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Analysis of cross-correlations between financial markets after the 2008 crisis☆,☆☆

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HIGHLIGHTS

- We use RMT to analyze the cross-correlations between worldwide stock markets.
- We find that the majority of the cross-correlation coefficients arise from randomness.
- We analyze the connection structure of markets before and after the crisis using network theory.
- Key financial markets are revealed.

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ABSTRACT

We analyze the cross-correlation matrix **C** of the index returns of the main financial markets after the 2008 crisis using methods of random matrix theory. We test the eigenvalues of **C** for universal properties of random matrices and find that the majority of the cross-correlation coefficients arise from randomness. We show that the eigenvector of the largest deviating eigenvalue of **C** represents a global market itself. We reveal that high volatility of financial markets is observed at the same times with high correlations between them which lowers the risk diversification potential even if one constructs a widely internationally diversified portfolio of stocks. We identify and compare the connection and cluster structure of markets before and after the crisis using minimal spanning and ultrametric hierarchical trees. We find that after the crisis, the co-movement degree of the markets increases. We also highlight the key financial markets of pre and post crisis using main centrality measures and analyze the changes. We repeat the study using rank correlation and compare the differences. Further implications are discussed.

1. Introduction

The global financial system is composed of a large variety of markets that are positioned in different geographic locations and in which a broad range of financial products are traded. Despite the diversity of markets, index movements often respond to the same economic announcements or market news [1–3] which implies that financial time series can display similar characteristics and be correlated. Since the work of Markowitz [4], correlations of financial time series are constantly

☆ The views expressed in this work are those of the authors and do not necessarily reflect those of the Borsa Istanbul or its members.

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a subject of extensive studies both at the theoretical and practical levels. It is important not only for understanding the collective behavior of a complex system but also for asset allocation and estimating the risk of a portfolio.

In particular since the recent 2008 financial crisis, which originated in the US and then spread to almost all markets in the world, many economists have been studying the correlation structure between financial markets and the transmission of volatility from one to another. One of the major difficulties in these studies are the complicated unknown underlying interactions of the financial markets. Besides correlations between markets need not be just pairwise but may rather involve clusters of markets and relationship between any two pair may change in time [5].

In earlier times, physicists experienced similar problems. The problem became popular by Wigner's work in the 1950s for application in nuclear physics, in the study of statistical behavior of neutron resonances and other complex systems of interactions [6]. He tried to understand the energy levels of complex nuclei, when model calculations failed to explain experimental data. To overcome this problem, he assumed that the interactions between the constituents comprising the nucleus are so complex that they can be modeled as random [5]. Based on this assumption, he derived the statistical properties of very large symmetric matrices with i.i.d. entries and the results were in remarkable agreement with experimental data.

More recently random matrix theory (RMT) has been applied to analyze the financial time series [5,7–37]. In particular, correlation matrices are computed for the empirical data and quantities associated with these matrices are compared to those of random matrices. The extent to which properties of the correlation matrices deviate from random matrix predications clarifies the status of the information derived from the computation of covariances [12]. The literature focuses on the correlations between individual stocks in a market; however, in this study we will analyze the cross-correlations between 87 main financial markets in the world by tools of RMT.

The rest of the paper is organized as follows; in Section 2, we give a brief description of the methodology. Section 3 describes the data and contains several results of our analysis; in particular Sections 3.1, 3.2 and 3.4 present the eigenvalue and eigenvector analysis of the correlation matrix with discussion of the relation between volatility and correlation of financial markets. In Section 3.6, we construct a correlation based market network and compare the structure before and after the 2008 financial crisis by tools of graph theory. In Section 4, we use an alternative approach to the construction of the correlation matrix, present the related results and discuss possible further studies. Finally, Section 5 contains some concluding remarks.

2. Methodology

Let $P_i(t)$ be the index of the stock market $i = 1, 2, \ldots, N$ at time t and $t = 0, 1, \ldots, T$. The logarithmic index return of the ith market index over a time interval Δt is given by

$$R_i(t, \Delta t) \equiv \ln P_i(t + \Delta t) - \ln P_i(t). \quad (1)$$

We consider the normalized returns

$$r_i(t) \equiv \frac{R_i - \langle R_i \rangle}{\sigma_i} \quad (2)$$

where $\sigma_i \equiv \sqrt{\langle R_i^2 \rangle - \langle R_i \rangle^2}$ is the standard deviation of R_i and $\langle \cdots \rangle$ is the time average over the considered period. Then the equal time cross-correlation matrix C is the matrix with elements

$$c_{ij} \equiv \langle r_i r_j \rangle. \quad (3)$$

In matrix notation, the interaction matrix $\mathbf{C}$ can be written as

$$\mathbf{C} = \frac{1}{T} \mathbf{R}\mathbf{R}^t \quad (4)$$

where $\mathbf{R}$ is an $N \times T$ matrix with entries $r_{im} \equiv r_i(m\Delta t)$ with $i = 1, 2, \ldots, N$; $m = 1, \ldots, T$ and $\mathbf{R}^t$ denotes the transpose of $\mathbf{R}$.

We will compare the properties of the interaction matrix $\mathbf{C}$ with those of a random cross-correlation matrix.

Let $x_i(t)$; $i = 1, 2, \ldots, N$ where $x_i(t)$ are independent, identically distributed random variables. We define the $N \times T$ matrix $\mathbf{A}$ by elements $a_{it} \equiv x_i(t)$. The matrix $\mathbf{W}$ defined as

$$\mathbf{W} = \frac{1}{T} \mathbf{A}\mathbf{A}^t \quad (5)$$

is called a Wishart matrix [38–40]. Let each $x_i(t)$ be normally distributed and rescaled to have zero mean and constant unit standard deviation. Under the restriction of $N \to \infty$, $T \to \infty$ with $Q \equiv T/N > 1$ is fixed, the probability density function $\rho_{rm}(\lambda)$ of eigenvalues λ of the matrix $\mathbf{W}$ is [39,40]

$$\rho_{rm}(\lambda) = \frac{Q}{2\pi} \frac{\sqrt{(\lambda_{\max} - \lambda)(\lambda - \lambda_{\min})}}{\lambda} \quad (6)$$

and

$$\lambda_{\min}^{\max} = 1 + \frac{1}{Q} \pm 2\sqrt{\frac{1}{Q}} \tag{7}$$

where $\lambda_{\min}^{\max}$ are the maximum and minimum eigenvalues of **W**. For the rest of the paper, the analyzed eigenvalues are rank ordered i.e. $\lambda_i < \lambda_j$ for all $i < j$ and α_i denote the corresponding unfolded eigenvalues for all i.

The distribution of nearest neighbor eigenvalue spacing of **W** is given by Wigner–Dyson distribution [41,42]:

$$\rho_{\mathbf{W_{nn}}}(s) = \frac{\pi s}{2} \exp\left(-\frac{\pi}{4}s^2\right) \tag{8}$$

where $s = \alpha_{i+1} - \alpha_i$.

The distribution of next-nearest neighbor eigenvalue spacing of **W** is given by [41,42]:

$$\rho_{\mathbf{W_{nnn}}}(s) = \frac{2^{18}}{3^6\pi^3} s^4 \exp\left(-\frac{64}{9\pi}s^2\right) \tag{9}$$

where $s = (\alpha_{i+2} - \alpha_i)/2$.

The *number variance* $\sum^2$ is defined as the variance of the number of unfolded eigenvalues in the intervals of length l, around each α_i [41,42],

$$\sum\nolimits^2(l) = \langle [n(\alpha, l) - l]^2 \rangle_\alpha \tag{10}$$

where $n(\alpha, l)$ is the number of unfolded eigenvalues in the interval $[\alpha - l/2, \alpha + l/2]$ and $\langle \cdots \rangle_\alpha$ denotes an average over all α. For large values of l, the number variance for **W** behaves like $\sum^2 \sim \ln l$ and if the eigenvalues are uncorrelated then $\sum^2 \sim l$ [41,42].

Let v_k be the eigenvector corresponding to the eigenvalue λ_k. We denote the jth component of v_k as $v_{k,j}$. By construction we have $\sum_{j=1}^{N}[v_{k,j}]^2 = 1$. If we normalize the eigenvectors ($v_k \to v'_k$) such that $\sum_{j=1}^{N}[v'_{k,j}]^2 = N$ then the components of each normalized eigenvector v'_k have a Gaussian distribution with mean zero and unit variance [10],

$$\rho(v') = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{v'^2}{2}\right). \tag{11}$$

A useful quantity in characterizing the eigenvectors is the so-called Inverse Participation Ratio (IPR) [11]. For the eigenvector v_k, it is defined as

$$IPR_k \equiv \sum_{j=1}^{N}[v_{k,j}]^4. \tag{12}$$

For our purposes it is sufficient to know that the reciprocal of the IPR_k (called participation ratio) quantifies the number of significant components of the eigenvector v_k. In RMT, the expectation of IPR_k is $3/N$ since the kurtosis for the distribution of the eigenvector component is 3.

