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The Social Cost of Carbon When We Wish for Full-Path Robustness

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We compute the social cost of carbon (SCC) when decision makers want robust estimates in the face of deep (or "Knightian") uncertainty. We introduce the notion of full-path accumulated robust preferences from stochastic control theory to an integrated assessment model. Robust preferences are appropriate for analyzing climate-related problems because, given the large uncertainty in climate science, they enable decision makers to attain solutions that are robust to a wide range of climate change scenarios. We solve the resulting model, which includes uncertainty about climate change and about the ensuing economic damage, and show the existence of optimal solutions and time-consistent optimal deterministic Markov policies. Additionally, we also prove that the standard Hansen-Sargent recursive utility provides an upper bound of this full-path utility. In our baseline model specification, we find that the year 2020's optimal SCC is US\$162 per tCO_2 with an average annual growth rate of 2.5%—setting the world on a 1.37°C path, which requires full decarbonization by 2068. We introduce the notion of *SCC robustness premium*, which we define as the additional SCC price tag for robustness. For a plausible range of preference parameters the SCC robustness premium in 2020 is between US\$1.41 and US\$25.89 per tCO_2, with US\$2.20 per tCO_2 in our baseline calibration. Over time this premium grows significantly. The forecasts of our model facilitate managerial decision-making during the world's transition from a carbon- and emission-intensive economy to a regenerative economy. The high estimates for the SCC predict drastic rises in emission cost for high-emission industries.

Key words: full-path robustness, integrated assessment models, model uncertainty, risk-sensitive preferences, social cost of carbon, robustness premium.

1. Introduction

The social cost of carbon (SCC) is an attempt to capture the cost to society of an additional ton of carbon dioxide pollution in a single number. Estimates of the SCC are important for decision makers in private industry and government. The SCC directly affects how policy makers and regulators weigh the costs and benefits of any proposed regulation or public investments that in turn influence the economic environment for private businesses. Numerous studies have investigated the effects of economic and climatic uncertainties on the optimal SCC. In this paper, we examine the effects of robust preferences on the optimal SCC within a simplified integrated assessment model (IAM) of the global economy and the climate inspired by the hallmark DICE model of Nordhaus (1993). The objective of the analysis is to determine climate policies that are robust to a wide range of climate change scenarios.

Specifically, we compute values for the SCC when decision makers want robust estimates in the face of deep, or "Knightian," uncertainty with regard to both climate change and the resulting economic damage. To handle such uncertainties with a notion of "robustness," the climate economics literature addressing such setups has primarily followed an robustness approach using the risk-sensitive preferences formulated by Hansen and Sargent (1995) for the derivation of the optimal SCC (see, e.g., Li et al. (2016) and Brock and Xepapadeas (2020), among others). We call that approach the "per-stage recursive" robustness approach. The generic form of such a recursive approach was first introduced by Kreps and Porteus (1978, 1979) for the finite horizon and later extended by Epstein and Zin (1989) to the infinite horizon. In this paper we do not follow this approach. Instead, we build on and generalize the formulation of risk-sensitive preferences as studied by van der Ploeg (1993) for an analysis of precautionary savings in finance (see also Kihlstrom and Mirman (1974) for the initial idea), which is the standard formulation in stochastic control theory (see, e.g., Jacobson (1973), Di Masi and Stettner (1999), Bäuerle and Rieder (2014), and Whittle (1990), and references therein) as initiated by the seminal work of Howard and Matheson (1972) in this very journal. We call this formulation "full-path accumulated" robustness to distinguish it from the aforementioned per-stage recursive robustness approach.

The full-path accumulated approach represents utility as a certainty equivalent of the time-separable additive von Neumann–Morgenstern discounted utility on the entire time horizon, using a concave transformation thereof. The per-stage recursive framework models utility as a sum of the current (per-stage) utility and the one-step discounted certainty equivalent of the future (next-stage) continuation utility wherein this certainty equivalent is computed using a concave transformation. It is important to distinguish between these two forms of utility representations. As we explain in Section 2.3, the risk-sensitive full-path accumulated approach allows for time-consistency and monotonicity without compromising on the resolution between risk-aversion and the elasticity of intertemporal substitution (EIS). We prove that the per-stage recursive utility is always greater than or equal to the full-path accumulated utility (see Theorem 1 in Appendix A). This result implies that the former setup also leads to a larger optimal value function. Accordingly, by means of a numerical example, we illustrate that the per-stage model overestimates the SCC relative to the full-path accumulated approach.

To the best of our knowledge, the present paper presents the first application of full-path accumulated robustness in the climate change economics literature. Our concept herein is similar to van der Ploeg (1993),[1] but is more general. Bommier et al. (2015) follows a similar approach but without discounting in the risk-sensitive preference structure—thereby artificially forcing stationarity in the system. Their paper models a catastrophic collapse using climate damage estimates from Weitzman (2009) and risk-sensitive preferences where the risk-sensitive parameter $k > 0$ is interpreted as risk aversion rather than ambiguity aversion or aversion to mis-specification. Moreover, Bommier et al. (2015) does not provide any proof of existence of the value function as a solution to the corresponding Bellman equation nor any results on the existence or characterization of the corresponding optimal policies.

As mentioned above, the notion of risk-sensitive preferences was first conceived in Howard and Matheson (1972). These preferences have been studied intensively in recent years primarily in the context of their connection to mathematical finance, including in portfolio management: see, for example, Bäuerle and Jaśkiewicz (2018) and Bielecki and Pliska (2003). Unlike classical problems with von Neumann–Morgenstern (additive) discounted utilities, risk-sensitive preference maximization problems are much more difficult to handle as they are not technically equivalent to negative cost (or disutility) minimization problems because of their multiplicative nature, and hence the more abundantly available machinery for the latter problem type cannot be directly applied. Moreover, the maximization problem for risk-sensitive preferences can be interpreted as the lower value function of a zero-sum game in a setup of deep uncertainty, which corroborates the “robustness” tag associated with such preferences—that is, guaranteeing a maximum possible payoff under the assumption of some level of model mis-specification determined by a robustness parameter. A larger degree of risk aversion is identical to greater concern regarding robustness—that is, when the risk parameter $k > 0$ is large, the maximizer effectively tries to maximize against the worst-case minimizing strategy of the adversary, leading to a lower-value game, whereas when k approaches zero, the minimizing adversary is forced to play close to the chosen dynamics of the maximizer, thereby eventually resulting in a standard additive discounted maximization problem. In light of this interpretation, the choice of such a utility for our IAM makes our analysis most robust to worst-case scenarios.

[1] See van der Ploeg (1984a) and van der Ploeg (1984b) for the original technical framework.

We consider these preferences to be the best possible choice for the analysis of climate-related problems because, given the large uncertainty inherent in climate science, they enable decision makers to search for solutions that are robust to a wide range of worst-case climate change scenarios. We find that the year 2020's optimal SCC is US$162 per tCO_2 and that the expected SCC increases to about US$1,214 per tCO_2 in 2120. Our baseline calibration implies full decarbonization by 2068, resulting in an expected global average long-run temperature change of around 1.37°C above preindustrial levels, thus well in line with the Paris Accord. We also introduce the *SCC robustness premium*, which we define as the excess SCC for different levels of risk sensitivity. We find that over a plausible range of preference parameters the SCC robustness premium in 2020 ranges from US$1.41 to US$25.89 per tCO_2, with US$2.20 per tCO_2 in our baseline calibration. Moreover, we find that the SCC robustness premium increases significantly over time and goes to zero as the risk-sensitivity parameter approaches zero. Our model's findings have significant managerial implications: Companies will have to anticipate inevitably rising carbon taxes and rising prices of carbon permits in emission trading systems, and plan accordingly. Similarly, forward-thinking companies in emission-heavy industries will want to anticipate these higher costs by proactively planning with higher internal carbon prices. Otherwise they will risk investing in assets that may become stranded, either because regulators prohibit them or simply because carbon-intensive projects will become outright unprofitable. The drastically rising costs of emissions pose significant business risks to such companies in such industries.

The remainder of this paper is organized as follows. Section 2 describes the motivation of our work and explains the novelty of the full-path accumulated robustness concept in the context of IAMs. Section 3 presents our stochastic integrated assessment model with risk-sensitive preferences. In Section 4 we describe solutions from our model and perform comparative as well as sensitivity analyses. Section 5 concludes. The statements and proofs of the main results are provided in Appendix A.

2. Preferences for Robustness

In this section we motivate our work in this paper and explain how the full-path accumulated robustness approach is novel to the IAM literature. We first contrast the notions of per-stage recursive and full-path accumulated robustness. Next, we provide a literature overview of works employing per-stage recursive robustness in the context of climate change economic models. Finally, we provide details on full-path accumulated robustness for our analysis in this paper.

In the following discussion, we denote a consumption stream for some finite time horizon, $N > 0$, by $\{c_t\}_{t=0}^N$, an instantaneous (per-stage) utility function by $u(\cdot)$, and a per-period discount factor by $0 < \beta < 1$. The underlying randomness in the system at time $t \geq 0$ is captured by the expectation operator $E_t[\cdot]$ on the future (random) events starting at and from t.

2.1. Two Notions of Robustness

The standard additive expected utility framework of von Neumann–Morgenstern for maximizing the time-0 discounted utility

$$E_0\left[\sum_{t=0}^{N}\beta^t u(c_t)\right] \tag{1}$$

lacks the flexibility to explore the role of risk aversion while disentangling it from the intertemporal elasticity of substitution (IES). However, this distinction between IES and risk aversion can be achieved within the von Neumann–Morgenstern framework of expected utility analysis through a concave transform of the above time-separable objective function (1), as demonstrated in Kihlstrom and Mirman (1974); that is, by maximizing the full-path certainty equivalent functional

$$\mathcal{U}_k^{-1}\left(E_0\left[\mathcal{U}_k\left(\sum_{t=0}^{N}\beta^t u(c_t)\right)\right]\right) \tag{2}$$

over all admissible consumption plans $\{c_t\}_{t=0}^N$, where $U_k(\cdot)$ is a concave increasing function dependent upon some given risk aversion parameter $k>0$. As pointed out by Epstein and Zin (1989), such preferences are non-stationary and preference orderings generally depend on past consumption values (except when $\mathcal{U}_k(\cdot)$ is exponential, which we discuss later) implying both time-dependent attitudes (tastes) with regard to future gambles and time-inconsistent consumption plans. To address these problems, Epstein and Zin (1989), Svensson (1989), and Weil (1990, 1993) moved away from the intertemporal expected utility framework and instead used the per-stage recursive utility framework developed by Kreps and Porteus (1978, 1979) in order to achieve a separation between intertemporal substitution and risk aversion. In this framework, the agent maximizes at time t an overall continuation utility V_t given as the sum of the current utility $u(c_t)$ and the one-step β-discounted time-t certainty equivalent $\beta\mathcal{E}_t(V_{t+1}) \stackrel{def}{=} \beta\mathcal{U}_k^{-1}\left(E_t\left[\mathcal{U}_k\left(V_{t+1}\right)\right]\right)$ of the future (next-stage $t+1$) continuation utility V_{t+1}; namely,

$$V_t = u(c_t) + \beta\mathcal{U}_k^{-1}\left(E_t\left[\mathcal{U}_k\left(V_{t+1}\right)\right]\right) = u(c_t) + \beta\mathcal{E}_t(V_{t+1}), \tag{3}$$

which proposes to resolve the problems of time inconsistency and time-varying attitudes. Note that the time-t expectation operator $E_t[\cdot]$ captures the underlying future (for time $\geq t+1$) randomness of the system conditional on the events that have unfolded up to and including time t. The preference structure (3), however, generally does not satisfy the natural monotonicity property, which stipulates that an agent will not take an action if another available action is preferable in all circumstances; this lack of monotonicity eventually leads to conclusions that are counterintuitive in the analysis of risk aversion. As shown in Bommier et al. (2017), the only solution to this anomaly is to use exponential functions (or affine transforms thereof) for $\mathcal{U}_k(\cdot)$. Bommier et al. (2017) use, for example, $\mathcal{U}_k(\cdot) \stackrel{def}{=}$

$\frac{1-e^{-k(\cdot)}}{k}$ implying $\mathcal{E}_t^{HS}(\cdot) \stackrel{def}{=} -\frac{1}{k}\ln\left(E_t\left[e^{-k(\cdot)}\right]\right)$ ('HS' for Hansen–Sargent, as explained below), which transforms Equation (3) to the following form:

$$V_t = u(c_t) + \beta\mathcal{E}_t^{HS}(V_{t+1}) = u(c_t) - \frac{\beta}{k}\ln\left(E_t\left[e^{-kV_{t+1}}\right]\right) = -\frac{\beta}{k}\ln\left(E_t\left[e^{-\frac{k}{\beta}\left(u(c_t)+\beta V_{t+1}\right)}\right]\right). \quad (4)$$

Hansen and Sargent (1995) introduced this exponential type of per-stage recursive robustness—so, risk-sensitive preferences—for the study of optimal decision-making under deep uncertainty. Thereafter, it was adopted both in the finance literature (see, e.g., Anderson et al. (2012), Bommier and Le Grand (2019), Bäuerle and Jaśkiewicz (2018), and Tallarini Jr (2000), among others) and in the climate change literature (see, e.g., Hennlock (2009), Anderson et al. (2016), Lemoine and Traeger (2016), Li et al. (2016), Barnett et al. (2020), Brock and Hansen (2018), Rezai and van der Ploeg (2017), Berger and Marinacci (2020), Chari (2018), Cai (2021), Brock and Xepapadeas (2020), Millner et al. (2013), Berger et al. (2017), and Rudik (2020), among others) because it is the only Kreps–Porteus-type of recursive preferences that admits the separation of risk aversion from intertemporal substitution while maintaining monotonicity, as explained above.

