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Intraday downward/upward multifractality and long memory in Bitcoin and Ethereum markets: An asymmetric multifractal detrended fluctuation analysis[☆]

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ABSTRACT

This study examines high-frequency asymmetric multifractality, long memory, and weak-form efficiency for two major cryptocurrencies, namely, Bitcoin (BTC) and Ethereum (ETH), using the asymmetric multifractal detrended fluctuation analysis method to consider different market patterns. Our results show evidence of structural breaks and asymmetric multifractality. Moreover, the multifractality gap between the uptrend and downtrend is small when the time scale is small, and it increases as the time scale increases. The BTC market is more inefficient than ETH. The inefficiency is more (less) accentuated when the market follows a downward (upward) movement. The efficiency level varies based on each subperiod.

1. Introduction

Multifractality is a major stylized fact in financial time series. The source of multifractality is known to be the long-range memory and fat tails in the return distribution (Baruník et al., 2012; Green et al., 2014). This implies that financial time series with high multifractality show sharp rises and drops, the volatility of price movements is clustering, and the price dynamics follow a pattern and have a certain predictability. Moreover, the multifractal behaviour in international financial markets may be time-varying, and differ in upside and downside market conditions; if this be the case, investment risk may also be time-varying, and price movements show asymmetric responses to good or bad market news. Thus, changes in multifractality and efficiency have important implications vis-à-vis asset allocations and portfolio management.

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The multifractality of equity prices closely relates to the long-memory feature of the market, and it contradicts the efficiency market hypothesis (EMH) in finance theory. On the other hand, asymmetric correlations between financial assets have implications on asset allocations and portfolio risk assessment, on account of asymmetric risk imposed by market conditions or the upward or downward movement of asset prices. Thus, enhancing the general understanding of multifractal behaviour and time-varying efficiency in financial markets helps market participants optimize their investment decision-making process. Additionally, policy-makers are interested in handling these stylized facts, as they may inform the stimulation of economic development: by better understanding these facts, they may craft new policy that enhances the efficiency of the financial market and optimizes resource allocations.

Since their creation in 2008, cryptocurrencies—in particular, Bitcoin (BTC) and Ethereum (ETH)—have attracted special attention. More than 100 cryptocurrencies are traded in the market, but BTC and ETH remain the most widely traded ones: together they account for more than 60% of the total cryptocurrency market capitalization. Due to their exponentially increasing popularity, an emerging body of literature has addressed the cryptocurrency market. For example, a first strand of research analyses the informational efficiency of Bitcoin, and finds that Bitcoin is inefficient (Urquhart, 2016; Nadarajah and Chu, 2017; Cheah et al., 2018; Al-Yahyaee et al., 2018; Tiwari et al., 2018; Vidal-Tomás and Ibañez, 2018). Some studies investigate other cryptocurrency market features (Koutmos, 2018; Katsiampa, 2018; Chaim and Laurini, 2018).

The current study aims to examine the asymmetric multifractality and dynamic market efficiency of the world's two largest cryptocurrencies in terms of market capitalization, BTC and ETH. To do this, we use the generalized Hurst exponent and the asymmetric multifractal detrended fluctuation analysis (A-MF-DFA) approach with high-frequency data (5, 10, and 15 min). The advantage inherent in the A-MF-DFA method of Cao et al. (2013) is that it discerns the scaling properties in two different market trends; uptrends and downtrends. We thus acquire considerable information on asset allocation when we consider separately the upward and downward market movements. More precisely, the portfolio gains are maximized when the market goes up, and the portfolio loss is reduced when the market goes down. Thus, addressing the price movements during two different market situations is important to understanding the persistence of prices in cryptocurrency markets. We follow Lee et al. (2018) and quantify the generalized Hurst exponent by simultaneously discerning the overall long-range correlation for overall trends, uptrends, and downtrends; this procedure allows us to analyse the asymmetric EMH by exploring the differences among the various trends in terms of the asymmetric generalized Hurst exponent. To the best of our knowledge, the current study is the first to use the A-MF-DFA to investigate the intraday dynamics of BTC and ETH prices for overall, upward, and downside trends.