3. Data and the results

We analyze daily closing values of 87 main benchmark indexes in the world between 01/01/2009 and 31/07/2012 (data are obtained from Bloomberg). To reflect the market dynamics better, index values are not converted to a single currency. Markets in some countries do not operate on Fridays; in that case Saturdays' values are considered as Fridays'. If a market is closed on a business day, we carry over the last value. The list of indexes is in the Appendix.

3.1. Eigenvalue analysis

We take $\Delta t = 1$ day and compute the 87×87 cross-correlation matrix **C**. We have $N = 87$ and $T = 933$ giving $Q \approx 10.73$, with theoretical lower and upper limits $\lambda_{\min} \approx 0.48$ and $\lambda_{\max} \approx 1.71$ for the eigenvalues of **C**. First, eigenvalues of **C** are compared with the theoretical distribution $\rho_{rm}(\lambda)$ (see Fig. 1).

One immediate thing to note is that the largest eigenvalue of **C** is ≈23.8 which is 14 times larger than the theoretical upper limit and stands out from all others. Also a first view suggests the presence of a well-defined bulk of eigenvalues. Although ≈52% of the eigenvalues fall into the theoretical interval, ≈93% of the eigenvalues are smaller than $\lambda_{\max}$.[1]

[1] The high percentage of eigenvalues below $\lambda_{\min}$ may be attributed to the fact that many of the less liquid markets behave independently relative to the rest of the others [12], and also theoretical results are also valid in the infinite limit; hence there is always a small probability of finding eigenvalues above $\lambda_{\max}$ and below $\lambda_{\min}$ [10].

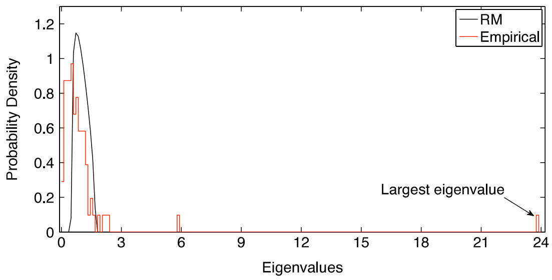


Fig. 1. Empirical vs. theoretical eigenvalue distribution.

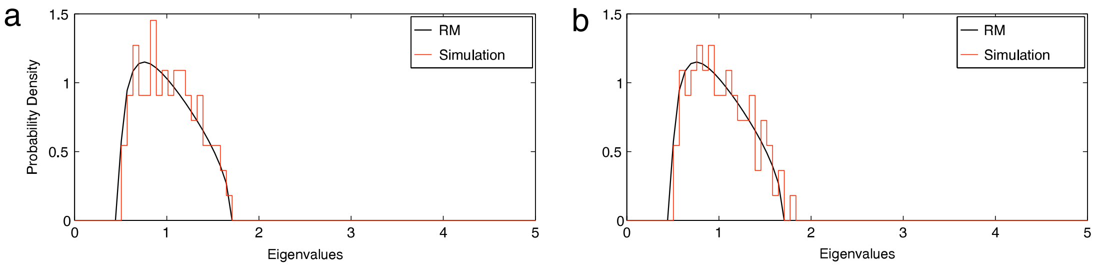


Fig. 2. Simulation (finite size (a)–fat tails (b)) vs. theoretical eigenvalue distribution.

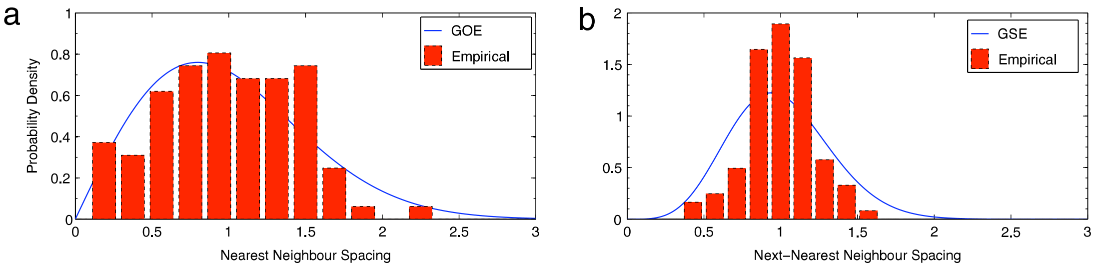


Fig. 3. Nearest and next-nearest neighbor spacing distribution of eigenvalues.

Since the theoretical distribution is valid strictly for $N \to \infty$ and $T \to \infty$, we must test that the deviations for the largest few eigenvalues are not finite size effects [11]. First, we construct $N = 87$ mutually uncorrelated time series generated to have (a) standard normal distribution (as in theory), and (b) identical power-law tails (as in empirical examples [43]) each having length $T = 933$. Then we compare eigenvalue densities of their cross-correlation matrices with the theoretical distribution (see Fig. 2). We find good agreement with the theory suggesting that the deviations of the few largest eigenvalues from RMT in Fig. 1 are not caused by finite size effects or the fact that returns are fat tailed.[2]

We apply further RMT tests to strength our claim. The first independent test is the comparison of the distribution of empirical nearest neighbor eigenvalue spacing $\rho_{nn}(s)$ with $\rho_{\mathbf{W}_{\mathbf{nn}}}(s)$ (see Fig. 3). The agreement suggests that the positions of two adjacent empirical unfolded eigenvalues at the distance s are correlated similar to the eigenvalues of **W**.

The next test is the comparison of the distribution of empirical next-nearest neighbor eigenvalue spacing $\rho_{nnn}(s)$ with $\rho_{\mathbf{W}_{\mathbf{nnn}}}(s)$. We demonstrate this correspondence in Fig. 3 which shows a nice agreement between empirical data and the theory.

To test for long-range two point eigenvalue correlations, we consider the number variance. It is clear that the number variance of empirical data agrees well with the theory (see Fig. 4).

It can be concluded that the bulk of the eigenvalue statistics of the empirical cross-correlation matrix **C** are consistent with those of the real symmetric random matrix **W** and the deviations from the RMT contain *genuine* information about the correlations in the system.

2 The simulation procedure is repeated many times; in each case similar results as in Fig. 2 are obtained.

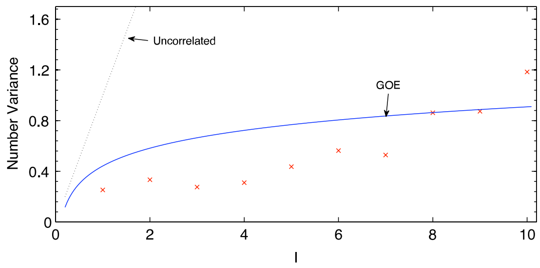


Fig. 4. Number variance.