In the next section, we briefly elaborate on the contributions made by the aforementioned works—which use the per-stage recursive approach—to better illustrate the main differences between this literature and our work in this paper. In particular, the discussion clearly differentiates this literature from the full-path accumulated approach employed in this paper.

2.2. The Literature on Per-stage Recursive Robustness

Li et al. (2016) and Anderson et al. (2016) solve a standard additive von Neumann–Morgenstern utility model with no tipping and artificially introduced model uncertainty. Lemoine and Traeger (2016) handle 0-1 tipping (i.e., tipping or no tipping; two states) with uncertainty artificially introduced by a hazard rate that defines a tipping probability, whereas we solve a multi-state model with tipping.[2] Moreover, in Lemoine and Traeger (2016) the certainty equivalent is handled as a convex combination of tipping/no-tipping values averaged over all risk disentangling from uncertainty and is closely related to Epstein–Zin–Weil models, whereas our model combines risk with uncertainty in certainty equivalence as a natural consequence of our exponential framework. Hennlock (2009) uses Epstein–Zin–Weil-type utilities with artificially introduced uncertainty. Berger and Marinacci (2020) review, as opposed to our dynamic approach, certain static models of choices under uncertainty, and try to provide an introductory framework for such setups; unlike our framework, however, they do not actually solve any model or provide any existence results. Cai (2021) provides a general nontechnical survey of Epstein–Zin–Weil-type models, but none cover our approach.

[2] Because of our exponential formulation, large deviations theory implies that our model can be transformed into a max–min game. In this game, the minimization is performed by an adversarial external opponent ("Nature") on the entire set of probability measures over the disaster states.

Berger et al. (2017) solves a two-period climate model with one scalar decision variable as opposed to our long-horizon model with a long-term policy decision, and addresses robustness across a finite set of models (distributions over risks), whereas our analysis leads to generic robustness across all possible model distributions. Brock and Hansen (2018) illustrates how risk-sensitive preferences can be applied to climate change economic models to account for three kinds of uncertainty that arise naturally in these models—namely, risk, ambiguity aversion, and model mis-specification. Millner et al. (2013) also studies robustness across a finite set of equally likely (uniformly distributed) models but with power utility preferences as opposed to our generic (across all possible risk distributions) utility maximization with exponential preferences. They model ambiguity aversion regarding climate sensitivity and analyze the effect of such aversion on the stationary equivalent; they do not, however, compute the social cost of carbon. Brock and Xepapadeas (2020) solves a deterministic robust control problem with quadratic "per-stage" utilities, which is quite limited in scope compared to our setup. Chari (2018) presents a nontechnical discussion on uncertainty in climate change economics and does not propose or solve any model.

Klibanoff et al. (2005) again solves a static model of ambiguity aversion wherein ambiguity is modeled as a chosen (by the Decision Maker) "subjective" belief or probability measure over the set of probabilities characterizing the underlying systemic risk or uncertainty[3]. Skiadas (2003) and Chen and Epstein (2002) handle the continuous-time counterpart of Hansen–Sargent recursive utilities—namely, stochastic differential utilities as introduced by Duffie and Epstein (1992)—where Skiadas (2003) makes no assumption regarding the Markov structure of the underlying dynamics and Chen and Epstein (2002) handles multiple priors in such continuous-time setup. Epstein and Schneider (2003) addresses intertemporal utility with multiple priors and ambiguity aversion under a crucial dynamic consistency assumption, unlike the present paper where dynamic consistency follows from the structure of the optimal policies. Hansen and Miao (2018) studies generic per-stage recursive representations of discrete-time intertemporal preferences that allow for ambiguity aversion to (subjective) uncertainty and their corresponding heuristic continuous-time limiting Hamilton–Jacobi–Bellman equations.

Rudik (2020) applies per-stage robustness and solves a model with a stochastic damage function that incorporates parameter learning and uses risk-sensitive preferences to account for misspecification. Although there is an interesting effect of learning on the optimal carbon tax, the author does not find a significant effect from the risk-sensitive preferences. The lack of such an effect

[3] It should be noted here that our approach is different in the sense that we handle a dynamic model, in which the probabilities of the underlying systemic risk are assumed to be known. It is possible to add an additional outer maximization layer involving an ambiguity distribution on these risk measures (characterizing our model). Such a model adjustment would not change the core analysis. We leave this extension for future work.

stems from the narrow choice of values of the penalty parameter θ, which corresponds to the reciprocal of the robustness parameter k that we will use in our model with risk-sensitive preferences. In fact, the values of the robustness parameter in Rudik (2020) vary only between 0 and 0.25. And so, because the preferences are very close to the standard von Neumann–Morgenstern expected utility, risk sensitivity appears to have little influence. The author also reports that his algorithm did not converge for other values of k.

Barnett et al. (2020) handles robustness by using a differential asset-pricing approach to deal with the model uncertainty that enters the corresponding Hamilton–Jacobi–Isaacs second-order semi-linear elliptic PDE as a modified drift with additional quadratic and Kullbach–Leibler divergence penalty terms for model mis-specification, but the authors' derivation and analysis is only heuristic. In this light, it should be noted here that in the abovementioned papers there is neither any existence or regularity proofs of the solutions to the corresponding dynamic programming equations nor any verification proofs of these solutions as the corresponding optimal value functions. Some of these papers only provide a heuristic derivation of one functional form of such a solution but again provide no uniqueness results, thereby leading to some degree of vagueness in the subsequent inferences based on such solutions.

2.3. Full-Path Accumulated Robustness

Bommier et al. (2017) demonstrates that although the use of exponentials or Hansen–Sargent-type risk-sensitive preferences introduces the much-needed monotonicity property to such per-stage recursive preferences, the condition of stationarity of the optimal policies is still not generally satisfied unless $\{\mathcal{E}_t(\cdot)\}_{t=0,1,\ldots,N-1}$ satisfies the following condition:

$$\mathcal{E}_{t+1}(\cdot) = \beta^t \mathcal{E}_1\left(\frac{\cdot}{\beta^t}\right). \tag{5}$$

As can be easily checked now, enforcing such a condition when $\mathcal{E}_t(\cdot) = \mathcal{E}_t^{HS}(\cdot)$ leads to very counterintuitive restrictions on the structure of this certainty equivalent—that is, stationarity in this Hansen–Sargent setup can now only be maintained by using an artificial "amplification mechanism" that compensates for the decrease in risk caused by discounting; namely, $\mathcal{U}_k(\cdot)$ at time t is now implicitly t-dependent and defined as $\mathcal{U}_k^{(t)}(\cdot) \stackrel{def}{=} \mathcal{U}_{k\beta^{-t}}(\cdot) \stackrel{def}{=} \frac{1-e^{-k\beta^{-t}(\cdot)}}{k\beta^{-t}}$. This implies that these preference functions, $\mathcal{U}_k^{(t)}(\cdot)$, decrease with time—thereby inducing a preference for early resolution of uncertainty as expected under risk aversion. In fact, stationarity of optimal policies cannot be generically guaranteed for such preferences given the non-homogeneity of the functional $\mathcal{E}_t^{HS}(\cdot)$ due to its entropic nature.

Also, to the best of our knowledge this per-stage recursive approach does not have closed-form solutions in general in the form of some well-defined value functions (of related dynamic programming equations) for the stage-t continuation utility V_t or even existence results for optimal V_t given some generic $\mathcal{U}_k(\cdot)$. Thus, studying the analytic properties of V_t under various model scenarios is either quite difficult or even not possible. Only recently, Bäuerle and Jaśkiewicz (2018) proved an existence result in this direction and derived some properties of the solution for risk-sensitive exponential $\mathcal{U}_k(\cdot)$ utilities in a specific type of optimal portfolio growth model, only under strong growth and Lyapunov-type stability assumptions. Thereby, Bäuerle and Jaśkiewicz (2018) generalizes, in some sense, the setup of Hansen and Sargent (1995), which studies a more restricted problem with linear state process evolution perturbed by Gaussian noise and quadratic per-stage utility (LQG). Corresponding results for generic $\mathcal{U}_k(\cdot)$ are still open problems (as far as we know).

The preceding discussion points to certain inherent weaknesses of the per-stage recursive framework. However, the literature on climate economics has, to date, adopted this approach. A serious examination of the "full-path accumulated robustness" framework for analyzing such problems along the lines of Bommier et al. (2015) and van der Ploeg (1993) is therefore required. This alternative approach is technically supported by the highly developed machinery of stochastic control theory (see, e.g., Jacobson (1973), Di Masi and Stettner (1999), Bäuerle and Rieder (2014), and Whittle (1990), and references therein). As we show in our derivation of Theorem 2 in Appendix A, this machinery can be used to arrive at concrete existence results as well as closed-form (optimal) solutions to the recursion as value functions of the corresponding dynamic programming (Bellman) equations as well as (weakly) non-stationary deterministic optimal policies. Thus, our approach addresses the methodological weaknesses of the per-stage recursive framework and enables us to provide a sound theoretical foundation for the application of full-path accumulated robustness in climate change economic models.

The present paper is the first application of the full-path accumulated robustness approach in the context of an IAM. The continuation utility V_t at time $t \geq 0$ corresponds to a concave exponential transform (see Formula (2) in Section 2.1) of the von Neumann–Morgenstern utility function (1) and is similar to, but more general than, Bommier et al. (2015) and also the work of van der Ploeg (1993) on precautionary savings in finance. Defining $\mathcal{U}_k(\cdot) \overset{def}{=} -\frac{1}{k}e^{-k(\cdot)}$ and $\mathcal{U}_k^{(t)}(\cdot) \overset{def}{=} \mathcal{U}_{k\beta^t}(\cdot)$, let $\mathcal{E}_t^{FP}(\cdot) \overset{def}{=} \left(\mathcal{U}_k^{(t)}\right)^{-1}\left(E_t\left[\mathcal{U}_k^{(t)}(\cdot)\right]\right) = -\frac{1}{k\beta^t}\ln\left(E_t\left[e^{-k\beta^t(\cdot)}\right]\right)$. We consider the problem of maximizing the full-path accumulated preference V_t (again using Formula (2)) starting at time $t \geq 0$,

$$V_t \overset{def}{=} -\frac{1}{k\beta^t}\ln E_t\left[e^{-k\beta^t\sum_{s=t}^{N}\beta^{s-t}u(c_s)}\right] = u(c_t) + \mathcal{E}_t^{FP}(\beta V_{t+1}), \tag{6}$$

over all admissible consumption plans $\{c_t\}_{t=0}^N$ where the second equality follows from Proposition 1 in Appendix A. A careful look at formulae (4) and (6) reveals that the two notions of robustness discussed in this paper match exactly under no discounting (i.e., $\beta = 1$). In fact, we prove something more fundamental for risk-sensitive preferences in Theorem 1 in Appendix A—namely, that the per-stage recursive utility is always greater than or equal to the full-path accumulated one. This result implies that the former overestimates the optimal value function for $\beta < 1$.

An important point to note here is that whereas van der Ploeg (1993) handles a specific instance of such a problem—similar to what Hansen and Sargent (1995) did for the per-stage recursive setup; namely, the linear evolution of the state space perturbed by Gaussian noise with quadratic per-stage utility functions (LQG)—we make no such assumptions on either our state process evolution or the exact form of our per-stage utilities for our proof technique. For our computational results, however, we have to use specific forms of the per-stage utility and the transition kernel (operator), which we in turn use in the proof of our results in Appendix A (for the sake of continuity and direct relevance).

Another important point to note is the crucial difference between the Arrow–Pratt coefficient of absolute risk aversion $\mathcal{A}^{FP}(\cdot)$ of our preferences and $\mathcal{A}^{EZ}(\cdot)$ of the more familiar Epstein–Zin (EZ) preference described in Section 3.3. In particular, the functional form of the EZ preference is $\mathcal{U}_{(k,\psi)}(\cdot) \stackrel{def}{=} (\cdot)^{\frac{1-k}{1-\frac{1}{\psi}}}$, where $\frac{1}{\psi}$ is the elasticity of intertemporal substitution (EIS). Thus, $\mathcal{A}^{FP}(\cdot) = -\frac{\mathcal{U}_k''(\cdot)}{\mathcal{U}_k'(\cdot)} = k$ whereas $\mathcal{A}^{EZ}(\cdot) = -\frac{\mathcal{U}_{(k,\psi)}''(\cdot)}{\mathcal{U}_{(k,\psi)}'(\cdot)} = \frac{k-\frac{1}{\psi}}{1-\frac{1}{\psi}}\frac{1}{(\cdot)}$, implying different types of attitudes toward risk aversion. In fact, the Arrow–Pratt coefficient of relative risk aversion $\mathcal{R}^{EZ}(\cdot) = (\cdot)\,\mathcal{A}^{EZ}(\cdot) = \frac{k-\frac{1}{\psi}}{1-\frac{1}{\psi}}$ is constant whereas for our preferences the coefficient of absolute risk aversion $\mathcal{A}^{FP}(\cdot)$ is constant. Of course, both of these result in the corresponding Arrow–Pratt coefficient of value 0 for the standard additive von Neumann–Morgenstern utility, since, for this value, $k = \frac{1}{\psi}$ for EZ and $k = 0$ for FP.

