Nonlinearity-tailored fiber laser technology for low-noise, ultra-wide-band tunable femtosecond light generation

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The emission wavelength of a laser is physically pre-determined by the gain medium used. Consequently, arbitrary wavelength generation is a fundamental challenge in the science of light. Present solutions include optical parametric generation, requiring complex optical setups; and spectrally-sliced supercontinuum, taking advantage of simpler fiber technology: a fixed-wavelength pump laser pulse is converted into a spectrally very broadband output, from which the required resulting wavelength is then optically filtered. Unfortunately, this process is associated with inherently poor noise figure, which often precludes many realistic applications of such supercontinuum sources. Here, we show that by adding only one passive optical element · a tapered photonic crystal fiber · to a fixed-wavelength femtosecond laser, one can in a very simple manner resonantly convert the laser emission wavelength into an ultra-wide and continuous range of desired wavelengths, with very low inherent noise, and without mechanical realignment of the laser. This is achieved by exploiting the double interplay of nonlinearity and chirp in the laser source and chirp and phase-matching in the tapered fiber. As a first demonstration of this simple and inexpensive technology, we present a femtosecond fiber laser continuously tunable across the entire red-green-blue spectral range.

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1. INTRODUCTION

Wavelength-tunable ultrafast light sources, with tunability far beyond the gain bandwidth of (a limited number of) naturally-available laser media, enable a great variety of applications ranging from bio-spectroscopy [1–3] to safety inspection [4]. Presently available solutions such as optical parametric generation (OPG) can provide the laser emission extending from ultraviolet (UV) to the mid-infrared (MIR) [5, 6]. However, the need of (intra-cavity) mechanical readjustment for wavelength tuning results in operational reliability issues, limiting the applications of OPG sources outside dedicated optical labs. On the other hand, supercontinuum (SC) fiber lasers usually have simple configuration and are operationally robust [7, 8]. However, for many applications, efficient power conversion in SC technology usually conflicts with the strict requirement on low noise [9], affecting negatively the system stability and precluding acceptably-fast measurements.

Fiber-optic Cherenkov radiation (FOCR), also known as dispersive wave generation [10] or non-solitonic radiation [11], is a remarkably efficient process, in which one can convert the wavelength of a standard mode-locked pump laser (e.g., Yb-, Er- or Tm-fiber laser) to almost any target wavelength. Unlike SC generation, the in-fiber Cherenkov process allows for wavelength conversion in a spectrally-isolated, resonant fashion, at modest pump pulse energies, and with low noise [12]. In this process, soliton compression of sub-picosecond pump pulses in a fiber with anomalous dispersion at the pump wavelength leads to the emission of a dispersive wave, or FOCR, at a wavelength determined by the phase-matching to the soliton wavenumber [13]. As illustrated in Fig. 1(a), varying the PCF dispersion profile, and thus the FOCR phase-matching condition by controlling the lateral dimension of a PCF structure, one can easily tune the resulting FOCR wavelength over a broad spectral range. A number of groups have utilized this flexibility to obtain efficient generation of relatively narrow-band, spectrally-isolated FOCR pulses in the visible and UV ranges [14–22].

Owing to its straightforward implementation and good noise figure, FOCR generation has been successfully used in bio-photonics, including such demanding applications as biomedical imaging [23]. However, the ultimate challenge of achieving the continuous ultra-broadband laser tunability still stands: in all systems demonstrated to date, the FOCR wavelength was fixed to within a few tens of nm [22] by the specific dispersion of the fiber and the wavelength of the pump laser. However, for many applications, a much wider tunability is desired, e.g. for fluorescence spectroscopy one might want a single source spanning the entire visible region. Discrete wavelength switching could be achieved by manually changing between the PCFs
with different dispersion profiles [15] [16] [19], or, equivalently, by switching between the pump lasers with different pump wavelengths [18] - a demanding task for a practical implementation of this technique. The true arbitrary wavelength generation – the spectrally continuous wide-range laser tunability, was still unattainable [14,24].

In this work, we demonstrate a novel widely-tunable femtosecond fiber laser based on FOCR in tapered photonic crystal fibers (PCF) pumped by an Yb-based fs laser whose power and pulse duration could be widely tuned. The location of the soliton compression point in the fiber depends on the pump pulse parameters, and the FOCR wavelength depends on the position of said compression point due to the fiber taper. Given that the laser parameters themselves are intrinsically interrelated [25], it is essential to regard the entire laser-tapered fiber setup as a single strongly coupled nonlinear dynamical system. As will be shown below, our design allows generation of continuously tunable FOCR spanning the wavelength range 414-612 nm using a single nonlinear fiber and a single pump source with a fixed central wavelength. The principal idea behind our scheme is applicable to other wavelength ranges and waveguide types, and we believe the technique has considerable potential for improvement with optimized fiber and laser design. In particular, it may allow for very rapid wavelength tuning by purely electric control of the pump source, without any need for mechanical rearrangements.