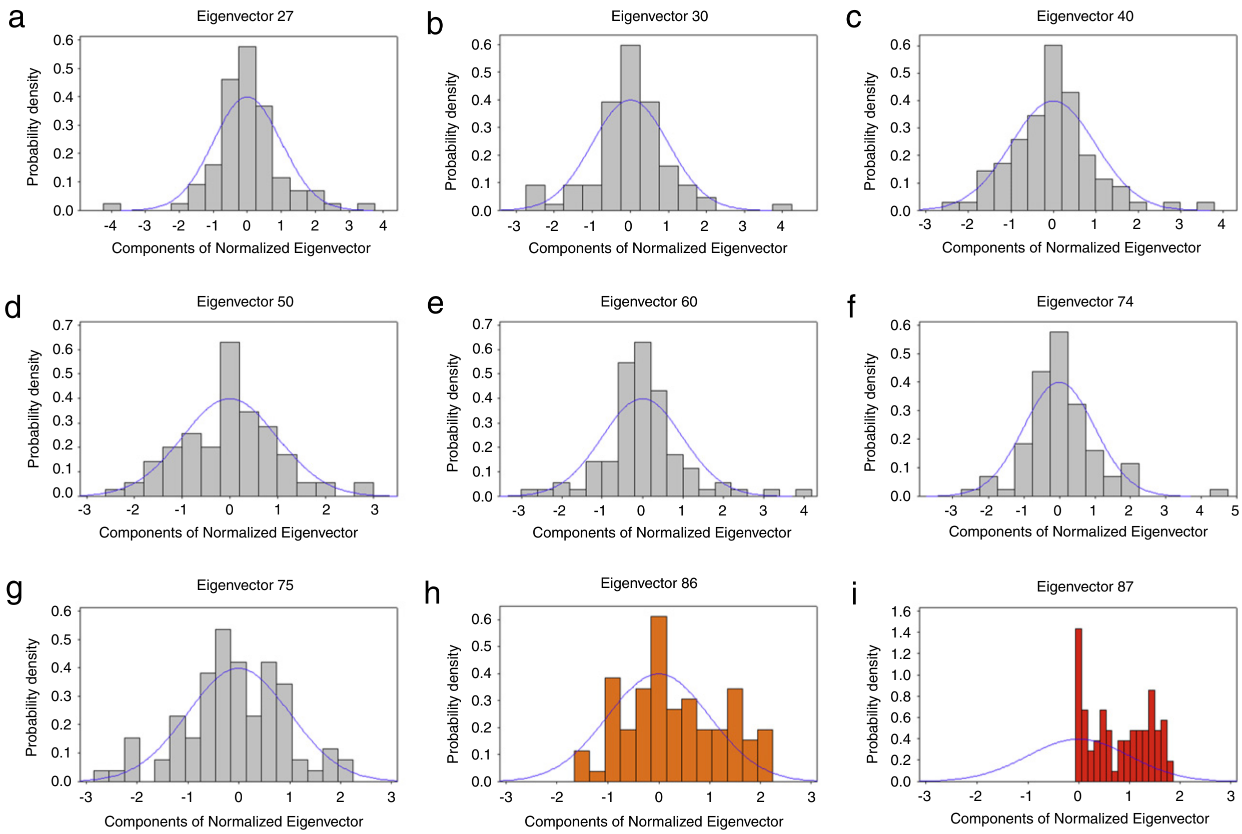


Fig. 5. Density of components of the normalized eigenvectors.

3.2. Eigenvector analysis

The deviations of eigenvalue statistics from the RMT results suggest that these deviations should also be displayed in the statistics of corresponding eigenvector components [43]. First, we choose some of the normalized eigenvectors and display their component distribution in Fig. 5 which shows that the probability density of eigenvector components corresponding to eigenvalues in the bulk agrees well with the RMT. However, the component distribution of $\lambda_i > \lambda_{\max}$ shows significant deviation from the theory. In particular $\rho(v'_{87})$ is almost uniform.

The kurtosis and skewness of the components of each v'_i are given in Fig. 6. For the bulk, kurtosis and skewness fluctuate around 3 and 0 respectively which is consistent with normal behavior.

Components of the v_{87} suggest that most of the financial markets participate in this eigenvector. In addition, almost all components are positive. To have a clear picture, we look at Fig. 7 showing the contributions of the stock markets to the eigenvector corresponding to (a) the largest eigenvalue, (b) an eigenvalue from the bulk and (c) the smallest eigenvalue. For the largest eigenvalue, the majority of the markets have positive representations which is an indicative of a common factor that affects almost all markets with the same bias. This gives us a reason to believe that v_{87} represents a global market itself, that is, the result of the interactions between markets [37].

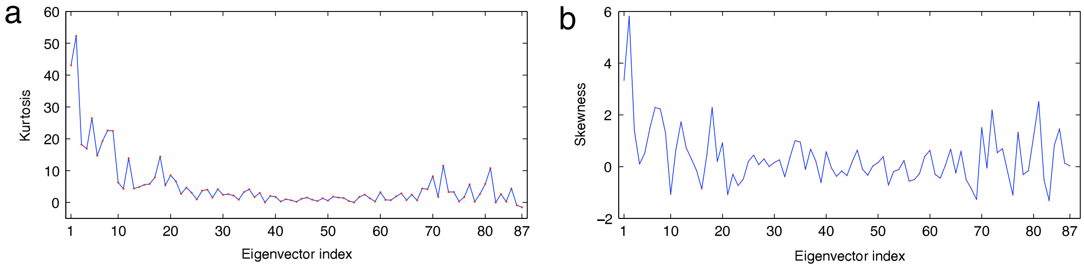


Fig. 6. Kurtosis and skewness of the components of each eigenvector.

3.3. Global market mode

To see if v_{87} represents a global market itself, we take the projection of the time series $R_i(t)$ on the v_{87} and compare it with a standard measure of a global performance. In our case, the most related global index is the Morgan Stanley Capital International (MSCI) All Country World Index (ACWI). It is a free-float weighted equity index which includes both emerging and developed world markets. The projection of the time series $R_i(t)$ on v_{87} is given by the following,

$$R_{v_{87}}(t) = \sum_{i=1}^{87} v_{87,i} R_i(t) \tag{13}$$

$R_{v_{87}}(t)$ is usually called the *market mode* [11,44] (in this study we will call it the *global market mode*). Fig. 8 shows a comparison of the global market mode and the returns of the MSCI index (we standardize both series to have zero mean and unit variance).

We find remarkable similarity between the two return series. The empirical correlation coefficient between them is 0.93. The good agreement shows that v_{87} corresponds to a global market factor showing the general trend of all markets and quantifies the worldwide influence on them [13].

Considering components of v_{87} we see that the top six contributers are from European countries. On the other hand, the majority of the markets have very small contributions that are proportional to the size and liquidity of these markets. An interesting case is the very small contributions of the big and liquid markets of South Korea, India and Russia. Since the market eigenvector can be considered as a general trend of all markets, this situation can be identified as the positive diversification of these emerging markets from others after the 2008 crisis.

For each eigenvector, the number of markets with a significant participation can be accurately quantified by the IPR. Fig. 9 shows the IPR and PR as functions of the eigenvalue index. Eigenvectors corresponding to the bulk have participation ratios around RMT prediction $N/3 = 29$. However, v_{87} has the highest number ($\approx$44) of significant participants which is far from the suggested value. We also see that eigenvectors corresponding to the smallest eigenvalues have the lowest number of significant participants.[3]

3.4. Relation between volatility and correlation

In order to examine the evolution of the correlations in the financial system, we investigate the mean correlation of returns by a rolling window approach. We pick window length $l = 22$ (business month) and roll the time window through the data one day at a time. Explicitly, the mean correlation $\bar{c}_l(t)$ for the correlation coefficients $c_{ij}^l(t)$ in a time window $[t - l + 1, t]$ is defined as

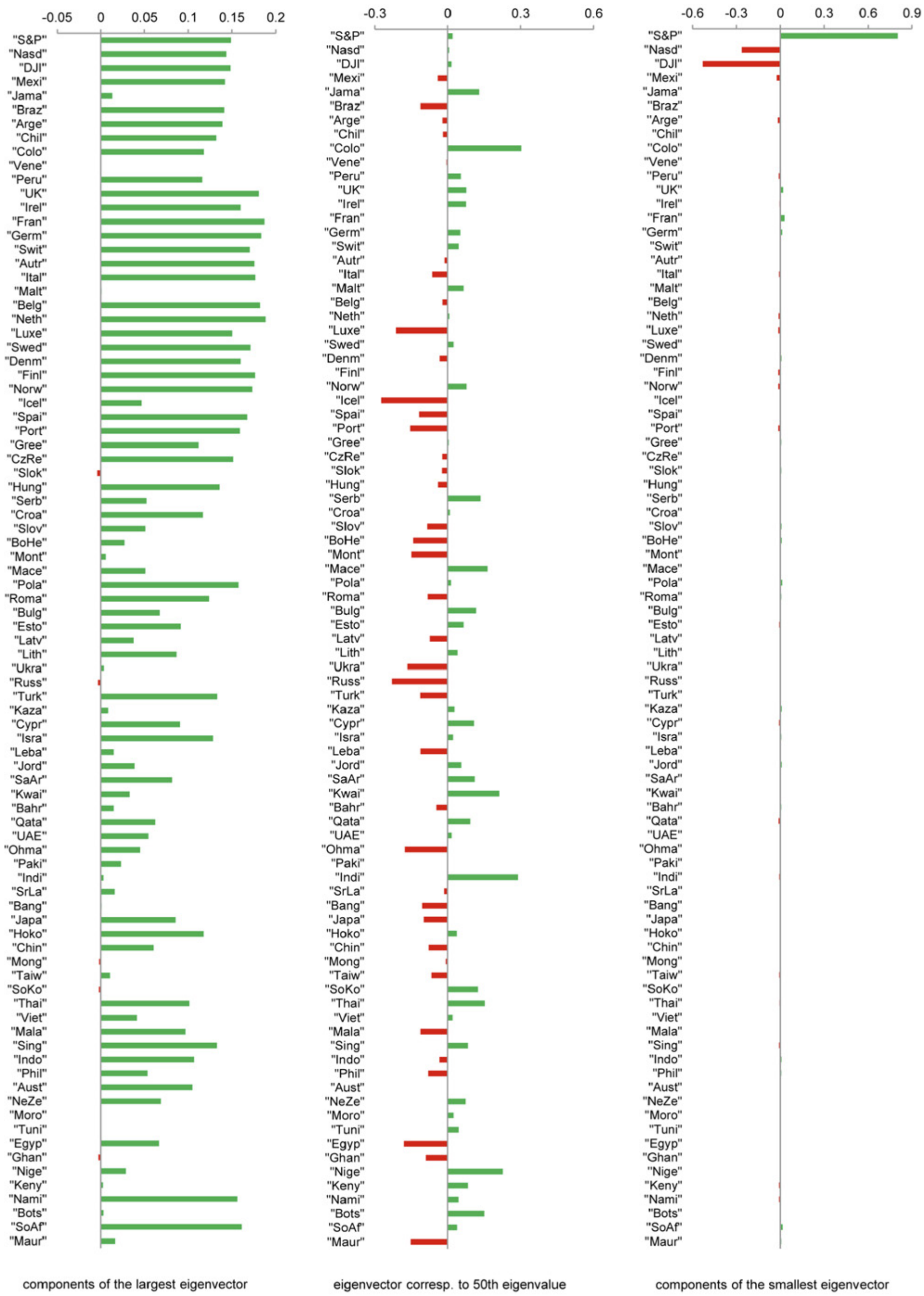
$$\bar{c}_l(t) = \frac{2}{N(N-1)} \sum_{i<j} c_{ij}^l(t). \tag{14}$$

