \alpha_0 + \alpha_1 \|z_{m,k}\|$ and $\|Y_{s,k}\| \leqslant \beta_0 + \beta_1 \|z_{s,k}\|$ for some constants α_0, α_1, β_0, and β_1. Using (57) and the sector condition on $Y_{m,k}$ and $Y_{s,k}$, the key technical lemma guarantees that $z_{m,k} \to 0$ and $z_{s,k} \to 0$ as $k \to \infty$. □

Similarly, the convergence of the adaptative controller error $\delta_{e,k}$ can be summarized in the following theorem

Theorem 2
Consider a discrete-time LKF given by

$$V_k = \sum_{j=1}^{l} \left(\sum_{i=k-d_2}^{k} \tilde{\Theta}_{e_j,i}^T P_{e,i}^{-1} \tilde{\Theta}_{e_j,i} \right) \tag{58}$$

where $\tilde{\Theta}_{e,i} = \left[\tilde{\Theta}_{e_1,i}^T, \ldots, \tilde{\Theta}_{e_l,i}^T\right]^T$ and $\tilde{\Theta}_{e_l,i} \in \Re^{2l}$. The parameter adaptation error $\tilde{\Theta}_{e,k}$ is bounded, and the contact force error $\tilde{f}_{e,k}$ is asymptotically stable.

Proof
The proof is identical to the one in *Theorem 1,* and, therefore, it can be concluded that $\tilde{\Theta}_{e,k}$ is bounded and the contact force error $\tilde{f}_{e,k}$ is asymptotically stable. □

From *Theorem 1* and *Theorem 2,* it is concluded that the tracking errors converge to zero and consequently $e_{s,k} \to 0$ as $k \to \infty$.

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6. SIMULATION RESULTS

In this section, we present simulation results that show the ability of the proposed method to provide position and force tracking in the presence of time delays and system uncertainties. In the simulations, the operator has a certain trajectory in his/her mind, and he or she applies an appropriate force input to realize that trajectory where the force input is generated by modeling the operator as a proportional-derivative (PD) controller.

Consider the two-link planar master and slave manipulators shown in Figure 2, whose dynamics are given as

$$\begin{aligned}\tau_1 &= \left(\frac{m_1}{2} + m_2\right) g L_1 \cos\theta_1 + \frac{1}{3}\left(m_1 + 3m_2\right) L_1^2 \ddot{\theta}_1 + \frac{1}{2} m_2 L_1 L_2 \ddot{\theta}_2 \cos\left(\theta_1 - \theta_2\right) \\ &\quad + \frac{1}{2} m_2 L_1 L_2 \dot{\theta}_2^2 \sin\left(\theta_1 - \theta_2\right) \\ \tau_2 &= \frac{m_2}{2} g L_2 \cos\theta_2 + \frac{1}{3} m_2 L_2^2 \ddot{\theta}_2 + \frac{1}{2} m_2 L_1 L_2 \ddot{\theta}_1 \cos(\theta_1 - \theta_2) - \frac{1}{2} m_2 L_1 L_2 \dot{\theta}_1^2 \sin\left(\theta_1 - \theta_2\right)\end{aligned} \tag{59}$$

where the link lengths are $L_1 = L_2 = 0.2$ meters and masses are $m_1 = m_2 = 0.2$ kg.

The operator hand force is generated from a PD model of the form $f_h = K_h(x_h - x_m) + B_h(\dot{x}_h - \dot{x}_m)$ where $K_h = 5$ and $B_h = 0.7$. The hand displacement in the x-direction x_h is assumed to be a step displacement of 0.2 m, while the displacement in the y-direction is maintained at zero.

An obstacle is placed at $x = 0.14$ m on the slave side. *The obstacle stiffness and damping values in the simulation are set as* $k_e = 30$ N/m *and* $b_e = 1$ Ns/m *(Note that the obstacle information is uncertain from the controller's point of view.) The impedance at the end effectors of the master and slave robots are selected to have a mass of m = 0.1 kg and b = 5 Ns/m. The communication time delay between master and slave is set to be* $\tau = 0.4$ s, *both forward and backward (0.8 s total), and a* 50% *uncertainty is assumed in master and slave masses.*

The overall system described earlier is simulated using the proposed method and the method given in [13], and the results are presented in Figure 3. In Figure 3(a), it is seen that the slave position follows the master position until the obstacle at $x = 0.14$ m. Once at the obstacle, the external force F_e is reflected back to the operator in the form of F_m, which can be seen in Figure 3(c). Finally, the hand force F_h and slave force F_s are shown in Figure 3(e) where during free motion, their variations are similar and at the contact, they converge to each other. As it can be seen from Figure 3(b), 3(d) and 3(f), the method in [13] produces good position tracking; however, the created forces, especially during free motion are much higher compared with the proposed approach. Also during contact with