Our results provide evidence of asymmetric multifractality for upward and downward BTC and ETH markets. The ETH market is less inefficient than that of the BTC, regardless of the frequency of the overall, upward, and downward trends. Finally, we find that the inefficiency is lower when the BTC and ETH markets follow an upward movement (save for the upward ETH with the 15-min frequency). Investors should consider the upward and downward trends when forecasting BTC and ETH prices. The ability to 'beat' these markets and generate abnormal returns is more evident in downward market conditions. In addition, investors and portfolio managers can leverage asymmetric multifractality to forecast future prices and generate profits.

2. Data and methodology

The current study investigates two major cryptocurrency markets, BTC and ETH. We use the closing price derived from high-frequency intraday data obtained from the Bitfinex Exchange, at three different frequencies (i.e. 5, 10, and 15 min). The sample data of the BTC (ETH) spans the period from June 1, 2013 (June 1, 2016) to June 23, 2018. Returns are calculated by taking the difference in the logarithm of two consecutive prices.

The current study leverages the A-MF-DFA method. By using this unique method, we can explore the asymmetric multifractal scaling behaviour among different trend types. Let it be time series $X = \{x(t)\}_{t=1}^{N}$, where N is the length of the series. When starting with the A-MF-DFA method, we construct the profile $Y = \{y(t)\}_{t=1}^{N}$ of $\{x(t)\}$, where $y(t) = \sum_{j=1}^{t} (x(j) - \bar{x})$ for $j = 1, 2, \cdots, N$ and $\bar{x}$ represents the mean of X.

We then divide the time series X and its profile Y into nonoverlapping sub-time series of length n that are selected from 5 to $N/4$, based on the recommendations of Peng et al. (1994). Since N may not be a multiple of n, the length of the last segment may be shorter than n. To consider the remainder of X, we also divide by starting from the other end of X. Thus, we obtain a $2N_n$ ($N_n = N/n$) sub-time series $\{X_j\}_{j=1}^{2N_n}$ for X. The sub-time series $\{Y_j\}_{j=1}^{2N_n}$ for Y can be obtained in the same manner. The j^{th} sub-time series of X is denoted by $X_j = \{x_{j,k}\}_{k=1}^{n}$, where $x_{j,k}$ indicates the k^{th} element of X_j.

For X_j and Y_j, we estimate the linear fit $\bar{X}_j(k) = a_{x_j} + b_{x_j} k$ and $\bar{Y}_j(k) = a_{y_j} + b_{y_j} k$, which represent the linear trends for the j^{th} sub-time series. $\bar{X}_j(k)$ is used to determine the direction of the trend of X_j via slope b_{x_j}, while $\bar{Y}_j(k)$ is used to detrend Y_j. We then determine the fluctuation functions as follows:

$$F_j(n) = \frac{1}{n} \sum_{k=1}^{n} (y_{j,k} - \bar{Y}_j(k))^2. \tag{1}$$

The directional q-order average fluctuation functions are calculated by:

$$F_q^{+}(n) = \left(\frac{1}{M^{+}} \sum_{j=1}^{2N_n} \frac{sig(b_{x_j}) + 1}{2} [F_j(n)]^{q/2} \right)^{1/q}, \quad M^{+} = \sum_{j=1}^{2N_n} \frac{sign(b_{x_j}) + 1}{2} \tag{2}$$