We want to compare the mean correlation of the financial markets with the system's volatility. We take the absolute value of the *global market mode* as the daily volatility proxy of the financial system. A comparison of mean correlation and volatility is given in Fig. 10 which shows that high levels of global volatility and correlation are strongly linked.[4] Furthermore, after the times of high volatility, markets still stay highly correlated for some period[5] although we have to keep in mind that the procedure of shifting the window by one data point is partially responsible in this case.

[3] That differs from the observations on the US stock market [11] where large values of PRs have been found at both edges of the theoretical distribution.

[4] Other studies find similar results by empirical analyses [45–47] and agent based model simulations [48]. For example, Ref. [46] reveals that cross-correlations between nine highly developed markets fluctuate strongly with time and increase in periods of high market volatility. Moreover, based on this phenomenon, [49] constructs an indicator of systemic risk by principle component analysis.

[5] In Ref. [50], such a situation is explained as the effect of the belief that market movement connectedness turns into a self-fulfilling prophecy after the crisis.



(a) Contributions of stock markets. (b) Contributions of stock markets. (c) Contributions of stock markets.

Fig. 7. Contributions of stock markets corresponding to different eigenvectors.

The three highest volatility values are observed in the weeks of:

- 10/05/2010: European Union finance ministers agreed an emergency loan package that with IMF support could reach 750 billion euros to prevent a sovereign debt crisis spreading through the eurozone,
- 08/08/2011: Credit rating agency SP downgraded the US federal government rating from AAA (outstanding) to AA+ (excellent),
- 22/09/2011: Moody's downgraded three US banks: Bank of America, Citigroup and Wells Fargo; SP downgraded seven Italian banks and Fed announced significant downside risks to US economy,

where each week above, global correlation also takes its highest values.

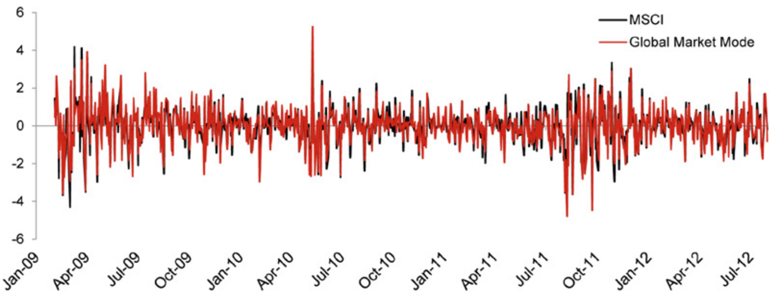


Fig. 8. MSCI All Country World Index returns vs. global market mode.

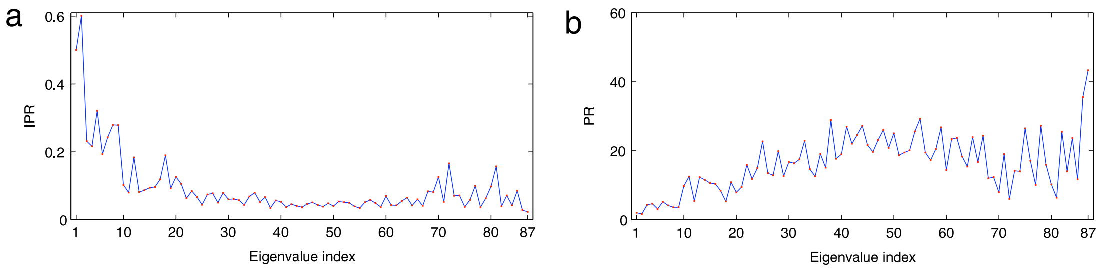


Fig. 9. IPR and PR of the eigenvectors.

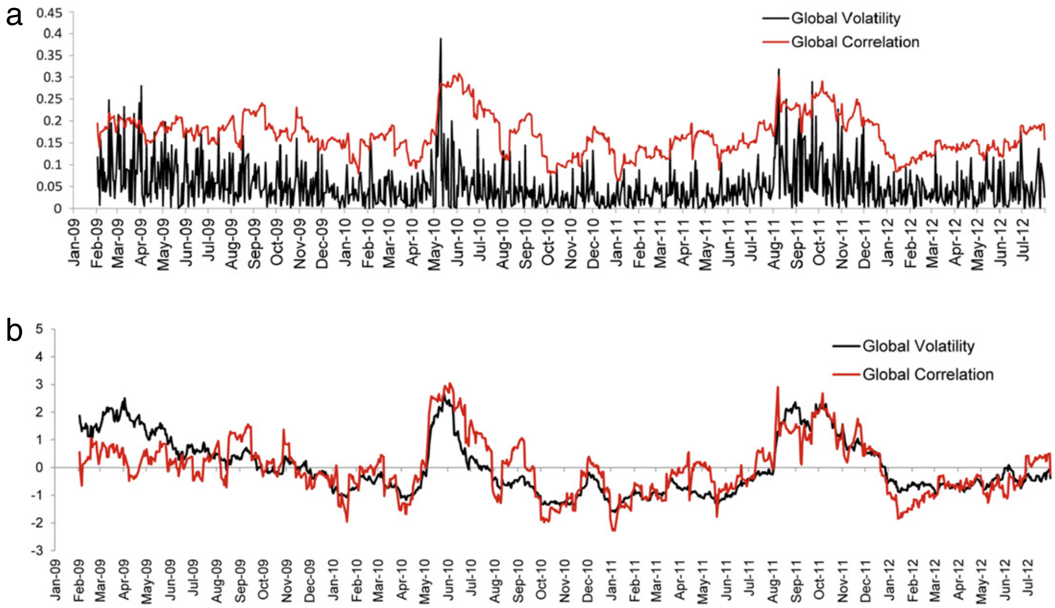


Fig. 10. (a) Global volatility vs. global correlation and (b) global volatility (moving window) vs. global correlation.

We also analyze the skewness and kurtosis of return distributions by rolling the time window of length $l = 264$ (business year) through the full data set (see Fig. 11, fat tails are demonstrated by excess kurtosis).

3.5. Time-varying largest eigenvalue

After revealing that the largest eigenvalue carries true information, we apply a similar approach of Ref. [51] to our data. With a one-year length rolling window, we obtain the time-varying largest eigenvalue of the return correlation matrix and observe its characteristics.

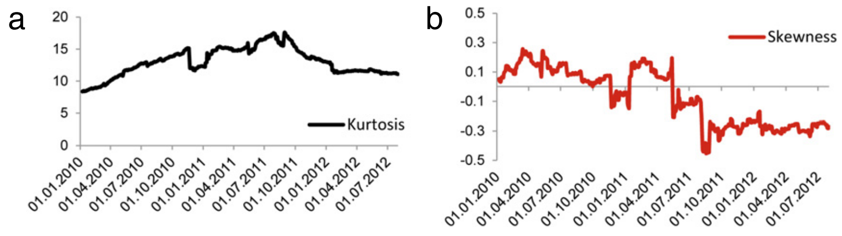


Fig. 11. Kurtosis and skewness of returns obtained from a moving window.