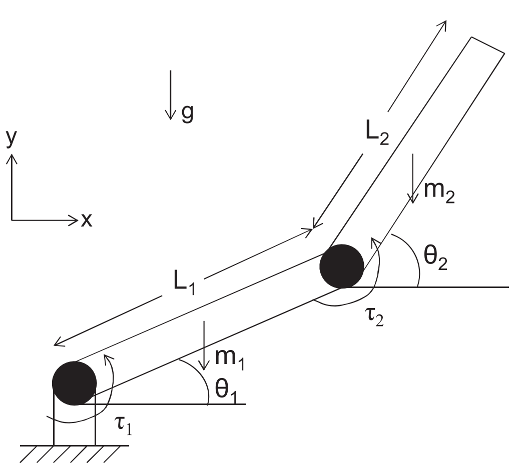

Figure 2. Two-link manipulator.

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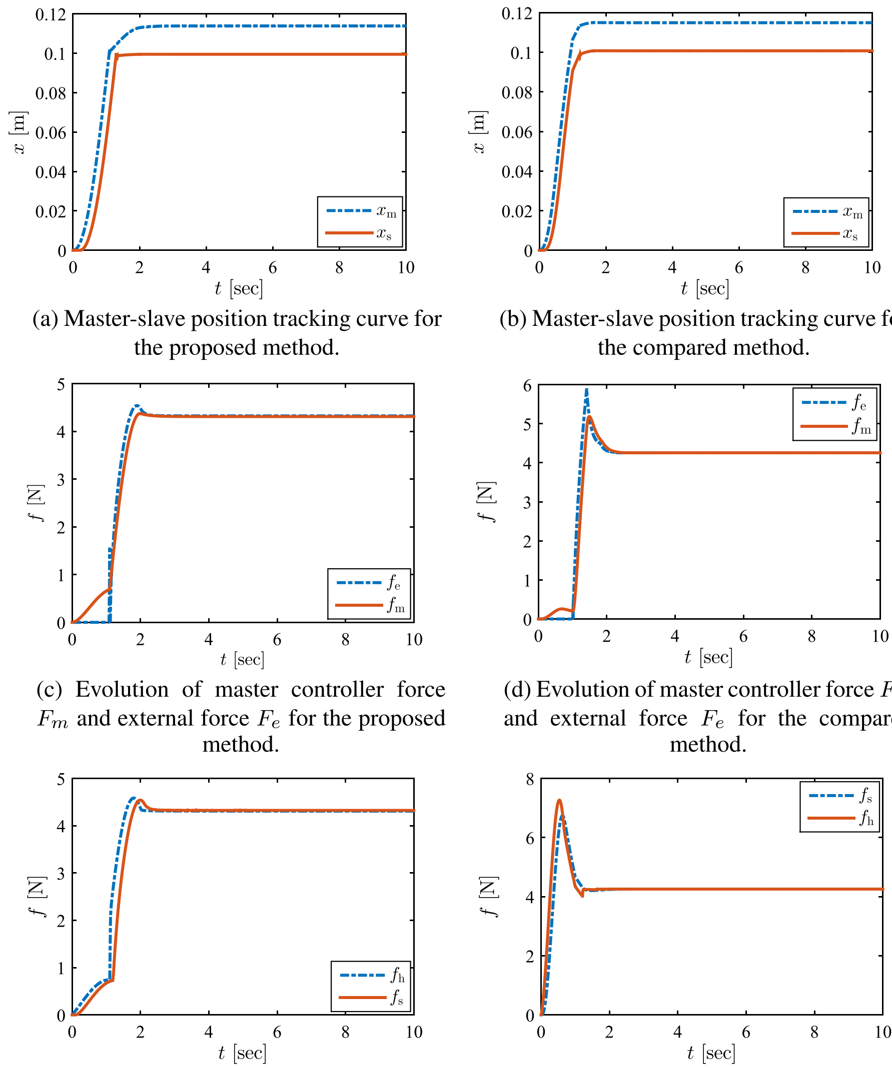

(a) Master-slave position tracking curve for the proposed method.

(b) Master-slave position tracking curve for the compared method.

(c) Evolution of master controller force F_m and external force F_e for the proposed method.

(d) Evolution of master controller force F_m and external force F_e for the compared method.

(e) Evolution of slave controller force F_s and hand force F_h for the proposed method.

(f) Evolution of slave controller force F_s and hand force F_h for the compared method.