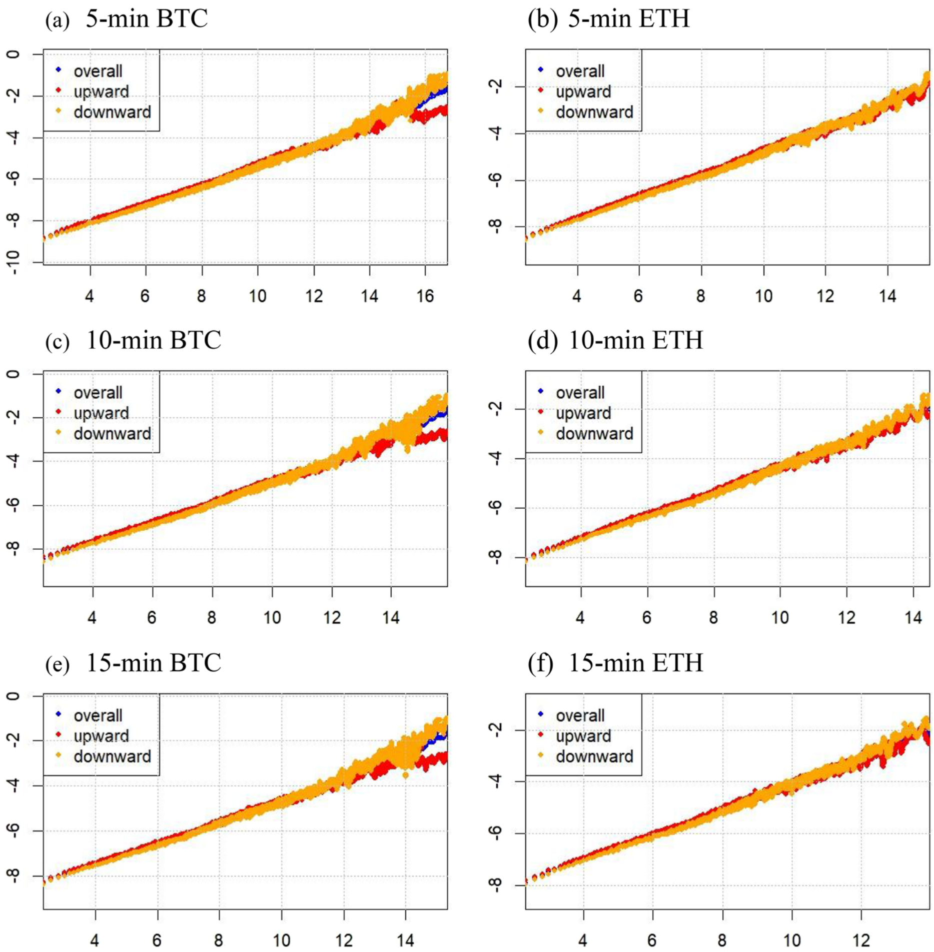

Fig. 1. Asymmetric MF-DFA functions $F_2(n)$ versus the time scale n in a log-log plot of BTC and ETH returns.
Note: The vertical axis represents $\log_2(F_2(n))$, while the horizontal axis represents $\log_2(n)$.

$$F_q^-(n) = \left(\frac{1}{M^-}\sum_{j=1}^{2N_n}\frac{-[sign(b_{x_j})-1]}{2}[F_j(n)]^{q/2}\right)^{1/q},\quad M^- = \sum_{j=1}^{2N_n}\frac{-[sign(b_{x_j})-1]}{2}, \tag{3}$$

where $F_q^+(n)$ and $F_q^-(n)$ denote the upward and downward q-order average fluctuation functions, respectively. Assuming that $b_{x_j} \neq 0$ for all $j = 1, \cdots, 2N_n$, then $M^+ + M^- = 2N_n$.

3. Results

3.1. Asymmetric multifractality analysis results

Fig. 1 presents the asymmetric MF-DFA functions $F_q(n)$ versus the time scale n in a log-log plot of the BTC and ETH returns. The asymmetry in the fluctuation function is discovered within a single unit of the time scale, where the distinctions between the uptrend and downtrend values are observed throughout most of the time scale. As seen in this figure, the trajectories for the asymmetric MF-DFA functions $F_q(n)$ versus the time scale n for different frequencies are similar. The A-MF-DFA discerns straight upward and downward trends pivoting on the dots. Further, the graphical evidence shows that deviations from symmetry are greater, in particular, at higher time scales (i.e. of about 15); this implies that investors with a log horizon should pay close attention to asymmetric long-range correlation. This symmetry deviation is more apparent for BTC than ETH.