Some researchers may consider the existence of non-stationary (instead of stationary) optimal policies—a result of the time-dependent preferences as in (6)—a drawback of our model (and its optimal solution). However, this is not a problem because our optimal policies are still time-consistent in the sense of Johnsen and Donaldson (1985) (see also Hansen et al. (2006)) when viewed as optimal deterministic policies on an extended state space.[4] In fact, these are (weakly) stationary on that extended state space as, by construction, they are sub-path optimal therein (see, again, Theorem 2 and also Proposition 1 in Appendix A). We note that since $\mathcal{E}_t^{FP}(\cdot)$ is an entropic risk measure it is not necessarily homogeneous as a functional of its argument and hence the optimal solution will be time-dependent in any case. This feature is naturally realized in the structure of our optimal policies and cannot be removed by any artificial "amplification mechanism," unlike in the per-stage recursive case; that is, our full-path accumulated approach reflects the true non-stationary nature of such systems in a "hard-coded" way that is not amenable to artificial parametric perturbations

[4] The optimal policy retains the same functional over time on this extended state space as explained in Theorem 2 in Appendix A. Therefore, the original optimal policy is carried through whatever the current state of the system at any time t. Hence, the dynamic preferences of the policy maker admit time-consistency in the sense of Johnsen and Donaldson (1985).

to retain "false" stationarity. Also, as explained in van der Ploeg (1993), in contrast to the per-stage recursive utility approach the customary assumption of indifference to the temporal resolution of uncertainty is retained (see Kreps and Porteus (1978, 1979)). More importantly, the distinction between IES and risk aversion is maintained in our approach without compromising on monotonicity and time-consistency.

As a final remark, we note that our proof can be easily adapted to any general setup by suitably adjusting the kernels, functions, and their domains and then following our proof technique in this paper, which uses the highly developed machinery of stochastic control theory from, for example, Di Masi and Stettner (1999) or Bäuerle and Rieder (2014). Although Bommier et al. (2015) followed a similar approach, the authors neither incorporate discounting in the risk-sensitive preference structure nor provide any proof of existence of the value function as a solution to the corresponding Bellman equation, nor any proof of existence of optimal policies.[5] In the present paper, we provide a novel proof technique for more general climate change problems involving full-path accumulated robustness. In the process, we extend and formalize both the ideas in Bommier et al. (2015) and the LQG setup for such systems studied by van der Ploeg (1993) in precautionary savings, just as Bäuerle and Jaśkiewicz (2018) extended the similar LQG setup for per-stage recursive systems studied by Hansen and Sargent (1995).

3. A Stochastic Integrated Assessment Model

Our stochastic integrated assessment model builds on the concepts of the Ramsey–Cass–Koopmanns growth model (Ramsey (1928), Cass (1965), Koopmans et al. (1963)) for the economic aspect, and interconnects them with a climate module in the spirit of Nordhaus (1993) but replaces the carbon cycle with a linear relationship between temperature and cumulative emissions; see Matthews et al. (2009) and Matthews et al. (2012). The integrated assessment model, then, incorporates both the impact of consumption and mitigation choices on the climate and reversely that of climate damage on production. In all these interconnections lie both risks and deep uncertainties. Climate tipping points in stochastic integrated assessment models have been assessed, for example, in Lontzek et al. (2015) and van der Ploeg and de Zeeuw (2017). Cai et al. (2016) models a complex system of interacting climate tipping points and Cai and Lontzek (2019) also studies multilayered catastrophic risks and uncertainty quantification.[6] In the present paper, we introduce persistent endogenous discrete disaster states that model the risk of climate change–induced disasters that would have a grave effect on the economy. For the better assessment of this risk we use risk-sensitive preferences

[5] Because of the absence of discounting (i.e., $\beta = 1$), the approach in Bommier et al. (2015) is effectively the same as the per-stage recursive framework.

[6] A methodological overview of decision-making under uncertainty and risk in economic models of climate change is presented in Brock and Hansen (2018).

as discussed in detail in Section 2. In our model, we view risk-sensitive preferences as a means of making robust decisions by deliberately putting a greater emphasis on worst-case scenarios. The risk-sensitive parameter k can then be interpreted as the robustness parameter rather than as a risk-aversion or ambiguity-aversion parameter. This approach offers a sensible means of uncertainty quantification that leads to well-balanced, robust decisions, in contrast to simply comparing the results to the worst-case scenario itself.

First, we specify our general model with risk-sensitive preferences and discuss the existence of solutions to our model. Then, we briefly introduce alternative preferences, which we use for the sensitivity analysis. Subsequently, we choose functional forms and a baseline parametrization for the model. Finally, we provide details on the calculation of the social cost of carbon in our model.

3.1. The General Model with Risk-Sensitive Preferences

We denote the world capital stock in trillions of dollars at time t by $K_t \in [0,\bar{K}], \bar{K} > 0$. We write $T_t \in [0,\bar{T}]$, $\bar{T} > 0$ for the rise in the global average temperature compared to preindustrial levels in °C at time t. The stochastic disaster state $\vartheta_t \in \mathcal{S} \equiv \{1,2,3,4\}$ represents the climate disaster–induced persistent damage to production at time t and follows a discrete-time temperature-dependent Markov process. At time t, $C_t \in [0, K_t]$ is world consumption in trillions of dollars, and $\mu_t \in [0,1]$ is the abatement rate. The parameter $k > 0$ denotes the robustness parameter for the risk-sensitive preferences. Capital, K_t, evolves according to the capital accumulation model $C^1(D_{K,t}) \ni g_{K,t} : D_{K,t} \to \mathbb{R}$ where $D_{K,t} = [0,\bar{K}] \times [0,\bar{T}] \times \mathcal{S} \times [0,K_t] \times [0,1]$. Analogously, temperature, T_t, evolves according to the climate model $C^1(D_{T,t}) \ni g_{T,t} : D_{T,t} \to \mathbb{R}$ where $D_{T,t} = [0,\bar{K}] \times [0,\bar{T}] \times [0,1]$. We assume that the utility function $\mathcal{C}^1([0,K_t]) \ni U_t : [0,K_t] \to \mathbb{R}_+ \equiv [0,\infty)$ is strictly increasing and continuous with $U_t(0) = 0$ for $t = 0,\dots,N-1$. We define the expectation operator $\mathbb{E}^{\cdot}[\cdot|\cdot]$ as follows:

$$\mathbb{E}^{(C_t,\mu_t)}\left[e^{-k_t\beta V_{t+1}(K_{t+1},T_{t+1},\vartheta_{t+1})}|K_t,T_t,\vartheta_t\right] = \sum_{\hat{\vartheta}\in\mathcal{S}} p_{\vartheta_t,\hat{\vartheta}}(T_t) e^{-k_t\beta V_{t+1}(g_{K,t}(K_t,T_t,\vartheta_t,C_t,\mu_t),g_{T,t}(K_t,T_t,\mu_t),\hat{\vartheta})}, \tag{7}$$

using the function $C^1([0,\bar{T}]) \ni p_{\vartheta_t,\hat{\vartheta}} : [0,\bar{T}] \to [0,1]$, where $p_{\vartheta_t,\hat{\vartheta}}(T_t)$ denotes the time-t transition probability from $\vartheta_t \in \mathcal{S}$ to $\hat{\vartheta} \in \mathcal{S}$ when the rise in the global average temperature is T_t. Note, we denote $k_t \equiv k\beta^t$ and hence $k_0 = k$. Now, using Equation (6) and putting $E_t[\cdot] = \mathbb{E}^{(C_t,\mu_t)}[\cdot|K_t,T_t,\vartheta_t]$ therein, the social planner proposes the following Bellman optimality equation for our model:

$$\begin{aligned} V_t(K_t,T_t,\vartheta_t) &= \max_{C_t,\mu_t} \left\{ U_t(C_t) - \frac{1}{k_t}\ln\left(\mathbb{E}^{(C_t,\mu_t)}\left[e^{-k_t\beta V_{t+1}(K_{t+1},T_{t+1},\vartheta_{t+1})}|K_t,T_t,\vartheta_t\right]\right)\right\} \\ \text{s.t.} \quad & K_{t+1} = g_{K,t}(K_t,T_t,\vartheta_t,C_t,\mu_t), \\ & T_{t+1} = g_{T,t}(K_t,T_t,\mu_t), \end{aligned} \tag{8}$$

for $t = 0,\dots,N-1$ (where each period t is equal to one year) with $V_N(\cdot) \equiv h(\cdot)$ for some $C^1([0,\bar{K}]) \ni h : [0,\bar{K}] \to \mathbb{R}$ ($h(\cdot)$ may be a constant; possibly 0).

3.2. Existence of Solutions

The existence of a value function and an optimal policy as a solution to the social planner's problem with time-separable additive utility functions is well understood. However, these results do not apply to our risk-sensitive preference framework because of its recursive and multiplicative structure. In order to solve the social planner's dynamic programming problem (8) with standard numerical methods, however, we need to show the existence of a solution to the Bellman equation (8) and the existence of a corresponding optimal (deterministic) Markov strategy (as opposed to policy correspondences), and verify that the full-path accumulated robust utility functional $\mathcal{E}_t^{FP}(\cdot)$ is indeed optimized at this solution for each $t \geq 0$, attaining such optimality by this optimal Markov policy. For completion therefore, we state and prove such an existence theorem (Theorem 2 in Appendix A) for our model under the stated assumptions on the utility functions U_t and the constraint functions $g_{K,t}$ and $g_{T,t}$ for all $t = 0, 1, \ldots, N-1$, and the transition probability function $p_{\vartheta_t,\hat{\vartheta}}$. Using results from stochastic control theory, as proved in Bäuerle and Rieder (2014) (see also Di Masi and Stettner (1999)), Theorem 2 asserts the existence of a value function solving the Bellman equation (8) that maximizes the full-path accumulated robust utility functional (6) at each time $t \geq 0$ under the corresponding optimal deterministic (weakly) non-stationary Markov policy, thereby proving sub-path optimality, and hence time-consistency for our full-path robust preference.

3.3. Alternative Preferences

In our sensitivity analysis, we also solve the integrated assessment model with time-additive CRRA preferences and with Epstein–Zin preferences and compare the results from these model specifications to those of the model using risk-sensitive preferences. For these preferences, the social planner's problem is stated as follows:

$$\begin{aligned} V_t(K_t, T_t, \vartheta_t) \;&=\; \max_{C_t,\mu_t} \left\{ U_t(C_t) + \beta\phi^{-1}(\mathbb{E}\left[\phi(V_{t+1}(K_{t+1}, T_{t+1}, \vartheta_{t+1}))\right]) \right\} \\ \text{s.t.} \qquad & K_{t+1} = g_{K,t}(K_t, T_t, \vartheta_t, C_t, \mu_t), \\ & T_{t+1} = g_{T,t}(K_t, T_t, \mu_t), \end{aligned} \tag{9}$$

where $\phi(v) = v$ corresponds to time-additive preferences using CRRA-type utility and

$$\phi(v) = \begin{cases} v^{(1-k)/(1-1/\psi)} & \text{if } v \geq 0 \\ (-v)^{(1-k)/(1-1/\psi)} & \text{if } v < 0 \end{cases}$$

represents Epstein–Zin preferences (Epstein and Zin 1989).[7] For Epstein–Zin preferences, when the risk-aversion parameter, k, is equal to $1/\psi$, then we obtain the standard time-additive CRRA utility function. Epstein–Zin preferences disentangle the two parameters and allow modelers to choose any two positive values.

[7] See Cai and Lontzek (2019) for the value function transformation from utility U_t to the value function V_t.

Epstein–Zin preferences have been successfully applied in management science and economics. The IAM of Cai and Lontzek (2019) employs a social planner with such preferences. Westermann (2018) uses a continuous-time version of these preferences (with $k = 10$ and $\psi = 1.5$) in her analysis of the impact of manager–shareholder agency conflicts and macroeconomic risk on corporate policies and firm value. Cai et al. (2018) also uses these preferences, in a continuous-time portfolio selection model with capital gains taxes. The authors choose $k = 3$ and $\psi = 1.43$. In recent years, researchers in decision analysis have repeatedly tried to estimate the average EIS from data. Brown and Kim (2014) presents an experiment designed to elicit subjects' preferences regarding risk, time, intertemporal substitution, and uncertainty resolution. Results reveal that most subjects prefer an early resolution of uncertainty, which means that their relative risk aversion exceeds the reciprocal of the elasticity of intertemporal substitution, $k > 1/\psi$. That is, the results are consistent with the properties of recursive preferences and refute CRRA preferences. Kapoor and Ravi (2017) reports results from a natural experiment and estimates the elasticity of intertemporal substitution to be 2.2. Burgaard and Steffensen (2020) estimates the parameters of Epstein–Zin utility based on a questionnaire, simultaneously estimating risk aversion, the subjective discount rate, and the EIS. The authors find substantially larger values for the EIS than traditionally used.

Even though Epstein–Zin preferences have been very popular, some papers question whether they have been used appropriately. Epstein et al. (2014) argues that the parameter values used in the long-run risk literature in finance—$k = 10$ and $\psi = 1.5$; see, for example, Bansal and Yaron (2004)—imply that agents are willing to give up an unrealistically large portion of their future consumption stream for an early resolution of risk. Put differently, the preference for an early resolution of risk is much too strong compared to people's actual behavior. Thus, such Epstein–Zin-type utilities are not so effective in preference specification for early uncertainty resolution separately from fluctuations in risk aversion and intertemporal substitution. For example, as shown in de Groot et al. (2018), this Epstein–Zin type preference specification violates an economically meaningful restriction on the weights in its time-aggregator in the context of valuation risk. Consequently, again as shown therein, when the corrected preference specification is combined with Bansal–Yaron long-run risk (see Bansal and Yaron (2004)), then valuation risk plays a significantly smaller (compared to the original Epstein–Zin framework) role in determining asset prices. A different line of criticism, notably brought forth in Bommier et al. (2017) and Bommier and Le Grand (2019), argues that Epstein–Zin preferences are non-monotone and therefore could lead to counterintuitively large precautionary savings.