![Fig. 1. Principles of continuously tunable femtosecond fiber laser. (a) Phase-matching curves for fiber-optic Cherenkov radiation (FOCR) generation for different fiber pitch dimensions (see Appendices Fig. S1), allowing for FOCR generation at different wavelengths in the visible range from a pump pulse of 1035 nm central wavelength. Circles indicate phase-matched FOCR wavelengths in the limit of weak pump power, while squares indicates the phase-matching points for a typical peak power of 100 kW. Inset: representative image of a PCF structure. (b) Peak power evolution of the pump pulse (left) and simulated spectrum (right) along a tapered PCF for FOCR generation at 580 nm from a transform-limited Gaussian input pump pulse at 1035 nm. FOCR is generated around the point of maximum soliton compression of the pump pulse in the fiber as indicated by the green arrow. (c) Illustration of continuous FOCR tunability in a tapered PCF by a combination of power and pulse duration control of a fixed-wavelength pump pulse. Such a control of the pump pulse determines the point of maximum pump pulse compression, which is the FOCR generation point within the taper, as shown in (a). The FOCR wavelength, in its turn, is defined by the local dispersion of the tapered PCF at this generation point, according to the pump-to-FOCR phase-matching condition such as shown in (a). (d) Average visible wavelength for numerically simulated FOCR spectra, as a function of pump pulse energy and duration. Well-defined FOCR peaks appear close to the FOCR generation threshold, whereas for increasing pulse energy/decreasing duration, continuum formation ensues. Circles are the experimental results. The error bars represent the uncertainty in deconvolution factor of the pulse duration from the measured autocorrelation, as explained in the Appendices Section 2.]

2. GENERAL PRINCIPLE OF WIDELY-TUNABLE CHERENKOV FIBER LASER

Our scheme ultimately relies on the following facts. First, the FOCR generation requires the pump-to-FOCR phase-matching dictating the resulting FOCR wavelength based on the pump pulse central wavelength and the fiber dispersion, as illustrated in Fig 1(a). Second, the FOCR generation process is strongly power-dependent, and is hence spatially localized around the point of maximum soliton compression of the pump pulse in the fiber, as seen from the numerical simulation in Fig 1(b). In a tapered fiber, with the dispersion profile varying along the fiber length, the resulting FOCR wavelength will thus be determined by the local fiber dispersion properties at this maximum-compression point within the fiber. In its turn, the actual position of this point along the fiber length, and hence the resulting output wavelength of the Cherenkov laser, can be unambiguously and continuously determined simply by varying the duration and power of the input pump pulse, as illustrated in Fig 1(c).

Fig. 1(d) illustrates the parameter space for continuous ultra-wide tuning of central wavelength of the fiber taper – based Cherenkov laser demonstrated in this work, which is defined by the energy and the duration of the pump pulses derived form a modelocked Yb-fiber laser. As the figure shows, for a given pulse duration there is a
threshold energy for FOCR generation. Close to this threshold, low-noise spectrally isolated FOCR peaks appear, whereas a further increase of pump pulse energy leads to continuum formation and increased noise. Since our goal is to achieve low-noise FOCR peaked around a chosen central wavelength, it is clear from the figure that we need to control both pump pulse energy and duration. This is because one needs to achieve a pump pulse energy close to the FOCR generation threshold (which itself depends on the FOCR wavelength), while at the same time ensuring that the soliton compression happens at the right position within the taper. Further details on the calculation and the experiment are described below in the text.

While many variations on the above general principle can be envisioned, in the present paper we focus on the generation of femtosecond pulses continuously tunable over the visible spectrum in the 400 - 600 nm range. We chose this spectral range for its special importance for bio-photonics applications, which we foresee as one of the main beneficiaries of our new technology. There, such a wide tuning range will allow the user to easily match the central wavelength of the laser to both one- or two-photon absorption peaks of molecules of interest [26]. As a result, both single-photon [1,26,27] and multi-photon [26–32] microscopy can be conveniently performed with the same laser system and microscope without any setup readjustment.

3. RESULTS AND DISCUSSION

![Experimental set-up. Simplified schematic of the tunable femtosecond Cherenkov fiber laser.](image)

Fig. 2. Experimental set-up. Simplified schematic of the tunable femtosecond Cherenkov fiber laser. By adjusting the power and compression settings, or the power alone, of a fixed-wavelength pump pulse provided by a standard mode-locked fiber laser, the output FOCR wavelength from a PCF taper is continuously tuned in a wide spectral range. Inset: the designed (solid line) and its practical realization (dots) of a PCF taper profile.