Fig. 2 plots the overall, upward, and downward generalized Hurst exponent ($h(q)$, $h^+(q)$, and $h^-(q)$) values in the BTC and ETH return dynamics for different q values ranging from –4 to 4. The Hurst exponent values show a downside trend with an increase in q for all cases, thus providing evidence of multifractality. More interestingly, the value decrease in $h^+(q)$ is greater than that of $h(q)$ or $h^-(q)$. Moreover, the trends are similar across all frequencies. These results suggest that the BTC and ETH return series exhibit a multifractal process at all frequencies, regardless of the nature of the trend. Interestingly, the gap between the uptrend and downtrend is small for both BTC and ETH when q is small (i.e. small fluctuation), and it increases as q increases (i.e. large fluctuation). The

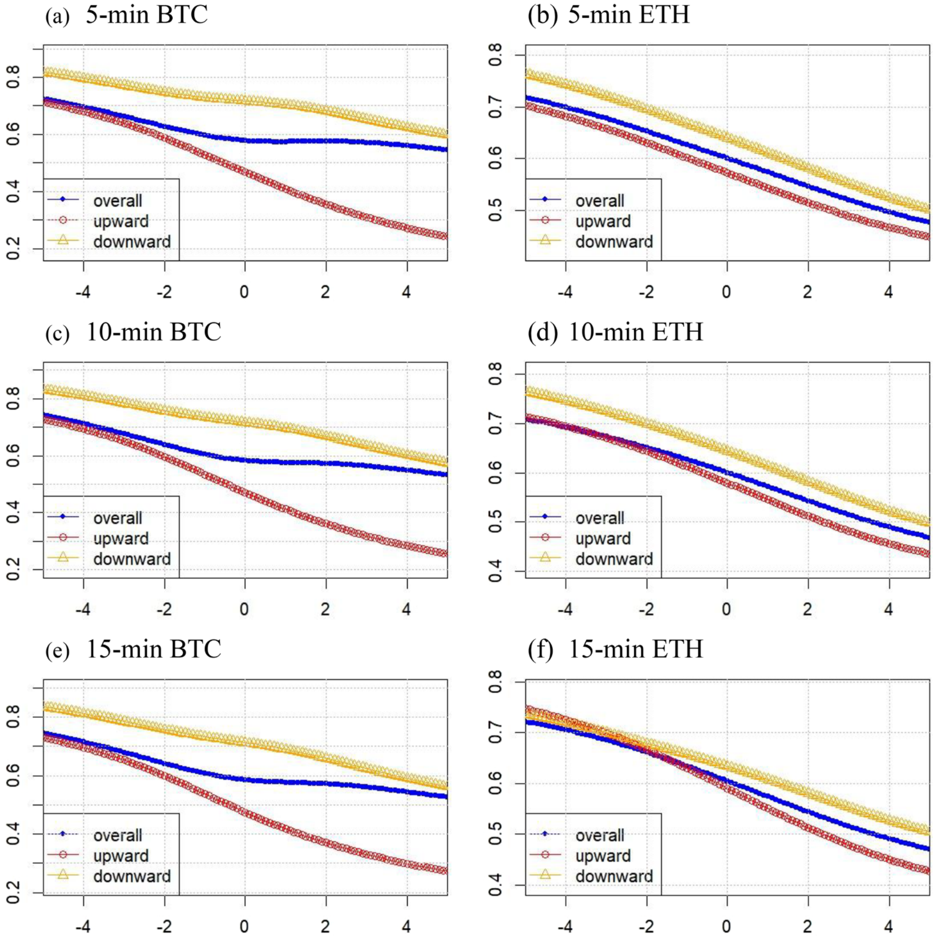

Fig. 2. The $h(q)$, $h^+(q)$, and $h^-(q)$ functions in the BTC and ETH return dynamics versus q.
Note: The x-axis represents q, which varies from –4 to 4; the y-axis represents $h(q)$, $h^+(q)$, and $h^-(q)$.

gap in this figure is larger and more important for BTC than for ETH, indicating that the multifractality and its asymmetry are more apparent in the BTC market. This result could stem from the fact that BTC was the first cryptocurrency market, and so individual investors are more interested in the BTC market.

