Real Term Structure and New Keynesian Models*

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Abstract

Recently some authors have argued that a new Keynesian model with simple modifications can match the nominal term structure of interest rates. In this paper, I investigate how well these models do in matching the term structure of real rates using TIPS data. I find that a standard new Keynesian model that is successful in matching nominal term structure properties cannot match real yield curve features. Then I investigate the model’s relative success in fitting the nominal term structure and show that the model generates implausibly volatile inflation expectations and implausibly high inflation risk premium to fit the nominal yield curve to compensate for the lack of fit to real yields. I study various potential extensions of the benchmark model, and find that incorporating labor market frictions, long run productivity risks, and preference shocks are not helpful in matching real term structure features.

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1 Introduction

Dynamic new Keynesian models are widely used by academics and policy makers. There are two main reasons for their popularity: They tell an internally consistent story about the dynamics of the macroeconomy and they can match the moments of the aggregate variables fairly well (e.g. Smets and Wouters (2007)). These two points are used as a first pass test for the success or failure of the models with the new Keynesian core, which are examined in great detail in the literature.

One of the main problems with this test is that many models fit to a small number of macroeconomic stylized facts well (Taylor and Wieland (2012), Gürkaynak and Wright (2013)). Hence, these models are put to a different test: Can they match asset prices, in particular the nominal yield curve properties? Specifically, can these models generate a time varying and positive term premium? This is an important question because if the models are indeed successful in matching the key moments of the yield curve, then both policy makers and researchers would have an internally consistent model about the connection between the macroeconomic fundamentals and yields. Furthermore it would be a leap forward in the macro-finance literature because as Duffee (2013) suggests, asset pricing models are incapable of explaining this relationship. On the other hand, if these models fail to explain the connection between asset prices and the economic fundamentals, then this shows that the models are flawed in some dimension and need refining.

In recent contributions De Graeve et al. (2009), Rudebusch and Swanson (2012) and Van Binsbergen et al. (2012) show that new Keynesian models with standard extensions have the potential to match the properties of the nominal term structure. Even though these studies use different approaches, they attribute the positive nominal term premium to the combination of negative technology shocks and long run inflation risk. The intuition behind their result is that if shocks to inflation are bad news for future consumption growth, the negative long run covariance must be compensated by higher returns. This result is related to the empirical evidence in Piazzesi and Schneider (2007), who show that bad times are associated with low consumption growth and high inflation. They argue that consumers are worried about stagflation and they want to be compensated for
holding long term bonds. Even though Duffee (2013) and Albuquerque et al. (2016) show that the long-run consumption-inflation correlation is poorly identified, the models that try to match the nominal yield curve stick to explanations in this vein, and by doing so match the average shape of the nominal yield curve.

In this paper I ask whether a canonical new Keynesian model is consistent with real term structure properties or not. Specifically I ask how well these models can match the mean and volatility of the slope, the excess returns, and the real term premia of the TIPS term structure. A yield curve is not only characterized by its shape, but also by the properties of term premium and excess returns. Differently from the literature (e.g. Piazzesi and Scheneider (2007), Swanson (2016)) the focus of this paper is not only the average shape of the real yield curve or the volatility of the real yields, but on top of these moments, I focus on the size and volatility of real term premium and the excess returns as well.

First, I show that a standard new Keynesian model with Epstein-Zin preferences and long run inflation risk (following Rudebusch and Swanson (2012)) under standard calibration has difficulty in jointly matching macroeconomic, nominal, and real term structure properties for the US. Then I estimate the benchmark model and show that the model requires a high coefficient of relative risk aversion (CRRA) and a low intertemporal elasticity of substitution (IES) to match nominal term structure moments and to get the model implied real term structure moments closer to data. Even then the model is unable to match the real term structure features.

Armed with this result, I investigate why new Keynesian models are relatively more successful in matching the nominal term structure. I show that the fit to nominal term structure moments come at the cost of unreasonably volatile long term inflation expectations and high inflation risk premia. Since the model is incapable of generating enough real risk, inflation risk has to be high to make sure that nominal bonds are risky. Finally, I search for extensions that have the potential to increase quantity of real risk. Specifically, I incorporate labor market frictions, long run real risk and preference shocks in turn and show that these extensions are not helpful in matching real term structure features.

This failure of the canonical macro-finance model in explaining ex-ante real rates is
worrisome, especially given the correlation between ten-year nominal and real zero coupon bond yields in the US is more than 90% (as shown in figure 1). Moreover, there is high frequency evidence that monetary policy and macroeconomic announcements generate movements in the real term structure that accounts for most of the variation in nominal yield curve in the US (Beechey and Wright (2009), Hansen and Stein (2012)). At lower frequencies, there is similar evidence that the variation in nominal bond yields can be partly explained by the variation in the real rates (Pflueger and Viceira (2011), Haubrich et al. (2012), Duffee (2018)). For these empirical results to be reconciled, either expected real rates or the real risk premia (or both) needs to be volatile. Because having volatile long term real rate expectations is not plausible (Blue Chip survey implies that the long term real rate expectations are very close to 2% at all times), almost all variation in real yields should be a result of the volatility of the real risk premia. Hence, we would like our macro models to capture the dynamics of real risk premia but this paper shows that they do not.

This question also has policy implications because monetary policy works through real interest rates. Analyzing this relationship in a structural model will enable researchers to have a better understanding of monetary policy transmission mechanism through its impact on the yield curve, particularly the real yield curve. Given the high frequency evidence (Hansen and Stein (2012) for instance), a successful model should explain why monetary policy has effects on the real term premium on top of its affects on average expected real short rates.

The main goal of this paper is to show that even though the literature made some headway in making models compatible with nominal yield curve moments, it seems that implausible expected inflation and inflation risk premium dynamics are the crucial components of these results. The usual mechanism that would help the model to match the upward sloping nominal yield curve would imply a downward sloping real yield curve. In an economy with well anchored long run inflation expectations, economic fundamentals will affect real short and long term yields and in turn long term nominal bond yields. If these models cannot match the real bond yields, then we should not be comfortable with their success in matching the nominal yield curve as a major component must be misspec-
ified. In this sense, this paper is a call for action for further study of the macroeconomic drivers of yield curve dynamics.

This paper is organized as follows. Section 2 presents the benchmark model, Section 3 calculates the US TIPS yield curve moments that the model is required to match. In Section 4 I take the model to the data and show that this model is not capable of reproducing real yield curve properties. In Section 5 I explore the reasons of the model’s inability to fit the real term structure and ask why the model is more successful in matching the nominal term structure. Section 6 considers different extensions of the benchmark model and examines their implication on the real term structure. In this section I show that these extensions are not helpful either. Section 7 concludes.

2 The Benchmark Model

The benchmark model is similar to Rudebusch and Swanson (2012) which is the generalization of dynamic new Keynesian model of Woodford (2003) with Epstein and Zin (1989) preferences and time varying inflation target. Rudebusch and Swanson (2012) show that a model with such characteristics can match nominal term structure properties. An important reason why I chose a small scale new Keynesian model is to be able to clearly examine the role of nonlinearities and the contribution of various mechanisms to fit term structure properties. Without properly understanding why the models are unsuccessful in matching the real term structure properties, it will not be possible to built better models.

1 There are medium scale models in the literature that try to reconcile macroeconomic dynamics and the properties of nominal/real yield curve. For instance, Hsu, Li and Palomino (2016) aims to match the average excess returns and real/nominal, and first and second moments of macroeconomic variables by approximating the model to a second-order. Dew-Becker (2014) has a medium scale DSGE model (similar to Smets and Wouters (2007)) with time varying relative risk aversion that tries to match the nominal yield curve moments. In his model risk aversion and labor-neutral technology shocks are crucial for the nominal term structure. Latter is important essentially because it generates inflation risk. In such a model, the real term structure would be downward sloping. Therefore, it is not clear whether a medium scale DSGE model would help with matching the term structure of real rates given that stagflation risk is the main driver in these models.
2.1 Households

The representative agent maximizes the discounted life time expected utility by choosing consumption, $c_t$ and labor, $l_t$. The maximand of the lifetime expected utility is denoted by $V_t$ and given by:

$$V_t = u(c_t, l_t) + \beta E_t V_{t+1}$$

(1)

where $u(c_t, l_t)$ is the instantaneous utility at time $t$ and $\beta$ is the discount factor.

An important ingredient of this model is Epstein-Zin or recursive preferences. This specification allows for the separation between intertemporal elasticity of substitution (IES) and coefficient of relative risk aversion (CRRA). The former parameter governs smoothing over time and the latter parameter governs smoothing over states. There is no reason for these two parameters to be tightly linked to each other like expected utility suggests. Standard new Keynesian models that use expected utility specification need risk aversion to be very high to match risk premia (see Rudebusch and Swanson (2008)) implying very low intertemporal elasticity of substitution. If the IES is very low, this implies consumption to be too smooth and short term real interest rates to be too volatile. By generalizing the preferences, I can change the coefficient of relative risk aversion to match the real term structure moments and keep the intertemporal elasticity of substitution parameter the same, which is crucial for matching macro moments.