a. Experimental layout

Our experimental setup is illustrated in Fig. 2 (see Appendices Section 3 for details). The pump laser is a standard-design mode-locked Yb-fiber master oscillator - power amplifier (MOPA) laser operating at 42 MHz repetition rate, with a central wavelength of 1035 nm, maximum pulse energies in excess of 10 nJ, and pulse durations tunable upwards from 90 fs [33]. The pump pulses are easily reshaped in terms of peak power, duration and chirp by tuning the fiber amplifier and/or the compressor (shadowed box in Fig. 2). The structural profile of the PCF used in both the experiments and the simulations is shown in the Appendices Section 4.

b. Modeling

Our numerical simulations are based on a generalized nonlinear Schrödinger equation adapted to tapered fibers [34], parameterized by plane-wave based calculations of the PCF dispersion and effective area properties. In the simulations we assume that the fiber structure is preserved through the taper, except for an overall scaling which follows the outer diameter of the fiber, which we can experimentally measure. Furthermore, we consider single-mode propagation of linearly polarized transform-limited Gaussian pulses, whose duration and power is varied. Birefringence may be included, but has been neglected in the presented results, because the birefringence of the PCF used in the experiments is unintentional and not well characterized. In order to obtain the desired spectrally-smooth narrow-bandwidth FOCR pulses, the precise taper shape is found to play an important role. From theory, we expect the accelerated down-taper as shown in Fig. 2 to yield the cleanest FOCR pulse shapes, as explained in Appendices Section 5, and such a taper was used in this work.

c. Ultra-wide continuous laser tunability

The upper panel of Fig. 3(a) shows the simulated FOCR spectra by pumping the designed down-tapered PCF with transform-limited Gaussian pulses at 1035 nm, and varying the duration and power of the pump pulses for FOCR tunability. The simulation results are also shown in Fig. 1(d), where the parameter space for the resulting laser output tunability - the 1035 nm pump pulse duration and energy, are presented. The experimentally obtained spectra between 414 and 612 nm are depicted in the lower panel of Fig. 3(a) and the corresponding experimental conditions (Appendices Section 6, the power and duration of the input pump pulse) are shown in Fig. 1 (d). As one can see, the broad laser tunability within ~ 200 nm spectral range is conveniently achieved by the electrical control of the fiber amplifier power and the pulse compressor, with the experimental results closely matching the theoretical predictions.

In the above experiments the laser output wavelength was tuned in a matter of seconds, by manually adjusting the amplifier gain and the compressor setting (see shadowed box in Fig. 2). Remarkably, even by adjusting the pump power alone, and keeping the compressor setting fixed, we could tune the laser wavelength over the 420 – 560 nm range, covering much of the visible spectrum. This constitutes a further simplification of the practical implementation of our technology, drastically speeding up the laser tuning process. Indeed, the electronic control of pump power allows its adjustment on ~ 10 μs timescale (Appendices Section 7), and in principle, can be scaled down even to nanoseconds with complementary modulation of the laser signal [35]. Such a capability to achieve fully electrical ultra-wide band tunability of a laser on a microsecond, eventually even sub-microsecond timescale can enable a new class of demanding applications where high laser tuning rate is of essence.

Fig. 3(b) shows the far-field mode profiles of the measured FOCR signals, which were observed to be stable and with no signs of higher-order mode admixture, although the PCF used was multi-mode at visible wavelengths. Fig. 3(c), left panel, shows the experimental autocorrelations (AC) of the FOCR output, measured in the operational range of our autocorrelator 500 – 600 nm. Just as expected from the simulations, we obtain smooth pulse spectral profiles with moderate variations in duration. In Fig. 3(c), right panel, the AC full width at half maximum (FWHM) of Cherenkov pulses is shown as a function of their wavelength (blue dots), along with the corresponding values for the input pump pulses from Yb-fiber laser at 1035 nm (red squares). For comparison, in the same figure the AC FWHM of the simulated FOCR pulses is shown (dashed blue line), being corrected for dispersive effects in the optical elements between the PCF and the autocorrelator in order to match the experimental conditions (Appendices Section 8). We note that the simulated pulse durations at the fiber end facet were below 100 fs at all CR wavelengths, including the spectral range below 500 nm, for which the experimental data could not be obtained due to the limitations of our autocorrelator and related optics. These generated pulses are much shorter than what is typically produced by currently most-used spectrally-sliced supercontinuum technology [32]. This means that higher pulse peak power can be achieved with comparable pulse energy, benefitting such applications of tunable ultrafast lasers as nonlinear spectroscopy and microscopy.

Our simulations quite accurately reproduce both the measured spectra of the Cherenkov output in the entire range of laser tunability 414 – 612 nm (Figs. 1(d) and 3(a)), and the measured autocorrelations within the accessible experimental range above 500 nm, while
predicting the output pulse duration with reasonable accuracy (Fig. 3 (c), right panel). This fact strongly supports the presented physical picture of widely-tunable FOCR generation in fiber tapers. We note that our simulations are performed using an idealized Gaussian pump laser pulse shape, while in reality the pump pulses may have a more complicated time structure. Nevertheless, even using such a simplification in the modeling, a close agreement is achieved between the experimental data and the theoretical predictions.