3.4. Parametrization and Calibration

We assume that gross world output before climate damage is modeled by the Cobb–Douglas function

$$f(K, L, A) = AK^{\alpha}L^{1-\alpha}, \tag{10}$$

where $\alpha = 0.3$ is the capital share of production and A is the factor productivity. We specify gross world output according to the DICE 2016-R calibration in Nordhaus (2017) and set the present-time value of capital, K_0, to 318.85 trillion dollars,[8] and the present-time total factor productivity, A_0, to 6.2529.[9] The path of world population (in billions),

$$L_{t+1} = (1 + 0.0115e^{-0.03t})L_t, \text{ where } L_0 = 7.795, \tag{11}$$

is calibrated such that it matches the median pathway (Median PI) from United Nations and Social Affairs (2019) until the year 2100 and asymptotically reaches just under 11 billion people after 300 years. This is close to the calibration in Nordhaus (2017), which goes to 11.5 billion asymptotically.

Climate damage to production depends on the temperature T and the disaster state ϑ, where $\vartheta = 1$ denotes the state with no additional disasters and higher ϑ represent higher disaster damage. Thus, we define the damage function as

$$\Omega(T, \vartheta) = (1 - \pi_2 T^2)(1 - w_\vartheta(T)), \tag{12}$$

where $\pi_2 = 0.00236$ and $w_\vartheta(T) = a_\vartheta T^{b_\vartheta}$, with parameter values shown in Table 1. The damage function is the rate of production that remains after climate damage; that is to say, $\Omega(T, \vartheta) f(K, L, A)$ would be the GDP after climate damage.

	$\vartheta = 1$	$\vartheta = 2$	$\vartheta = 3$	$\vartheta = 4$
a_ϑ	0	0.01	0.015	0.02
b_ϑ	0	0	1	2

Table 1 **Disaster Damage Parameters**

We assume that the world starts in state $\vartheta = 1$, which is calibrated to match Nordhaus's DICE 2016-R model (Nordhaus 2017). Thus, we define $w_1(T) = 0$. The remaining parameters for the damage $w_\vartheta(T)$ and the transition probabilities $p_{ij}(T)$, $i, j \in \mathcal{S}$ are calibrated such that the worst case matches the upper 95% damage function in Howard and Sylvan (2020). Additionally, the transition probabilities are chosen such that the disaster states are fairly persistent and that the probability of evolving into a worse state rises with the temperature. More precisely, the transition probabilities between disaster states ϑ are given by

$$p(T_t) = \begin{pmatrix} 0.96 & 0.02 & 0.02 & 0.00 \\ 0.03 & 0.93 & 0.02 & 0.00 \\ 0.00 & 0.03 & 0.95 & 0.02 \\ 0.01 & 0.03 & 0.03 & 0.93 \end{pmatrix} + \begin{pmatrix} -\Delta p(T_t) & \Delta p(T_t) & 0 & 0 \\ 0 & -\Delta p(T_t) & \Delta p(T_t) & 0 \\ 0 & 0 & -\Delta p(T_t) & \Delta p(T_t) \\ 0 & -0.1\Delta p(T_t) & -0.1\Delta p(T_t) & 0.2\Delta p(T_t) \end{pmatrix} \tag{13}$$

[8] We are denoting the present value of capital in 2020 US dollars.

[9] Although we are using the exact same formulation for the time path of A as Nordhaus (2017), an alternative, still accurate description of the first 200 periods is $A_t = 6.2529 + 0.1123 + 0.00047t^2$.

with

$$\Delta p(T_t) = 1 - \exp(-p_{Hazard} T_t),$$

and transition probability parameter $p_{Hazard} = 0.015$.

The long-run expected damage function (12) is calibrated to roughly match the median damage function, even though for real pathways the damage function depends not only on the current temperature but also on the history of temperatures because of the endogeneity in the transition law.

We use the calibration for the ratio of mitigation cost-to-output from Nordhaus (2017):

$$\Lambda_t(\mu) = \theta_1(t)\mu^{\theta_2}, \tag{14}$$

where $\theta_1(t) = 0.55(1-0.025)^{\frac{t+5}{5}} \frac{\sigma(t)}{\theta_2}$ and $\theta_2 = 2.6$. Capital K evolves according to

$$K_{t+1} = (1-\delta)K_t - C_t + (1-\Lambda_t(\mu_t))\Omega(T_t, \vartheta_t) f(K_t, L_t, A_t) \equiv g_{K,t}(K_t, T_t, \vartheta_t, C_t, \mu_t), \tag{15}$$

where $\delta = 0.1$ is a constant rate of depreciation of physical capital stock.

The carbon intensity,

$$\sigma(t) = (1+g_\sigma)^{t-1}\sigma_0, \tag{16}$$

with parameters $\sigma_0 = 0.3255$ and $g_\sigma = -0.0145$, denotes the rate of emissions to output in $GtCO_2$/\$ trillion (see Nordhaus (2017)). The emissions E_t in $GtCO_2$ are then defined as

$$E_t = \sigma(t)(1-\mu_t) f(K_t, L_t, A_t). \tag{17}$$

Thanks to advances in climate modeling, we can use a simple climate model that makes use of the finding that there is a linear relationship between temperature and cumulative emissions; see Matthews et al. (2012) and Matthews et al. (2009).[10] In fact, Dietz et al. (2021) argues that this simple model performs even better than the much more involved carbon cycle model in DICE 2016-R (Nordhaus 2017), and the statistical analysis in Miftakhova et al. (2020) finds significant explanatory power of the simple linear relationship. We model the temperature equation as

$$T_{t+1} = \eta E_t = \eta\sigma(t)(1-\mu_t) f(K_t, L_t, A_t) =: g_{T,t}(K_t, T_t, \mu_t), \tag{18}$$

where $\eta = 0.002 \times (12/44)$ is the transient response to cumulative CO_2 emissions in °C/$GtCO_2$.[11]

[10] This relationship has already been used in integrated modeling. See, for example, Barnett et al. (2020) and Anderson et al. (2016).

[11] Because our model does not account for exogenous emissions from land-use change and forestry, our choice for η is roughly 1.15 times the mean estimate of Matthews et al. (2012). We adopt this linear scaling factor motivated by the analysis in Simmons and Matthews (2016).

The utility U_t is given by

$$U_t(C_t) = \frac{(C_t/L_t)^{1-1/\psi}}{1-1/\psi}. \tag{19}$$

The assumptions of Theorem 2 on the utility function require $\psi > 1$, which is in line with the $1 < \psi \leq 2$ range assumed in the literature on long-run risk and asset pricing (see, among many other works, Bansal and Yaron (2004), Pohl et al. (2018), and Epstein et al. (2014)) and the values found in the management science literature on estimating this elasticity (see Brown and Kim (2014) and Kapoor and Ravi (2017)).

Finally, we assume that the utility discount rate is given by $\beta = 0.985$ and that the terminal value function h is equal to some constant value.

3.5. The Social Cost of Carbon

In the language of mathematical optimization the SCC is a relative shadow price of the atmospheric stock of carbon. Measured in units of the numeraire good (in our model, capital or consumption) it allows us to put a price tag on the negative externality caused by an additional unit of carbon emitted. We define the social cost of carbon in units of dollars per ton of carbon as

$$SCC_t = -1000\eta \left(\frac{\partial V_t}{\partial T_t}\right) / \left(\frac{\partial V_t}{\partial K_t}\right), \tag{20}$$

where η is the transient response to cumulative CO_2 emissions (measured in $GtCO_2$). Because the SCC is expressed in US$/$tCO_2$, we use $-1,000$ as a scaling factor to adjust for the fact that K is measured in 10^{12} US$ and cumulative CO_2 emissions are measured in $GtCO_2$, thus, 10^9 tCO_2. The formulation of the SCC as a relative shadow price implies that the path of the SCC will depend on the marginal value of capital and the marginal value of the degree of global warming. These marginal values will reflect the expectation of the risks associated with climate disasters.

3.6. Numerical Solution Method

We solve the social planner's problem specified in (8) using value function iteration. The value function has three arguments, a single discrete state variable, ϑ, and two continuous state variables, K and T. First, we estimate a reasonable deterministic model so that we can build a time-varying grid of Chebyshev nodes around the solution paths that satisfy the condition that all simulation paths of the stochastic model stay well within these grids. We then iteratively solve the Bellman equation on these discrete Chebyshev nodes and approximate the value function for the two continuous state variables with multivariate complete Chebyshev polynomials while imposing shape preservation constraints, including a positive first derivative and a negative second derivative with respect to capital; see Appendix B for further numerical details and an analysis of the numerical approximation error.[12]

[12] See Judd (1998) for an extended discussion of the mathematical and computational details. See Cai (2019) for a discussion on approximations with complete Chebyshev polynomials.

4. Results

We solve the 300-period stochastic dynamic programming problem and obtain the optimal decision rules for consumption and mitigation in any time period.[13] We use these optimal decision rules, the dynamic equation for the two continuous state variables, and the Markovian transition rules for the discrete-state disaster variable to simulate 50,000 paths of the model economy—starting in 2020. The large number of simulations allows us to study the statistical distribution of the dynamic model variables. We first report numerical solutions for the baseline version of the model and interpret them in the context of our IAM. Subsequently, we provide some sensitivity analysis on a few key model parameters.

4.1. The Baseline Model

The results for the benchmark model with $\psi = 1.5$ and $k = 5$ computed with 50,000 Monte Carlo simulations together with the range from worst to best case are shown in Figure 1.[14] We also compute both the deterministic worst-case scenario by setting $\vartheta \equiv 4$ and the deterministic best-case scenario with $\vartheta \equiv 1$ as standard robustness checks. The dashed blue lines represent the pathways for the deterministic model where the disaster state is constantly set to $\vartheta = 1$, which represents the Nordhaus (2017) damage calibration.

The dashed red lines are the deterministic pathways in the worst disaster state $\vartheta = 4$. The orange pathways show the mean path out of 50,000 Monte Carlo simulations of the stochastic model with parameters $\psi = 1.5$ and $k = 5$. The light gray area represents the range for these simulations whereas the dark gray area represents the 75% interval range. As shown in Figure 1, the social cost of carbon in year 2020 is US\$162/tCO_2 and this grows to a mean social cost of carbon of US\$1,233 /tCO_2 in year 2120; the SCC ranges from US\$969/tCO_2 to US\$1,554/tCO_2 by that time. If we only had to expect little damage as in Nordhaus (2017), which corresponds to permanently being in disaster state 1 in our model, the SCC would only amount to US\$67/tCO_2 in 2020 and grow to US\$520/tCO_2 in 2120. Conversely, if we would assume a permanent disaster state 4—which still corresponds to a much lower damage function than the upper 95% confidence interval in Howard and Sylvan (2020)—the SCC would start at US\$371/tCO_2, and grow to US\$2,462/tCO_2 in 2120. The expected temperature, starting at 1°C above preindustrial levels, would grow roughly linearly until 1.28°C in 2045 and then asymptotically reach its steady state of 1.37°C in 2065. The temperature in our model after 2065 ranges between 1.28°C and 1.40°C. This is rather narrow compared

[13] We run the model for 300 years but only report results for the first 100 years. The terminal value function has only a negligible effect on the results in the first 100 years.

[14] Although there is a rich body of literature to draw from for calibrating the EZ parameters (see Section 3.3) there are no microfounded analyses suggesting an appropriate choice of k, the robustness parameter. We address that issue in Subsection 4.3 by solving our model for a broad range of k values, and validate our choice of k by checking the endogenous discount rate of damage arising from our model.