Fig. 3 illustrates the excess asymmetry in multifractality for BTC and ETH, for 5, 10, and 15-min frequencies. The excess asymmetry is calculated as $\Delta h(q) = h^+(q) - h^-(q)$. The greater the absolute value of this measure is, the more pronounced the asymmetric behaviour of the market. If $\Delta h(q)$ is equal or close to 0, then the multifractality is symmetric for different trends. If the calculated value of $\Delta h(q)$ is positive, this indicates that the cross-correlation exponent is higher when the time series has a positive trend than when it has a negative trend. In this figure, we can see the presence of significantly excessive asymmetry in multifractality, for both BTC and ETH; the implication here is that the use of the A-MF-DFA approach is valid. For BTC, this excess asymmetry in multifractality shows a negative value in most periods, regardless of frequency, thus implying that the multifractality is much stronger in downward price movements. We also see that $\Delta h(q)$ is relatively close to 0 for ETH, compared to BTC. These results indicate that the BTC market is more inefficient than the ETH market, especially in a bear market.

3.2. Time-varying efficiency analysis and robustness tests

In Fig. 4, we see that the efficiency of BTC and ETH (under all frequencies) is dynamic and changes over time. The trajectories of the dynamic Hurst exponent for each market do not change under different frequencies. All markets experience phases of efficiency and of deficiency; they also exhibit phases of persistence and anti-persistence, regardless of frequency. Looking for an example for the BTC, we see that the mode Hurst value reaches 0.526 (with a maximum value of 0.86 in 2016 and a minimum value of 0.169 in 2013), 0.57 (with a maximum value of 0.849 in 2015 and a minimum value of 0.188 in 2013), and 0.55 (with a maximum value of 0.852 in 2016 and a minimum value of 0.186 in 2013) for the 5, 10, and 15-min frequencies, respectively. In comparing the efficiency of BTC to that of ETH, we find that variation therein is comparatively higher for the former; this implies that the BTC market is less efficient but more dynamic, volatile, and risky than the ETH market, and that while investors in the BTC market can derive great