We comment that the actual limits to wavelength tunability of the Cherenkov fiber laser are defined by the interplay between the pump wavelength and soliton peak power, as well as the fiber dispersion at the pump and target wavelengths [12], all of which can be engineered.

d. Laser output power and noise figure

Fig. 4(a) shows the generated FOCR output power, and the pump-to-FOCR power conversion efficiency of our laser system. The output power range 1 - 5.5 mW corresponds to the pulse energies of ~25 - 130 pJ, easily satisfying the energy requirements for e.g. nonlinear microscopy experiments [36]. The observed conversion efficiency of our widely tunable FOCR laser of a few percent is quite typical for this wavelength-conversion process [37,38].

The intensity noise of the light source is another key parameter for the laser performance evaluation. Fig. 4(b) shows the signal-to-noise ratio (SNR) of the output of our laser, which is 1-3 orders of magnitude better than the typical picosecond-pumped supercontinuum source (ps-SC) spectrally sliced to ~10 nm bandwidth [38]. We note that spectral slicing of our Cherenkov laser to the same ~10 nm bandwidth around its peak wavelength will only reduce its SNR by an additional factor of 2. Therefore, using the tunable Cherenkov laser, the spectroscopy or imaging data of the same quality can be obtained up to 2-4 orders of magnitude faster, as compared to the ps-SC source of the same ~10 nm bandwidth, clearly demonstrating a significant practical advantage of widely tunable Cherenkov laser technology. The details of laser noise characterization are presented in Appendices section 9. With regards to the spectral stability, we did not observe any noticeable spectral changes during the measurement periods typically lasting up to 10 minutes.

Generally, the FOCR noise is found to increase as the wavelength is shortened, which is theoretically expected because the short-wavelength generation requires higher pump soliton numbers, thus translating into higher noise [7]. By the same argument, for a given wavelength the SNR is found to decrease with increasing output FOCR power (Appendices Fig. S10).
4. CONCLUSIONS

We have presented a novel technology for widely tunable and highly-stable ultrafast fiber lasers, based on fiber-optic Cherenkov radiation in fiber tapers. Adding only one passive new element, a fiber taper, to a standard pump pulsed laser, allows one in a very simple manner to continuously and resonantly convert the fixed pump laser wavelength to any desired wavelength within an ultra-wide spectral range, in a single-mode output, with a power conversion efficiency of a few percent, and with excellent noise characteristics as compared to existing approaches such as spectrally-sliced supercontinuum. Based on this idea, we demonstrated a femtosecond fiber laser continuously tunable over the 200 nm - wide red-green-blue spectral range, emitting 100 - 200 fs pulses with pulse energies ~25 - 130 pJ. Our measurements are in good agreement with parameter-free simulations.

Due to its modest requirements on pump pulse energy and duration, monolithic all-fiber integration of a widely-tunable Cherenkov laser [22] is a straightforward technological perspective. The demonstrated basic principle of combining tailored PCF taper structures with agile femtosecond pump lasers, such as widely commercially available Yb-, Er- or Tm-fiber lasers, could be implemented in many ways, to target the generation within different spectral and intensity ranges, as well as discrete wavelengths, multiple pulses etc. For example, when pumped with a standard Er-fiber laser at 1550 nm, the Cherenkov femtosecond output with higher energy [39] as well as broader tunability could be generated. Our findings thus pave the way to practical applications of simple and inexpensive Cherenkov femtosecond fiber lasers with wide continuous tunability in industry, science, and medicine.

As one of the key applications for the demonstrated laser technology we foresee bio-photonics, making use of one- and many-photon excitations, Raman processes etc in biological tissues as well as in specially added markers [see e.g. [27,40]]. Currently, substantial effort is being put into translating bio-spectroscopy techniques, demonstrated and tested in research labs, into clinical environments. Here, the widely-tunable Cherenkov fiber lasers will satisfy most stringent requirements on enabling light sources such as: adjustment-free long lifetime operation, compactness and robustness, high operational stability, sufficiently fast wavelength tuning, and very low noise. This will drastically, by several orders of magnitude, minimize the acquisition time of high-quality imaging and spectroscopy data, as compared to existing laser technologies [14,24], thus enabling the measurements which were unfeasible before. Potentially, a single widely tunable Cherenkov laser will be able to access the whole range of fluorophores (~ 400 - 800 nm) used both in confocal microscopy and super-resolution microscopy, thereby providing a complete multimodality laser scanning solution in translational environment.

The tapered fiber used in this work is based on the small-core nonlinear PCF NL-3D-850 (NKT Photronics A/S), which has a zero-dispersion wavelength (ZDW) around 850 nm. The definition of fiber pitch A is shown in Fig. S1.