Following Rudebusch and Swanson (2012), the recursive utility specification is given by:

$$V_t = u(c_t, l_t) + \beta (E_t V_{t+1}^{1-a})^{1/a}$$

(2)

where the period utility function is given by:

$$u(c_t, l_t) = \frac{c_t^{1-\varphi}}{1-\varphi} + \chi_0 \frac{(1-l_t)^{1-\chi}}{1-\chi}$$

(3)

where $\varphi$, $\chi$ and $\chi_0$ are positive.\(^2\) In this specification, intertemporal elasticity of substitution is given by $1/\varphi$ and Frisch labor supply elasticity is given by $(1-l)/\chi l$, where $l$

\(^2\)Rudebusch and Swanson (2012) adds stochastic trend for productivity to make the preferences consistent with the balanced growth path. Moreover, they use the stochastic trend in their model of long run real risk. Having this feature of the model is not crucial for the results that will be presented below.
is the steady state labor supply. When $\alpha$ is 0 recursive utility specification reduces to expected utility specification.

Households maximize the Epstein-Zin functional with respect to a flow budget constraint, given by:

$$p_t a_t + P_t c_t = w_t l_t + d_t + p_t a_{t-1}$$

(4)

where $p_t$ is the price of the real bond at time $t$, $a_t$ is the amount of the asset that the household chooses to hold, $w_t$ is the wage rate, $d_t$ is the lump sum transfer from the firms owned by the households, $P_t$ is the aggregate price index at time $t$.\(^3\)

The optimality conditions are given by:

$$\frac{-\chi_0 (1 - l_t)^{-\gamma}}{c_t^{-\varphi}} = \frac{w_t}{P_t}$$

(5)

$$c_t^{-\varphi} = \beta E_t \left( (E_t V_{t+1}^{1-\alpha})^{\alpha/1-\alpha} V_{t+1}^{1-\alpha} c_{t+1}^{-\varphi} \frac{p_{t+1}}{p_t} \frac{P_{t+1}}{P_t} \right)^{-\varphi}$$

(6)

Further manipulation of the Euler equation gives nominal stochastic discount factor from period $t$ to $t + 1$:

$$m_{t,t+1}^{nom} = \frac{V_{t+1}}{(E_t V_{t+1}^{1-\alpha})^{1/1-\alpha}} \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\varphi} \frac{P_t}{P_{t+1}}$$

(7)

Since the nominal stochastic discount factor is a combination of the real stochastic discount factor and one period ahead inflation, we can rewrite it as:

$$m_{t,t+1}^{nom} = \frac{m_{t,t+1}^{real}}{\pi_{t+1}}$$

(8)

where $\pi_{t+1} = \frac{P_{t+1}}{P_t}$ is the one period ahead inflation and

$$m_{t,t+1}^{real} \equiv \frac{V_{t+1}}{(E_t V_{t+1}^{1-\alpha})^{1/1-\alpha}} \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\varphi}$$

(9)

The first term in the stochastic discount factor implies that news at $t + 1$ about

\(^3\)All the variables in this model are state contingent. For notational simplicity the states are dropped from the equations.
consumption in $c_{t+2}, c_{t+3}, \ldots$ affect marginal utility of $c_{t+1}$ relative to marginal utility of $c_t$, where all of the marginal utilities are with respect to agent’s time $t$ life time utility function $V_t$.\footnote{The net effect of good news on the discount factor is ambiguous: Due to intertemporal consumption smoothing, this positive shock raises the marginal utility of $c_{t+1}$ and this effect increases with $\varphi$. On the other hand, because of intratemporal risk aversion, high certainty equivalent term lowers marginal utility of $c_{t+1}$. The net effect of good news on the discount factor depends on the parameter values of $\varphi$ and $\alpha$. If $|\alpha| > |\varphi|$, then positive news about future consumption growth is a negative shock to the real stochastic discount factor. If the inequality is reversed, the relationship between consumption growth and real discount factor is reversed as well.}

The key parameter that I am interested in is the coefficient of relative risk aversion, given its implications for asset prices. Following Swanson (2012) and Rudebusch and Swanson (2012), I define the “effective” coefficient of relative risk aversion which includes endogenous labor choice as:

$$CRRA = \frac{\varphi}{1 + \frac{\varphi}{\chi} \frac{1}{1 - \psi}} + \alpha \frac{1 - \varphi}{1 + \frac{1 - \varphi}{1 - \chi} \frac{1}{1 - \psi}}.$$ (10)

Note that coefficient of relative risk aversion is no longer the inverse of intertemporal elasticity of substitution due to the recursive utility specification. This “effective” coefficient of relative risk aversion is inversely related with labor supply since the household can endogenously change her labor supply to hedge against unexpected changes in her income in a production economy. On the other hand, in an endowment economy, households have fixed labor supply and their consumption is equal to their endowment. As Swanson (2012) shows, in a production economy the endogenous choice of labor supply changes how one defines coefficient of relative risk aversion. Furthermore, this new definition of the risk aversion parameter helps to match the equity premium. Thus the “effective” coefficient of relative risk aversion will be our focus below.

### 2.2 Firms

Following Woodford (2003), I assume that the economy contains a continuum of monopolistically competitive intermediate goods firms indexed by $f$ that set prices according to Calvo contracts. Firms hire labor from households and capital is firm specific. Woodford (2003, chapter 5, subsection 3) shows that a model with firm-specific fixed capital and a
model with endogenous capital stock and investment adjustment costs have very similar business cycle dynamics.\footnote{Altig et al.(2011) show that firm-specific capital stock does help to generate empirically relevant inflation persistence, which is important in matching the standard deviation of inflation.} I follow this specification because adding endogenous capital with investment adjustment costs would make estimation computationally inefficient (Rudebusch and Swanson (2012)). Therefore, I assume that $k_t(f) = \bar{k}$.

All the firms in the economy have identical Cobb-Douglas production functions. The production function of a given firm $f$ is given by:

$$y_t(f) = A_t k_t(f)^{1-\eta}(l_t(f))^{\eta} \quad (11)$$

where $A_t$ is a stationary aggregate productivity shock that affects all the firms and follows an AR(1) process:

$$\log(A_t) = \rho A \log(A_{t-1}) + \varepsilon_A \quad (12)$$

where $|\rho_A| < 1$ and $\varepsilon_A \sim N(0, \sigma_A^2)$.

The prices are set by Calvo contracts and the duration of these contracts are determined by $\xi$. The firm chooses a new price $p_t(f)$ when it is its turn to update the prices. These prices are set such that they would maximize the expected discounted profits of the monopolistic firm. The objective function of this optimization problem is given by:

$$E_t \sum_{j=0}^{\infty} \xi^j m_{t,t+j}^n \left[ p_t(f) y_{t+j}(f) - w_{t+j} l_{t+j}(f) \right] \quad (13)$$

where $m_{t,t+j}^n$ is the nominal discount factor from period $t$ to $t+j$.

The firm’s optimality condition is given by:

$$p_t(f) = \frac{(1 + \theta) E_t \sum_{j=0}^{\infty} \xi^j m_{t,t+j}^n c_{t+j}(f) y_{t+j}(f)}{E_t \sum_{j=0}^{\infty} \xi^j m_{t,t+j}^n y_{t+j}(f)} \quad (14)$$

The marginal cost of the firm $f$ is given by:

$$m_{c_t}(f) = \frac{w_t l_t(f)}{\eta y_t(f)} \quad (15)$$
2.3 Nominal and Real Bonds

The price of a default-free $n$-period nominal zero-coupon bond that pays 1 dollar at maturity satisfies:

$$p_{nom,t}^{(n)} = E_t[m_{t,t+1}^{nom}(n-1)p_{nom,t+1}^{(n)}]$$ (16)

In a model with nominal stochastic discount factor, one can price any asset, including real bonds. Using the real stochastic discount factor and the no-arbitrage condition we can price a real bond that pays 1 unit of consumption good:

$$p_{real,t}^{(n)} = E_t[m_{t,t+1}^{real}(n-1)p_{real,t+1}^{(n)}]$$ (17)

Then the continuously compounded nominal and real yield to maturity on a $n$-period nominal and real zero-coupon bond are given as:

$$y_t^{(n)} = -\frac{1}{n} \log p_{nom,t}^{(n)}$$ (18)

$$r_t^{(n)} = -\frac{1}{n} \log p_{real,t}^{(n)}$$ (19)

where $y_t^{(n)}$ is the $n$-period nominal zero coupon bond and $r_t^{(n)}$ is the $n$-period real zero coupon bond.