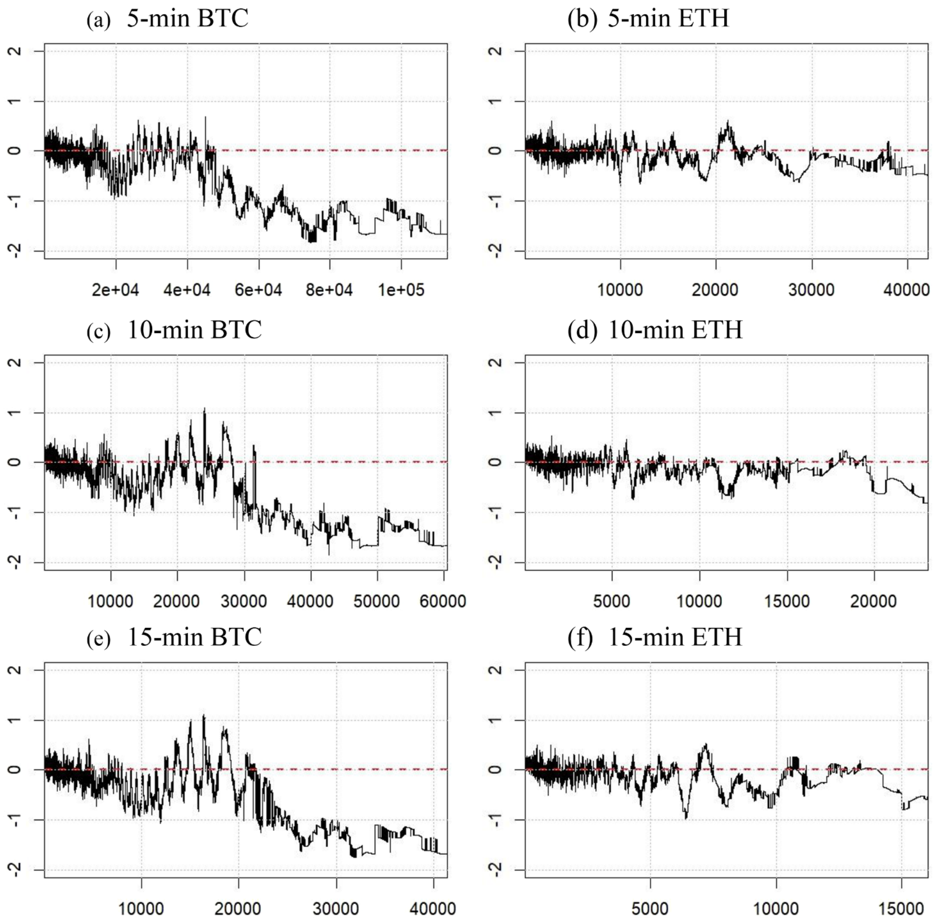

Fig. 3. Excess asymmetry in multifractality.
Note: The *x*-axis represents the time scale *n*, which varies from 5 to $N/4$ (where N is the number of observations in the time series). For (a) 5-min BTC and (b) 5-min ETH, the largest n values are 132,419 and 13,409, respectively. The *y*-axis represents the difference between $\log_2(F_2^+(n))$ and $\log_2(F_2^-(n))$.

profits, they are also exposed to a high level of risk.

To measure and compare the market efficiency of BTC and ETH, we calculate the market deficiency measure (*MDM*) as follows (Wang et al., 2009): $MDM = \frac{1}{2}(|h(-4) - 0.5| + |h(4) - 0.5|)$. If an asset market is efficient, then all kinds of fluctuations, including those both small ($q = -4$) and large ($q = +4$), will follow a random walk process. Therefore, in an efficient market, the *MDM* value will be 0; in an inefficient market, however, it will be high.

For traders and portfolio managers alike, it is critical to detect instabilities (i.e. structural breaks) in financial time series. In the literature, ignoring structural breaks can lead to an overestimation of volatility persistence, and to spurious correlations. To consider the structural breaks in the price dynamics of the BTC and ETH markets, we perform a structural break test and identify two structural break points, by undertaking a cumulative sum (CUSUM) test.[1] We then divide the whole sample period into three subperiods (Fig. 5). The whole BTC sample is from June 1, 2013 to June 23, 2018; that of ETH, is smaller, spanning June 1, 2016 to June 23, 2018. From the results of the BTC structural break test, we find that the first and second structural break points of the price dynamics are at September 20, 2017 and January 30, 2018, respectively. The first and second structural break points of the ETH series, on the other hand, are at November 23, 2017 and March 13, 2018, respectively.

Table 1 presents the measurement results of market efficiency using *MDM*, for the whole period and subperiods. Looking at the first row (i.e. the whole period), we see that the BTC market is more inefficient than the ETH market in terms of overall, upward, and downward trends, across all frequencies. In addition, the inefficiency is dissimilar between upward and downward market movements: specifically, we find the inefficiency to be less pronounced when the market follows an upward movement for the BTC and ETH markets (except for the upward ETH trend in the 15-min frequency). This result also justifies the appropriateness of using the A-MF-DFA method, relative to the symmetric MF-DFA method. The change in efficiency level during upward and downward movements has implications for investors and portfolio managers, in terms of both asset allocations and portfolio risk management. In the

[1] For more details on the test method, see the ordinary least squares-based CUSUM test in the 'strucchange' package of R (Zeileis et al., 2002).