2. The error bars in Fig. 1d

The error bars represent the estimated uncertainty in deconvolution factor of the actual pulse FWHM duration from the measured autocorrelation. As an error we chose the 8.7% relative difference of the deconvolution factors for a Gaussian pulse (c = 1.141), used for the deconvolution of experimental AGs, and another common pulse shape - the sech® pulse (c = 1.545).

3. Experimental setup

The experimental setup of the tunable FOCR laser is shown as Fig. S2. The laser system consists of two main parts: a home-made femtosecond Yb-fiber laser with master-oscillator power-amplifier configuration at 1035 nm wavelength and 42 MHz repetition rate used as the pump source and a tapered PCF used for tunable FOCR wave conversion. The pump source is comprised of four main stages: a passively mode-locked oscillator using nonlinear polarization rotation, a stretch fiber, an Yb-fiber amplifier and a grating pair as the pulse compressor. The oscillator setup is similar to the laser described in Ref. [41]. After the amplification, the stretched pump pulse signal can be compressed down to 90 fs by a grating pair with the energy up to 50 nJ. We note here that for the FOCR generation experiments, pump pulse energy only up to 11.2 nJ is needed. The amplifier configuration and the associated dynamics were originally explored in Ref. [33]. The coupling efficiency from the pump laser into the tapered PCF is around 50%.

We also note here that due to wavelength limitation of the bandpass filter inside the autocorrelator in our setup, FOCR pulse duration with the FOCR wavelength shorter than 500 nm could not be measured. The typical spectrum of the pump laser is shown as the inset of Fig. S2.

During the experiments, both the output pump pulse energy and pulse duration were controlled by two parameters: control of the pump signal amplifier power and the grating pair distance.

Appendices

1. Photonic crystal fiber (PCF)

Figure S1. The representative image of the photonic crystal fiber (PCF). A is the PCF fiber pitch.

Figure S2. The experimental setup of the tunable FOCR laser. OSC: pump oscillator; AMP: pump amplifier; ISO: pump isolator; G: grating; M: high reflection mirror; HWP: half-wave plate; PBS: polarization beam splitter; FM: flip mirror; AC: autocorrelator; PM: photometer; OSA: optical spectrum analyzer; PCF: tapered photonic crystal fiber NL-3D-850; DM: dichroic mirror; F: optical band pass filter; ESA: electrical spectrum analyzer with photodiode. Inset: typical spectrum of the pump laser output.
4. Tapered PCFs preparation

As for the tapered PCF, a Vytran LDS1250 splicing workstation is used to prepare all the tapers needed for the experiments. The taper profiles are then characterized with a microscope. The tapered PCF used both in the experiments and simulations presented in the main text has an initial 25 mm uniform ‘lead’ section and a following 16 mm down-tapering part with decreasing diameter. The taper ratio (taper waist divided by the initial PCF diameter) is 60%. The taper rate increases towards the waist, as shown in the inset of Fig. 2 in the main text.

Besides the taper mentioned in the main text, some other examples of different designs and their measured taper profiles are shown in Fig. S3. More than 40 linear and 20 non-linear tapers with tapering ratios up to 60% have been prepared. All of them fit well with the calculated profiles, which demonstrate the high level of tapering accuracy and repeatability.

5. Theory and numerical simulations

In this work, the simulations of pulse propagation in short tapered PCFs relies on a formulation of 1+1-dimensional propagation equations for tapered fibers developed in [34]. The electric field is expanded into a sum of local waveguide modes according to

\[
E(z, t) = \sum_m \int d \omega \beta_m(z, \omega) \frac{e^{i \omega t}}{\beta_m(z, \omega)} \psi_m(z, \omega) e^{i \omega t} - \frac{\beta_m(z, \omega)}{\beta_m(z, \omega)}
\]

where \(\beta_m(z, \omega)\) are the frequency-dependent propagation constants of the different waveguide modes, indexed by \(m\). Although the fiber structure under consideration is multimode at both pump and FOCR wavelengths, only propagation in the fundamental mode is considered here. This is justified by the short length of the tapers, and the feasibility of selective coupling into the fundamental mode. In the simulations we also assume propagation in a single linear polarization state, which may not be in full accordance with the experimental situation. However, the PCF is not intentionally birefringent, and although it is likely it has some unintentional birefringence, we cannot estimate this property with an accuracy that would allow for more detailed modeling of polarization properties.

