Costly Preparations in Bargaining

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Abstract
We model costly preparations in negotiations and study their effect on agreements in a bilateral bargaining game. In our model, players bargain over a unit pie, where each player needs to pay a fixed cost in the beginning of every period $t$, if he wants to stay in the game in period $t+1$ in case a deal has not been reached by the end of $t$. Whether a player has paid this cost (i.e., prepared for negotiations in $t+1$) is his private information. If only player $i$ stops paying, then player $j$ receives the entire pie. We characterize a “war of attrition” equilibrium, which is a symmetric equilibrium. We do not know whether the game has other symmetric equilibria, but we show that if such an equilibrium exists, its payoff converges to zero as the frictions (discounting and preparation cost) vanish. Efficiency can be obtained by asymmetric play. Specifically, with asymmetric strategies every Pareto-efficient payoff vector can be approximated in equilibrium, provided that the cost of preparations is sufficiently small and that the discount factor is sufficiently close to one.

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JEL classification: C72; C78; D74; D80

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I. Introduction

The alternating offers bargaining game of Rubinstein (1982) is one of the cornerstones of non-cooperative game theory, with applications in a vast range of topics. In this game, two players alternate roles as proposer and responder, and they exchange proposals on the division of a surplus until a proposal is accepted. Though this game is not symmetric – there is a first mover and a second mover – it can be symmetrized easily, as follows. In the beginning of every period (prior to which no agreement has been reached), a proposer and responder are selected randomly with equal probabilities; the proposer makes an offer, which is implemented if it is accepted by the responder, and otherwise there is a one-period delay and the game starts all over again.¹ Similarly to Rubinstein’s game, this “random proposer game” has a unique subgame perfect equilibrium, and this equilibrium has many nice properties: it is symmetric, stationary, and its outcome is efficient. Our goal in the current paper is to investigate whether the random proposer game is robust to the following perturbation: we introduce small participation costs that the players need to pay as long as the bargaining process is ongoing.²

Specifically, our game is as follows. Two players need to divide a pie of size one. In the beginning of every period there is an initial stage in which each player chooses whether or not to take a costly preparatory action (“invest”); the cost of this action is \(c > 0\). Whether a player has paid the cost (i.e., whether he/she is prepared) is unobservable to his/her opponent.³ Once these choices are made, bargaining takes place in the second stage of the period, in the following fashion. A proposer is selected at random by nature, with each player being equally likely to be selected, and then the proposer offers a split of the pie. If the offer is accepted by the responder, then it is implemented and the game ends; otherwise, the game moves one period forward, but a player who is not prepared (i.e., did not take the costly action) drops out of the game. If only one player is prepared, he receives the entire pie. If both players are prepared, then both move forward to the next period, and the above story repeats itself.

Three central assumptions in our game are the following: (1) a player who stops investing in preparations drops out and receives a null payoff; (2) once a player drops out, his opponent, if not dropped out himself, receives

¹This game was analyzed in Binmore (1987).
²We use the terms “participation costs” and “preparation costs” interchangeably.
³For the sake of brevity/readability, throughout the paper the masculine pronoun is used to refer to both male and female.
the entire pie; (3) preparation for $t+1$ has to be made in $t$. Assumption (1) means that unless one is prepared for negotiations, one has to leave the table; alternatively, this can be interpreted as “staying at the table”, but without any ability to make persuasive arguments or produce effective moves. Assumption (2) means that in bargaining between a prepared agent and a non-prepared agent, the former can force whatever terms of trade he wishes. Assumption (3) reflects the fact that preparations take time, and thus have to be made in advance. In other words, we model situations where following a disagreement today, there is not enough time to prepare for the next round of negotiations.

Strictly speaking, the $c=0$ version of our game is not the random proposer game but, from a practical standpoint, there is no difference between the two. To see why with $c=0$ our game is still not equivalent to the random proposer game, consider an arbitrary split $(x^*, 1-x^*)$, which is different from the one of the random proposer game’s equilibrium, and under which the responder receives no more than $\delta$. In the $c=0$ version of our game, this split can be sustained in equilibrium: a player always invests with certainty in the beginning of each period, offers $(x^*, 1-x^*)$, and accepts an offer if and only if it gives him $1-x^*$ or a pie-slice larger than $\delta$; any other offer is rejected, and this rejection is supported by the belief that the deviating proposer did not invest. Thus, the $c=0$ version of our game has infinitely many equilibria, whereas the random proposer game has just one. In particular, these are different games. Note, however, that the beliefs that sustain the $(x^*, 1-x^*)$ equilibrium assign positive probability to the event that the opponent is playing a weakly dominated strategy. Once such beliefs are ruled out, there is practically no difference between the $c=0$ version of our game and the random proposer game. More precisely, under the assumption that the players do not play weakly dominated strategies – and with the accompanying condition that no belief assigns positive probability to the play of such strategies, even

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4Given Assumptions (1) and (2), it is fair to say that we model costly preparations in a reduced-form fashion, which exacerbates the consequences of being (un)prepared. For instance, it is plausible to argue that being (un)prepared is a source of (dis)advantage in negotiations. Our assumptions point toward an extreme form of such (dis)advantage: an unprepared bargainer facing a prepared bargainer loses his bargaining power and becomes fully vulnerable, whereas a prepared bargainer facing an unprepared bargainer can practically capture the whole surplus. We can consider models where the consequences of being (un)prepared are less severe. We discuss such variations in Appendix C.

5An obvious alternative to Assumption (3) is that the players decide whether to prepare or not after today’s negotiation. We discuss a variation of our model in this direction in Appendix C.
off the path – our game with $c = 0$ boils down to the random proposer game.\textsuperscript{6,7}

Our modification of the random proposer game is not merely a technical robustness check; instead, we view participation costs as having economic merit in their own right. These costs model conveniently the various ways in which preparations-for-negotiations require resources, time, and effort. As widely discussed in the literature – and this includes the popular and professional literatures in addition to academic writing – these costs are of critical importance for getting the most out of negotiations. For example, in their best-selling book, *Getting to YES*, Fisher et al. (1992) argue that gaining negotiation power “requires hard work in advance to bring your resources to bear on being persuasive in a particular situation. In other words, it requires preparation”. Collins (2009) mentions pre-negotiation preparations in his “ten commandments of negotiation”, and argues that “[…] the best negotiator is the best-prepared negotiator […] This is as true in negotiating a multimillion-dollar labor contract as it is making a purchase from a flea market vendor.” Thompson (2013) labels pre-negotiation preparations as “the magic bullet” and she argues (in Thompson, 2009) that about 80 percent of a negotiator’s effort should be spent in preparations. Practically, every book on negotiation strategies spares a chapter for pre-negotiation preparations. Our motivation lies in incorporating such preparations into a canonical bargaining model and studying the consequences of this addition.

\textsuperscript{6}Ignoring beliefs that assign positive probability to the event that a player is playing a dominated strategy has a similar flavor to a solution concept called essentially perfect Bayesian equilibrium (EPBE), due to Blume and Heidhues (2006). EPBE differs from PBE in that it does not require optimal behavior after every history: if there are some histories that can (for a good enough reason) be ignored, then the solution concept does not take them into account. For example, consider the following two-stage interaction. First, player 1 chooses between two options, $a$ or $b$. This choice is publicly observable and, after it is made, a continuation game is played, either $G(a)$ or $G(b)$, depending on the choice. Suppose that if player 1 starts with $a$, his supremum possible payoff is strictly below the worst payoff that results after choosing $b$. EPBE says that histories that start with $a$ can be ignored. Ignoring histories with non-investment in the $c = 0$ version of our game is of course not identical to the requirement of EPBE, but expresses a similar idea – namely, dominated actions (in our case, weakly dominated) should not be taken into account. (EPBE is not restricted to the exclusion of dominated behavior; it is based on a general partition of the set of histories into relevant histories and irrelevant histories, and demands optimal behavior only following relevant histories; excluding dominated behavior is just one example of a sorting-criterion for relevant versus irrelevant histories.)