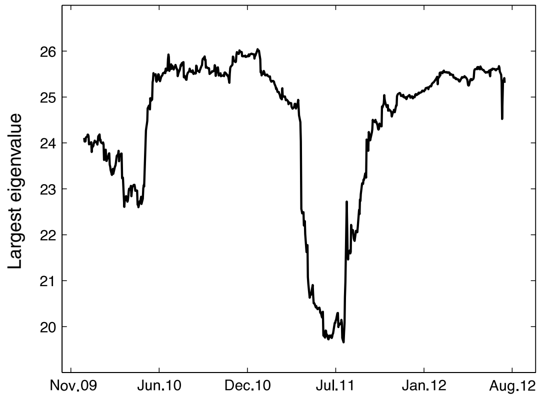


Fig. 12. Time-varying largest eigenvalue of the correlation matrix obtained from a 1-year length rolling window.

Fig. 12 shows that the largest eigenvalue peaks during the highest global volatility levels and there is a strong link between the magnitude of this eigenvalue and the volatility levels in general.[6]

3.6. Correlation-based financial network analysis

Financial markets around the world can be regarded as a complex system. This forces us to focus on a global-level description to analyze the interaction structure among markets which can be achieved by representing the system as a network. During the recent years networks have proven to be a very efficient way to characterize and investigate a wide range of complex systems including stock, commodity and foreign exchange markets [45,52–76]. In this study, we are interested in identifying the connection structure and hierarchy in the network of financial markets formed with cross-correlations of returns. In order to do that we construct the minimal spanning tree (MST) and the ultrametric hierarchical tree (UHT) associated with it [44,45,53–80].

To create a network based on return correlations, we use the metric defined by Mantegna [53],

$$d_{ij} = \sqrt{2(1 - c_{ij})}. \tag{15}$$

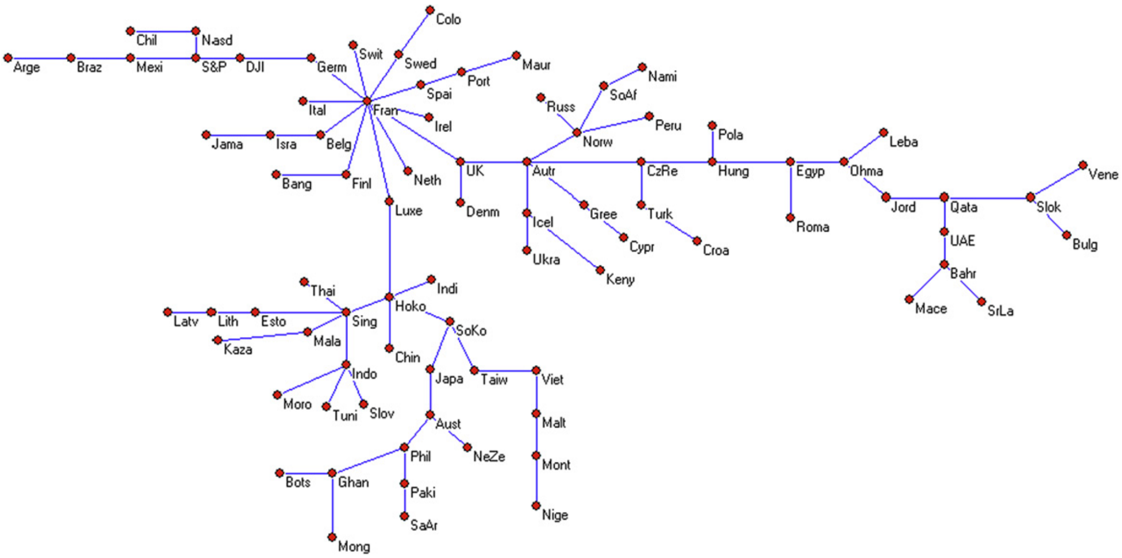
It is a valid euclidean metric since it satisfies the necessary properties; (i) $d_{ij} \geq 0$, (ii) $d_{ij} \Leftrightarrow i = j$, (iii) $d_{ij} = d_{ji}$ and (iv) $d_{ij} \leq d_{ik} + d_{kj}$. This transformation creates an $N \times N$ distance matrix D from the $N \times N$ cross-correlation matrix $\mathbf{C}$. The distance d_{ij} varies from 0 to 2 with small distances corresponding to high correlations and vice versa.

MST is constructed as follows: start with the pair of elements with the shortest distance and connect them; then the second smallest distance is identified and added to the MST. The procedure continues until there are no elements left, with the condition that no closed loops are created. Finally we obtain a simply connected network that connects all N elements with $N - 1$ edges such that the sum of all distances is minimum.[7]

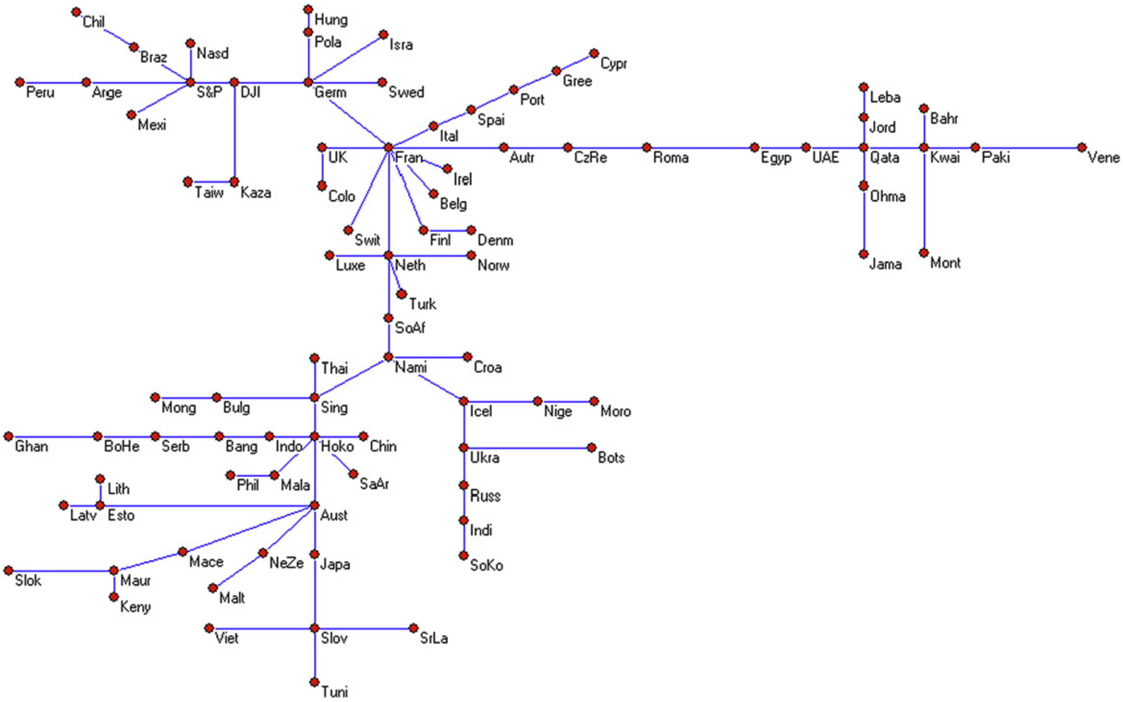
After defining the euclidean space of financial markets, we next move to the ultrametric space. An ultrametric space is the space where all distances within it are ultrametric. The ultrametric distance d^*_{ij} is understood as a regular distance with properties (i)–(iii) and property (iv) is replaced by a stronger condition; (iv)* $d^*_{ij} = \max(d^*_{ik}, d^*_{kj})$. Ultrametric distances are important to hierarchical clustering since they redefine the distance between two elements as the distance between their

6 Which coincides with the findings of Ref. [51]: The authors study 1340 time series with 9 year daily data and investigate how the maximum singular value λ changes over (time lags) for different years and find that it is greatest in times of crises.

7 This can be seen as a way to find the $N - 1$ most relevant connections among a total of $N(N - 1)/2$ connections which is especially appropriate for extracting the most important information concerning connections when a large number of markets is under consideration. In terms of financial markets, MSTs can also be considered as filtered networks enabling us to identify the most probable and the shortest path for the transmission of a crisis.



(a) 2005–2007 (period 1).