Figure 3. Force and position tracking results of the closed loop system where the proposed method results are provided in (a), (c), and (e), and the compared method results are presented in (b), (d), and (f).

the surface, although the external and master forces converges to the same values as the proposed method, during the transients the external force acting on the slave robot have both higher magnitude and higher frequency, compared with the presented approach in this study, which is practically less favorable. *When the tracking errors are compared, both in position and force following, it is seen that the two approaches are similar. However, as pointed out earlier, the created forces are higher both in terms of amplitude and frequency, in the case of compared approach, which is a disadvantage during real implementations.*

Finally, the time delay τ is increased to 0.8 s, and both the proposed approach and the compared method are simulated, and the results are provided in Figure 4. As it can be seen from Figure 4(a), the proposed approach provides good tracking performance of the slave robot as well as good force tracking performance (Figure 4(c) and Figure 4(e)). On the other hand, increased oscillations due to the increase in time delay can be observed with the compared approach (Figures 4(b), 4(d), and 4(f)).

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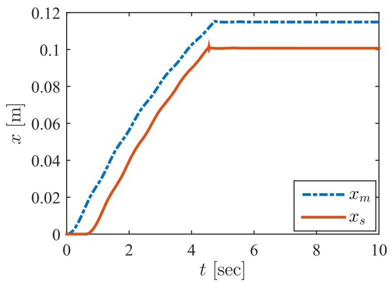
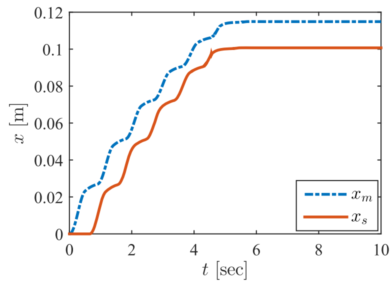
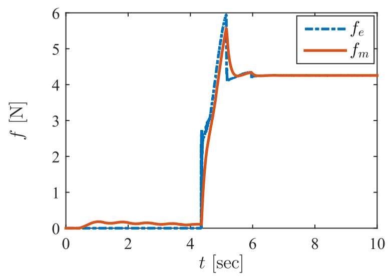


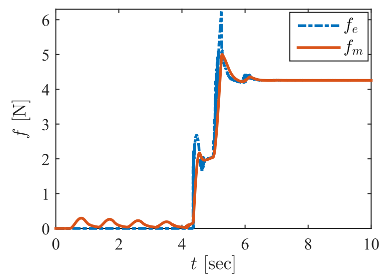
(a) Master-slave position tracking curve for the proposed method, in the presence of a higher delay value.



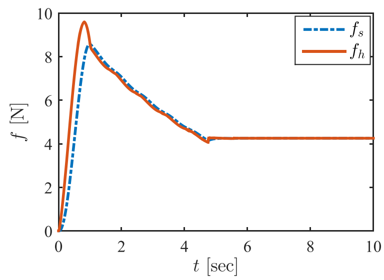
(b) Master-slave position tracking curve for the compared method, in the presence of a higher delay value.



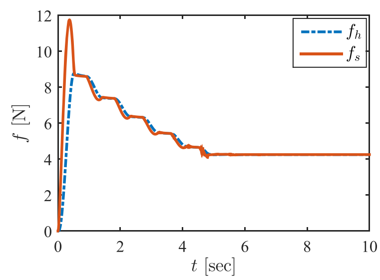
(c) Evolution of master controller force F_m and external force F_e for the proposed method, in the presence of a higher delay value.



(d) Evolution of master controller force F_m and external force F_e for the compared method, in the presence of a higher delay value.



(e) Evolution of slave controller force F_s and hand force F_h for the proposed method, in the presence of a higher delay value.



(f) Evolution of slave controller force F_s and hand force F_h for the compared method, in the presence of a higher delay value.

Figure 4. Force and position tracking results of the closed loop system, when the delay value is increased to $\tau = 0.8$. The proposed method results are provided in (a), (c), and (e), and the compared method results are presented in (b), (d), and (f).

Remark 4
As it can be seen from the results, the increase in the time delay leads to an increase in the overshoot in the forces. The overshoot can be attenuated by retuning the controller parameters, but care must be taken not to slow down the closed loop response to unacceptable levels.

7. SUMMARY

In this paper, a new adaptive approach was proposed for the stable operation of a telerobotic systems with force feedback, in the presence of communication time delays and parametric uncertainties. A rigorous stability proof is provided, and controller performance is verified via simulations where both master and slave robots are assumed to be two-link manipulators with full nonlinear system dynamics.

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