\textsuperscript{7}We can show that with $c = 0$ both players necessarily invest with certainty in the beginning of every period in any strongly symmetric equilibrium – namely, an equilibrium in which behavior is symmetric after every history. The equilibrium of the random proposer game has this property, so the only way to embed it in the augmented framework where the players privately decide whether or not to (freely) invest is to have everybody invest with certainty in every period (for brevity, we omit the details).
In Proposition 1, we present a necessary and sufficient condition for a war of attrition equilibrium – an equilibrium in which each player always mixes between investing and not (according to a history-independent probability), demands the entire pie if he is called by nature to be the proposer, and rejects the offer when he is the responder, unless he did not invest, in which case he “surrenders” and leaves the game empty handed. The force that sustains this equilibrium is the temptation to try to remain the “last man standing”, which is hard to resist when the preparation cost is small and the discount factor, \( \delta \), is close to one. As the frictions vanish – namely, as \( (\delta, c) \) approaches \( (1, 0) \) – the equilibrium’s investment probability increases, and payoffs converge to zero.

The war of attrition equilibrium is symmetric. We do not know whether our game has other symmetric equilibria, but we show (in Proposition 2) that if such an equilibrium exists, then it looks approximately like the war of attrition. In particular, as frictions vanish, the payoffs that can be obtained in a symmetric equilibrium converge to zero.

By contrast, with asymmetric strategies, efficiency can be obtained. We construct a simple stationary strategy that can approximate any efficient payoff vector in equilibrium, provided that \( (\delta, c) \) is sufficiently close to \( (1, 0) \). The construction is as follows. Suppose that the target (Pareto-efficient) payoff vector is \( (u_1, u_2) \), with \( u_2 \geq u_1 \). Take \( x \in (1/2, 1) \), arbitrarily close to \( u_2 \).\(^8\) We let each player invest with certainty in the beginning of each period, and always demand \( x \) for himself whenever he is called by nature to be the proposer. We let player 2 play aggressively, and always reject the opponent’s offer while player 1 follows a compromising strategy, under which he accepts the opponent’s offer. Thus, with probability one, player 2’s proposal is implemented, and he ends up with a pie slice of size \( x \); consequently, when \( (\delta, c) \) is close to \( (1, 0) \) his payoff is approximately \( x \) and that of player 1 is approximately \( 1 - x \).\(^9\) For \( u_1 \geq u_2 \), the analogous construction, under which player 1 is aggressive and player 2 is compromising, delivers the desired result. In this way, any point on the Pareto frontier can be approximated.

The activity around which our model revolves is pre-bargaining investment. It should be noted that there are many types of such investment, and our focus is just on one such type: investments that strengthen a bargainer’s position in the conflict, the prototypical example of which is “lawyering up”. Other types of investment are not covered in our study;

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\(^8\)It is possible to take \( x = u_2 \), unless \( u_2 \in \{1/2, 1\} \).

\(^9\)\( x > 1/2 \) is necessary for making player 2 willing to reject player 1’s offer and trigger another bargaining period. However, \( x < 1 \) stems from player 1’s incentive constraint: \( x = 1 \) would mean that player 1 receives a zero share of the pie, in which case investment is suboptimal, no matter how small \( c \) is.
specifically, we do not consider investments that increase the parties’ joint surplus. More interestingly, one can think of models in which two types of investment are possible – one that enhances the bargaining positions and one that is socially productive – and there is a trade-off between the two. Such a trade-off has been studied by Anbarci et al. (2002) and Karagözoglu and Keskin (2015) in the context of cooperative one-shot bargaining problems. Addressing the issue in the framework of an infinite-horizon extensive form is a topic for future research.

Modeling the costs of delay in extensive form bargaining games as fixed costs goes back to Rubinstein (1982). In Rubinstein’s game with fixed costs, the cost is unavoidable: in case of a failure to reach an agreement, each player pays the cost of moving to a new bargaining period. A slightly different role of fixed costs was considered by Perry (1986), who takes the cost to be associated with making an offer. Hence, if player $i$ makes an offer that is rejected by $j$ – in which case play moves to the next period and the roles alternate – only $i$ pays a cost but $j$ does not. Perry shows that the player with a smaller cost makes the first offer and the bargaining is terminated immediately, either by an acceptance or a rejection by the other player. A follow-up paper on Perry’s work by Cramton (1991) combines (similar to our specification) both fixed costs and discounting in the players’ utility functions. In Cramton’s model, as in Rubinstein’s, a player pays the fixed cost in every period, no matter whether he was the proposer or responder in that period. Cramton reports three different types of equilibrium outcomes depending on the magnitude of gains from trade (on which the information is asymmetric) and bargaining costs: immediate agreement, delayed agreement, and immediate termination. Another related paper is by Anderlini and Felli (2001), who study a bargaining model with periodic participation costs. They show the following. (i) For some values of participation costs, players never reach an agreement although doing so would be efficient. (ii) Whenever there exists an equilibrium where players reach an agreement, there also exists one where they never reach an agreement or one where the agreement is arbitrarily delayed. (iii) The only way to reach an agreement in equilibrium is using inefficient punishment strategies. Contrary to these models, our periodic preparation cost must be paid in advance, and, more importantly, whether or not a player pays the cost is his private information.

Another group of related papers consists of those that study bargaining models in which the players compete for a favorable position in the bargaining process. For example, Yildirim (2007, 2010) studies a model in

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10In Perry’s model, the identity of the first proposer is endogenously determined and a player can unilaterally enforce perpetual disagreement.
which the players compete for the proposer’s position by exerting effort that influences the selection of the proposer through a contest success function. In both papers, there is no delay in equilibrium but just inefficiencies from costly effort the players exert to become the proposer. Board and Zwiebel (2012) study a bargaining game in which two players compete, in every period, for the proposer’s position, and this competition is through a first-price auction; the players have budget constraints, and the horizon is finite. They show that the game, generically, ends in one period. Ali (2015) studies a similar game, in a framework that allows for both finite and infinite horizons, in which \( n \) players compete in every period for the right to make a proposal through an all-pay or first-price auction. He shows that the game ends immediately and the proposer captures all the surplus. Auster et al. (2019) study a one-period, buyer–seller bargaining game where the players invest to increase the probability with which they make an offer. An important difference from earlier papers mentioned above is that such an investment may signal their private information to their opponents, who can use it against them. They show that when the buyer can be of two types, the seller’s profit is non-monotonic in the share of high-type buyers in the society. Finally, an earlier related paper is by Evans (1997), who studies a coalitional bargaining game in which, at each stage, the players compete for the right to make a proposal. In all these papers, the players compete for being the proposer, which is a favorable position in the extensive form. In our game, the costly investments guarantee another kind of favorable position – the ability to continue bargaining in case it is necessary.

The rest of the paper is organized as follows. In Section II, we formally describe our model. Section III is dedicated to the war of attrition. In Section IV, we show how it is avoided under asymmetric play. In Section V, we conclude. Appendix A collects proofs that are omitted from the main text. In Appendix B, we consider two alternatives regarding the observability of the investment decisions. Specifically, we consider the case where these decisions are publicly known (exogenous transparency) as well as the case where their observability is a strategic variable whose value is determined in equilibrium (endogenous transparency). Roughly,\(^{11}\) we find the intuitive results one would expect for those cases: under exogenous transparency, the game becomes a standard bargaining game with a unique equilibrium, and when transparency is endogenous the players choose to make their investments observable, in order to signal their strength; consequently, transparency emerges, and the symmetric efficient equilibrium materializes. In Appendix C, we discuss further variants concerning the

\(^{11}\)The discussion in Appendices B and C is slightly informal.
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timing of investment and the consequences of not investing. We show that if investment is made at the end of the period, then it almost does not matter whether it is observable or not – the equilibrium is similar to the random proposer’s equilibrium, and the difference between the two vanishes as $\delta \to 1$. The result of the random-proposer equilibrium can also be obtained if the consequences of non-investment are “softer”, so that a non-investing player does not have to leave the game upon disagreement. In summary, these additional analyses show that the various assumptions we make in the base model are essential for our results.