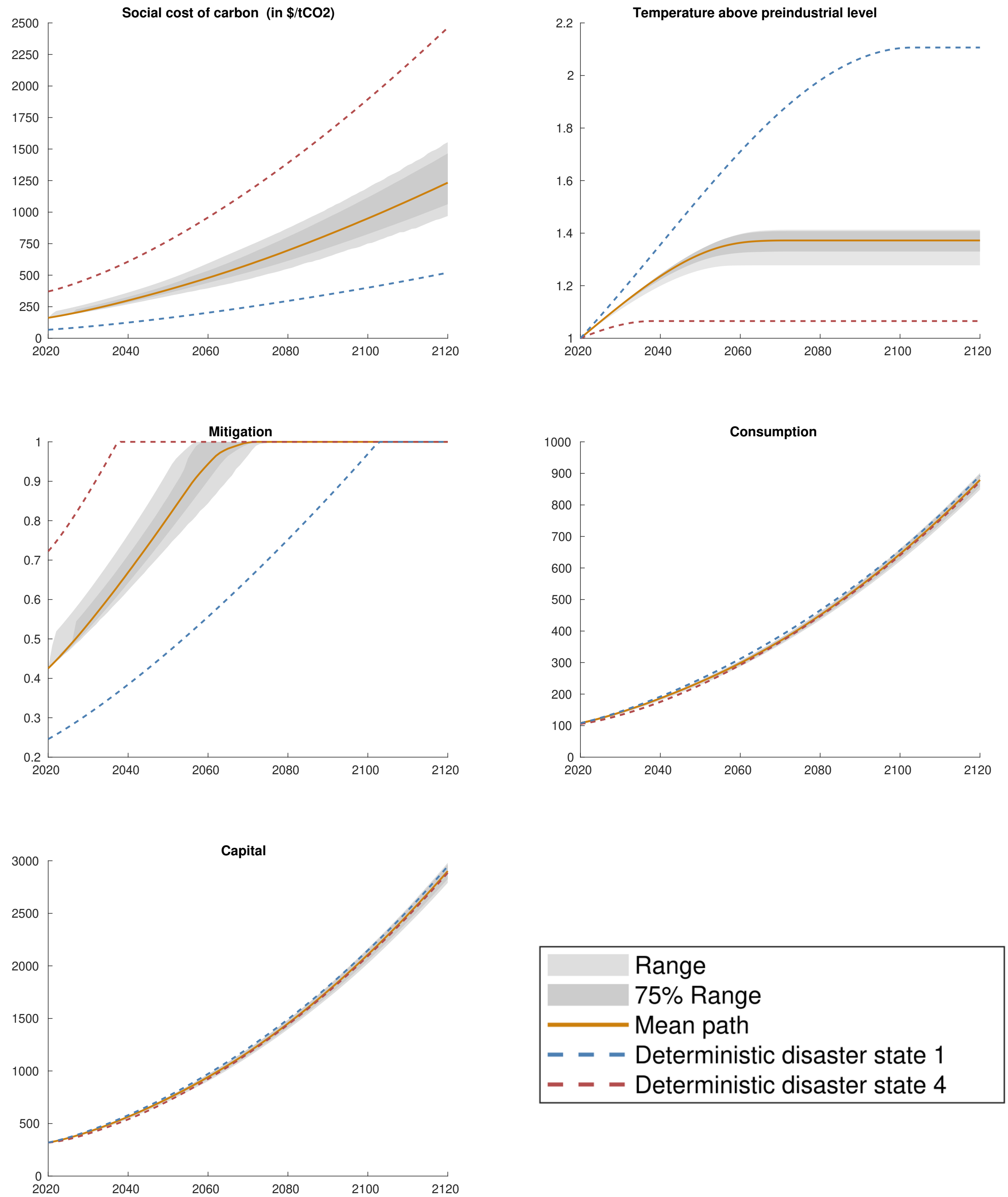

Figure 1 Results of the Benchmark Parameter Case

to the optimal long-run temperature of 1.07°C if we expect the permanent high damage in disaster state 4 and to the optimal long-run temperature of 2.11°C if we assume only little damage as in disaster state 1. Figure 1 shows that the optimum abatement rate in 2020 would have been 43% and that the world should reach zero emissions between the year 2057 and the year 2080. Analogously to the temperature, there is also a very wide range of optimal abatement efforts for the highest and the lowest deterministic damage functions. If we permanently expect the worst case—that is, the damage rate from state 4—we have already seen that the optimal temperature path would not exceed 1.07°C. In order to ensure this, one would have to start with a high abatement effort of 72% and reach carbon neutrality in 2038. On the other hand, if damage becomes as small as assumed in Nordhaus (2017), it would be sufficient to abate at only 25% in 2020 and reach zero emissions in 2103.

Interestingly, the variation in consumption and capital is rather low because the social planner chooses the mitigation effort optimally such that climate damage is kept in a range where it does not significantly reduce consumption. In fact, the consumption paths of our stochastic model range from US$849 trillion to US$901 trillion in 2120, whereas the two deterministic consumption paths for the best disaster and worst disaster states are even much closer, where consumption for the deterministic disaster state 1 is US$893 trillion and that for the deterministic disaster state 4 is US$873 trillion. So, we see that even in the case of permanent high climate damage, as in disaster state 4, consumption in 2120 would still be significantly higher than in the worst pathway of the stochastic damage model—as long as we have correctly anticipated the higher damage and chosen the high abatement efforts beforehand.

4.2. Damage

Figure 2 shows the ratio for each disaster state from the year 2020 to the year 2120 in 50,000 Monte Carlo simulations. We observe a sharp rise in occurrences of the disaster states 2 and 3 early in this period whereas the occurrence of disaster state 4 emerges rather slowly. In the period 2060 to 2080, the occurrence shares of the disaster states have approximately reached the steady state. Now we can plot the damage rate (as a share of GDP) in each disaster state and also the abovementioned steady state and compare it to damage rates given in the literature. This plot is shown in Figure 3. In fact, we have calibrated the damage states and the transition probabilities such that the steady-state damage rate matches the median damage rate from Howard and Sylvan (2020) quite well. All four damage rate functions for the four different damage states lie well within the 95% confidence interval shown in Howard and Sylvan (2020). More precisely, the highest damage rate, in state 4, is 20% of GDP at a temperature increase of 3°C whereas the Howard and Sylvan (2020) upper bound of the 95% confidence interval shows damage of 40%. Given that the occurrence of damage state 4 is 19% in the steady state, our calibration seems sensible.

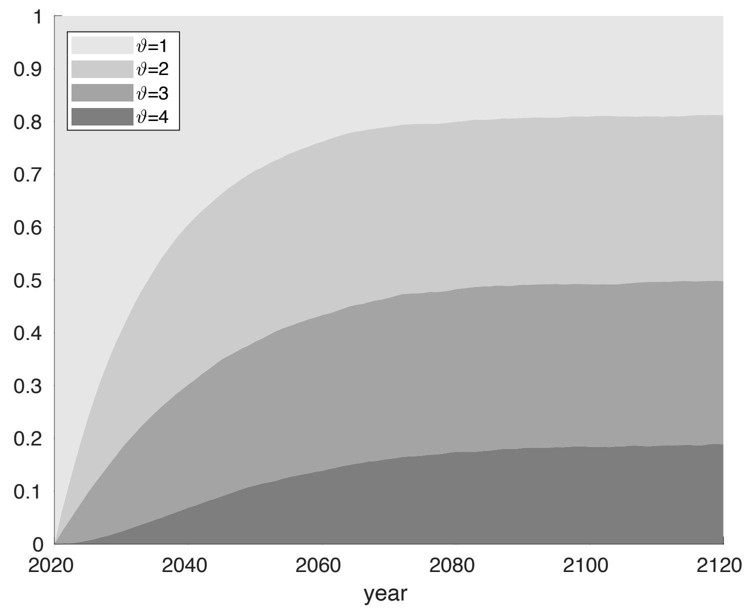

Figure 2 Occurrence Share of each Disaster State

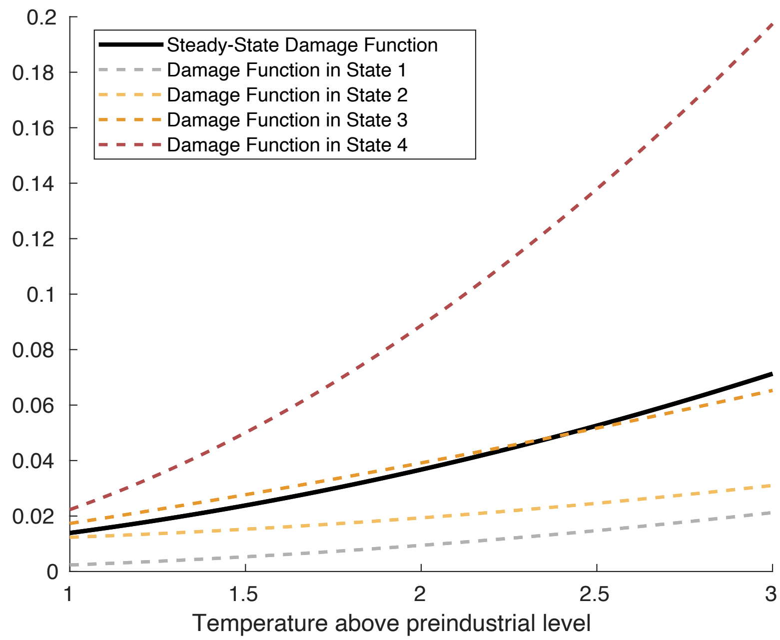

Figure 3 Damages as a Share of GDP

4.3. Discounting Damage

In order to verify that our choice of the range for the robustness parameter k is sensible, we compute the endogenous discount rate of climate damage using the methodology applied in Cai and Lontzek (2019). Although we compute the social cost of carbon as the negative marginal value of emissions over the marginal value of capital, one can also compute the SCC by discounting the expected extra damage from emitting an additional unit of CO_2 and using a given discount rate. So, we can write the present SCC as

$$SCC_0 = \sum_{t=0}^{\infty}(1+\rho_\Delta)^{-t}\Delta_t, \tag{21}$$

where ρ_Δ denotes the social discount rate and Δ_t the expected additional damage from adding one unit of CO_2 at time $t=0$. So, given the social cost of carbon and the expected additional damage computed from our model in (8), we can compute the discount rate of damage ρ_Δ using (21). Table 2 shows that the discount rate for our model with risk-sensitive preferences and $\psi = 1.5$ ranges from 2.2% to 2.5% and decreases slightly with higher k. We observe that the implied discount rate of

Preference parameter k	0.1	5	10	20	30
Discount rate of damage (in %)	2.5	2.4	2.4	2.3	2.2

Table 2 Discount Rate of Damage for Risk-Sensitive Preferences with $\psi = 1.5$

damage is rather robust to values of k in a range between 0 and 30. Moreover, the value of the discount rate fits well to the values chosen by the Interagency Working Group on the Social Cost of Greenhouse Gases, which uses a discount rate of 3% as a baseline for its calculations of the SCC (and rates of 2.5% and 5% for sensitivity analyses). Put differently, if we view this exercise as a calibration approach for the robustness parameter k, then our chosen values for k in the range $(0,30)$ appear reasonable.

4.4. The SCC Robustness Premium

We can also analyze and compare the evolution of the mean SCC over time for different robustness parameters k in the stochastic model with risk-sensitive preferences and for different risk aversion parameters k for the model with Epstein–Zin preferences. The mean SCC pathways for $\psi = 1.5$ for both risk-sensitive and Epstein–Zin preferences are shown in Figure 4.[15]

Whereas the SCC in year 2020 approximately ranges from US\$162/tCO_2 to US\$174/tCO_2 both for risk-sensitive preferences with robustness parameters k between 5 and 30 and for Epstein–Zin preferences with risk aversion k between 5 and 30, the gap widens much more in the case of Epstein–Zin preferences in later periods compared to the case of risk-sensitive preferences. In 2120, the SCC for risk-sensitive preferences with $k = 30$ is only US\$1,248/tCO_2, compared to US\$1,357/tCO_2 for Epstein–Zin preferences with $k = 30$. For $k \to 0+$ and $k = 1/\psi$, both preferences coincide and are the same as simple additive time-separable CRRA preferences, so that the SCC in 2120 is the same

[15] Tables 7 and 8 in Appendix C provide numerical values for the SCC in the model with risk-sensitive preferences in the years 2020 and 2120 for four different levels of the EIS, $\psi \in \{1.25, 1.5, 1.75, 2\}$. Tables 9 and 10 report the corresponding SCC values for Epstein–Zin preferences. The values in the rows for $\psi = 1.5$ are represented in Figure 4.

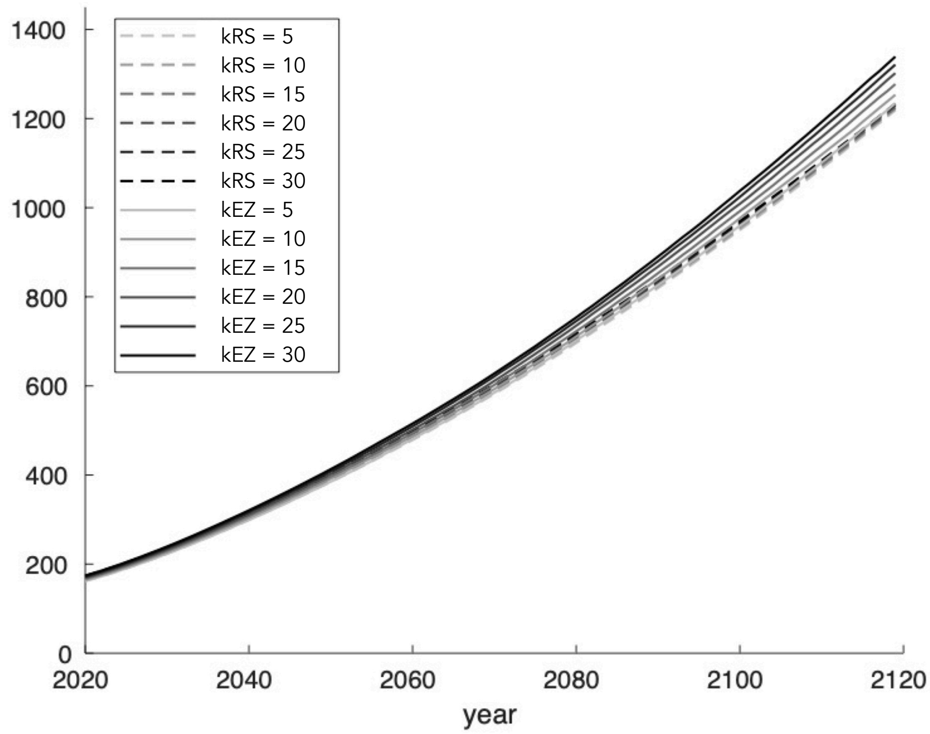

Figure 4 **Mean SCC Values (US\$/tCO_2) for both the Model with Risk-Sensitive Preferences and the Model with Epstein–Zin Preferences**

as well, and amounts to US\$1,233/tCO_2. Therefore, the range for risk-sensitive preferences in 2120 is approximately US\$15/tCO_2, whereas the range for Epstein–Zin preferences is US\$124/tCO_2.

Next we analyze how the robustness parameter k in risk-sensitive preferences and the risk aversion parameter k in Epstein–Zin preferences—under different values for the EIS ψ—influence the social cost of carbon at initial time $t = 1$. We know that there is one case for which the risk-sensitive and the Epstein–Zin preferences coincide—namely, for $k \to 0+$ and $k = 1/\psi$. Moreover, these two preferences become equivalent to the time-additive CRRA preferences. In the following we study the optimal SCC in 2020. In Figure 5, the farthest left value—that is, $k \to 0+$—therefore also denotes the SCC for CRRA preferences.