Under these assumptions, the mode index, \(m\), in Eq. (1) may be dropped, and the propagation equation then reads,

\[
\frac{\partial \psi_m(z, \omega)}{\partial z} = \frac{1}{c} \frac{\partial}{\partial \omega} \int d \omega_1 \int d \omega_2 A_m(z, \omega) A_{\omega_1}^{*}(z, \omega_1) A_{\omega_2} \psi_m(z, \omega_2) A_{\omega_1}(z, \omega - \omega_1 + \omega_2) e^{i \omega_1 t} [1 - f_0] + \frac{f_0}{\nu} R(t, w - w_0),
\]

where \(n_1\) is the nonlinear coefficient of silica, and \(f_0\) is the fraction of the silica nonlinear response function contributed by the delayed (Raman) response, described by the response function \(R(t, w - w_1)\). The necessary input parameters for the model are then the fiber propagation constant \(\beta(w)\) and the effective area, \(A_{eff}(z, \omega)\), as a function of frequency for the fundamental mode. These parameters are determined by using the freely available MIT Photonic bands software package [42], which solves the Maxwell equations for the guided fiber modes by expanding the dielectric function and magnetic field vector in plane waves. This software is used in combination with a perturbative technique for including the effects of material dispersion [34,43], which is crucial in determining the FOCR phase-matching wavelengths.

The pure-silica PCFs are modeled as a cobweb geometry, as illustrated in the inset Fig. S4. The structural parameters defined in the figure were chosen as \(r_c = 0.2, r_e = 0.45\), \(w_0 = 0.05\), \(\Lambda = 3.2\) \(\mu \text{m}\). With these parameters, experimental results for FOCR generation in untapered fibers were reproduced reasonably well. When describing tapered fibers, it is assumed that the taper simply scales \(\Lambda\), while leaving the parameters \(r_c, r_e, \Lambda\) and \(w_0\) invariant. The dispersion curves of fibers tapered to various dimensions are displayed in the main panel of Fig. S4. The tapering leads to an overall increase of the dispersion coefficient over the wavelength range of interest, and shifts the ZDW from 888 nm to 725 nm.

![Figure S4. Dispersion curves for three different \(\Lambda\) values of the PCF structure shown in the inset.](image)

The Cherenkov phase-matching condition is given by [13]

\[
\beta(w) - \beta(w_0) - \beta_1(w_0)(w - w_0) = \frac{\nu P_0}{\omega A_{eff}(w_0)} \quad (4)
\]

where \(w_0\) is the pump frequency, \(\beta_1 = \nu P_0 / \omega A_{eff}(w_0)\) and \(P_0\) is the peak power of the pump pulse at the point where the Cherenkov radiation is generated. Typical pump peak powers attained in our simulations are on the order of 100 kW. Fig. 1a in the main text illustrates how the down-tapering of the PCF allows tuning the FOCR phase-matching point from about 625 nm down to 400 nm, for \(P_0 = 100\) kW, by changing the linear dispersion of the fiber. On the other hand, the nonlinear term only affects minor shifts of the phase-matching point, especially at short FOCR wavelengths, where the phase-matching curve is steep.

The phase-matching criterion in Eq. (4) may be rewritten as

\[
\int w_0 dw [\beta_1(w_0) - \beta_1(w_0)] = \frac{\nu P_0}{2} \quad (5)
\]

Since the pump should be in the anomalous dispersion regime, \(D > 0\) (otherwise soliton compression will not occur), and \(\beta_1\) is equal to the inverse group velocity, it follows that the left hand side (LHS) of Eq. (5) will initially be negative as \(w\) increases from \(w_0\). This is because for anomalous dispersion, the group velocity increases with frequency.
In order for the LHS to eventually become positive as \( w \) increases, \( \beta_1(w) - \beta_0(w_0) \) must reverse its sign, i.e., the FOCR phase-matching will necessarily occur at a frequency where the FOCR has a lower group velocity than the pump. On the other hand, as the PCF is down-tapered, the ZDW moves to higher frequencies, and the group velocity at a given visible frequency (not necessarily the FOCR frequency) increases relative to the pump group velocity. These observations are of crucial importance to the following discussions.

Fig. S5A and the upper left inset plot of Fig. S5E show an example of a simulated spectral evolution for a linear taper profile. The PCF has a straight lead section of 2.5 cm, with \( \Lambda = 3.2 \) \( \mu \)m, and is then down-tapered to \( \Lambda = 1.92 \) \( \mu \)m, i.e. to 60\% of its initial diameter; over a distance of 1.6 cm, in which \( \Lambda \) varies linearly with distance. A final straight tail section of 0.8 mm is also included. The input pump pulse has a peak power of 7370 W and a duration (full width at half maximum, FWHM) of 105 fs, which ensures maximal compression, and FOCR generation, at the end of the lead section, \( z = 2.5 \) cm.

Visualization 1.mov
Visualization 2.mov

Figure S5. Simulations of FOCR generation in linear and non-linear taper profiles. Spectrum (A, C) and temporal power profile (B, D) versus propagation distance in a linear (A, B) and non-linear (C, D) taper profile, with taper and input pulse parameters as described in the text. Both spectral density (pJ/THz) and power (W) is plotted logarithmically. (E) and (F) are corresponding Supplementary Visualization files [Visualization 1 and Visualization 2] for simulations of FOCR generation in linear and non-linear taper profiles, respectively.