II. Model

Let $G$ be the following game. Two players, 1 and 2, need to divide a pie of size one. They share a common discount factor, $\delta \in (0, 1)$. In the first period of the game, $t = 1$, both players are active. In every period $t$ in which both players are active, play is as follows. First, each player privately decides whether or not to take a costly preparatory action (i.e., invest). If a player invests, then he pays a cost, $c > 0$, upfront. Not investing has no cost. The preparation (or investment) decisions are not publicly observable. After these decisions have been made, nature randomly selects a proposer and a responder with equal probabilities. The proposer offers a split of the pie, $(x_1, x_2)$, where $x_i$ is $i$’s proposed share and $x_1 + x_2 = 1$.12 If the responder accepts the offer, the game ends. If he rejects, the game moves to period $t + 1$, but a player who did not invest in preparations in the beginning of period $t$ drops out of the game and does not reach period $t + 1$. If neither player invested in the beginning of $t$, then the game ends and nobody receives anything. If only one player invested, then he is the sole active player at $t + 1$, and he obtains the entire pie then. If both invested, then both reach $t + 1$, and both are active at $t + 1$.

The payoff of player $i$ from an agreement on $(x_1, x_2)$ in the first period is $x_i$ if he did not invest and $x_i - c$ if he did. His payoff from an agreement on $(x_1, x_2)$ at $t + 1$, evaluated at the beginning of the game, is

$$\delta^t x_i - \sum_{i=0}^{\tau-1} \delta^i c,$$

where $\tau \in \{t, t + 1\}$. The value $\tau = t$ corresponds to the case where he did not invest in the last period, and $\tau = t + 1$ corresponds to the case

12We often write an offer as $(x, 1 - x)$, with the understanding that the proposer asks $x$ for himself. Note that the restriction to efficient splits is standard in bilateral bargaining games. That said, whether inefficient splits could be implemented in equilibrium is left for future research.

where he did invest in that period. The utility from perpetual disagreement is \(-\sum_{i=0}^{\infty} \delta^i c\).

Let \(t + 1\) be a period in which both players are active. A history leading to \(t + 1\) is a list \([(i_1, x_1), \ldots, (i_t, x_t)]\), meaning that in period \(k \in \{1, \ldots, t\}\) the proposer was player \(i_k\), he demanded \(x_k\) for himself, and the opponent rejected his offer. The set of those histories is denoted \(H_{t+1}\). The initial history is \(\emptyset\), and we set \(H_1 \equiv \{\emptyset\}\). The set of histories is \(H \equiv \cup_t H_t\).

A strategy for \(i\) is a triplet of functions, \(\sigma^i = (I^i, f^i, g^i)\). The function \(I^i: H \rightarrow [0, 1]\) prescribes the investment probability in the beginning of every period in which both players are active, as a function of the history leading to this period. The function \(f^i: H \times \{0, 1\} \rightarrow [0, 1]\) assigns an offer as a function of the history and the player’s investment decision; a player uses this component of the strategy whenever he is called by nature to be the proposer. The function \(g^i: H \times \{0, 1\} \times [0, 1] \rightarrow \{\text{Accept, Reject}\}\) assigns a response to the opponent’s offer as a function of the history, the player’s investment decision, and the opponent’s offer; a player uses this component of the strategy whenever he is called by nature to be the responder. A pair of strategies is denoted by \(\sigma = (\sigma^1, \sigma^2)\).

A system of beliefs for player \(i\) is a function \(\mu^i: H \rightarrow [0, 1]\). Given a history leading to \(t + 1\), \(h_{t+1}\), the number \(\mu^i(h_{t+1})\) is the probability that \(i\) attaches to the event that \(j\) invested in the beginning of \(t + 1\). A pair of belief systems is denoted by \(\mu = (\mu^1, \mu^2)\).

Given a strategy \(\sigma\) and a history \(h \in H\), let \(Pr_{\sigma}(h)\) be the probability of \(h\) under \(\sigma\).

The pair \((\sigma, \mu)\) is a perfect Bayesian equilibrium (PBE) if:13

1. for every \(i\) and every history after which both players are active, each component of \(\sigma^i\) assigns an optimal action for \(i\), given \(\sigma^j\) and the beliefs \(\mu^i\);

2. the beliefs \(\mu\) obey the Bayes rule in every history \(h\) that satisfies \(Pr_{\sigma}(h) > 0\).

Throughout the paper, “equilibrium” means PBE. Given a history \(h\), let \(\sigma^i(h)\) be the periodic strategy that \(\sigma^i\) induces in the specific period that follows \(h\). An equilibrium \((\sigma, \mu)\) is symmetric if

\[Pr_{\sigma}(h) > 0 \Rightarrow \sigma^1(h) = \sigma^2(h).\]

That is, symmetry means that in every period the players utilize the same periodic strategy, provided that no deviation occurred in the history that preceded that period. This definition has a strong aspect and a weak aspect.

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Its weak aspect is that it only applies on the path. Its strong aspect is that the requirement $\sigma^1(h) = \sigma^2(h)$ is imposed also for (on-path) histories $h$ along which the players behaved differently. It is similar to, but weaker than, the notion of “strong symmetry” (Abreu, 1986), which requires the players to behave identically, independent of history.

Consider a period in a symmetric equilibrium, in which both investment and non-investment occur on the path. This period is pooling if the proposer’s offer does not reveal whether he invested; otherwise, it is separating. A symmetric equilibrium is pooling (separating) if, under this equilibrium, every period is pooling (separating). A player who invested is called “strong” (in the period he invested); otherwise he is called “weak”. An equilibrium $(\sigma, \mu)$ is stationary if $\sigma$ is history-independent.

III. Symmetric Play Implies a War of Attrition

In this section, we show that under symmetric play, and when $(\delta, c)$ is close to $(1, 0)$, our game is, effectively, a war of attrition. Specifically, we call an equilibrium “a war of attrition equilibrium” if it is a pooling and stationary equilibrium in which a player mixes in his investment decision and the proposer demands the entire pie, no matter whether he invested or not.\(^{14}\) The following result provides a necessary and sufficient condition for the existence of such an equilibrium.

**Proposition 1.** A war of attrition equilibrium exists if and only if:

\[
1 - \delta^2 \leq \frac{4c}{\delta} < 1. 
\]

Note that equation (1) implies that:

(i) given any $c \in (0, 1/4)$, a war of attrition equilibrium exists for all sufficiently large values of $\delta$;

(ii) if $c \geq 1/4$, then a war of attrition equilibrium does not exist for any value of $\delta$.

The first part says that for a small enough participation cost, the players can rationally play a destructive war of attrition if they are sufficiently patient. The second part says that if participation is sufficiently costly, then no matter how patient the players are, a war of attrition cannot come about. In this case, being highly patient is not conducive for a war of attrition,\(^{14}\)

\(^{14}\)In several places in the paper (e.g., in the Introduction) we use the term “war of attrition” informally. Here, we use it to denote a specific equilibrium. We hope that this “double usage” does not cause any confusion.
because the patient players realize the substantial future investment that is involved in a war; as they care about the future, this war is prevented.