In the case of Epstein–Zin preferences we observe that for a given value of ψ the 2020 SCC increases linearly with higher levels of the risk parameter k. Moreover, higher values of ψ lead to a parallel upward shift of the linear k–SCC mapping. By contrast, risk-sensitive preferences induce a higher increase in the 2020 SCC for higher ψ values. For $\psi = 1.5$, the robustness parameter k in the model with risk-sensitive preferences and the risk aversion parameter k in the model with Epstein–Zin preferences seem to have a strikingly similar effect on the SCC. This can be explained by the fact that the higher the EIS is, the higher the influence from possible future disasters is.

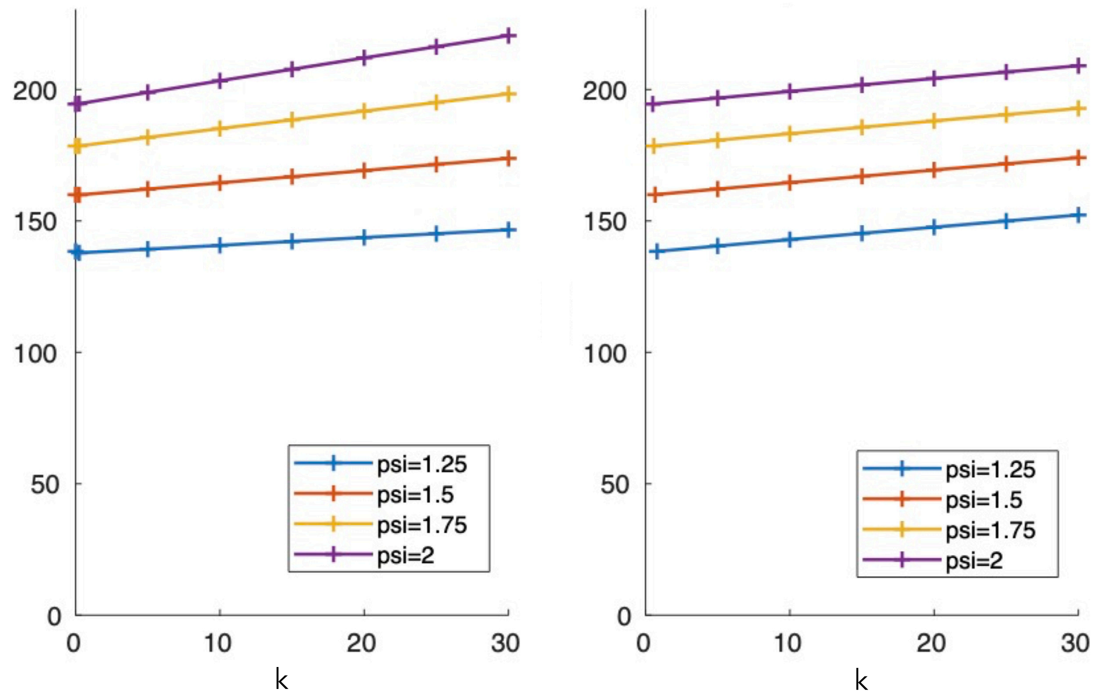

Figure 5 **The 2020 SCC (US$/tCO$_2$) in Relation to the Robustness Parameter k for Risk-Sensitive Preferences (left) and to Risk Aversion k for Epstein–Zin Preferences (right)**

Clearly, the socially optimal carbon tax (that is, the SCC) is increasing with the preference for robustness. We introduce the term "SCC robustness premium" to describe the excess SCC with risk-sensitive preferences (for any level of k) over the SCC level under CRRA. The intuition behind the SCC robustness premium is straightforward: given the disaster risk, an aversion to uncertainty, and thus stronger preferences for robustness, will lead to much higher emission reductions in 2020 and thus put a premium on the SCC over standard CRRA preferences. Table 3 summarizes the SCC robustness premium in the ψ-k parameter space of our sensitivity analysis.

	$k=5$	$k=10$	$k=15$	$k=20$	$k=25$	$k=30$
$\psi=1.25$	1.41	2.86	4.37	5.86	7.31	8.78
$\psi=1.50$	2.20	4.57	6.93	9.25	11.60	13.89
$\psi=1.75$	3.24	6.59	9.92	13.25	16.53	19.79
$\psi=2.00$	4.27	8.70	13.08	17.43	21.67	25.89

Table 3 **SCC Robustness Premium in US$/tCO$_2$ in Year 2020**

Our baseline calibration implies a SCC robustness premium of US$2.20, but within our plausible range of preference parameters the SCC robustness premium may be as high as US$25.89 in 2020 (for $\psi=2$ and $k=30$). This US$25.89 premium is in excess of the US$194.48 SCC in 2020 for CRRA preferences—a relative premium of 13.31%. Moreover, as expected, this robustness premium goes to zero as the robustness parameter $k \to 0+$, as in such a situation the model reduces to the standard CRRA setup, implying no premium for robustness (see Figure 6).

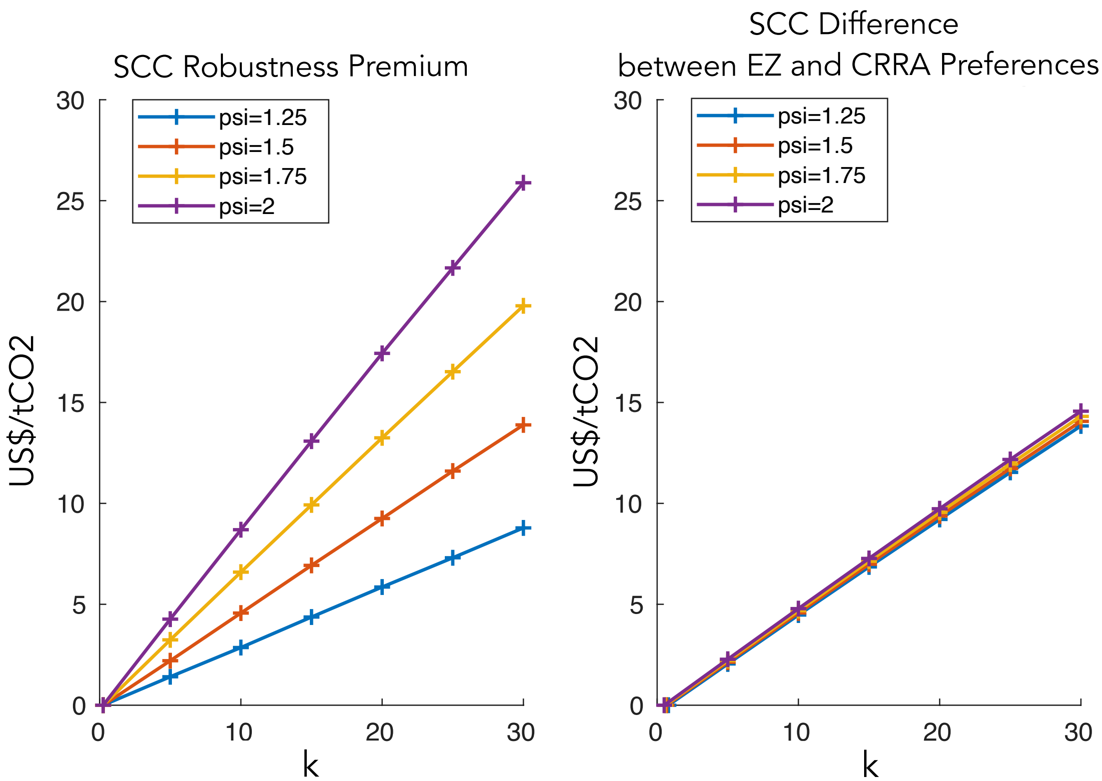

Figure 6 **SCC Robustness Premium (left, in US\$/tCO$_2$) and SCC Difference between EZ and Time-Additive Utility (right, in US\$/tCO$_2$) in 2020**

4.5. Sensitivity Analysis

In this section, we provide some sensitivity analysis for our stochastic integrated assessment model. First, we compare the SCC of the per-stage recursive to the full-path accumulated approach for different values of the discount factor. Subsequently, we describe the effect of larger tail risks in the model on the SCC and the robustness premium.

4.5.1. Comparison of Robust Approaches Recall the notation from Section 2: the indices HS and FP refer to quantities from the models with per-stage recursive and with full-path accumulated preferences, respectively. We now compare the SCC for the HS and FP models. Table 4 reports the SCC for different values of the discount factor, β.

We observe that the SCC in the HS model exceeds the corresponding value for the FP model. This is not an accident. Theorem 1 in Appendix A states that, at any time $t \geq 0$, the HS maximum utility $V_t^{*HS} \geq V_t^{*FP}$ (the FP maximum utility)[16]. We therefore expect the corresponding SCCs to

[16] Note that, in our case here, it follows from Theorem 2 in Appendix A that V_t^{*FP} is attained by a (weakly) non-stationary time-consistent deterministic Markov (maximizing) policy.

β	SCC(HS)	SCC(FP)	SCC(FP)/SCC(HS)
0.98	121.02	117.49	0.971
0.985	166.79	162.12	0.972
0.99	246.22	240.35	0.976
0.995	409.31	401.16	0.980
0.999	728.79	723.21	0.992
0.9999	853.08	852.30	0.999
1	868.18	868.18	1

Table 4 SCC Comparison for HS and FP in US\$/tCO$_2$ in Year 2020 for $k = 5$

satisfy a similar monotone relation. We further observe that the ratio $SCC(FP)/SCC(HS) \uparrow 1$ as $\beta \uparrow 1$. Recall from the discussion in Section 2—see (4) and (6)—that the two robustness approaches should indeed produce identical results in the limit $\beta = 1$. Finally, note that the SCC values increase substantially as $\beta \uparrow 1$. The larger β, the less the long-horizon per-stage utility values are discounted. As a result, both the values of the utility and the SCC become larger as well. (We hesitate to provide an economic discussion of these large values of the SCC, since for very large values of β close to 1 the time horizon T of the model has a strong effect on the results.)

4.5.2. Toward Tail-Risk Events Here we study the sensitivity of our model results with respect to less frequent but more extreme disaster events. For this purpose we modify the Markov transition matrix to

$$p(T_t) = \begin{pmatrix} 0.96 & 0.02 & 0.02 & 0.00 \\ 0.03 & 0.93 & 0.02 & 0.00 \\ 0.00 & 0.03 & 0.965 & 0.005 \\ 0.01 & 0.03 & 0.03 & 0.93 \end{pmatrix} + \begin{pmatrix} -\Delta p(T_t) & \Delta p(T_t) & 0 & 0 \\ 0 & -\Delta p(T_t) & \Delta p(T_t) & 0 \\ 0 & 0 & -0.5\Delta p(T_t) & 0.5\Delta p(T_t) \\ 0 & -0.1\Delta p(T_t) & -0.1\Delta p(T_t) & 0.2\Delta p(T_t) \end{pmatrix} \tag{22}$$

where we have reduced the probability of moving from disaster state 3 to disaster state 4. At the same time, we increase the disaster intensity in state 4: this new parameterization roughly doubles the

	$\vartheta = 1$	$\vartheta = 2$	$\vartheta = 3$	$\vartheta = 4$
a_ϑ	0	0.01	0.015	0.025
b_ϑ	0	0	1	2.5

Table 5 Disaster Damage Parameters

disaster intensity of state 4 and at the same time halves its occurrence likelihood. As a result, while the SCC levels increase only slightly, by about 3 percent for our benchmark parameterization (Tables 7 and 11 in Appendix C), the SCC robustness premium increases sharply, by about 37 percent to 6.27 US\$/tCO$_2$ in our benchmark case and by about 30–40 percent for other ψ–k parameter combinations. This result hints at a high sensitivity of the SCC robustness premium with respect to tail risk and should be explored in further research with a refined formulation of tail-risk events.

	$k=5$	$k=10$	$k=20$	$k=30$
$\psi=1.25$	1.92	3.86	7.82	11.76
$\psi=1.50$	3.12	6.27	12.55	18.83
$\psi=1.75$	4.37	8.86	17.72	26.39
$\psi=2.00$	5.76	11.62	23.08	34.31

Table 6 **SCC Robustness Premium with Risk-Sensitive Preferences and Higher Tail Risks in US$/t$CO_2$ in Year 2020**

5. Conclusion

This paper seeks to contribute to the ongoing public discussion about appropriate values of the social cost of carbon. For this purpose, the paper analyzes an integrated assessment model in the tradition of the seminal work of Nordhaus (1993). To account for the large uncertainty in physical climate change processes and their effects on economic activity, the model includes an exogenous shock process and risk-sensitive preferences. Unlike the entire literature on the economics of climate change, we do not use the Hansen and Sargent (1995) robustness framework but instead apply, for the first time in this context, a generalized version of the robustness notion studied in van der Ploeg (1993), using ideas from stochastic control theory and entropic risk measures. We state and prove an existence theorem for the value function of the social planner's problem in our model as well as for the existence of a (sub-path) optimal time-consistent deterministic Markov policy. This result serves as a theoretical foundation for the numerical solution of the parameterized model. In the process, we also prove something fundamental for risk-sensitive preferences—namely, that the per-stage recursive utility is always greater than or equal to the full-path accumulated one. This result implies that the former overestimates the optimal value function for $\beta < 1$. This result serves as a theoretical foundation for the sensitivity analysis of our model.

A standard specification of the integrated assessment model suggests values for the social cost of carbon that exceed standard estimates based on CRRA or Epstein–Zin preferences as well as those SCC values used by policy makers. In addition, the SCC model estimates exceed the carbon credit prices observed in the most liquid carbon markets and the internal carbon prices used by many companies. We have introduced the notion of an SCC robustness premium, which we define as the excess SCC for different levels of uncertainty aversion compared to standard CRRA preferences. This SCC robustness premium is always positive, is increasing both over time and in the level of risk sensitivity, and decreases to zero as the latter goes to zero (as expected).