I calculate $n$-period term premium as the difference between the $n$-period yield and average expected short rates:

$$\psi_{nom,t}^{(n)} = y_t^{(n)} - \frac{1}{n} E_t \left( \sum_{j=0}^{n-1} i_{t+j} \right)$$ (20)

$$\psi_{real,t}^{(n)} = r_t^{(n)} - \frac{1}{n} E_t \left( \sum_{j=0}^{n-1} r_{t+j}^{(1)} \right)$$ (21)

where $\psi_{nom,t}^{(n)}$ is the nominal term premium and $\psi_{real,t}^{(n)}$ is the real term premium.\(^6\) Differently from the affine term structure literature, this definition of real and nominal term

\(^6\)This definition of the term premium is consistent with Rudebusch and Swanson (2012) and Swanson (2016), where in those studies term premium is calculated as the difference between the risk-neutral yield and yield to maturity.

\(^7\)This definition of nominal and real term premium is consistent with Rudebusch and Swanson (2012).
premium includes Jensen’s inequality term as a part of real term premium. Empirically, the inequality term is very small to make a difference in model’s predictions below (see D’Amico, Kim and Wei (2018)).

2.4 Aggregation and Resource Constraints

Aggregate output and price index is given by:

\[ Y_t = \left[ \int_0^1 y_t(f)^{1/1+\theta} df \right]^{1+\theta} \]  
(22)

\[ P_t = \left[ \int_0^1 p_t(f)^{-1/\theta} df \right]^{-\theta} \]  
(23)

Next I define cross sectional price dispersion in this economy. This is useful since one of the channels that inflation affects real economy is by creating a dispersion of output. Introducing cross sectional dispersion into the model, helps the model to match the business cycle moments better. Following Rudebusch and Swanson (2012), I define cross sectional dispersion as:

\[ \Delta_t^{1/\eta} = (1 - \xi) \sum_{j=0}^{\infty} \xi^j p_t(f)^{-1+(1+\theta)/\theta\eta} \]  
(24)

Aggregate labor in the economy is:

\[ L_t = \int_0^1 l_t(f) df \]  
(25)

Then the aggregate production function is:

\[ Y_t = \Delta_t^{-1} A_t K_t^{1-\eta}(Z_t L_t)^{\eta}, \]  
(26)

where \( K_t = \bar{k} \) is the aggregate capital stock.

I assume a fiscal authority that levies lump sum taxes \( G_t \) on households and destroys
the resource it collects. Government consumption follows a stationary AR(1) process:

\[
\log(G_t/G) = \rho \log(G_{t-1}/G) + \varepsilon^G_t,
\]

where \(\varepsilon^G_t \sim N(0, \sigma^2_G)\).

Then the aggregate resource constraint becomes:

\[Y_t = C_t + I_t + G_t,\]

where \(I_t = \bar{k}(1 - \delta)\), where \(\delta\) is depreciation.

To close the model, I have to specify how the nominal interest rates are determined in the economy. There is a monetary authority that sets the short-term nominal interest rates following a Taylor rule:

\[
i_t = \rho_i i_{t-1} + (1 - \rho_i)[r^* + \log \bar{\pi}_t + g_y(Y_t - Y^* / Y^*)]
+ g_c(\log \bar{\pi}_t - \log \pi^*_t) + \varepsilon^i_t
\]

where \(r^*\) denotes the steady-state real interest rate, \(Y^*\) is the steady state level of output, \(\pi^*_t\) is the long run inflation and \(\varepsilon^i_t\) is an i.i.d monetary policy shock with variance \(\sigma^2_i\). The variable \(\bar{\pi}_t\) denotes a geometric moving average given by:

\[
\log \bar{\pi}_t = \theta\pi \log \bar{\pi}_{t-1} + (1 - \theta\pi) \log \pi_t,
\]

where inflation is \(\pi_t = P_t / P_{t-1}\) and \(\theta\pi = 0.7\) so the geometric average has a duration of four quarters. I assume that the central bank has a time varying inflation target, which has the following specification (Rudebusch and Swanson (2012)):

\[
\pi^*_t = \rho\pi \pi^*_{t-1} + \nu\pi(\bar{\pi}_t - \pi^*_t) + \varepsilon^*_t
\]

where \(\varepsilon^*_t \sim N(0, \sigma^2_{\pi^*})\). Gürkaynak, Sack, and Swanson (2005) show that this specification is crucial in matching the high frequency responses of nominal bond yields to macroeconomic news and monetary policy announcements.
Finally, the ex-ante short term real interest rate is given by:

\[ E_t(r_{t+1}) = i_t - E_t(\pi_{t+1}) \] (32)

### 3 Macroeconomic, Nominal and Real Yield Curve Moments

A natural question is whether the benchmark model presented in the previous section can jointly match the properties of macroeconomic variables and the term structure of nominal and real interest rates. To answer this question, I calibrate and estimate the model to generate unconditional moments from the benchmark model and compare them with their empirical counterparts.

The macroeconomic moments that I am going to match are the second moments of consumption, hours worked, real wage, inflation, nominal and real short term interest rates.\(^8\) I use Edge, Gürkaynak and Kıscakoğlu (2013) data set for macroeconomic moments.\(^9\) Standard deviations for consumption, hours worked and real wage were computed for the (quarterly) deviations from the Hodrick-Prescott trend. For inflation, nominal and real short term rates the (annualized) standard deviations are calculated from the level of these variables.\(^10\) Moments for macro variables are calculated using quarterly data from 1985 to 2007.

I calculate the mean and the standard deviation of nominal term premium using the end-of-quarter values of nominal term premium estimates provided by Adrian, Crump and Moench (2013). The mean and the standard deviation of ten-year nominal bond, three month nominal excess returns and the slope of the nominal yield curve is calculated using Gürkaynak, Sack and Wright (2007) data set. I use end-of-quarter values of nominal yields to calculate these moments. The sample for nominal term structure moments is 1985-2007.

The real yield curve moments that I am interested in are the first and second moments

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\(^8\)In this model, consumption and output have similar dynamics.  
\(^9\)The details of the macroeconomic data used in this paper can be found in the data appendix of Edge, Gürkaynak and Kıscakoğlu (2013).  
\(^10\)Here the short term nominal rate is the 3-month nominal Treasury bill yield. The fed funds rate and three month nominal Treasury bill yields have a correlation around 99%. From the model’s perspective, these are indistinguishable. I follow Rudebusch and Swanson (2012) and use Treasury bill yields.
of slope of the TIPS term structure, one period real excess returns and the estimates of the real term premium, which is estimated using a VAR in real yield curve factors (more on this below). TIPS moments are calculated using end-of-quarter TIPS yields from 1999 to 2007.\footnote{All the results presented below are similar if I used 1999-2007 sample for macroeconomic and nominal yield curve moments.}

In order to be able calculate real term premium, slope of the real yield curve and real excess returns, I have to measure short-term ex-ante real rates (i.e. three month TIPS yields), which do not exist.\footnote{The reason why Treasury never issues real bills is because the indexation lag would be overwhelmingly important.} Hence I measure the short term real rate as the short term nominal bond yield minus one quarter ahead headline CPI forecast from Survey of Professional Forecasts.\footnote{Unfortunately, core CPI forecasts in Survey of Professional Forecasters do not go back until 1999. So I used headline CPI forecasts instead. However, using headline CPI forecasts makes the implied short rates to be consistent with the TIPS since they are indexed by the headline CPI.}

Using one period ex-ante real rates and one period nominal interest rates, I can compute nominal and real excess returns:

\[
\begin{align*}
    x_{\text{nom},t}^{(n)} &= \ln \left( \frac{p_{\text{nom},t}^{(n-1)}}{p_{\text{nom},t-1}^{(n)}} \right) - y_{t-1}^{(1)} \quad \text{(33)} \\
    x_{\text{real},t}^{(n)} &= \ln \left( \frac{p_{\text{real},t}^{(n-1)}}{p_{\text{real},t-1}^{(n)}} \right) - r_{t-1}^{(1)} \quad \text{(34)}
\end{align*}
\]

where \(y_{t-1}^{(1)} = i_{t-1}\).