The FOCR generation is seen to occur over a distance of 1–2 mm (note that it is the logarithm of the spectral density which is plotted), with a slight blueshift occurring immediately after the generation point. Hereafter, the FOCR spectrum is relatively constant until around \( z = 3.8 - 4 \) cm where the spectrum suddenly breaks up with a strongly blue-shifted component appearing, having a spectral density comparable to the remaining peak at the original FOCR wavelength. Such a composite spectrum is incompatible with our purpose, which is to generate a single spectral peak whose central wavelength is tunable.

The observed spectral evolution may be rationalized by considering the evolution of the temporal power profile, depicted in Fig. S5B and the right two plots of Fig. S5E. The FOCR initially moves slower than the pump pulse, and is seen to emerge at later times, to the right of the figure. The overlap between pump and visible wavelengths is evidenced by strong interference fringes. The initial blueshift of the FOCR can then be interpreted as a cross-phase modulation (XPM) effect resulting from being on the trailing edge of the stronger pump pulse. As the fiber down-tapers, the relative velocity of pump and FOCR pulses reverses its sign, and the FOCR pulse is seen to catch up with the pump, with a re-collision occurring around \( z = 3.8 \) cm. As the FOCR passes through the pump pulse, it is initially blueshifted by XPM, but subsequently redshifts as it emerges on the leading edge of the pump pulse, since XPM spectral shifts are proportional to the time derivative of the pump pulse profile. However, part of the FOCR pulse becomes trapped behind the pump, and is continually blue-shifted in the manner described by Skryabin and Gorbach [44,45]. As a result of this effect, the blue spectral peak is formed.

A straightforward mitigation scheme for this re-collision effect is to introduce a non-linear taper profile, as depicted in the inset of Fig. 2 in the main text. In this taper, the initial taper rate is reduced, so that the FOCR pulse propagates a longer distance with slower velocity than the pump pulse. The taper rate gradually increases, so that the final value of \( \Lambda \) is the same as before.

The difference in taper rate from start to end of the tapered section is a factor of 10. Fig. S5C, D and the inset plot of Fig. S5F show spectral and temporal evolutions of the same pump pulse as described above in the non-linear taper. It is clearly seen that the re-collision between pump and FOCR is now delayed until the very end of the taper, and the resulting spectral modifications are much weaker. It should be noted, however, that as the pump power is increased, the emerging pump soliton is increasingly redshifted, and thereby slowed down due to the anomalous dispersion at wavelengths above 1 \( \mu \)m. Therefore, pump power is ultimately limited by the re-collision effect, even in non-linear tapers, and the power of the input pulse used to generate the inset plots in Fig. S5 is around the maximal useful power for generation of a clean spectrum above 600 nm in the non-linear taper profile.

It should also be noted that the re-collision effect is absent if the FOCR is generated in an up-tapering fiber. This approach suffers from other limitations which will be discussed in detail elsewhere. However, our main reason for choosing the down-tapering profile can be understood from the dispersion curves in Fig. S4. It can be seen that the dispersion coefficient reaches values of around \(-1000 \) ps/nm/km at a wavelength of 400 nm, regardless of the value of \( \Lambda \). In contrast, the dispersion at 600 nm is around \(-200 \) ps/nm/km, and decreases significantly in magnitude with \( \Lambda \). If we imagine generating blue FOCR at the beginning of an up-tapering section having a length of 1.6 cm, the dispersive broadening of a FOCR pulse with 10 nm bandwidth can be estimated to be around 160 fs. A similar estimate for a down-tapering profile, where light at 600 nm is generated at the start of the taper, would predict a broadening of around 30-35 fs. As shown below, we have indeed found that visible FOCR pulses from our down-tapered design emerge from the fiber end facet with FWHM durations below 50 fs. In calculations for up-tapering fibers (not shown), we correspondingly find that the short-wavelength pulses are broadened.
to 100-200 fs, leading to an order-of-magnitude difference between pulse durations at different wavelengths.

To investigate the potential of the proposed FOCR generation scheme, a large number of simulations were performed for the non-linear taper profile, varying both power and duration of the Gaussian pump pulse, in order to obtain compression and FOCR emission at different points in the taper. A selection of spectra and temporal pulse profiles at the fiber end facet are shown in Fig. 56. The selection criterion was to obtain the highest FOCR energy without compromising spectral isolation, which is evaluated in a purely subjective and non-quantitative way. It can be seen that spectral energy densities around 1.7-2.4 pl/nm are obtained for few-nJ pump pulses with durations in the 100-300 fs range. As the pump pulse duration is shortened, and/or the power is increased, the compression point moves towards earlier sections of the taper, leading to a longer FOCR wavelength being generated.

The temporal pulses are found to have FWHM durations between 14 and 34 fs, with the blue spectra having the shortest duration, as they are generated near the end of the taper, and therefore see little dispersion. Total FOCR energies range between 29 and 42 pl, which in a 42-MHz laser, as was used in the experiments, would correspond to average visible powers between 1.22 and 1.76 mW.