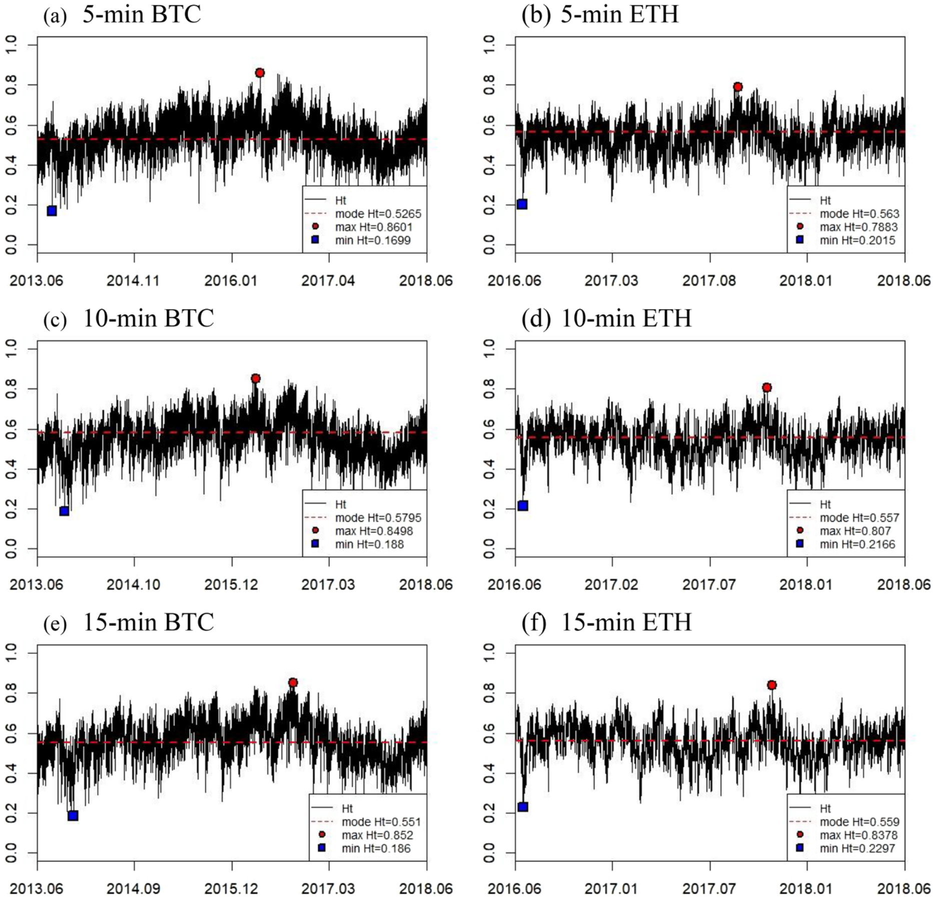

Fig. 4. Time-varying dynamics of Hurst exponent $H(q = 2)$ in the case of $n = 48$.

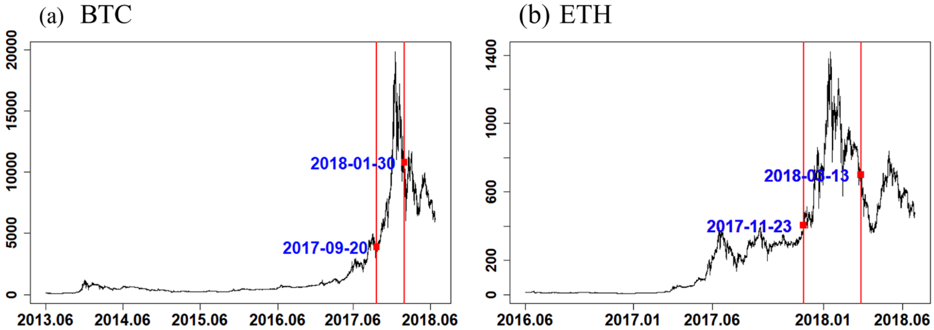

Fig. 5. Structural breaks in BTC and ETH price dynamics.