One reasonable concern about the TIPS yield curve moments is the liquidity premium associated with the TIPS. Gürkaynak, Sack and Wright (2010), Pflueger and Viceira (2011) and D’Amico, Kim and Wei (2018) show that when TIPS were first introduced, the yields had a substantial liquidity premium but it decreased as the market for the TIPS grew. Moreover, they show that liquidity premium increased again during the Great Recession. Pflueger and Viceira (2011) attribute this increase to high uncertainty in the economy and the disruption of the financial system. Since I am not explicitly modeling the liquidity premium, I have to adjust the TIPS yields accordingly. I follow Gürkaynak, Sack and Wright (2010) to estimate the liquidity premium and use ten-year
TIPS liquidity adjusted measure.\textsuperscript{14}

3.1 Estimating the Real Term Premium

As equation (21) shows, there are two ingredients that are needed to calculate the real term premia: ten-year TIPS yields and the average expected future short term real interest rates. I follow the methodology of Joslin, Le and Singleton (2012) to construct average expected real short rates.\textsuperscript{15} Subtracting average expected real short term rates from the liquidity adjusted ten-year TIPS gives the real term premium, which has an average of 1.3%. This estimate of real term premium is in line with the literature. Kim and Wright (2005) find real term premium to be 1.4% on average, whereas Haubrich, Penacchi and Ritchken (2012) and D’Amico, Kim, and Wei (2018) find it to be 1.25% and 2% on average, respectively.\textsuperscript{16}

Although this paper focuses only on the US real term structure, US is not the only country that issues index-linked bonds. A prominent example is United Kingdom, a country that has been issuing index-linked bonds since 1983, a longer time span compared to other countries these countries (and US) with similar markets. The slope of the UK index gilt yield curve in different time periods has been subject to a debate (Anderson and Sleath (2001)), where differences in sample periods may play a crucial role in the differences among yield curve slope estimates. Evans (1998) estimates the zero curve to be downward sloping for 1983-1995, whereas Bank of England data shows that for the 1985-1995 sample, the curve is on average upward sloping (average slope is around 50 basis points).\textsuperscript{17} However, after 1997 the UK real yield curve is, on average, downward sloping (consistent with the evidence of Piazzesi and Schneider (2007)) with a slope of

\textsuperscript{14}The details can be found in the online appendix.
\textsuperscript{15}The details can be found in the online appendix.
\textsuperscript{16}For comparison, a model-free estimate of the ten-year real term premia can be calculated using survey forecasts. One can subtract 5-by-5-year real interest rate forecasts from the 5-by-5-year real forward rates to have a gauge about the magnitude of the real term premia. To do so, I used median Blue Chip long term (6 to 10 years) nominal short rate and inflation forecasts to calculate long term real rate forecasts, which I use as a proxy for average expected real rates in 5 to 10 years. After correcting for the liquidity premium which is found to be around 40 basis points on average in the literature (D’Amico, Kim and Wei (2018) and Gürkaynak, Sack and Wright (2010)) model free risk premia is around 50 basis points for the US. This naive estimate shows that positive real term premium survives in the data.
\textsuperscript{17}Swanson (2016) shows that the UK real yield curve is slightly upward sloping (34 basis points) for 1985-2017 sample and almost flat (slope with 1 basis point) for 1990-2007 sample.
-30 basis points.

Some studies in the literature, explain the reversal in the shape of the UK real yield curve to the pension fund reforms undertaken by the UK, which increased the demand for real bonds by the pension funds and decreased real yields through preferred-habitat effects. This point has been brought up by (see Bank of England May 1999 inflation report, McGrath and Windle (2006), Campbell, Shiller and Viciera (2009), Andreasen (2012), Shen (2012), Joyce et al. (2010), Vayanos and Vila (2010)) among others. However, the points brought up by the literature are about the average slope of the UK index-gilt yield curve implying that the market could be sufficiently segmented in the past 20 years to generate downward sloping real yield curve. Moreover, the literature finds sizable and volatile real term premium in the UK (Joyce et al. (2010), Andreasen (2012), Abrahams et al. (2016)). The uncertainty about the estimates of the real yield curve slope justifies to focus on other important features of the real term structure such as excess returns and real term premium. Even if one were to take the average slope to be negative and find that the baseline model matches this, as will be evident below for the US, the model grossly misses the other main moments (such as size and the volatility of the real term premium and excess returns\textsuperscript{19}) that should be matched to understand

\textsuperscript{18}Pension fund reform in the UK proceeded in two steps. First step was in 1995 (effective in April 1997) and second one was in 2004. Vayanos and Vila (2010) point out that the funding requirements instated with Pension Act of 1995 coincided with accounting reforms implying higher demand from pension funds for long-term UK bonds, including inflation-indexed bonds. Bank of England Inflation Report in February 1999 mentions Institutional factors, such as the minimum funding requirement for pension funds, may also have put upward pressure on the price of UK index-linked bonds by raising demand relative to supply. (p. 9). Joyce et al. (2010) mention the act in 1995 as a major contributor the fall in the longer term real yields in the UK due to an increase in the pension fund demand after the minimum funding requirements. Campbell, Shiller and Viceira (2009) indicate that pension reforms in the UK had effects on the real term structure.

For the Pension Act of 2004, Vayanos and Vila (2010) do an event study analysis where they show that after the reform, cumulative net purchases of long term bonds by the pension funds went up substantially compared to equities and short term assets. Higher demand from the pension funds made the ten-year/three-year spread negative and the real yield curve downward sloping. Similarly, Joyce et al. (2010) stress that there are reasons to believe that the index-linked gilt market in the UK might have become more segmented between 2005 and 2007 due to the Pension Act of 2004 which replaced the minimum funding requirement implemented in Pension Act of 1995. They show that this segmentation might result with lower long term yields than short term yields. Similar ideas are explored in McGrath and and Windle (2006) where they show that both the supply and the demand to index-linked gilts increased however, pension funds rebalanced their portfolios towards index-linked gilts which created a demand much higher than the supply.

\textsuperscript{19}Pflueger and Viceira (2016) show that the real excess returns for the UK is as volatile as the real excess returns for the US.
the indexed yield curve\textsuperscript{20}. A successful model is expected to generate sizable real term premium (and real excess returns) with enough volatility in these features of the real term structure. Therefore, the discussions below will mostly focus on the properties of real term premium.

There are recent studies for two of the other index-linked bond issuers, Australia and France, which show that in these countries indexed-linked bond yield curve is upward sloping on average with positive real risk premium. Hambur and Finlay (2018) estimates zero-coupon real term structure for Australia and shows that the average yield curve is slightly upward sloping with a slope (ten-year minus two-year) of 23 basis points. They show that ten-year real term premium is mostly positive in Australia between 1997-2011, then became negative after 2011. For France, Hördahl and Tristani (2014) show that the average real term premium is around 150 basis points for the sample 1999-2006. Though not conclusive, there is some evidence that (at least for these countries) real term premium is positive on average, implying an upward sloping real yield curve on average.\textsuperscript{21}

4 Taking the Model to Data

The baseline calibration is reported in Table 1 and are very standard in the literature. I set the discount factor $\beta$ is set to 0.99 (consistent with 4% annualized real interest rates), the depreciation rate $\delta$ 0.02, share of government spending in output at the steady state ($\bar{G}/\bar{Y}$) to 17 percent, and capital-output ratio at the steady state ($\bar{K}/\bar{Y}$) to 2.5 (Rabanal and Rubio-Ramirez (2005), Smets and Wouters (2007), Rudebusch and Swanson (2012), Swanson (2016)).

Following Rudebusch and Swanson (2012) and Del Negro et al. (2015), $\chi_0$ chosen such that at the steady state hours worked is equal to the one-third of the time endowment. I choose $\chi$ to have Frisch elasticity of 2/3, consistent with microeconomic estimates

\textsuperscript{20}To make this point more clear, I calculated mean (2.66 \%) and the standard deviation (12.86\%) of the UK excess returns using zero coupon index-linked gilt yields and asked the model to match it. The model cannot match the size and the volatility of the real excess returns. Therefore, the results below are robust to the UK

\textsuperscript{21}Obviously there are many other countries that issue index-linked bonds. Unfortunately these data are neither public (or do not have studies analyzing the properties) nor have a long time series to be able to make meaningful analysis.
The intertemporal elasticity of substitution, $\phi$ is calibrated to 0.5, which is consistent with the micro evidence (see Havránek (2015) for meta-analysis). Following Rudebusch and Swanson (2012) coefficient of relative risk aversion, $\gamma$, is calibrated to 75.

Firms’ elasticity with respect to labor $\eta$ is set to 2/3 and firms’ steady state markup $\theta_\pi$ is set to 0.2, which are standard in the literature. Calvo contract duration $\xi$ is set to 0.75, implying a duration of four quarters similar to the estimates of Rabanal and Rubio-Ramirez (2005), Altig et al. (2011), Del Negro et al. (2015) and Swanson (2016).

The monetary policy rule coefficients $\rho_i, g_\pi, g_y$ are taken from Rudebusch (2002). I normalize the steady state inflation rate $\pi^*$ to 0. Both technology shock persistence $\rho_a$, and government spending shock persistence $\rho_g$ are set to 0.95 where as shock standard deviations $\sigma_a, \sigma_g, \sigma_\pi$ are set to 0.005, 0.004, 0.003, respectively, consistent with the estimates in Smets and Wouters (2007) and Rudebusch (2002). For the long term inflation, following Rudebusch and Swanson (2012), I set $\nu_{\pi^*} = 0.01, \rho_{\pi^*} = 0.99$ and $\sigma_{\pi^*} = 5$ basis points.