6. Experimental conditions

Table S1 shows the generated FOCR wavelength, as a function of pump pulse energy and duration as shown in Fig. 1 (d). We note here that Gaussian pulse shape was assumed for the pump pulse duration measurements. We also note that in our system for the FOCR wavelength tunability, only the pump pulse energy and duration, are the key parameters. The chirp of the pulses only matters when the pump spectral bandwidth is much broader which is not the case in our work.

<table>
<thead>
<tr>
<th>Central wave [nm]</th>
<th>Pump energy [nJ]</th>
<th>Pump pulse FWHM [fs]</th>
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<td>380</td>
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Table S1. Experimental conditions of the pump pulses as shown in Fig. 1 (d) in the main text.

7. Response time of electronic control of pump power

As shown in Fig. S7, electronic control of pump power allows adjustment of power in few-µs timescale, which is limited by the driving electronics. In Ref. [35], we have characterized the response to pump modulation for the output power of a fiber amplifier, comprising of the same type of gain fiber and diode laser as pump as used in this work. As to be expected, the transfer of any modulation of the pump laser is limited in frequency due to the finite response of the gain medium. These results show that about a 10% modulation of the signal power is possible upon full modulation of the pump laser at frequencies >100 kHz. Therefore, a 10-µs response time is relatively easy to achieve with straightforward modulation of the laser. This level of modulation is achieved easily using simple, low-cost driver electronics. Even this is not a fundamental limitation, since it was demonstrated in the same reference that just as the transfer of the modulation of the pump rolls of at high frequencies, transfer of modulation of the signal increases concurrently. This leads to the possibility of complementary modulation of both the pump and signal of an ultrafast fiber amplifier without any evident frequency limitation. Modulation of the signal was obtained in that study using an acousto-optic modulator, which can also be implemented with an electro-optic modulator. Therefore, should the need arise, there is no laser-physics-based limitation to how fast the output power of an ultrafast fiber amplifier system can modulation. It would be limited in practice by the electronics, but we routinely achieve rise or fall times of <10 ns in other work [38]. Since the propagation of each pulse in the amplifier/compressor is physically independent of each other, future possibility of tuning our FOCR wavelength at a timescale of nanoseconds is evident.

Figure S7. Optical pump power response profile dependent on the electronic control of pump power.

8. Pulse duration

From the experimental setup of Fig. S2, we can see that a focusing lens (1.9 mm thick), a band-pass filter (2 mm thick, filtering out the residual pump signal), and a half wave plate (0.312 mm thick) together with one BBO crystal (0.3 mm thick) as part of the autocorrelator are used between the laser output and the autocorrelator for the FOCR pulse duration measurements. Therefore, the measured autocorrelation (AC) FWHM of FOCR pulses shown in Fig. 2(c) of the main text also include pulse chirp induced by the dispersion of the optical components mentioned above. In order to compare the simulated FOCR pulse duration with the experimental measurements, the AC FWHM of the simulated FOCR pulses shown in Fig. 2(c) in the main text are carefully corrected for dispersive effects in the optical elements between the PCF and the autocorrelator. The details are shown in Table S2. The right two columns of Table S2 show the corrected AC FWHM of the simulated FOCR pulses and the measured AC FWHM separately, as plotted in Fig. 3(c) in the main text.
9. Intensity noise measurements

The details of the laser noise measurements can be found in Ref. [38,46]. The noise spectra were measured in the baseband of the electrical frequency ranging from 1 MHz to 21 MHz (half of the repetition rate of the pump laser source).

Fig. S9A shows the optical spectra of the typical FOCR output from the laser system as measured with and without the bandpass optical filters. The FWHM bandwidth of the filters is 10 nm. The power of unfiltered CR in this case is 1.3 mW. The noise spectra of the filtered and unfiltered FOCR signals are shown in Fig. S9B. Fig. S9C shows the optical intensity spectra of the spectrally-unfiltered FOCR, and the SNR spectra measured from the bandpass-filtered CR signals such as shown in Fig. S9A. The SNR of the spectrally-unfiltered FOCR is calculated to be 931 while the SNR of the filtered FOCR with the central wavelength at 490 nm is 488.4. Therefore, as it is written in the main text, our measurements indicate that a filtering of the FOCR spectrum only reduces the SNR by a factor of 2 for a filter centered on the FOCR peak.

Fig. S10A shows the RIN spectra of the FOCR signal dependent on the output power at FOCR wavelength at 560 nm, while the corresponding SNR is shown in Fig. S10B. As written in the main text, the SNR decreases as the output power increases.

### Table S2

<table>
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Table S2. The simulated pulse duration and autocorrelation corrected for dispersive effects in the optical elements between the PCF and the autocorrelator.


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