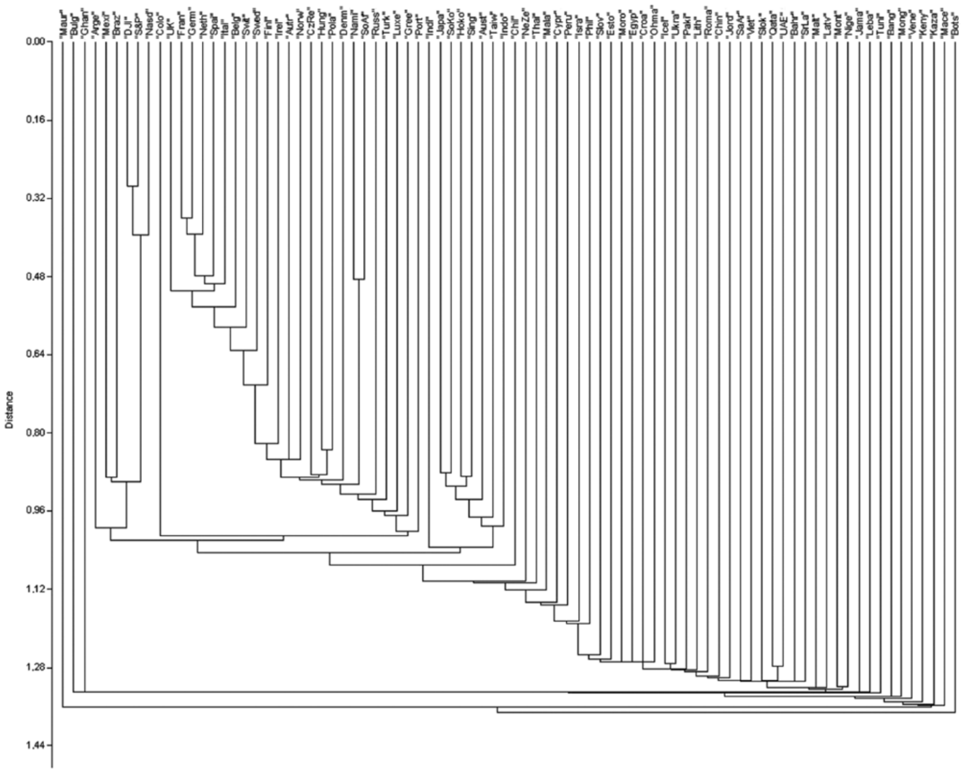
(b) 2009–2012 (period 2).

Fig. 13. Minimal spanning trees of (a) period 1 and (b) period 2.

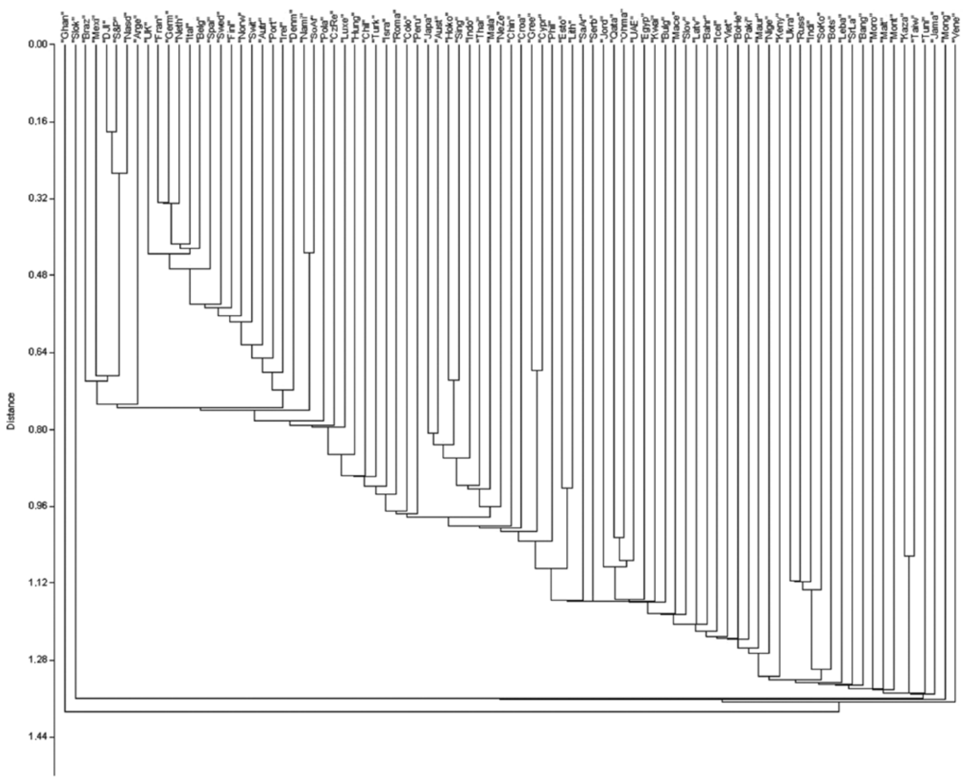
closest ancestor. The MST provides the sub-dominant ultrametric hierarchical structure of the markets into what is called UHT. The MST is associated with the single-linkage clustering algorithm [54], so we present the UHT using the same method.

To understand the effects of the 2008 financial crisis on markets' integration structure, we analyze the cases corresponding to two different time periods; period one: 01/01/2005–01/01/2007 and period two: 01/01/2009–31/07/2012. The MSTs and the UHTs of these periods are given in Figs. 13 and 14 respectively.[8]

[8] Since data were unavailable, we had to omit three markets; Bosnia and Herzegovina, Kuwait and Serbia in the analysis of period one.



(a) 2005–2007 (period 1).



(b) 2009–2012 (period 2).

Fig. 14. Ultrametric hierarchical trees of (a) period 1 and (b) period 2.

Table 1
Markets with highest node degrees.

(a) Period 1		(b) Period 2	
Market	Node degree	Market	Node degree
France	11	France	11
Hong Kong	5	Hong Kong	5
Singapore	5	SP	5
Austria	5	Germany	5
Indonesia	4	Netherlands	4
Norway	4	Australia	4
SP	3	Singapore	3
UK	3	Qatar	3
South Korea	3	Kuwait	3
Australia	3	Slovenia	3

Table 2
Markets with highest node strength.

(a) Period 1		(b) Period 2	
Market	Node strength	Market	Node strength
France	8.93286	France	7.88209
Austria	2.57976	SP	4.18860
SP	2.47343	Germany	3.79013
Hong Kong	2.26497	Netherlands	3.77997
Norway	2.13227	Hong Kong	3.77615
UK	2.10376	Australia	2.57149
Singapore	2.07421	Singapore	2.18214
Czech Republic	1.77035	Namibia	2.17670
South Korea	1.72229	Italy	1.80545
South Africa	1.47247	DJI	1.79151

For both periods, the three US market indexes are close together as expected and Germany serves as a hub for the connection between two main clusters; America and Europe. France seems to be the central node as it has the highest number of linkages in both cases and surprisingly the US market, which is usually accepted as the world's most important financial market, displays a somewhat looser connection with the others. The European Union (EU) seems to form the central trunk of the MSTs and the clusters appear to be organized principally according to a geographical position and historical and linguistic ties [81].

However, major changes are observed between two periods. In particular, the effect of the eurozone debt crisis shows itself on the financial markets in period 2. The problematic countries Greece, Italy, Portuguese, Spain and Cyprus are all tied together, showing that bond market connection results with stock market connection. UK, not a eurozone member, loses its importance in the network in period 2. Three important markets that are positively diversified from the others through the 2008 crisis (Russia, India and South Korea) stand isolated in the network.

3.6.1. Centrality measures

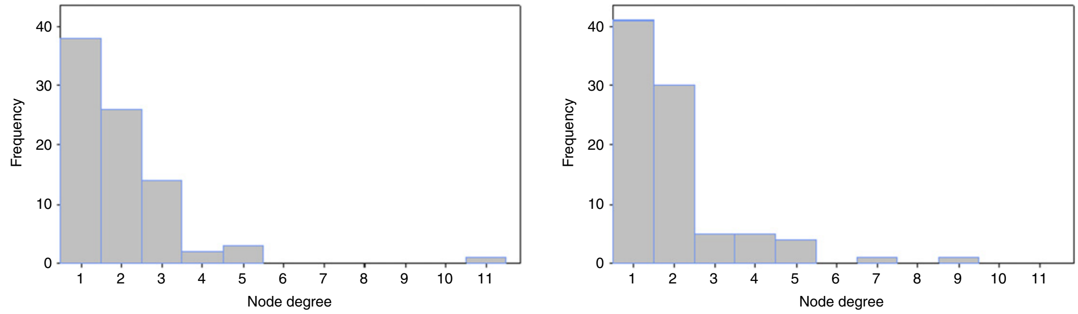
In network theory, the centrality of a node determines the relative importance of that node within a network. Next, we perform a detailed analysis on MSTs using different quantitative definitions of centrality.[9]

Node degree is the number of nodes that is adjacent to it in a network. In general the larger the degree, the more important the node is. The highest ten node degrees and the frequency distributions are given in Table 1 and Fig. 15 respectively.