Since the model is highly nonlinear, I use Perturbation AIM (Swanson, Anderson and Levin (2005)) algorithm to solve and estimate the benchmark model.\textsuperscript{22} I compute the model implied moments for nominal and real term premium by approximating the model up to third order around the non-stochastic steady state. In the first and second order approximations of the model, the nominal and real term premia are either zero or constant, respectively. Using a third order approximation gives a time varying and sizeable term premia because third order approximation incorporates endogenous conditional heteroskedasticity to nominal and real stochastic discount factor. In other words, this approximation generates time varying quantity of risk.\textsuperscript{23}

Results of the calibration exercise are given in the second column of Table 2. The model can match macroeconomics moments well, which is not surprising. This is an

\textsuperscript{22}Perturbation AIM package can be found in Eric Swanson’s website: http://www.ericswanson.us/perturbation.html
\textsuperscript{23}There is a large body of evidence that the expectations hypothesis fail for nominal bonds (see Campbell and Shiller (1991), Cochrane and Piazzesi (2005) among many others) implying time varying and sizeable nominal term premium. Recently, similar evidence emerged in the literature for real bonds (Pflueger and Viceira (2011)).
important reason why new Keynesian models are widely used. Under calibrated values, the benchmark model can generate a sizable and somewhat volatile positive nominal term premium and an upward sloping average nominal yield curve. Model implied mean nominal excess returns is close to their empirical counterpart, where as the standard deviation is high but still away from the data. Positive nominal term premium in the model is generated by negative technology shocks, and with Epstein-Zin preferences and time varying long run inflation target the effects of these shocks are magnified. A negative technology shock increases inflation and marginal utility of consumption, which in turn decreases long term nominal bond prices. Since low nominal bond prices coincide with high marginal utility, nominal bonds command a positive risk premium. For nominal term premium to be large, the covariance needs to be sufficiently positive over the life of the bond. This long run positive covariance is generated by the time varying inflation target. For a positive $\nu_\pi^*$, a negative technology shock raises inflation and in turn long term inflation. This mechanism generates positive covariance between marginal utility and inflation in the long run, increasing riskiness of long term bonds implying higher risk compensation. Epstein-Zin preferences play a crucial role in this model by making high marginal utility in the future being priced in the nominal bond yields today. Therefore, the model can generate high nominal risk premium, and nominal excess returns and positive slope for the nominal term structure.

However, the benchmark model is not as successful in matching the real term structure. Model implied real term premium is negative with a standard deviation of nearly 0. The model also does badly in matching other real term structure moments. The model implied average real yield curve is slightly downward sloping. Real excess returns are far too small and way too stable compared to their empirical counterparts. From the model’s perspective these results are intuitive. Bad news about current consumption is also bad news for future consumption. This bad news about future consumption increases real stochastic discount factor over the course of the bond’s maturity. Since real rates are low in bad state, the price of the real bond will be high. So the bond pays well when consumption growth is low, or when the real stochastic discount factor is high due to high future marginal utility. Thus the long term investors see these bonds as a hedge and are
willing to pay more to have these bonds.

Next I ask if there is a different parameter combination that would match the nominal and real term structure moments without sacrificing the fit of the macroeconomic moments. To answer this question I find the values that minimize the following GMM objective function:

$$
\hat{\Phi} = \arg\min_{\Phi} [\Psi(\Phi) - \Psi_D]^{\Sigma_D^{-1}} [\Psi(\Phi) - \Psi_D]
$$

(35)

where $\Psi(\Phi)$ is the vector of unconditional first and second moments computed from the model, $\Psi_D$ is the vector of unconditional first and second moments of the data and $\Sigma_D^{-1}$ is the weighting matrix. I search the parameter values for intertemporal elasticity of substitution (IES), risk aversion, Frisch labor supply elasticity, Calvo contract duration, technology shock persistence and variance (I vary the monetary policy rule parameters in subsection 6.1). Varying other parameters, in particular, the steady state markup, standard deviations to shocks to government spending, persistence in government spending and geometric average parameter given in equation (27) do not make a consequential change for the fit to the real term structure. The third column of Table 2 gives the results for the model implied moments and the parameter values that are needed to match the data.

Estimation results show that macroeconomic and nominal term structure moments are matched fairly well. The benchmark model performs better in matching the real term structure moments compared to the baseline calibration. But the fit is still poor; the model matches the sign of the averages but not the magnitudes. Model implied real term premium is low and stable compared to the data. Slope of the real term structure is positive but the real yield curve is too flat compared to data. Real expected excess returns are too low and too stable than their empirical counterparts. To jointly

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24 I follow Rudebusch and Swanson (2012) and divide the weight of nominal and real excess returns by 10. I use equal weights for all other moments.

25 Treating these parameters as calibrated and varying them makes a difference in the fit, but does not improve over the estimated parameters.

26 I experimented with varying parameters individually and in groups in calibration and estimation, but did not improve over the original fit to the real term structure except for policy parameters, which were discussed above. To be sure that my calibration does not driving the results in a way that hurts the models fit, I estimated the entire model without calibrating any parameters. The estimated parameters were close to the values for calibration I had picked, hence the model fit was not affected due to estimation either.
match macroeconomic, nominal and real yield curve moments the model requires a high coefficient of relative risk aversion (125) and low intertemporal elasticity of substitution (0.07) relative to their calibrated values.

The finding that a high coefficient of relative risk aversion is needed to match asset prices in a production economy is an established result in macro-finance. In matching the risk premium, Epstein-Zin preferences improve standard models’ fit but these models still need a high coefficient of relative risk aversion to match the properties of asset prices (Tallarini (2000), Weitzman (2007), Barillas, Hansen and Sargent (2010)). The reason is that standard macroeconomic models cannot generate enough quantity of risk. Malloy et al. (2009) and Campanale et al. (2010) show that high consumption volatility (or high quantity of risk) is needed to fit the risk premium on assets. If the model cannot generate the required quantity of risk, to match the risk premium, the price of risk (risk aversion) needs to be very high. The results show that, this conclusion extends to bond pricing.27

In power utility models, a high relative risk aversion implies that the households are extremely unwilling to substitute current consumption with future consumption. However, the results show that the estimated intertemporal elasticity of substitution is low even with Epstein-Zin preferences. An important reason for this result is that real risk free rates are volatile with a low covariance with the expected aggregate consumption growth (Duffee (2013)). To match this empirical result, model estimate of intertemporal elasticity of substitution is substantially below one, implying very smooth consumption and volatile short term real risk free rates. Even though the estimated value is in contrast with the typical value for the IES used in macroeconomic models (which is between 0.5 and 1), the estimated value of the IES may not be inconsistent with the literature given the uncertainty around its estimates. For instance, Hall (1988) and Yogo (2004) find it to be very small (close to 0), Vissing-Jorgensen (2002) finds it to be between 0.8-1 for bond

27For robustness, I considered using five-to-ten-year and three-to-ten-year nominal and real forward premia as targeted moments instead of the real term premium calculated using the ex-ante real rate. 5-to-10 year average real forward premium is 1.85% with standard deviation of 0.34%, where as the average of 5-to-10 year nominal forward rate is 2.31% with a standard deviation of 2.44%. I used the benchmark model both under calibration and with the best fit estimates. Benchmark model generates 5-to-10 year nominal and real forward premia around 28 basis points. Then I estimated the model by removing yield term premia from the target moments and plugging in 5-to-10 year forward nominal and real term premium and their standard deviations. The results show that model can match nominal forward rates but have trouble in matching real forward rates. Results for 3-to-10 year forward premia are similar.
holders, where as Attanasio and Weber (1995) and Hansen and Singleton (1982) find it to be well above 1.5. Braun and Nakajima (2012) show ways making a macroeconomic model consistent with estimates between 0.35-0.5. Havránek (2015) performs a meta-analysis of the literature and find that the mean of the estimates is around 0.6 (for the US). Therefore, the estimated intertemporal elasticity of substitution lies in the battery of estimates found in the literature.

In summary, to jointly match macroeconomic and term structure moments the model requires a high coefficient of relative risk aversion and low intertemporal elasticity of substitution. Coefficient of relative risk aversion is relevant for the nominal and real term structure moments, especially for real and nominal term premium, where as intertemporal elasticity of substitution is crucial for matching the macroeconomic moments through its effects on consumption. Even though, this parameter combination is helpful in matching macroeconomic and nominal term structure moments, it is not as helpful in matching real term structure moments. In the following subsection I analyze this issue in more detail.

5 Decomposing the model fit

This section explores the benchmark model’s inability to fit the real term structure properties and the reasons behind its relative success in matching the nominal term structure features. Nominal bond yields have three components: real yields (measured using TIPS term structure), expected inflation and nominal term premium. If the model can match the nominal term structure moments without matching the real term structure features, this has to be because the model misses other components of nominal yields. I first explain why the model is not capable to match the real term structure properties, then taken this results as given, I ask what helps the model to match the nominal term structure.

5.1 Why the model fails to match the real term structure?

The results in the previous section showed that even with high coefficient of relative risk aversion, model implied real term premium and real excess returns are low and stable.
Due to low model implied real term premium the real yield curve is flatter than the data suggests. An important reason for this result is that conditional heteroskedasticity in the real stochastic discount factor is low, implying a low and very stable real term premium.