At the technical level, both (i) and (ii) address the following question. For a fixed $c$, is there a value of $\delta$ for which there is a war of attrition equilibrium? The analogous exercise is to take a fixed $\delta$ and ask about the values of $c$ for which there is a war of attrition equilibrium. Here, the answer is an interval: $c$ must lie in

$$\left[\frac{(1-\delta^2)\delta^2}{4}, \frac{\delta}{4}\right].$$

The fact that $c$ cannot be too large is clear. The cost $c$ cannot be too small either, because of a combination of “first-order” and “second-order” effects: as $c$ becomes smaller, the equilibrium investment probability increases and approaches one (this is the first-order effect). However, if this probability is too close to one, then it would not be possible to sustain non-investment as an equilibrium action. This is because when a player contemplates whether or not to invest, he takes as given that his opponent will almost surely be strong, so investment dominates non-investment (this is the second-order effect). It follows that every player must invest with certainty in the beginning of every period, but this cannot be sustained in equilibrium.\(^{15}\)

To prove Proposition 1, we make use of the following lemma.

**Lemma 1.** In a war of attrition equilibrium, a responder who did not invest (i.e., a weak responder) agrees to the proposer’s demand.

**Proof**: Assume by contradiction that this is not the case, namely the weak responder rejects the offer. It is clear that a strong responder rejects the offer, because accepting it means that the equilibrium’s continuation value is zero, which is not true.\(^{16}\) Therefore, if the lemma’s claim is false then the proposer’s offer is rejected with certainty. Thus, it is suboptimal for a weak proposer to make this offer - in contradiction to equilibrium.\(^{17}\) Therefore, the weak responder agrees to the greedy demand, and he leaves the game empty-handed.

**Proof of Proposition 1**: In such an equilibrium, each player is indifferent between investing or not. The expected utility from not investing is

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\(^{15}\)If everybody invests with certainty in every period, the only way to have an equilibrium is if there is immediate agreement, but if there is immediate agreement, no one will find it worthwhile to invest.

\(^{16}\)For example, the strong responder can reject the offer and play as follows: if he is called by Nature to be the proposer in the next period, he makes the other player an offer that “he cannot refuse,” specifically offers a share $x \in (\delta, 1)$ of the pie. It is clear that this course of action gives a strictly positive expected payoff.

\(^{17}\)For example, he would be better off with an offer to which the responder “cannot refuse.”
(1 − p)/2, where p is the equilibrium investment probability. The reason is the following. Consider a non-investing player. With probability half, he becomes the responder, in which case his utility is zero (by Lemma 1). With probability half, he becomes the proposer and demands the entire pie, and the opponent will agree to this greedy demand if and only if he did not invest (again by Lemma 1), which happens with probability 1 − p. Thus, the expected utility from not investing is (1 − p)/2. However, the utility from investing is −c + (1/2)[pδV + (1 − p)] + (1/2)δV, where V is the equilibrium’s continuation value.\(^{18}\) Indifference implies that they are equal:

$$\frac{1 - p}{2} = -c + \frac{1}{2}(p\delta V + (1 - p)) + \frac{1}{2}\delta V.$$  

Multiplying both sides by 2 and rearranging yields

$$V = \frac{2c}{\delta(1 + p)}.$$  

This V also equals the expected utility from not investing, i.e., V = (1 − p)/2. Hence, the following equation needs to hold:

$$\delta p^2 + 4c - \delta = 0.$$  

The solution to this equation, \(p = \sqrt{1 - (4c/\delta)}\), is in (0, 1) if and only if \(4c < \delta\). Also, a weak proposer should find it optimal to follow the equilibrium and demand the entire pie, rather than offering the responder \(\delta\), which is an offer the responder “cannot refuse”. Therefore \(1 - p \geq 1 - \delta\), or \(\delta \geq p\). With the aforementioned p, this simplifies to \((4c/\delta) \geq 1 - \delta^2\). Therefore, equation (1) is necessary.

Conversely, suppose that equation (1) holds. Consider the following symmetric stationary profile. On the path, behavior is as follows. A player invests with probability \(p = \sqrt{1 - (4c/\delta)}\), demands the entire pie if he is the proposer, and rejects the opponent’s offer if he invested; a responder who did not invest accepts the opponent’s offer. Off the path, behavior is as follows. Any offer different from zero is interpreted as a signal that the opponent did not invest, and is therefore accepted by a strong responder if and only if it gives the responder at least \(\delta\). In the beginning of every subgame that follows a deviation, the deviation is ignored, and play is as in the first period.

\(^{18}\)A war of attrition equilibrium is stationary, so V is the ex ante value of starting to play the game in an arbitrary period, and \(\delta V\) is tomorrow’s continuation value, discounted to the present. The value V is the solution to the following equation:

$$V = \max \left\{ \frac{1 - p}{2}, -c + \frac{1}{2}[p\delta V + (1 - p)] + \frac{1}{2}\delta V \right\}.$$
We argue that this is an equilibrium. If a strong proposer demands \( x < 1 \), his payoff is \( (1 - p)x + p\delta V < (1 - p) + p\delta V \), where \( V \) is the value; note that the right-hand side of this strict inequality is the payoff from following the strategy. Hence, demanding the entire pie is optimal. Now consider a proposer who did not invest. Note that the demand \( x \in (1 - \delta, 1) \) is accepted if and only if the opponent is weak,\(^{19}\) and the demand \( x < 1 - \delta \) is accepted with certainty. Therefore, the only candidates for a best response are the demands \( 1 - \delta \) and \( 1 \). The inequality \( 1 - \delta \leq 1 - p \) guarantees the optimality of the latter. It is easy to see that the prescribed responses are optimal. □

Proposition 1 talks about “a” war of attrition equilibrium but, in fact, as seen in the proof, only one such equilibrium is possible. Therefore, when a war of attrition equilibrium exists, it is unique.

One may wonder about the possibility of other equilibria, besides the war of attrition: do they exist, how do they look, and are they substantially different from the war of attrition? The following result says that, actually, the war of attrition equilibrium “tells most of the story” in the sense that under the restriction to symmetric strategies any equilibrium (even a non-stationary equilibrium) looks, more or less, like the war of attrition equilibrium.

**Proposition 2.** Let \( \epsilon > 0 \). Then there exists \( c_\epsilon > 0 \) and \( \delta_\epsilon < 1 \) such that if \( c < c_\epsilon \) and \( \delta \in (\delta_\epsilon, 1) \) then in every period \( t \) in every symmetric equilibrium the following hold:

(a) each player invests with probability at least \( 1 - \epsilon \);

(b) if the common investment probability is strictly smaller than one, then a strong proposer demands for himself at least \( 1 - \epsilon \); and

(c) the utility from starting the period-\( t \) subgame is at most \( \epsilon \).

Note that, as opposed to Proposition 1, Proposition 2 is not an existence result. In particular, it refers (among other things) to parameter specifications for which we know (by Proposition 1) that the war of attrition equilibrium does not exist; for example, fix \( \delta \in (\delta_\epsilon, 1) \) and then set \( c \) to be sufficiently small so that equation (1) is violated. Moreover, we do not know whether, given such parameter specifications, the game has any symmetric equilibrium. Yet, the proposition is meaningful also for such specifications: it says that if an equilibrium exists – and it can be a hard-to-find, not-well-behaved equilibrium – then it resembles the war of attrition equilibrium, despite the fact that it does not coincide with it completely.

\(^{19}\)This deviation is interpreted as saying that the proposer is weak, so the strong responder will prefer to wait one period and obtain the entire pie then. This is preferable to a present-period payoff, which is smaller than \( \delta \).
The proof of Proposition 2 is based on the following results, the proofs of which are relegated to Appendix A.