second to fourth rows of Table 1, the market efficiency results calculated by using the *MDM* of the three subperiods are mixed, indicating both the complexity and dynamics of the efficiency. From this table, we see that ETH is less inefficient than BTC during the first subperiod, regardless of frequency, whereas in the second subperiod, BTC is more efficient than ETH, regardless of frequency. We see that the efficiency is higher during the upward trend for the first subperiod, while it is lower for the second subperiod. For the final subperiod, we see that BTC is more efficient than ETH for the overall and downward trends, whereas for the upward trend, ETH is more efficient. The upward and downward efficiency results in the final subperiod are mixed.[2]

[2] For a robustness analysis, see Appendix 3.

Table 1
Measurement of market efficiency using *MDM* with $h(-4)$ and $h(4)$.

	BTC 5-min	10-min	15-min	ETH 5-min	10-min	15-min
Whole period	*June 1, 2013 to June 23, 2018*			*June 1, 2016 to June 23, 2018*		
Overall	0.1273	0.1301	0.1290	**0.1013**	**0.1015**	**0.1072**
Upward	0.2041	0.2051	0.1986	**0.1074**	**0.1193**	**0.1377**
Downward	0.2101	0.2062	0.2013	**0.1341**	**0.1345**	**0.1227**
Subperiod #1	*June 1, 2013 to September 20, 2017*			*June 1, 2016 to November 23, 2017*		
Overall	0.1452	0.1469	0.1461	**0.0811**	**0.0920**	**0.0983**
Upward	0.2114	0.2046	0.1961	**0.0970**	**0.1115**	**0.1375**
Downward	0.2011	0.1789	0.1770	**0.1140**	**0.1363**	**0.1258**
Subperiod #2	*June 1, 2013 to January 30, 2018*			*November 24, 2017 to March 13, 2018*		
Overall	**0.0889**	**0.0901**	**0.0878**	0.1453	0.1527	0.1541
Upward	**0.1228**	**0.1251**	**0.1224**	0.1901	0.1999	0.2021
Downward	**0.0896**	**0.0900**	**0.0894**	0.1129	0.1175	0.1191
Subperiod #3	*January 31, 2018 to June 23, 2018*			*March 14, 2018 to June 23, 2018*		
Overall	0.0998	**0.1027**	**0.1047**	**0.0995**	0.1039	0.1057
Upward	0.1105	0.1122	0.1133	**0.1020**	**0.1054**	**0.1070**
Downward	**0.0997**	**0.1039**	0.1065	0.1004	**0.1054**	0.1079

Note: The bold values indicate the least inefficient market in each subperiod.

4. Conclusion

This study leveraged intraday data at three different frequencies (5, 10, and 15 min) and the asymmetric multifractal detrended fluctuation analysis approach to examine asymmetric multifractality, long-range memory, and time-varying efficiency for overall, downside, and upside trends in the price dynamics of the world's two largest cryptocurrencies (i.e. BTC and ETH).

We found evidence of multifractality in the movement of cryptocurrency prices, implying that these prices do not follow a random walk and that there is a pattern in the price dynamics of these two cryptocurrencies; ultimately, the suggestion here is that the BTC and ETH markets are inefficient. We also found that multifractality in cryptocurrency markets is asymmetric during upward and downward-trending markets. Furthermore, the multifractality gap between the uptrend and downtrend is small when the time scale is small, and it increases as the time scale increases. The ETH market is less inefficient than that of the BTC, in terms of overall, upward, and downward trends. For both the BTC and ETH markets, inefficiency is lower when they are moving upward, relative to when they are moving downward.

Finally, we also found evidence of structural breaks in both markets. There were mixed results over the three subperiods: in the first and third subperiods, ETH was less inefficient than BTC, while in the second subperiod, BTC was less inefficient than ETH. The implication here is that the level of efficiency varies with each subperiod. These results will be of interest and importance to investors, portfolio managers, and policy-makers, as these results can readily inform their decision-making.

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