In light of the tremendous uncertainty surrounding climate change and carbon emissions, our analysis suffers from several limitations. We model uncertainty in a rather simple manner using a four-state Markov state. The functional form of the dependence of the transition probabilities on temperature changes is rather ad hoc. Furthermore, in the tradition of many IAMs we abstract from agent heterogeneity and the local effects of climate change. Because of this level of aggregation, the model is a gross simplification of the interaction between climate change and economic activities across the globe. Despite these limitations, we believe our work provides a new direction and methodology for the estimation of the social cost of carbon—an important figure in the discussion on climate change.

In sum, the social cost of carbon is a moving target. The global community requires ongoing research on improving integrated assessment models to continuously update estimates for the SCC, because the SCC will play a crucial role in guiding governments, regulators, and private industry in their efforts to mitigate climate change and to adapt to the irreversible damage it may cause. While the predictions for the SCC in the IAM literature vary greatly, there appears to be widespread agreement that the world will see steadily rising values of the SCC—just as our model predicts. The 6th IPCC assessment report (IPCC 2021) predicts that the goals of the celebrated 2015 Paris Agreement—to keep the rise in the global average surface temperature to 2°C and ideally to 1.5°C above preindustrial temperature levels—will not be reached, which strongly suggests an increased likelihood of severe climate disasters in the near-term future. Companies in countries with carbon taxes must thus prepare for such taxes to rise.[17] Similarly, companies using internal carbon pricing will need to plan ahead for higher internal carbon prices.[18] Both carbon taxes and internal carbon pricing affect the accounting and finance functions of companies. So, for example, in the calculation of net present values (NPVs) or internal rates of return (IRRs) for the evaluation of new investments, corporate finance divisions must plan for rising carbon costs.[19] Fluctuating values of the SCC will likely result in additional risks for companies' strategic and financial planning.

Acknowledgement

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[17] According to a report by the World Bank—see World Bank (2020)—governments raised US$45 billion in carbon pricing revenues in 2019.

[18] The World Bank (2020) reports that in 2019 about 1,600 companies claimed to use internal carbon pricing or anticipated implementing such a mechanism within the next two years. The same report also states that more than 400 companies used a carbon price as a shadow price of the GHG emissions from their economic activities.

[19] Obviously, many carbon-intensive projects will become outright unprofitable. According to IRENA (2017) and the Group of Thirty (2020), more than 50% of gas and 35% of oil will have to be left in the ground if we are to have a 50% probability of limiting global warming to 2°C. Estimates of the value of such "stranded" assets vary from US$900 billion for proven oil reserves to over US$10 trillion if existing buildings, industry, and energy infrastructure are included. Stranded assets have already resulted in huge write-offs for companies' balance sheets.

Appendix A: Theoretical Results

We first prove the Equality (6) in Section 2.3 as the following Proposition 1.

PROPOSITION 1. *Equality (6) in Section 2.3 holds; i.e.,*

$$V_t = u(c_t) + \mathcal{E}_t^{FP}(\beta V_{t+1}),\ \ t \geq 0. \tag{23}$$

Proof: Note that, using the definition of V_t in (6), we get

$$\begin{aligned} V_t \ &\equiv -\frac{1}{k\beta^t}\ln E_t\left[e^{-k\beta^t\sum_{s=t}^N \beta^{s-t}u(c_s)}\right] = -\frac{1}{k\beta^t}\ln E_t\left[e^{-k\beta^t u(c_t) - k\beta^t\beta\sum_{s=t+1}^N \beta^{s-(t+1)}u(c_s)}\right] \\ &= -\frac{1}{k\beta^t}\ln E_t\left[e^{-k\beta^t u(c_t)}E_{t+1}\left[e^{-k\beta^{t+1}\sum_{s=t+1}^N \beta^{s-(t+1)}u(c_s)}\right]\right] = -\frac{1}{k\beta^t}\ln E_t\left[e^{-k\beta^t u(c_t)}e^{-k\beta^{t+1}V_{t+1}}\right] \\ &= u(c_t) - \frac{1}{k\beta^t}\ln E_t\left[e^{-k\beta^t(\beta V_{t+1})}\right] = u(c_t) + \mathcal{E}_t^{FP}(\beta V_{t+1}), \end{aligned} \tag{24}$$

where the third equality follows from conditioning on the event filtration, the fourth equality follows from the definition of V_t (and hence V_{t+1}) in Equality (6), and the last equality follows from the definition of $\mathcal{E}_t^{FP}(\cdot)$ in Section 2.3. □

We now state and prove a new result in stochastic control that has important applications in our paper (see Section 4.5.1) as well as in the generic analysis of risk-sensitive preferences in the economics literature. For notational convenience (see also Sections 2.1 and 2.3 for details), at any time $t \geq 0$ we denote the HS total utility thereof as V_t^{HS} and the FP total utility thereof as V_t^{FP} with V_t^{*HS}, V_t^{*FP} denoting the corresponding maximums (optimals).

THEOREM 1. *Given our conditions of finite horizon analysis, i.e., compactness of control spaces and continuity/positivity of per-stage/terminal utilities, at any time $t \geq 0$ and any given $k > 0$, $V_t^{HS}, V_t^{FP}, V_t^{*HS}, V_t^{*FP}$ exist. Moreover, $V_t^{HS} \geq V_t^{FP}$ and, in particular, $V_t^{*HS} \geq V_t^{*FP}$.*

Proof: A closer look at equalities (4), (6), and (24) reveals that, at any time $t \geq 0$, the effective continuation CE can be written as $\mathcal{E}_t^{eff}(\cdot) \stackrel{def}{=} -\frac{1}{k_t^{eff}}\ln\left(E_t\left[e^{-k_t^{eff}(\cdot)}\right]\right)$ with the corresponding utility iteration as $V_t = u(c_t) + \beta\mathcal{E}_t^{eff}(V_{t+1})$ for both HS and FP, where the time-t effective k-value k_t^{eff} for HS is $k_t^{eff}(HS) = k$ and that for FP is $k_t^{eff}(FP) = k\beta^{t+1}$. Therefore, starting at a given terminal (per-stage) utility and a given control (possibly a vector) sequence $\{c_t\}_{t=0}^N$, it follows by backward induction that both V_t^{HS} and V_t^{FP} exist and hence, by Weierstrass's Theorem, so do V_t^{*HS} and V_t^{*FP} by the compactness of the control spaces and the continuity of the per-stage/terminal utilities. Now, as before, we obtain $\mathcal{E}_t^{eff}(\cdot) \equiv \mathcal{U}_{k_t^{eff}}^{-1}\left(E_t\left[\mathcal{U}_{k_t^{eff}}(\cdot)\right]\right)$ with $\mathcal{U}_{k_t^{eff}}(\cdot) \stackrel{def}{=} -\frac{1}{k_t^{eff}}e^{-k_t^{eff}(\cdot)}$ implying $\frac{\partial \mathcal{E}_t^{eff}(\cdot)}{\partial k_t^{eff}} > 0$ since $\frac{\partial \mathcal{U}_{k_t^{eff}}(\cdot)}{\partial k_t^{eff}} = -\mathcal{U}_{k_t^{eff}}(\cdot)\left((\cdot) + \frac{1}{k_t^{eff}}\right) > 0$. As $k_t^{eff}(HS) \geq k_t^{eff}(FP)$, this further implies that, at any time $t \geq 0$, $V_t^{HS} \geq V_t^{FP}$ under any given control sequence $\{c_t\}_{t=0}^N$. Now, let $\{c_t^{*HS}\}_{t=0}^N$ be a maximizing selector sequence for HS and $\{c_t^{*FP}\}_{t=0}^N$ be correspondingly the same for FP. Then we have, for any time $t \geq 0$,

$$V_t^{*HS} \equiv V_t^{HS}|_{\{c_t^{*HS}\}_{t=0}^N} \geq V_t^{HS}|_{\{c_t^{*FP}\}_{t=0}^N} \geq V_t^{FP}|_{\{c_t^{*FP}\}_{t=0}^N} \equiv V_t^{*FP}, \tag{25}$$

where the first inequality follows from the maximizing selector property and the second inequality is just proved above for any given control sequence. □

Next we state and prove here the main theorem, Theorem 2, of this paper. Since we use bounded per-stage utilities we can apply the results from Bäuerle and Rieder (2014) to prove the existence results for our model.[20] The choice of utility functions with $\psi > 1$ guarantees that the utility function is well defined for zero consumption. As a consequence the optimal value function is well defined at $K = 0$. We conjecture that the conclusion of Theorem 2 holds more generally, for example for utility functions that are unbounded below. In addition, we also surmise that certain regularity properties hold for the value functions and policies in our model. The proofs of such more general results, however, would require us to first extend the results of Bäuerle and Rieder (2014). Therefore, such extensions need a separate body of work (beyond the current scope of this paper) as suitable growth and Lyapunov-type stability assumptions need to be made (see, e.g., Bäuerle and Jaśkiewicz (2018)) and accordingly the existing stochastic control results need to be extended. Moreover, whether such assumptions are reasonable for these climate scenarios needs to be economically justified first.

THEOREM 2. *Under the assumptions in Section 3.1, our optimization problem (8) has a bounded solution $V_t^*(\cdot), 0 \le t \le N-1,\ V_N^*(\cdot) \equiv h(\cdot)$ to the Bellman equation (8) and a corresponding maximizing (non-stationary) Markov policy $\{\phi_t^*(K_t, T_t, \vartheta_t)\}_{0 \le t \le N-1}$ satisfying*

$$\begin{aligned} \phi_t^*(K_t, T_t, \vartheta_t) \;&=\; \arg\max_{C_t, \mu_t} \; [U_t(C_t) \; - \\ &\frac{1}{k_t} \ln \left(\sum_{j \in \mathcal{S}} p_{\vartheta_t j}(T_t) \exp\left(-k_t \beta V_{t+1}^*(g_{K,t}(K_t, T_t, \vartheta_t, C_t, \mu_t), g_{T,t}(K_t, T_t, \mu_t), j)\right) \right) \Bigg] \end{aligned} \tag{26}$$

such that

$$V_0^*(K_0, T_0, \vartheta_0) = -\frac{1}{k} \ln E^{\{\phi_t^*(K_t, T_t, \vartheta_t)\}_{0 \le t \le N-1}} \left[\exp\left(-k \left(\sum_{t=0}^{N-1} \beta^t U_t(C_t) + \beta^N h(C_N) \right) \right) |K_0, T_0, \vartheta_0 \right], \tag{27}$$

where $E^{\{\phi_t^*(K_t, T_t, \vartheta_t)\}_{0 \le t \le N-1}}[\cdot | K_0, T_0, \vartheta_0]$ denotes the expectation with respect to the probability measure induced on $\{(K_t, T_t, \vartheta_t)\}_{0 \le t \le N}$ by the corresponding process dynamics as described in (7) and (8) and by the optimal deterministic (non-stationary) Markov policy $\{\phi_t^*(K_t, T_t, \vartheta_t)\}_{0 \le t \le N-1}$ starting at (K_0, T_0, ϑ_0). Moreover, $\phi_t^*(\cdot) \equiv \phi^*(k_t, \cdot)$ for some function $\phi^* : [k\beta^N, k] \times \mathbb{R}_+ \times \mathbb{R}_+ \times \mathcal{S} \mapsto \mathbb{R}_+ \times [0,1]$ where, as before (8), $k_t = k\beta^t$; implying $\phi_t^*(\cdot)$ is (weakly) non-stationary and hence sub-path optimal (time-consistent).

Proof: The state (of the world) process X_t can be viewed as a "controlled" Markov chain $\{X_t \equiv (t, k_t, K_t, T_t, \vartheta_t)\}_{0 \le t \le N}, \infty > N \ge 1$ on an "effective" state space $\mathcal{X} \equiv [N] \times [k\beta^N, k] \times \mathcal{X}' \times \mathcal{S} \ni X_t$ where $\mathcal{X}' \equiv \mathbb{R}_+ \times \mathbb{R}_+$ and $[N] \equiv \{0, 1, 2, \ldots, N\}$. Putting $\mathcal{K} \equiv \mathbb{R}_+ \times [0,1]$, let $\mathcal{U}(x) \subset \mathcal{K}$ denote the measurable set of allowed (admissible) actions in state $x \in \mathcal{X}$. Starting at $t = 0$, at each time t the social planner observes the current state X_t of the system and then chooses action $a_t \equiv (C_t, \mu_t) \in \mathcal{U}(X_t) \stackrel{def}{=} [0, K_t] \times [0,1] \equiv \mathcal{U}_t \subset \mathcal{K}$. The

[20] We follow the approach usual to much of the economic-growth literature and choose the utility function and the capital accumulation function so that capital, K_t, remains in the interior of its domain. The Inada conditions of the utility function then imply that the consumption allocations in the optimal solution are positive.