**Lemma 2.** For every \( \epsilon > 0 \), there exists a \( \tilde{c}_\epsilon > 0 \), such that if \( c < \tilde{c}_\epsilon \), then the following holds in every pooling period of every symmetric equilibrium:

(a) if the common investment probability is strictly smaller than one, then the common offer, \((d, 1 - d)\), satisfies \( d \geq 1 - \epsilon \);

(b) the common investment probability, \( p \), satisfies \( p \geq 1 - \epsilon \).\(^{20}\)

**Lemma 3.** The following holds in every separating period in a symmetric equilibrium: if both players invest in the beginning of the period (i.e., if both are strong), then there is disagreement in this period.

**Lemma 4.** There exists \( c^* > 0 \) such that the following holds in every separating period in a symmetric equilibrium, provided that \( c < c^* \): the strong responder accepts the offer of the weak proposer.

The stage is now set for the proof of Proposition 2.

**Proof of Proposition 2:** Parts 2(a) and 2(b) were proved in Lemma 2 for the case of a pooling period. Consider, then, an arbitrary separating period. Let \((s, 1 - s)\) and \((w, 1 - w)\) be the offers made by the strong and weak proposers, respectively. From Lemmas 3 and 4, we can assume that the strong responder rejects the \(s\)-offer and accepts the \(w\)-offer. Because he accepts the latter, \(1 - w \geq \delta.\(^{21}\) By subgame perfection, the inequality cannot be strict, and hence \( w = 1 - \delta \). Indifference between investing and not investing in the beginning of the period implies

\[
\frac{1 - \delta}{2} + \frac{1}{2} \left[p(1 - s) + (1 - p)\delta\right] = -c + \frac{1}{2} \left[p\delta V + (1 - p)s\right] + \frac{1}{2} \left[p\delta V + (1 - p)\delta\right], \tag{2}
\]

where \( V \) is the continuation value.\(^{22}\) Rearranging this equation gives

\[
1 - \delta + p(1 - s) = -2c + 2p\delta V + (1 - p)s, \tag{3}
\]

or \(1 - \delta + p = -2c + 2p\delta V + s.\)

\(^{20}\)The word “common” is used with two different meanings here. The second appearance of the word in (a) refers to the equilibrium being pooling (i.e., the offer is common to the strong and the weak). The other appearances refer to the equilibrium’s symmetry (the investment probability is common to both players).

\(^{21}\)Here, \(1 - w < \delta\) means that a weak proposer offers to his opponent less than \(\delta\); but in this case the strong responder would prefer to wait one period and obtain the entire pie.

\(^{22}\)\(V\) is the continuation value after the particular period being analyzed here. As opposed to the war of attrition from Proposition 1, here we consider an arbitrary equilibrium that is not necessarily stationary, and hence the symbol \(V\) has a different meaning from the one it had in the proof of Proposition 1; in particular, it is not a time-invariant value.
Because the strong responder rejects the $s$-offer:

$$\delta V \geq 1 - s. \quad (4)$$

Therefore, the above equation implies $1 - \delta + p \geq -2c + s + 2p(1 - s) = -2c + s + 2p - 2ps$, and so

$$1 - \delta + ps \geq -2c + p + (1 - p)s.$$  

Taking $(\delta, c) \to (1, 0)$ and denoting by $p^*$ and $s^*$, the corresponding limit values of $p$ and $s$, we obtain

$$p^* s^* \geq s^* + (1 - s^*)p^* \geq p^*.$$  

We argue that $p^* > 0$. To see this, assume by contradiction that $p^* = 0$. Then, equation (3) implies that $s^* = 0$, and therefore equation (4) implies $V^* \geq 1$, where $V^*$ is the limit corresponding to $V$. This is impossible, because by symmetry $V^* \leq 1/2$.

Because $p^* > 0$, it follows that $s^* = 1$, and therefore $p^* = 1$. This completes the proof of parts 2(a) and 2(b) for the case of a separating period. Only part 2(c) remains to be proved.

As for part 2(c), consider an arbitrary period $t$ in a symmetric equilibrium. Let $V_t$ be the utility from playing the period-$t$ subgame. Suppose first that $t$ is a separating period. The value $V_t$ equals, in particular, the left-hand side of equation (2). Because $s^* = p^* = 1$, the value converges to zero as $(\delta, c) \to (1, 0)$.

Next, consider a pooling period. Let $p_t$ be the common investment probability in the beginning of the period-$t$ subgame. It is enough to prove that $V_t \leq \epsilon$ under the assumption $p_t < 1$. The reason is that if $t$ is a period with $p_t = 1$, then there is disagreement in period $t$. Therefore, the value of starting the subgame of period $t$, if not zero, is given by $\delta^k V_{t+k}$, where $t + k$ is the first period after $t$ in which the investment probability is smaller than one.

Consider, then, a period in which the investment probability is in $(0, 1)$ and the common offer is $d$. In the proof of Lemma 2 in Appendix A, it is shown that when these quantities are written explicitly as functions of $c$, we find that $(p(c), d(c)) \to (1, 1)$ as $c \to 0$. As $V_t$ equals, in particular, the utility from not investing, which is $(1/2)(1 - p)d + (1/2)(1 - d)$, it follows that $V_t$ converges to zero as $c \to 0$. \qed

---

23It is easy to see that if $p_t = 1$ and if there is agreement in $t$, then it is profitable to deviate and not invest in the beginning of $t$, because no matter who will be the proposer, his proposal is supposed to be accepted with certainty.
IV. Asymmetric Play

The war of attrition can be avoided under asymmetric play. Consider the following asymmetric strategy. Fix an \( x \in (1/2, 1) \). Each player invests with certainty in every period, and demands \( x \) whenever he is called to make an offer. Player 1 accepts player 2’s offer if and only if it gives him at least \( 1 - x \).\(^{24}\) Player 2 accepts an offer if and only if it gives him at least \( \delta \). The prescribed rejections are supported by the following beliefs: whenever a player sees an unexpected offer that the strategy instructs him to reject, he adopts the belief that the proposer did not invest. Hence, rejecting the offer is optimal, as it guarantees the entire pie in the next period. If a player does not invest (which is a deviation) he accepts any offer of the opponent. If player 1 did not invest and he is selected to be the proposer, he demands \( 1 - \delta \) for himself. If player 2 did not invest and he is selected to be the proposer, he demands \( x \) for himself. Denote this strategy by \( \sigma(x, i) \).

Proposition 3. Fix \( x \in (1/2, 1) \) and \( i \in \{1, 2\} \). There exist \( \delta(x) \in (0, 1) \) and \( c(x) > 0 \) such that the following holds: \( \sigma(x, i) \) is sustainable in a stationary equilibrium, provided that \( \delta \in (\delta(x), 1) \) and \( c < c(x) \).

Proof: Fix an \( x \in (1/2, 1) \). Without loss of generality, we suppose that \( i = 2 \); namely, player 2 is the aggressive player. We consider \( \delta = 1 \); the same arguments establish equilibrium existence for \( \delta \) sufficiently close to one.

Let \( v_i \) be player \( i \)’s value from the above-mentioned strategies. For player 1, we have \( -c + (1/2)(1 - x) + (1/2)v_1 = v_1 \Rightarrow v_1 = 1 - x - 2c \). In particular, \( 1 - x > v_1 \), so accepting player 2’s offer is better than rejecting it and triggering the next period.

The following condition guarantees that investing is better than not investing: \( [(1 - x)/2] \leq v_1 \), or

\[
2c \leq \frac{1 - x}{2}. \tag{5}
\]

Given that player 1 invested, asking for the equilibrium demand is optimal for him, as \( (1 - \delta) \leq \delta v_1 \) clearly holds for \( \delta = 1 \). Given that he did not invest, demanding \( 1 - \delta \) is optimal for him, as player 2 will reject any greedier offer.