This model implication can be understood in the context of Hansen-Jagannathan bounds. Using the no arbitrage condition for real bond prices, Hansen-Jagannathan bound for the real stochastic discount factor can be defined as (Backus, Boyarchenko and Chernov (2018)):

\[
\frac{E_t(x_{real,t+1}^{(n)})}{\sigma_t(x_{real,t+1}^{(n)})} \leq \frac{\sigma_t(m_{t,t+1}^{real})}{E_t(m_{t,t+1}^{real})}
\]

where \( E_t(x_{real,t+1}^{(n)}) \) is the conditional expectations of one-period real excess returns, \( \sigma_t(x_{real,t+1}^{(n)}) \) is the conditional standard deviation of one period real excess returns, \( E_t(m_{t,t+1}^{real}) \) is the conditional expectation of one period real stochastic discount factor and \( \sigma_t(m_{t,t+1}^{real}) \) is the conditional standard deviation of one period real stochastic discount factor. Note that the right hand side of this inequality is mainly determined by \( \sigma_t(m_{t,t+1}^{real}) \), since \( 1/E_t(m_{t,t+1}^{real}) \) equals to the return of the one period real bond. Hence to match the observed risk premium (or the Sharpe ratio), conditional standard deviation of the real stochastic discount factor needs to be high enough. The magnitude and volatility of \( \sigma_t(m_{t,t+1}^{real}) \) depend on coefficient of relative risk aversion, intertemporal elasticity of substitution and volatility of macroeconomic variables, which in turn determine the size and the volatility of the real term premium. In essence, real risk premium is a function of the price of risk (coefficient of relative risk aversion) and quantity of real risk determined by the volatility of macroeconomic variables and intertemporal elasticity of substitution (affecting real stochastic discount factor through consumption smoothing). The model cannot generate enough quantity of real risk therefore requires high coefficient of relative risk aversion to increase volatility in the real stochastic discount factor to get a better, but still a poor fit for the real term premium and in turn real term structure features.

### 5.2 What helps the model to fit the nominal term structure?

If the model is not capable of matching the real term structure properties, then it should be compensating this poor performance through the other components of nominal bond
yields such as long term inflation expectations and inflation risk premium. The no-
arbitrage condition implies that inflation expectations matter for bond pricing through
nominal stochastic discount factor, where as inflation risk premium matters for nominal
term premium. To derive model implications, I treat the moments of these components
as untargeted moments in the model, i.e. the model is not asked to make an effort to
match them. Therefore, the properties needed by the model can be directly compared
with their data counterparts.

In the benchmark model $\pi^*_t$ proxies for model implied long term inflation expecta-
tions. In a standard new Keynesian model, if the inflation target is known and credible,
long term inflation expectations will be formed around the long term inflation target of
the central bank. An implication of this result is that time variation in the target cap-
tures the time variation in long term inflation expectations.\textsuperscript{28} For the same reason, in
macro-finance, time varying inflation target is used as a proxy for time varying long term
inflation expectations. In particular, Kozicki and Tinsley (2001) show that time vary-
ing “endpoints” (or limiting conditional forecasts) of inflation, which are proxied by long
term inflation expectations, is a crucial mechanism to reconcile long term short rate fore-
casts generated from the reduced form models with the long term short rate expectations
implied by the bond yields. Following this result, time varying endpoint for inflation is
modeled as time varying inflation target to better fit short rate expectations, which cap-
tures the dynamics of long term inflation expectations.\textsuperscript{29} An alternative way of modeling
long term inflation expectations is to link observed long term inflation expectations to
the average of one period forecasts over the forecast horizon as in Aruoba, Cuba-Borda
and Schorfheide (2018). I leave this interesting and viable alternative for future work
to keep the model tightly linked to the macro-finance literature by keeping the standard
mechanisms common to the macro-finance models.

As discussed in the previous subsection, the benchmark model cannot generate enough

\textsuperscript{28}See Smets and Wouters (2007), Carvalho et al. (2017), Cogley and Sbordone (2008), Del Negro and
Schorfheide (2013), Campbell et al. (2016). There is game theoretic justification for this mechanism as
well. Demertzis and Viegi (2008) describe monetary policy as an information game and show that explicit
quantitative objectives provide better anchors for coordinating agents’ expectations.

\textsuperscript{29}See Rudebusch and Wu (2007, 2008), Hördahl, Tristani and Vestin (2005), Rudebusch and Swanson
(2012). Del Negro and Schorfheide (2013) show, having time varying inflation target improves long term
interest rate forecasts of standard DSGE models, consistent with shifting endpoints interpretation.
quantity of real risk, implying low real term premium. However, with the estimated coefficient of relative risk aversion, the model can match nominal term premium and other nominal term structure features. This implies that the model should generate high quantity of nominal risk through high conditional heteroskedasticity of the nominal stochastic discount factor. To achieve high conditional heteroskedasticity, the model requires counterfactually high volatility in inflation expectations. Table 3 shows this result, where model implied moments are derived from the estimated model (referred as “Best Fit” in Table 2). Table 3 panel A shows the standard deviation of model implied long term inflation expectations and its empirical counterpart. The model implied standard deviation of long term inflation expectations is 236 basis points, which is around 80 basis points between 1983-2007 in Blue Chip (around 8 basis points between 1999-2007 in the Blue Chip surveys and, 11 basis points based on TIPS (Aruoba (2018)) in the data). Generating long run inflation risks, one way or another is the standard way in the literature to make the model based nominal term structure slope upwards. Since the model implied real risk is very small, inflation risk has to be unreasonably high to make nominal bonds risky assets.

If the inflation risk should be very high to make nominal bonds risky, agents holding long term nominal bonds should require high compensation to bear that high inflation risk. The compensation for inflation risk is measured by inflation risk premium, which is defined as the difference between nominal and real term premium. Table 3 panel B, shows model implied average risk premium (i.e. implied by the estimated model referred as “Best Fit” in Table 2) and estimates of average ten-year inflation risk premium in the literature. The model implied inflation risk premium is around 155 basis points, which is high compared to the estimates of the premium based on TIPS breakeven inflation, inflation expectations from surveys and inflation swaps. As the table shows, these estimates

30 In a consumption based model, inflation risk premium is determined by the covariance between real stochastic discount factor and inflation times risk premium. For positive inflation risk premium, the covariance needs to be negative. As Andreasen (2012) shows, nominal term premium is equal to real term premium plus inflation risk premium.

31 The moments that are shown in this table (except for Aruoba (2018)) are given on Table 8 of D’Amico, Kim and Wei (2018). Moments for Aruoba (2018) are calculated using the data available on Federal Reserve Bank of Philadelphia website. The sample for the benchmark model is given as 1985-2007, which is the sample that covers all the targeted moments used in estimation. However, the results are very similar if the model is estimated for 1999-2007 sample.
are in the range of -12 bps to 70 bps for different sample periods. Studies that include Great Recession in the sample tend to estimate lower inflation risk premium (due to high deflation probability) where as studies with 1970s and early 1980s in the sample tend to estimate high inflation risk premium due to hyperinflation risks (D’Amico, Kim and Wei (2018)). Considering that the sample period used in this paper coincide with Great Moderation, one would expect inflation risk premium estimates to fall between these various estimates. However, the model implied inflation risk premium is very high; twice as large the highest estimate.

Intuitively, long run inflation risk increases the riskiness of long term nominal bonds through higher volatility in the nominal stochastic discount factor. However, due to the stability of the real stochastic discount factor, the model requires very volatile long run inflation, which in turn implies counterfactually high average inflation risk premium. The reason is, higher the uncertainty about future inflation, riskier the nominal bonds are going to be and the more compensation households will ask for inflation risk. The model makes up for the missing real risk through counterfactually volatile inflation expectations and high inflation risk premium to make the nominal term premium sizable.

6 Robustness and Extensions of the Model

Previous section demonstrated that the key issue with the benchmark model is that the real stochastic discount factor is too smooth, implying a low quantity of real risk. Therefore to match nominal term structure, the model requires very high inflation risk through counterfactually volatile inflation expectations, which in turn implies high inflation risk premium required by investors.

In this section, I search for extensions that might increase the quantity of real risk, therefore decrease the inflation risk needed to match nominal term structure properties. I first analyze the effects of monetary policy on real rates. Then I analyze the effects of

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32 The studies shown in the table (except for Aruoba (2018)) are from D’Amico, Kim and Wei (2018).
33 Recently, Gomez-Cram and Yaron (2018) show that in the past two decades inflation related risk factors did not play a dominant role in determining the nominal term premium and nominal excess returns.
34 Another missing feature can be uncertainty. Barillas, Hansen and Sargent (2010) show that a model with moderate uncertainty and low risk aversion and a model with high risk aversion are isomorphic. I do not take that route and leave to future research.
labor market frictions, long run real risks and preference shocks on the model’s fit for the real term structure. The bottom line of the results presented in this section is that none of these extensions are helpful and they imply very volatile long term inflation expectations along with high inflation risk premium. To make that point, I estimate the extended models in turn but only report the implications for the real term structure moments, standard deviation of inflation expectations and average inflation risk premium with the estimates for CRRA and IES. The full set of results (along with results under calibration) can be found in the online appendix.