Node strength is the sum of correlations of the given node with all other nodes to which it is connected. The highest ten node strengths and the frequency distributions are given in Table 2 and Fig. 16 respectively.

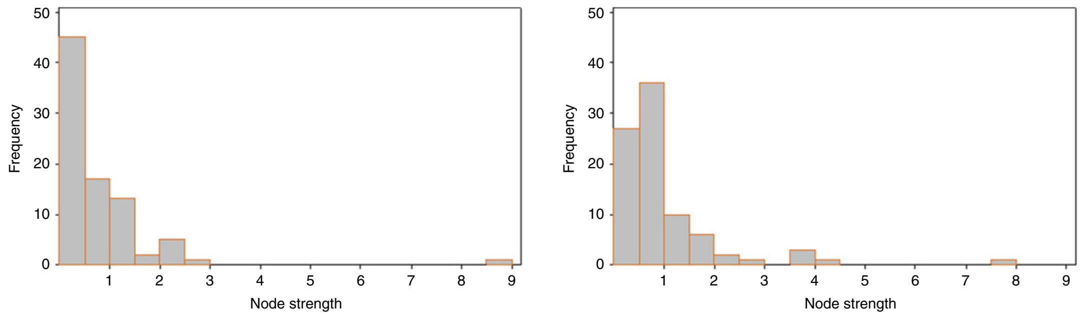
Eigenvector centrality is a measure that takes into account how important the neighbors of a node are. It is useful in particular when a node has a low degree but is connected to nodes with high degrees and thus the given node may influence others indirectly. It is defined as the ith component of eigenvector $\mathbf{v}$, where $\mathbf{v}$ corresponds to the largest eigenvalue λ of the *adjacency matrix* $\mathbf{A}$. The highest ten eigenvector centralities and the frequency distributions are given in Table 3 and Fig. 17 respectively.

[9] Before beginning the analysis, we point out an important observation: even we have an extra three edges in the network in period two, the total distances in the MST is 82.593 whereas this values is 87.651 in period one. This shows increased strength in the correlation of financial markets after the 2008 crisis. A similar conclusion is obtained by using the time-varying correlation data from the Section 3.4. In particular, we split the time-varying correlations into two sets as pre and post 2008. A non-parametric median comparison test reveals that the set of correlations in post 2008 has a significantly larger median.



(a) Period one. (b) Period two.

Fig. 15. Frequency distribution of node degree.



(a) Period one. (b) Period two.

Fig. 16. Frequency distribution of node strength.

Table 3
Markets with highest eigenvector centrality.

(a) Period 1		(b) Period 2	
Market	Eigenvec. centrality	Market	Eigenvec. centrality
France	0.666	France	0.624
UK	0.249	Germany	0.323
Luxembourg	0.224	Netherlands	0.310
Germany	0.213	Austria	0.212
Belgium	0.212	Italy	0.212
Spain	0.212	UK	0.209
Sweden	0.210	Finland	0.209
Finland	0.210	Switzerland	0.190
Switzerland	0.193	Belgium	0.190
Netherlands	0.193	Ireland	0.190

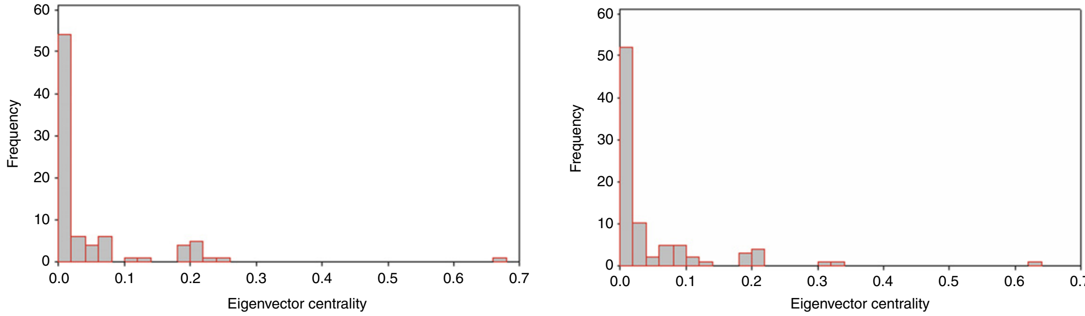
Betweenness centrality measures the importance of a node as an intermediate part between other nodes. For a given node k, it is defined as

$$B(k) = \sum_{i,j} \frac{n_{ij}(k)}{m_{ij}} \tag{16}$$

where $n_{ij}(k)$ is the number of shortest geodesic paths between nodes i and j passing through k, and m_{ij} is the total number of shortest geodesic paths between i and j.[10] The highest ten betweenness centralities and the frequency distributions are given in Table 4 and Fig. 18 respectively.[11]

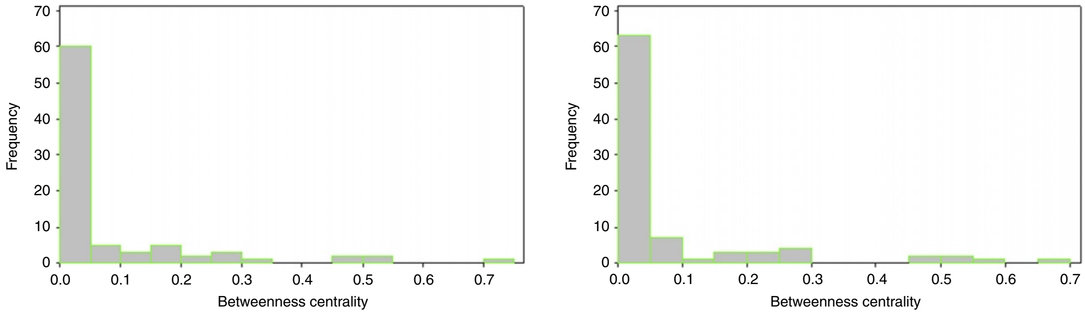
[10] MST is a fully-connected network so $m_{ij} \neq 0$.

[11] There are 38 indexes in period one and 41 indexes in period two with zero betweenness centrality i.e. for any two markets in the network, no shortest path passes through them.



(a) Period one. (b) Period two.

Fig. 17. Frequency distribution of eigenvector centrality.



(a) Period one. (b) Period two.

Fig. 18. Frequency distribution of betweenness centrality.

Table 4
Markets with highest betweenness centrality.

(a) Period 1		(b) Period 2	
Market	Btw. centrality	Market	Btw. centrality
France	0.7249486	France	0.68044
Hong Kong	0.5166030	Namibia	0.57264
Austria	0.5145460	Netherlands	0.53953
UK	0.4757567	South Africa	0.50150
Luxembourg	0.4601822	Singapore	0.46539
Czech Republic	0.3385248	Hong Kong	0.45253
South Korea	0.2970908	Australia	0.29521
Hungary	0.2876873	Germany	0.28810
Egypt	0.2535998	Austria	0.27579
Singapore	0.2248016	Czech Republic	0.25964

Closeness centrality is a measure of the average geodesic distance from one node to all others. This measure is high for strongly connected central nodes and large for poorly connected ones. For node i in a network with N nodes, it is defined as

$$C(i) = \frac{1}{\sum_{j=1}^{N} d(i,j)} \tag{17}$$

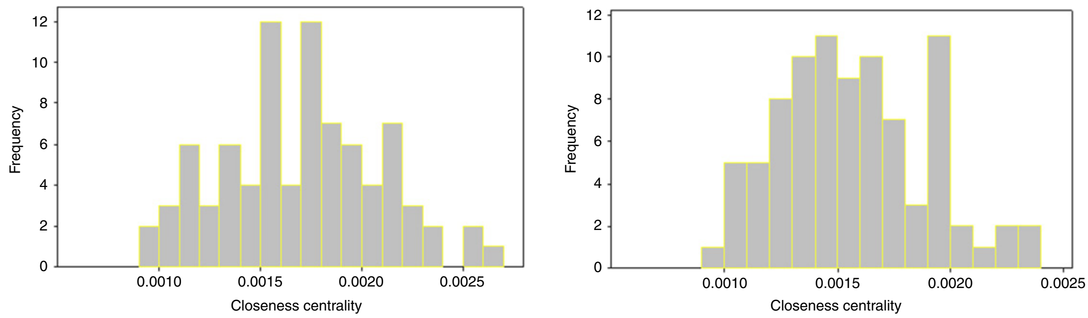
where $d(i,j)$ is the minimum geodesic path distance between nodes i and j. The highest ten closeness centralities and the frequency distributions are given in Table 5 and Fig. 19 respectively.