\(^{24}\)In equilibrium, a player “cannot refuse” an offer that gives him more than \( \delta \). As we are interested in equilibrium for large enough \( \delta \) (and small enough \( c \)), we can assume that \( 1 - x \leq \delta \).
For player 2, the value satisfies \((1/2)x + (1/2)v_2 - c = v_2\), or \(v_2 = x - 2c\). The following condition guarantees that investing is optimal: \((1/2)x + (1/2)(1 - x) = (1/2) \leq v_2\), or
\[
2c \leq x - \frac{1}{2}.
\]
Because player 2’s strategy prescribes him a rejection of \(1 - x\), it needs to be the case that \(1 - x \leq v_2\), or \(1 - x \leq x - 2c\). This condition is satisfied if \(c\) is sufficiently small as \(x > 1/2\).

Given that player 2 invested, demanding the equilibrium demand is optimal for him, as \(x \geq \max\{\delta v_2, 1 - \delta\}\) clearly holds for \(\delta = 1\). It is also easy to see that given that he did not invest, demanding \(x\) is optimal for him.

Combining equations (5) and (6), we obtain
\[
2c \leq \min\left\{\frac{1 - x}{2}, x - \frac{1}{2}\right\}.
\]
Clearly, the above requirement holds for all sufficiently small \(c\). □

The following is an immediate consequence of Proposition 3.

**Theorem 1.** For every Pareto-efficient payoff vector \(u\) and every \(\epsilon > 0\), there exist \(\delta(u, \epsilon) \in (0, 1)\) and \(c(u, \epsilon) > 0\) such that the following holds provided that \(\delta \in (\delta(u, \epsilon), 1)\) and \(c < c(u, \epsilon)\): \(G\) has a stationary equilibrium whose payoff vector is \(\epsilon\) – close to \(u\).

V. Conclusion

We have introduced an empirically relevant element – preparation costs – to an otherwise standard bargaining game (i.e., random proposer game or symmetrized Rubinstein game) and studied its effect on equilibrium predictions. In our model, one has to pay the cost at the beginning of every period \(t\), if one wants to stay in the game in \(t + 1\) in case a deal has not been reached by the end of \(t\). This reflects the fact that preparations take time and hence must be made in advance. This advance investment in preparations, when combined with private information, introduces an inference problem for the players: when making an offer (responding to an offer), the proposer (responder) does not know whether his opponent will be at the negotiation table tomorrow if they cannot reach a deal today.

Costly and unobservable preparations have significant implications. In particular, whereas the random proposer game has a symmetric, stationary,

\[\text{For a general } \delta, \text{ the corresponding condition is } 2c \leq \min\{1 - x + (x/2)(2 - \delta) - (1/2)(2 - \delta)^2, x + (\delta/2) - 1\}.\]
and efficient equilibrium, we have shown that when such preparations are added to the model, symmetric play becomes a war of attrition, and asymmetric strategies can approximate every efficient payoff vector in equilibrium, provided that the frictions (preparation cost and discounting) are sufficiently small.

Appendix A: Lemmas and Proofs

Lemma 1 was proved in the main text, and Lemma 2 will be proved later in this Appendix; we now proceed to the proof of Lemma 3.

Proof of Lemma 3: Assume by contradiction that if both players invest, then there is agreement. Let \( p \) denote the common investment probability. As the period is separating, \( p \in (0, 1) \).

Let \( s \) and \( w \) denote the shares of the pie demanded by a weak (non-investing) and strong (investing) proposer. As a player can secure the payoff \((1 - p) > 0\) by demanding the entire pie, \( w > 0 \).\(^{26}\) We argue that the strong responder rejects the \( w \)-offer. Otherwise (i.e., if he accepts it), then the \( w \)-offer is accepted for certain – namely, by both types of responders. However, the \( s \)-offer is also accepted by both types: it is clearly accepted by the weak responder, and as per our assumption it is accepted by the strong responder. Thus, each offer is accepted with certainty, which is clearly impossible in equilibrium (a proposer will choose the offer that maximizes his share of the pie). Therefore, the strong responder rejects the \( w \)-offer.

So, the weak proposer’s payoff is \((1 - p)w\). Because the weak proposer can guarantee the payoff \((1 - p)\), it follows that \( w = 1 \). Incentive compatibility for the weak proposer implies \((1 - p) \geq s\) (as, by our assumption, the strong responder agrees if the proposer asks for \( s \), the \( s \)-offer is accepted for certain if it is made). Incentive compatibility for the strong proposer implies \( s \geq (1 - p) + p\psi\), where \( \psi \) is the payoff he receives by making the weak proposal to a strong responder. Therefore, \((1 - p) \geq s \geq (1 - p) + p\psi\). Because \( p \in (0, 1) \), it is enough to prove that \( \psi > 0 \) in order to establish a contradiction. This is indeed the case because the player can, for example, avoid investing in the beginning of next-period’s subgame and then demand \( 1 - \delta \) if he is selected by nature to be the proposer. Therefore, \( \psi \geq [\delta(1 - \delta)/2] \). The lemma is therefore proved: if both players invest, there is disagreement. \( \square \)

Lemma A1. Consider a period in a symmetric equilibrium, at the beginning of which each player invests with probability \( p \). Then each player can secure a payoff of no less than

\[^{26}\text{If a player demands the entire pie and the opponent is weak, the opponent agrees to this proposal. The probability of the opponent being weak is } (1 - p).\]
\[ -c + \frac{(1-p)(1+\delta)}{2}. \] (A1)

Proof: Consider the following behavior. A player invests with certainty at the beginning of the period (instead of with probability \( p \)), demands the entire pie if he is called to be the proposer, and refuses anything short of the entire pie if he is called to be the responder. Clearly, this behavior brings a payoff no smaller than
\[ -c + \frac{1-p}{2} + \frac{(1-p)\delta}{2} = -c + \frac{(1-p)(1+\delta)}{2}. \]

\[ \square \]

Proof of Lemma 4: Consider such a separating period. Let \((s, 1-s)\) and \((w, 1-w)\) be the offers that are made by the strong and weak proposer, where \( s \neq w \). By Lemma 3, a strong responder rejects the \( s \)-offer. Assume by contradiction that he also rejects the \( w \)-offer. Therefore, the strong responder rejects both offers. This implies that \( w = 1 \). Let \( p \) denote the investment probability in that period. The indifference condition between investing and not investing is
\[ p\frac{(1-s)}{2} + \frac{1-p}{2} = -c + \frac{1}{2}[p\delta V + (1-p)s] + \frac{1}{2}\delta V, \] (A2)
where \( V \) is the continuation value. Therefore,
\[ p\frac{(1-s)}{2} + c = \left( \frac{1+p}{2} \right)\delta V + \frac{1-p}{2}(s-1), \]
or
\[ p(1-s) + 2c = (1+p)\delta V + (p-1)(1-s). \]
Therefore,
\[ (1-s) + 2c = (1+p)\delta V. \]

Because a strong responder rejects the \( s \)-offer, it must be that \( (1-s) \leq \delta V \). Therefore,
\[ 2c \geq p\delta V. \] (A3)

Now, fix a mapping that assigns for each \( c \) a separating period in a symmetric equilibrium of the game when the investment cost is \( c \), in which the strong responder rejects the weak proposer’s offer (and therefore rejects both offers). In particular, the indifference condition (A2) and its above implications apply. Let \((p(c), V(c))\) be the corresponding investment probability and continuation value in this period. Inequality (A3) implies \( p(c)V(c) \to 0 \) as \( c \to 0 \).