6.1 Monetary Policy and the Real Term Structure

Monetary policy affects macroeconomy through real interest rates. Therefore it is worth exploring the effects of monetary policy on the real term structure. If the central bank responds more aggressively to inflation and less so to output gap relative to the benchmark calibration, then a negative technology shock would increase real interest rates and decrease real bond prices. Hence real bonds pay less when the marginal utility is high implying a positive real term premium. Moreover, more aggressive monetary policy toward inflation implies higher volatility in real rates.

To approach the problem, I do a comparative statics exercise where I change the monetary policy rule parameters. I simultaneously change the response of monetary policy to inflation and output gap (keeping other parameters at their calibrated values) to investigate the effects on the real term structure moments. Second, to isolate the effects, I change the response to inflation and output gap individually. I compare the effects under high and low degrees of smoothing as well.

Table 4 panel A shows the results of this calibration exercise in terms of monetary policy’s implication on real term structure, where “Baseline” refers to the calibrated version of the benchmark model. In this exercise I choose extreme values for the responses to make the point clear. In particular, I double the total long run response to inflation from 1.53 (i.e. \( g_{\pi} = 0.53 \)) to 3 (i.e. \( g_{\pi} = 2 \)) and simultaneously halve the response to output gap from 0.93 to 0.45. Second column of the table shows the empirical moments and the third column shows the model implied moments under benchmark calibration.
Fourth column shows that high response to inflation and low response to output gap help the model to generate positive real term premium and an upward sloping real term structure. However, same improvement does not materialize with other model implied moments. Model implied real term premium and slope are too stable. Similarly, model implied excess returns are too small and too stable. Columns five and six show that positive real term premium and real yield curve slope is implied by higher response to inflation and lowering the response to output gap has minimal effects on the real term structure. An implication of this exercise is that the monetary policy response to inflation is a crucial determinant of the sign of the real yield curve slope. Stronger the response to inflation, more positive the real yield curve slope is.

Does varying interest rate smoothing improve model’s fit to real term structure? To explore this possibility I reduce the interest rate smoothing to 0.4 and increase to 0.95 in turn, and keep the high inflation and low output gap response (i.e. $g_\pi = 2, g_y = 0.45$). The results in the second column of Table 4 panel B show that lower smoothing combined with high response to inflation and low response to output gap generates slightly larger and slightly more volatile real term premium and real excess returns. However, the gains are modest. On the contrary, higher interest rate smoothing, as shown in the third column of Table 4 panel B worsens the model’s performance. For this model to generate plausible real term term premium and yield curve slope, monetary policy response to inflation has to be too high and interest rate smoothing has to be too low. Even then, it is doubtful that the model can generate enough volatility in real term premium and excess returns.35

6.2 Labor Market Frictions

One of the main differences between endowment and production economies is that in a production based economy, the households can freely choose their labor-consumption tradeoff. In a production economy, the agents have the ability to insure themselves against adverse shocks by endogenously varying their labor supply. As a result of this feature of the model, the benchmark model may not generate enough risk. To overcome this problem, I consider labor market frictions: quadratic labor adjustment costs (following Uhlig

35Estimation of the parameters do not alter the conclusion. The results can be found in online appendix.
(2007) and Rudebusch and Swanson (2008)) and real wage rigidities (following Blanchard and Gali (2007)). The results in the following two subsections are consistent with Rudebusch and Swanson (2008) that labor market frictions are not helpful in matching risk premia.

6.2.1 Quadratic Labor Adjustment Costs

For this extension, I assume that the households have to pay a quadratic cost to change their labor supply, \( \kappa (\log(l_t/l_{t-1}))^2 \), which is parameterized by \( \kappa \). In the benchmark calibration, I assume that \( \kappa = 50Y \), where \( Y \) is the steady state of aggregate output. This calibration implies that a 1\% change in labor supply from the previous quarter costs households 0.5\% of quarterly steady state output.

The third column of Table 5, shows the model implied moments for real term structure.\(^{36}\) The results are consistent with Rudebusch and Swanson (2008). Even though the model can generate slightly higher quantity of real risk by making fluctuations less desirable, the result found in the previous section cannot be overturned. That is, the volatility of real stochastic discount factor is still very low to match excess returns and real risk premium.

6.2.2 Real Wage Rigidities

Another way of introducing labor market frictions to the model is to consider real wage rigidities following Blanchard and Gali (2007). In particular, I assume that the real wage is no longer equal to the marginal rate of substitution between consumption and leisure, but rather it is a function of the lagged real wage, marginal rate of substitution (which is the frictionless real wage) and a steady state wedge between households marginal rate of substitution and the real wages. That is, the real wage is given by:

\[
    w_t = (w_{t-1})^\mu (mrs_t)^{1-\mu} \exp(\omega) \tag{37}
\]

\(^{36}\)See online appendix for the full set of estimates and the results under calibration.
where \( w_t \) is the real wage, \( mrs_t \) is the marginal rate of substitution between leisure and consumption derived from the household’s optimization problem, and \( \omega \) is the steady state wedge between \( w_t \) and \( mrs_t \). In this specification, \( \mu \) captures the sluggishness of the real wage: the closer \( \mu \) to one, higher the wage stickiness. Blanchard and Gali (2007) use this specification as a simple way of capturing the properties of wage bargaining.

The results are given in the fourth column of Table 5 for the estimated model. However, real wage rigidities do not change the conclusions about the model’s fit for the real term structure moments. Even this high coefficient of relative risk aversion is not enough to have a good fit to real term structure.

### 6.3 Long Run Productivity Risks

A promising extension is to introduce long run productivity risks in the spirit of Bansal and Yaron (2005). Epstein-Zin preferences play a crucial role in long run productivity risk models because under recursive preferences future uncertainty about the expected consumption growth is priced, which in turn may increase the conditional heteroskedasticity of the real stochastic discount factor.\(^{37}\)

Consistent with Bansal and Yaron (2005) and others, I assume that productivity shocks have two components: a persistent and predictable component and a transitory component.\(^{38}\) To this end, I replace technology process with the following:

\[
\log(A_t/A) = \rho A^\ast \log(A^\ast_{t-1}/A) + \sigma_{A^\ast} \varepsilon_{A^\ast_t} + \sigma_A \varepsilon_t
\]

The fifth column of Table 5 shows the implications of the model for the real term structure. As consistent with other extensions, adding long run real risk to the model is not helpful in matching the real term structure moments. An important reason is that

\(^{37}\)Ulrich (2011) explore the implications of long run real risk and ambiguity in an endowment economy for the real term structure. He shows that the interaction between ambiguity and long run risks is the crucial mechanism to generate upward sloping real term structure. I leave this extension for future work.

\(^{38}\)The long run real risk specification in this paper and the papers cited above is different. In this paper I assume that the technology follows a stationary process that is highly persistent, where as in the long run productivity risk literature the productivity is assumed to follow a random walk. In other words, the long run risk is incorporated to the models through shocks to the long run growth trend of the real variables. Since I do not have long run trends in the model I choose a stationary productivity specification. This specification follows the earlier working paper version of Rudebusch and Swanson (2012).
negative technology shocks increase marginal utility but decrease real interest rates. The positive covariance of marginal utility and real bond prices makes real bonds a hedge.

6.4 Preference Shocks

The previous subsection showed that long run productivity risks are not helpful in jointly matching the nominal and real yield curve properties. Recently, Albuquerque et al. (2016) showed that preference shocks help a model with endowment economy to match real term structure properties. I incorporate their specification to the production economy and ask the model to jointly match nominal and real term structure features along with macroeconomic moments.\textsuperscript{39}

With preference shocks, we can write real stochastic discount factor as:

\[ m_{t,t+1}^{\text{real}} \equiv \beta \lambda_{t+1} \lambda_t^{-\phi} \left( \frac{c_{t+1}}{c_t} \right)^{-\phi} \]

In this specification, a preference shock increases real rates and the marginal utility of consumption, generating a negative covariance between the real stochastic discount factor and real bond yields. If the preference shocks are very volatile, they can increase the conditional heteroskedasticity of the real stochastic discount factor. The last column of Table 5 shows the implications of the extended model for the real term structure properties. The model implied moments are slightly closer to their empirical counterparts compared to other extensions, however the gains are modest. Therefore, like other extensions, this extension is not helpful in a production economy.