Analysis reveals that France takes first place in almost all categories for both periods and other financial centers Germany, Hong Kong and Singapore keep their importance in the network after the crisis. However, the same cannot be said for the UK; while belonging to the top ten in all categories in period one, it belongs to the top ten only in 2 categories in period two.[12]

[12] Note that all frequency distributions of centrality measures (except for closeness centrality) are likely to decrease exponentially. One may say that these measures exhibit a power-law distribution; $p(x) \sim x^{-\beta}$ for measure value x and constant β which in the case the network is called scale-free [82].

Table 5
Markets with highest closeness centrality.

(a) Period 1		(b) Period 2	
Market	Closeness centrality	Market	Closeness centrality
France	0.002660	Netherlands	0.002336
UK	0.002513	France	0.002331
Luxembourg	0.002500	South Africa	0.002299
Austria	0.002358	Namibia	0.002252
Hong Kong	0.002347	Singapore	0.002114
Germany	0.002252	Germany	0.002058
Belgium	0.002203	Austria	0.002058
Spain	0.002203	Italy	0.001976
Sweden	0.002193	Hong Kong	0.001961
Finland	0.002193	UK	0.001953



(a) Period one. (b) Period two.

Fig. 19. Frequency distribution of closeness centrality.

4. Discussion

In our study, one concern about the correlations between financial markets is that the relationship may be non-linear; in that case rank correlation may capture the relations better. In order to see the difference, we use a rank correlation approach: for each index i, the returns $R_i(t)$ are ranked $R_i^{rank}(t)$ then normalized as

$$r_i^{rank}(t) \equiv \frac{R_i^{rank} - \langle R_i^{rank} \rangle}{\sigma_i^{rank}} \tag{18}$$

where $\sigma_i^{rank} \equiv \sqrt{\langle (R_i^{rank})^2 \rangle - \langle R_i^{rank} \rangle^2}$; then we repeat every analysis. The results are almost indistinguishable. The largest eigenvalue is $\approx$22.44 and $\approx$94% of the eigenvalues are smaller than λ_{max}. Considering components of the largest eigenvector, the top six positive contributers do not change.

For the network analysis, we define the metric as a linear realization of the rank correlation; $d_{ij} = 1 - c_{ij}$. The major difference is that (instead of Germany) France serves as a hub for connecting North America and Europe in period two.

4.1. Further study

A lagged relationship is a possible characteristic of many pairwise financial time series. First, it would not be a surprise if one series had a delayed response to another time series, or it had a delayed response to a common event that affects both series. Secondly, it may be the case that the response of one series to the other or to an outside event may spread in time, such that an event restricted to one observation elicits a response at multiple observations. Furthermore, series may not even be stationary in some cases. Equal-time cross-correlations are inadequate to characterize the relationship between time series in such situations. Some authors incorporated these facts into their studies and revealed interesting results using the concepts of detrended and long-range cross-correlations and time lag RMT [83–89].[13] In the near future, we are planning to apply these new approaches to our data set and analyze the differences in our findings.

13 For example, Kullmann et al. [83] showed that in many cases the maximum correlation appears at nonzero time shift, indicating directions of influence between the stocks. Similarly, Wang et al. [84] find long-range power-law cross-correlations in the absolute values of returns that quantify risk, and find that they decay much more slowly than cross-correlations between the returns. They find that when a market shock is transmitted around the world, the risk decays very slowly.

5. Conclusion

The global financial crisis of 2008 began in July 2007 when a loss of confidence by investors in the value of securitized mortgages in the US resulted in a liquidity crisis. In September 2008, the crisis deepened as stock markets worldwide crashed and entered a period of high volatility. This study compares before and after the 2008 crisis by analyzing the cross-correlations of financial markets using the tools of RMT and network theory.

In particular, we verified the validity of the universal predictions of RMT for the statistics of the eigenvalues and the corresponding eigenvectors of the cross-correlation matrix. Then the cross-correlations between markets not totally explainable by randomness were identified by computing the deviations of the empirical data from the RMT predictions. We showed the presence of a certain linear combination of indexes representing a global market itself that arises from interactions.

By using this particular combination, we observed that markets become highly correlated in times of high volatility (also the time-varying largest deviating eigenvalue peaks during the highest volatility); moreover when the volatility passes to its low levels, the increased degree of co-movement continues for a considerable amount of time. We also find that markets are more correlated after 2008 compared to the period of 2005–2007. These facts lower the diversification potential even if one constructs a widely internationally diversified portfolio of stocks.

We found the connection structure of financial markets for pre and post 2008 crisis using correlation based networks. We show that in an environment of increasing integration of trade and financial markets, geographical position and historical and linguistic ties still play an important role in co-movements of stock markets. Analysis also shows that eurozone debt crisis forces the stock markets of problematic countries to move together, revealing an interesting fact on how bond and stock markets of a country interact.

We identified key financial markets using several centrality measures. Analysis shows that centers like France, Germany and Hong Kong keep their importance in the financial system after the 2008 crisis. However, the same cannot be said for the UK.

To extract the information arising from non-linear relations between markets, we repeated each analysis using a rank correlation and found that the results are almost indistinguishable. Possible extensions for further research includes applications of the long-range cross-correlations and time lag-RMT to our data.

Acknowledgments

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Appendix. Analyzed markets

Country	Index	Symbol
Argentina	Merval	Arge
Australia	SP/ASX 200	Aust
Austria	ATX	Autr
Bahrain	Bahrain all share index	Bahr
Bangladesh	DSE general index	Bang
Belgium	BEL 20	Belg
Brazil	Ibovespa	Braz
Bosnia and Herzegovina	SASE 10	BoHe
Botswana	Gaborone	Bots
Bulgaria	SOFIX	Bulg
Chile	IPSA	Chil
China	Shangai SE composite	Chin
Colombia	IGBC	Colo
Croatia	CROBEX	Croa
Cyprus	CSE	Cypr
Czech Republic	PX	CzRe
Denmark	OMX Copenhagen 20	Denm
Egypt	EGX 30	Egyp
Estonia	OMXT	Esto
Finland	OMX Helsinki	Finl
France	CAC 40	Fran
Germany	DAX	Germ
Ghana	Ghana all share index	Ghan
Greece	Athens SX general index	Gree
Hong Kong	Hang Seng	HoKo

(*continued on next page*)

Country	Index	Symbol
Hungary	Budapest SX Index	Hung
Iceland	OMX Iceland all share index	Icel
India	SENSEX 30	Indi
Indonesia	Jakarta composite index	Indo
Ireland	ISEQ	Irel
Israel	Tel Aviv 25	Isra
Italy	FTSE MIB	Ital
Jamaica	Jamaica SX market index	Jama
Japan	Nikkei 225	Japa
Jordan	ASE general index	Jord
Kazakhstan	KASE	Kaza
Kenya	NSE 20	Keny
Kuwait	Kuwait SE weighted index	Kwai
Latvia	OMXR	Latv
Lebanon	BLOM	Leba
Lithuania	OMXV	Lith
Luxembourg	Luxembourg LuxX	Luxe
Macedonia	MBI 10	Mace
Malaysia	KLCI	Mala
Malta	Malta SX Index	Malt
Mauritius	SEMDEX	Maur
Mexico	IPC	Mexi
Mongolia	MSE TOP 20	Mong
Montenegro	MOSTE	Mont
Morocco	CFG 25	Moro
Namibia	FTSE/Namibia overall	Nami
Netherlands	AEX	Neth
New Zealand	NZX 50	NeZe
Nigeria	Nigeria SX all share index	Nige
Norway	OBX	Norw
Oman	MSM 30	Ohma
Pakistan	Karachi 100	Paki
Peru	IGBVL	Peru
Philippines	PSEi	Phil
Poland	WIG	Pola
Portugal	PSI 20	Port
Qatar	DSM 20	Qata
Romania	BET	Roma
Russia	MICEX	Russ
Saudi Arabia	TASI	SaAr
Serbia	BELEX 15	Serb
Singapore	Straits times	Sing
Slovakia	SAX	Slok
Slovenia	SBI TOP	Slov
South Africa	FTSE/JSE Africa all share	SoAf
South Korea	KOSPI	SoKo
Spain	IBEX 35	Spai
Sri Lanka	Colombo all-share index	SrLa
Sweden	OMX Stockholm 30	Swed
Switzerland	SMI	Swit
Taiwan	TAIEX	Taiw
Thailand	SET	Thai
Tunisia	TUNINDEX	Tuni
Turkey	ISE national 100	Turk
Ukraine	PFTS	Ukra
United Arab emirates	ADX general index	UAE
United Kingdom	FTSE 100	UK
United States of America	Dow Jones industrial	DJI

(*continued on next page*)

Country	Index	Symbol
United States of America	Nasdaq composite	Nasd
United States of America	SP 500	SP
Venezuela	IBC	Vene
Vietnam	VN-Index	Viet

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