If \( p(c) \to 0 \) as \( c \to 0 \), then \( V(c) \to 0 \). In this case, the periodic value, which, in particular, is given by the right-hand side of equation (A2), converges to \([((1-p^*)s^*)]/2\), where \((p^*, s^*)\) are the limit values of \( p \) and \( s \). However, by Lemma A1 a player can secure a payoff of approximately

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\[ \frac{(1 - p^*)(1 + \delta)}{2}. \]

If, however, \( p(c) \to 0 \) as \( c \to 0 \), then the behavior from Lemma A1 secures a payoff of approximately \( (1 + \delta)/2 \), which is impossible in a symmetric equilibrium. Therefore, there is \( c^* > 0 \) such that the following is true in every separating period in every symmetric equilibrium, provided that \( c < c^* \): the strong responder accepts the weak proposer’s offer.

\[ \square \]

**Lemma A2.** Suppose that \( c < \delta/2 \). Let \( p \) be the investment probability in some period in a symmetric equilibrium. Then \( p > 0 \).

**Proof:** Assume by contradiction that \( p = 0 \). Then each player’s payoff is \( 1/2. \) Therefore, each player has a profitable deviation: investing with certainty, demanding the entire pie, and refusing to anything short of the entire pie. This behavior gives the payoff \(-c + (1/2) + (\delta/2) > (1/2)\). \( \square \)

**Proof of Lemma 2:** Consider a pooling period in a symmetric equilibrium. Let \( p \) be the investment probability and let \( d \) be the common demand.\(^{28}\) Consider \( p < 1. \) By Lemma A2, \( p > 0 \), so a player is indifferent between investing and not investing in that period. Also, note that a strong responder rejects \( 1 - d \); otherwise, the offer \( 1 - d \) is accepted with certainty, which is impossible because \( p > 0. \)\(^{30}\) The expected utility from not investing is \((1/2)(1 - p)d + (1/2)(1 - d)\) and that of investing is \(-c + (1/2)\{p\delta V + (1 - p)d\} + (1/2)\delta V\), where \( V \) is the continuation value. Indifference implies that they are equal, and hence

\[ V = \frac{2c + 1 - d}{\delta(1 + p)}. \] \hspace{1cm} (A4)

The fact that a strong responder rejects \( 1 - d \) implies that \( \delta V \geq 1 - d \). Substituting the expression for \( V \) gives \((2c + 1 - d)/(1 + p)\) \( \geq 1 - d \), so

\[ 2c \geq p(1 - d). \] \hspace{1cm} (A5)

Now, fix a mapping that assigns for each \( c \) a pooling period in a symmetric equilibrium of the game in which \( p < 1 \), when the investment

\(^{27}\)If no one invests, then the proposer demands and receives the entire pie; as the probability of being the proposer is one-half, the expected utility in this case is also one-half.

\(^{28}\)Note that the equilibrium we are considering here is not necessarily stationary. We denote the probability of investment and the common demand by \( p \) and \( d \), without further subscripts to denote the period or the history, even though a more comprehensive notation could include those. Our thin notation is due to the fact that we are analyzing one (generic) period, so such additional notation is not necessary.

\(^{29}\)Note that if \( p = 1 \), then there is disagreement in this period. Had there been agreement, then clearly investment would have been suboptimal.

\(^{30}\)Had the common offer been accepted for certain, clearly there would be no reason to invest.
cost is \( c \). Let \((p(c), d(c))\) be the investment probability and common demand in this period. By equation (A5), \( p(c)(1 - d(c)) \to 0 \) as \( c \to 0 \).

We argue that \( p(c) \to 0 \) as \( c \to 0 \). Otherwise, Lemma A1 implies that a player can secure a payoff of approximately \((1 + \delta)/2\), which is impossible in a symmetric equilibrium. Therefore, \( d(c) \to 1 \) as \( c \to 0 \). This proves the first part of the proposition.

Now, for a fixed \( c \), a player’s expected utility is given by \((1/2)(1 - p)d + (1/2)(1 - d)\).\(^{31}\) This payoff has to be at least as large as the one described in Lemma A1, \(-c + [(1 - p)(1 + \delta)]/2\). That is, \((1/2)(1 - p)d + (1/2)(1 - d) \geq -c + [(1 - p)(1 + \delta)]/2\), or \( c + (1/2)(1 - d) \geq [(1 - p)(1 + \delta - d)]/2\). Taking \( c \to 0 \), we see that the limit value of \( p \) (call it \( p^* \)) must satisfy \(0 \geq [\delta(1 - p^*)]/2\); therefore, \( p^* = 1 \). This proves the second part of the proposition. \(\square\)

**Appendix B: Exogenous versus Endogenous Transparency**

Consider a game that is identical to our original game, but in which investment is publicly observable. We showed in a working paper version of this work that if \( c < \delta/2 \) and under the restriction to equilibria that are either stationary and/or symmetric, there is a unique equilibrium, and the equilibrium offer is\(^{32}\)

\[
\left(1 - \delta \left(-c + \frac{1}{2}\right), \delta \left(-c + \frac{1}{2}\right)\right).
\]

We now turn to the case where investment is not unobservable (as we have assumed throughout the greater part of the paper) nor is it publicly observable. Instead, consider the case where this observability is decided, strategically, by the players themselves. Intuitively, one would expect a player to disclose his investment, if such a move was possible. If this indeed was an option, then – in a richer model in which a player can choose the transparency/opacity of his investment decision – one would expect transparency to emerge endogenously.

This line of reasoning can be lent formal support. Prompted by the insight of an anonymous reviewer, we analyze a two-stage model in the first stage of which each player decides whether his investment decisions will be observable or not. Next, as a function of the first-stage choices, the corresponding extensive form bargaining game is played.

The first-stage \(2 \times 2\) game is given by the following table.

\(^{31}\)Recall that this is the utility from not investing.

\(^{32}\)The interested reader is welcome to contact the authors for details.
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<table>
<thead>
<tr>
<th>Player 1</th>
<th>Private</th>
<th>Public</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private</td>
<td>$\sim 0, \sim 0$</td>
<td>$a, b$</td>
</tr>
<tr>
<td>Public</td>
<td>$b, a$</td>
<td>$-c + \frac{1}{2}, -c + \frac{1}{2}$</td>
</tr>
</tbody>
</table>

When both players choose “Private”, then in the resulting game the payoffs (in any symmetric equilibrium) are approximately zero, and for simplicity we indicate this by “$\sim 0$”. The case where both choices are public leads to the expected payoff $-c + (1/2)$ for each player. It remains to fill in the off-diagonal cells, namely to pin down equilibrium values for $a$ and $b$. This calls for solving the asymmetric game associated with the off-diagonal cell, which is beyond our scope here. Instead, we do the following.

- First, we change the game slightly, and assume that if player $i$’s investment decisions are publicly observable, then player $j$ can condition not only his bargaining behavior on $i$’s investment decision, but also his own investment behavior on this information. In other words, for the off-diagonal cells, we assume that the investment decisions are not simultaneous, but sequential: if $i$’s investment decisions are public knowledge and $j$’s investments are not, then $i$’s investments are being made “one instant” before those of $j$.

- Within this amended framework we show that in any stationary equilibrium of this game, the following must hold (given that the frictions are small; i.e., $(\delta, c) \sim (1, 0))$:

$$a < -c + \frac{1}{2}.$$ 

Therefore, both playing “Public” is a Nash equilibrium. Moreover, if $b$ does not converge to zero as the frictions vanish, then the aforementioned Nash equilibrium is in strictly dominant strategies. To be brief, we proceed more informally.

Consider a stationary equilibrium of the game in which player 1’s investments are private and those of player 2 are public (i.e., the game corresponding to the upper-right cell). First, we argue that player 2 invests with certainty in this equilibrium. To see this, assume by contradiction that he does not. Then, conditional on player 2 being the (weak) proposer, he demands for himself either $1 - \delta$ or the entire pie, depending on the investment probability of player 1, $p$.