7 Conclusion

In this paper I have shown that a standard new Keynesian model with Epstein-Zin preferences has trouble in matching macroeconomic, nominal and, real term structure properties simultaneously. The dynamic new Keynesian model can match macroeconomic moments

\textsuperscript{39}See Albuquerque et al. (2016) for the details of the preference shocks considered in this subsection. See the online appendix for the details of how the preference shocks are incorporated to the baseline model.
and the nominal term structure, but it is unable to match the real term structure. The success of matching the nominal term structure, comes at the cost of counterfactually volatile inflation expectations and a very high inflation risk premium. I further shows that the mechanisms that are proposed in the asset pricing literature, which are argued to be helpful in matching asset pricing facts, not useful in matching the real term structure properties. An important reason is that these extensions cannot generate enough real risk to match the real term structure features. Either new Keynesian models and their natural extensions are misspecified, or there is a substantial mispricing of indexed-linked bonds.

What other mechanisms could have helped increase the real risk in the model? One approach could be rare disasters. Recently Kozlowski et al. (2018) use a similar idea to explain why riskless rates are depressed after the recession in developed countries. Contrary to Gabaix (2012) (who matches nominal term structure through inflation risk), in Kozlowski et al. (2018) tail risk is incorporated through capital quality shocks in a real business cycle model. The rise in tail risk makes agents invest and produce less, leading to lower output and capital because investing today is riskier. This negatively affects riskless rates due to precautionary savings (future consumption is riskier) and through higher demand for more liquid assets. Other approaches could be to analyze the effects of market segmentation (Fisher (2015), Andreasen et al. (2016)) or examine the implications of heterogeneous agents for the real term structure in the spirit of Constantinides and Duffie (1996). The macro-finance implications of these extensions are left for future research.

This paper has shown that there should indeed be further research as the existing canonical macro-finance models do not satisfactorily fit the key moments of the data.
8 References


9 Tables and Figures

Figure 1: Ten-year TIPS and Nominal Yield Correlation

| $\beta$ | 0.99 | $\lambda_0$ | 1.224 | $\eta$ | 2/3 | $\rho_i$ | 0.73 | $\sigma_a$ | 0.005 |
| $\delta$ | 0.02 | $\chi$ | 3 | $\theta_\pi$ | 0.2 | $g_\pi$ | 0.53 | $\sigma_g$ | 0.004 |
| $\bar{G}/\bar{Y}$ | 0.17 | $\phi$ | 0.5 | $\xi$ | 0.75 | $g_y$ | 0.93 | $\sigma_i$ | 0.003 |
| $\bar{K}/\bar{Y}$ | 2.5 | $\gamma$ | 75 | $\pi^*$ | 0 | $\rho_{\pi^*}$ | 0.99 | $\nu_{\pi^*}$ | 0.01 |
| $\sigma_{\pi^*}$ | 0.0005 |

Table 1: Baseline Calibration
<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Baseline</th>
<th>Best Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{sd}(C) )</td>
<td>0.79</td>
<td>1.63</td>
<td>0.81</td>
</tr>
<tr>
<td>( \text{sd}(L) )</td>
<td>1.29</td>
<td>1.79</td>
<td>2.29</td>
</tr>
<tr>
<td>( \text{sd}(w) )</td>
<td>1.02</td>
<td>1.44</td>
<td>1.34</td>
</tr>
<tr>
<td>( \text{sd}(\pi) )</td>
<td>0.93</td>
<td>2.81</td>
<td>3.59</td>
</tr>
<tr>
<td>( \text{sd}(i) )</td>
<td>1.97</td>
<td>2.68</td>
<td>4.07</td>
</tr>
<tr>
<td>( \text{sd}(r^{(3)}) )</td>
<td>1.51</td>
<td>1.17</td>
<td>2.08</td>
</tr>
<tr>
<td>( \text{sd}(y^{(40)}) )</td>
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<td>1.88</td>
<td>2.78</td>
</tr>
<tr>
<td>( \text{sd}(r^{(40)}) )</td>
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<td>0.25</td>
<td>0.90</td>
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<tr>
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<td>1.93</td>
<td>0.56</td>
<td>1.89</td>
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<td>( \text{sd}(\psi^{(40)}_{\text{nom}}) )</td>
<td>1.01</td>
<td>0.32</td>
<td>0.72</td>
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<tr>
<td>( \text{mean}(y^{(40)} - i) )</td>
<td>1.77</td>
<td>0.50</td>
<td>1.68</td>
</tr>
<tr>
<td>( \text{sd}(y^{(40)} - i) )</td>
<td>1.28</td>
<td>1.01</td>
<td>1.63</td>
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<tr>
<td>( \text{mean}(x^{(40)}_{\text{nom}}) )</td>
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<td>( \text{sd}(\psi^{(40)}_{\text{real}}) )</td>
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<td>( \text{mean}(r^{(40)} - r^{(3)}) )</td>
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<tr>
<td>( \text{sd}(r^{(40)} - r^{(3)}) )</td>
<td>1.13</td>
<td>1.15</td>
<td>1.93</td>
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<td>0.06</td>
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<td>IES</td>
<td>0.5</td>
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<td></td>
</tr>
<tr>
<td>CRRA</td>
<td>75</td>
<td>125</td>
<td></td>
</tr>
<tr>
<td>Frisch</td>
<td>0.66</td>
<td>0.23</td>
<td></td>
</tr>
<tr>
<td>( \xi )</td>
<td>0.75</td>
<td>0.71</td>
<td></td>
</tr>
<tr>
<td>( \rho_A )</td>
<td>0.95</td>
<td>0.95</td>
<td></td>
</tr>
<tr>
<td>( \sigma_A )</td>
<td>0.005</td>
<td>0.005</td>
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<td>( \nu_{\pi^*} )</td>
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<td>0.014</td>
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<tr>
<td>( \rho_{\pi^*} )</td>
<td>0.99</td>
<td>0.96</td>
<td></td>
</tr>
<tr>
<td>( \sigma_{\pi^*} )</td>
<td>0.0005</td>
<td>0.0008</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Macroeconomic, Nominal and Real Term Structure Moments Implied by the estimated benchmark New Keynesian Model
Panel A: $\text{std(Ten-year inf. exp.)}$

<table>
<thead>
<tr>
<th></th>
<th>Sample</th>
<th>Estimate</th>
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</thead>
<tbody>
<tr>
<td>Blue-Chip</td>
<td>1983-2007</td>
<td>80 bps</td>
</tr>
<tr>
<td></td>
<td>1999-2007</td>
<td>8 bps</td>
</tr>
<tr>
<td>Bench. Model</td>
<td>1985-2007</td>
<td>236 bps</td>
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</tbody>
</table>

Panel B: $\text{mean(Ten-year inf. risk prem.)}$

<table>
<thead>
<tr>
<th></th>
<th>Sample</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buraschi and Jiltsov (2005)</td>
<td>1960-2000</td>
<td>70 bps</td>
</tr>
<tr>
<td>Haubrich et. al. (2012)</td>
<td>1982-2010</td>
<td>45 bps</td>
</tr>
<tr>
<td>D’Amico et al. (2018)</td>
<td>1990-2013</td>
<td>29 bps</td>
</tr>
<tr>
<td>Ajello et al. (2016)</td>
<td>1985-2011</td>
<td>24 bps</td>
</tr>
<tr>
<td>Abrahams et al. (2016)</td>
<td>1999-2013</td>
<td>10 bps</td>
</tr>
<tr>
<td>Fleckenstein et al. (2017)</td>
<td>2004-2014</td>
<td>2.8 bps</td>
</tr>
<tr>
<td>Grischenko and Huang (2013)</td>
<td>2000-2008</td>
<td>-12 bps</td>
</tr>
<tr>
<td>Bench. Model</td>
<td>1985-2007</td>
<td>155 bps</td>
</tr>
</tbody>
</table>

Table 3: Model implied unconditional standard deviation of inflation expectations and mean of inflation risk premium. The estimates are in basis points. Panel A shows the model implied unconditional standard deviation of ten-year inflation expectations, standard deviation of 5-to-10 year inflation expectations calculated from the Blue-Chip and the standard deviation of ten-year inflation expectations from the data set of Aruoba (2018). Panel B shows the model implied unconditional mean of inflation risk premium and various estimates of inflation risk premium that are present in the literature (the listed papers except for Aruoba (2018) are from Table 8 of D’Amico et al. (2018)) for the US. Here “Bench. Model” refers to the estimated version of the benchmark model (referred as “Best Fit” in table 2).
Table 4: Effects of a Change in Policy Parameters on Real Term Structure. Panel A shows the model implied unconditional moments for the real term structure with different output gap and inflation responses. Panel B shows the effects of higher and lower interest rate smoothing conditional on high response to inflation and low response to output gap. “Baseline” refers to the moments implied by the calibrated version of the benchmark model.

![Table 4](image)

Table 5: Real term structure moments implied by the extensions of the benchmark model. Model implied average ten-year inflation risk premium and standard deviation of ten-year inflation expectations are given in basis points. Empirical moments for average ten-year inflation risk premium and standard deviation of ten-year inflation expectations represent the range of estimates given in Table 3.

![Table 5](image)