The following are easy to verify:
• if \( p = 0 \), then player 2, when he is the proposer, demands the entire pie, but then player 1 has a profitable deviation – to \( p = 1 \);

• if \( p = 1 \), then player 2, when he is the proposer, demands \( 1 - \delta \) for himself, but then player 1 has a profitable deviation – to \( p = 0 \);

• if \( p \in (0, 1) \), then player 2, when he is the proposer, demands either the entire pie or \( 1 - \delta \), but then player 1 has a profitable deviation – to \( p = 1 \) or \( p = 0 \), depending on which is the case.

Therefore, player 2 invests with certainty. We denote his offer (when in the position of the proposer) by \((d, 1 - d)\).

Now consider player 1. He cannot invest with certainty, because this would create incentives for cheating. Given that it is common knowledge that player 1 invests with certainty, he would be better off deviating and saving the investment cost. It is easy to check that if he invests with probability zero, his equilibrium payoff is \( a = \frac{1}{2} \), and this is clearly smaller than \(-c + \frac{1}{2}\) when the frictions are small. Lastly, consider the case in which player 1 invests with probability \( p \in (0, 1) \).

Consider first, the case that player 1 agrees to \( 1 - d \) (when he is the responder) regardless of whether he invested or not. Then, the following indifference condition must hold,

\[
\frac{1}{2}(1 - d) + \frac{1}{2}\pi^w = -c + \frac{1}{2}(1 - d) + \frac{1}{2}\pi^s, \tag{B2}
\]

where \( \pi^w \) (\( \pi^s \)) is the payoff conditional on being a weak (strong) proposer. This implies \((1/2)\pi^w < (1/2)\pi^s\). This inequality implies one of two options:

1. the weak player 1 and strong player 1 make different offers;
2. the weak player 1 and strong player 1 make the same offer, and it is rejected by player 2.

Option 2 is impossible because the weak player 1 needs to offer something to which player 2 cannot refuse. Therefore, option 1 is necessarily the case, and this means that \( \pi^w = 1 - \delta \). It therefore follows from equation (B2) that \( \pi^s \rightarrow 0 \) as the frictions vanish.

Now, note that player 2 rejects the offer made by the strong player 1, because he accepts the offer of the weak player 1, and it is impossible for two distinct offers to be accepted in equilibrium (if this were the case the proposer would have no reason to make the more generous offer). Therefore, \( \pi^s \) is the discounted continuation value, \( \delta a \); in particular, \( a < -c + (1/2) \).

Finally, consider the case where the strong player 1 rejects \( 1 - d \). This means that \( d = 1 \). Now the indifference condition between investment and non-investment is

\[
\frac{1}{2}\pi^w = -c + \frac{1}{2}\delta V + \frac{1}{2}\pi^s.
\]
where $V$ is player 1’s continuation value conditional on being a strong responder. If $(1/2)\delta V = c$, then $a < -c + (1/2)$, because $a = V = (2c/\delta) \rightarrow 0$. Otherwise $\pi^w \neq \pi^s$, and hence options 1 and 2 from above apply; in particular, $\pi^w = 1 - \delta$. Again, we have $a < -c + (1/2)$.

To sum, the profile (Public, Public) emerges as a Nash equilibrium, and the intuition that it is beneficial to signal preparedness receives formal support.

**Appendix C: Further Variants of the Model**

Consider the case where investment is made at the end of each period, in case there was disagreement in that period. The result from the publicly observable investment version – a unique equilibrium whose offer is described in equation (B1) – almost applies here: if $c < \delta/2$, then there is a unique equilibrium and the equilibrium offer is

$$a = \frac{2c}{\delta} \rightarrow 0.$$  

The only difference between equations (B1) and (C1) is the timing of investment. The proof of the result consists of a combination of arguments we have used throughout our paper, and stationarity arguments for bargaining games in the spirit of Shaked and Sutton (1984).

**Claim:** Suppose that $c < \delta/2$. Then, in every subgame perfect equilibrium, every player invests with certainty at the end of every period (i.e., following every disagreement).\(^{34}\)

**Sketch of proof:** Consider an arbitrary period in which there was disagreement, and now each player needs to decide whether to invest or not. The assumption $c < \delta/2$ implies that each player invests with a positive probability. Now suppose that player $i$ invests with probability $p \in (0, 1)$. This means that his net-of-investment-cost continuation payoff is zero. If this player $i$ was the responder in the period that just ended, then he was offered zero; otherwise, he would not have rejected the offer made to him. However, demanding the entire pie is not optimal for the proposer; he could have offered player $i$ some small $\epsilon > 0$, to which player $i$ would have agreed. If player $i$ was the responder, then it is even clearer that there cannot be disagreement following which his net continuation payoff is zero. A generous enough offer to player $j$ would secure acceptance, which, in turn, implies a positive payoff for player $i$. \(\square\)

\(^{33}\)Note that there is no need to impose symmetry or stationarity.

\(^{34}\)Because of the simpler informational structure, the solution concept here is subgame perfect equilibrium.

Claim: The unique equilibrium offer is as described in equation (C1).

Sketch of proof: Let $m$ and $M$ be the infimum and supremum subgame perfect payoffs payoffs for a player in the game. The definition of these numbers implies

$$M \leq \frac{1}{2} \left[1 + c - \delta m\right] + \frac{1}{2} \left[-c + \delta m\right] = \frac{1}{2} + \frac{\delta}{2} [M - m],$$

and

$$m \geq \frac{1}{2} \left[1 + c - \delta M\right] + \frac{1}{2} \left[-c + \delta m\right] = \frac{1}{2} + \frac{\delta}{2} [m - M].$$

These equations imply

$$M - \frac{\delta}{2} [M - m] \leq \frac{1}{2} \leq m + \frac{\delta}{2} [M - m].$$

Therefore, $M - m \leq \delta [M - m]$ and so $M = m$. This means that there is a unique equilibrium payoff; in particular, the equilibrium offer is as described in equation (C1).

Another alternative to the consequences of non-investment is this: if both are non-prepared, then there is delay; if $i$ is prepared and $j$ is not, then $i$ is selected to make an offer ($j$ does not drop out); if both are prepared, then the proposer is selected by a fair lottery. It is not hard to check that the following behavior can be made part of an equilibrium: each player invests with certainty in the beginning of every period, and makes an offer according to equation (B1) when he is the proposer. This play is described by the following table.

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Player 2 Invest</th>
<th>Player 2 Do not invest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Invest</td>
<td>$\left(\frac{1}{2} - c, \frac{1}{2} - c\right)$</td>
<td>$\left[1 - \delta \left(\frac{1}{2} - c\right), \delta \left(\frac{1}{2} - c\right)\right]$</td>
</tr>
<tr>
<td>Do not invest</td>
<td>$\left[\delta \left(\frac{1}{2} - c\right), 1 - \delta \left(\frac{1}{2} - c\right)\right]$</td>
<td>$\left[\delta \left(\frac{1}{2} - c\right), \delta \left(\frac{1}{2} - c\right)\right]$</td>
</tr>
</tbody>
</table>

Finally, another alternative to the “dropping out of the game” assumption is that a player who does not invest does not have to leave the game forever if there is disagreement, but instead leaves the game only for $T$ periods. One might then wonder whether our model is obtained in the limit, as $T \to \infty$. Without going into details, we believe that the answer to this question is negative. The reason is that with a finite suspension $T < \infty$, both players are punished: if only one player invested, then he is forced to wait until
his opponent can come back into the game. Thus, it is unlikely that in the limit the former player obtains the entire pie.

References


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