planner gets an immediate utility $U_t(C_t)$ and, now, the system moves randomly to the next stage, $t+1$, with state X_{t+1} as per the evolution described by the transition kernel

$$\begin{aligned} &Q_t\left(B_K \times B_T \times \{j\}|(K_t, T_t, \vartheta_t), (C_t, \mu_t)\right) \\ &\equiv Q\left(\{t'\} \times B_k \times B_K \times B_T \times \{j\}|(t, k_t, K_t, T_t, \vartheta_t), (C_t, \mu_t)\right) \\ &\equiv \delta_{t+1,t'} \otimes \mathbf{1}_{k_t\beta}(B_k) \otimes \mathbf{1}_{g_{K,t}(K_t,T_t,\vartheta_t,C_t,\mu_t)}(B_K) \otimes \mathbf{1}_{g_{T,t}(K_t,T_t,\mu_t)}(B_T) \otimes p_{\vartheta_t j}(T_t) \end{aligned} \tag{28}$$

where $B_k \in \mathcal{B}([k\beta^N, k])$, $B_K, B_T \in \mathcal{B}(\mathbb{R}_+)$ and $\delta_{t+1,t'}$ denotes the Kronecker delta. The whole process then repeats up to and including $t = N-1$. Utility $U_t(\cdot)$, which accumulates throughout the course of the evolution of the system, can now be denoted as $U(t, \cdot)$ repeating notation without loss of generality. Again without loss of generality, defining $V_t(\cdot) \equiv V_t(t, k_t, \cdot)$ by repeating notation as (t, k_t) is deterministic, and the optimization problem (8) can be reformulated, using (28), as

$$\begin{aligned} &V_t(t, k_t, K_t, T_t, \vartheta_t) = V_t(K_t, T_t, \vartheta_t) \;= \\ &\max_{C_t,\mu_t}\left[U_t(C_t) - \frac{1}{k_t}\ln\left(\sum_{j\in\mathcal{S}} p_{\vartheta_t j}(T_t)\exp\left(-k_t\beta V_{t+1}(K_{t+1}, T_{t+1}, j)\right)\right)\right] = \\ &\max_{C_t,\mu_t}\left[U_t(C_t) - \frac{1}{k_t}\ln\left(\sum_{j\in\mathcal{S}} p_{\vartheta_t j}(T_t)\exp\left(-k_t\beta V_{t+1}(t+1, k_{t+1}, K_{t+1}, T_{t+1}, j)\right)\right)\right] = \\ &\max_{C_t,\mu_t}\left[U_t(C_t) - \frac{1}{k_t}\ln\left(\sum_{j\in\mathcal{S}} p_{\vartheta_t j}(T_t)\exp\left(-k_t\beta V_{t+1}(t+1, k_t\beta, K_{t+1}, T_{t+1}, j)\right)\right)\right] = \\ &\max_{C_t,\mu_t}\,\Bigg[U(t, C_t) - \\ &\frac{1}{k_t}\ln\left(\sum_{t'\in[N]}\sum_{j\in\mathcal{S}} \delta_{t+1,t'} p_{\vartheta_t j}(T_t)\exp\left(-k_t\beta V_{t+1}(t', k_t\beta, g_{K,t}(K_t, T_t, \vartheta_t, C_t, \mu_t), g_{T,t}(K_t, T_t, \mu_t), j)\right)\right)\Bigg] = \\ &\max_{C_t,\mu_t}\,\Bigg[U(t, C_t) - \\ &\frac{1}{k_t}\ln\sum_{t'\in[N]}\sum_{j\in\mathcal{S}}\int_{[k\beta^N,k]\times\mathcal{X}'} \exp\left(-k'V_{t+1}(t', k', x', y', j)\right) Q\left(\{t'\} \times d(k', x', y') \times \{j\}|(t, k_t, K_t, T_t, \vartheta_t), (C_t, \mu_t)\right)\Bigg]. \end{aligned} \tag{29}$$

Because t is in a finite set $[N]$ and $U_t(\cdot), h(\cdot)$ are continuous on a compact set, there exists

$$M \equiv \max_{0\le t\le N}\max_{C_t\in[0,K_t]} |U_t(C_t)| \vee \max_{C\in[0,\bar{K}]} |h(C)| < \infty, \tag{30}$$

implying $U(t, \cdot) = U_t(\cdot)$ and $h(\cdot)$ are bounded. Also, by definition $\mathcal{U}_t$ is compact for all $0 \le t \le N$. Moreover, the effective state space $\mathcal{X}_N \equiv [N] \times [k\beta^N, k] \times [0, K_N] \times [0, T_N] \times \mathcal{S} \subset \mathcal{X}$ is also compact. Also, for each $t, t' \in [N]$, $i, j \in \mathcal{S}$, $B_k \in \mathcal{B}([k\beta^N, k])$, $B_K, B_T \in \mathcal{B}(\mathbb{R}_+)$, $Q\left(\{t'\} \times B_k \times B_K \times B_T \times \{j\}|\{t\}, \cdot, \{i\}\right)$ is jointly continuous (hence weakly continuous) in the other parameters—that is, $((k, K, T), (C, \mu))$. Therefore, together with our given boundary condition $V_N(\cdot) \equiv h(\cdot)$ and the fact that ln is a monotonically increasing function, it follows from Corollary 2 and Remark 2 of Bäuerle and Rieder (2014) and (29) that there indeed exists a (bounded) solution $V_t^*(\cdot), 0 \le t \le N$ to the DPE (29) (and hence to the original DPE (8)) and a corresponding deterministic maximizing (non-stationary) Markov policy $\{\phi_t^*(X_t)\}_{0\le t\le N-1}$ satisfying

$$\phi_t^*(K_t, T_t, \vartheta_t) \;=\; \arg\max_{C_t,\mu_t}\,[U(t, C_t) -$$

$$\frac{1}{k_t}\ln\left(\sum_{t'\in[N]}\sum_{j\in\mathcal{S}}\delta_{t+1,t'}p_{\vartheta_t j}(T_t)\exp\left(-k_t\beta V^*_{t+1}(t',k_t\beta,g_{K,t}(K_t,T_t,\vartheta_t,C_t,\mu_t),g_{T,t}(K_t,T_t,\mu_t),j)\right)\right)\Bigg]$$
$$= \arg\max_{C_t,\mu_t}\ [U_t(C_t)-$$
$$\frac{1}{k_t}\ln\left(\sum_{j\in\mathcal{S}}p_{\vartheta_t j}(T_t)\exp\left(-k_t\beta V^*_{t+1}(g_{K,t}(K_t,T_t,\vartheta_t,C_t,\mu_t),g_{T,t}(K_t,T_t,\mu_t),j)\right)\right)\Bigg] \tag{31}$$

such that

$$\begin{aligned}V^*_0(K_0,T_0,\vartheta_0) &= \max_{\{(C_t,\mu_t)\}_{0\le t\le N-1}} -\frac{1}{k}\ln E_Q^{\{(C_t,\mu_t)\}_{0\le t\le N-1}}\left[\exp\left(-k\left(\sum_{t=0}^{N-1}\beta^t U_t(C_t)+\beta^N h(C_N)\right)\right)|K_0,T_0,\vartheta_0\right]\\ &= -\frac{1}{k}\ln E_Q^{\{\phi^*_t(X_t)\}_{0\le t\le N-1}}\left[\exp\left(-k\left(\sum_{t=0}^{N-1}\beta^t U_t(C_t)+\beta^N h(C_N)\right)\right)|K_0,T_0,\vartheta_0\right],\end{aligned} \tag{32}$$

where $E_Q^{\{\phi_t(X_t)\equiv(C_t,\mu_t)\}_{0\le t\le N-1}}[\cdot|K_0,T_0,\vartheta_0]$ denotes the expectation with respect to the probability measure induced on $\{X_t\}_{0\le t\le N}$ by $Q(\cdot|\cdot)$—see (28)—and a given Markov policy $\{\phi_t(X_t)\}_{0\le t\le N-1}$ starting at (K_0,T_0,ϑ_0). The boundedness of $V^*_t, 0\le t\le N$ is obvious from the boundedness of $U_t(\cdot)$ and $h(\cdot)$. Note that the non-stationarity of $\phi^*_t(\cdot)$ is evident from its implicit dependence on $k_t=k\beta^t$ (see (31)) for each $t\in[N]$; i.e., $\phi^*_t(\cdot)\equiv\phi^*(k_t,\cdot)$ for some function $\phi^*:[k\beta^N,k]\times[0,K_N]\times[0,T_N]\times\mathcal{S}\mapsto[0,K_N]\times[0,1]$ implying it is (weakly) non-stationary and sub-path optimal, hence, time-consistent in the sense of Johnsen and Donaldson (1985). Defining $\overline{U}_t(C_t)\equiv U_t(C_t)+M\ge 0$ (see (30)), we have that

$$\begin{aligned}&-\frac{1}{k}\ln E_Q^{\{(C_t,\mu_t)\}_{0\le t\le N-1}}\left[\exp\left(-k\left(\sum_{t=0}^{N-1}\beta^t U_t(C_t)+\beta^N h(C_N)\right)\right)|K_0,T_0,\vartheta_0\right]\\ &=-\frac{1}{k}\ln E_Q^{\{(C_t,\mu_t)\}_{0\le t\le N-1}}\left[\exp\left(-k\left(\sum_{t=0}^{N-1}\beta^t\left(\overline{U}_t(C_t)-M\right)+\beta^N h(C_N)\right)\right)|K_0,T_0,\vartheta_0\right]\\ &=-\frac{1}{k}\ln E_Q^{\{(C_t,\mu_t)\}_{0\le t\le N-1}}\left[\exp\left(-k\left(\sum_{t=0}^{N-1}\beta^t\overline{U}_t(C_t)+\beta^N h(C_N)\right)\right)|K_0,T_0,\vartheta_0\right]-\frac{1-\beta^N}{1-\beta}M.\end{aligned} \tag{33}$$

Hence, by (32) a policy is optimal for the above utility using $\overline{U}(\cdot,\cdot)$ if and only if it is optimal for our original problem using $U(\cdot,\cdot)$, and therefore the results of Corollary 2 and Remark 2 of Bäuerle and Rieder (2014), which assume nonnegativity of the per-stage utility, can be used here without any problem. □

Appendix B: Numerical Approximation Error

In order to measure the numerical approximation error that occurs at each iteration step, we need to define an error measure for the value function approximation. As there is no inherent meaning to the absolute value of the value function, it is not sensible to simply compute the relative error. Instead, we use the notion of a unit-free error measure,

$$\xi_t(X_t)=\max_{\underline{x}\in X_t}\frac{\left|\hat{V}_t(\underline{x})-V^*_{t,\underline{x}}\right|}{\|\underline{x}\|_{K,T}\cdot\|\nabla_{K,T}\hat{V}_t(\underline{x})\|}, \tag{34}$$

where the set $X\subset[0,\bar{K}]\times[0,\bar{T}]\times\mathcal{S}$ contains a discrete number of points $(K^{(i)},T^{(i)},\vartheta^{(i)})$, $i\in\{1,\dots,n\}$ in the state space. The Chebyshev approximation of the value function $V_t(\cdot)$ is denoted by $\hat{V}_t(\cdot)$ and the numerical solution to the Bellman equation (8) at point $\underline{x}$ and time t computed using the optimization solver KNITRO is denoted by $V^*_{t,\underline{x}}$. The seminorm for $\|\cdot\|_{K,T}$ is defined as the Euclidean norm over capital and temperature; that is,

$$\|(K,T,\vartheta)\|_{K,T} \equiv \|(K,T)\|_2. \tag{35}$$

For each time period t, we choose X as a 20x20x4 grid spread evenly over the domain for the multivariate Chebyshev polynomials. The unit-free error measure $\xi_t(X_t)$ for the value function calculated with the numerical algorithm with 9x9 Chebyshev nodes and 4x4 degree Chebyshev polynomials are computed on the time-varying approximation domain and shown in Figure 7. As we can see, the errors are of a magnitude of about 10^{-4} and lower. Therefore, we conclude that the numerical approximation of the value function is sufficiently accurate.

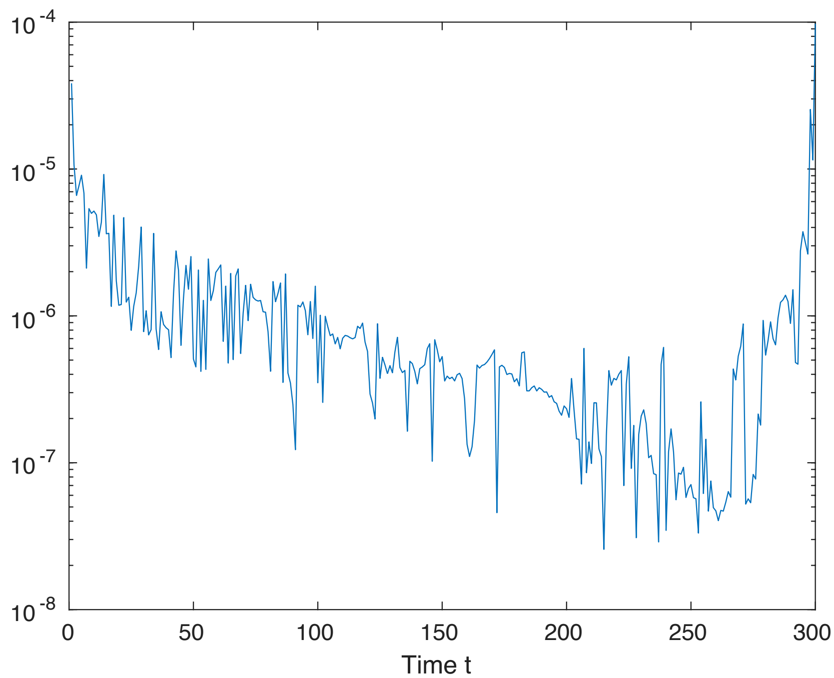

Figure 7 **Maximum Unit-Free Error Measure of the Value Function for Each Time Period** t

Appendix C: Additional Results

Tables 7 and 8 report the SCC for the model with risk-sensitive preferences in the years 2020 and 2120, respectively, for four different levels of the EIS, $\psi \in \{1.25, 1.5, 1.75, 2\}$. Subsequently, Tables 9 and 10 report the corresponding SCC values for Epstein–Zin preferences. Table 11 reports the SCC with risk-sensitive preferences and higher tail risks for the sensitivity analysis.

	$k=0.25$	$k=5$	$k=10$	$k=15$	$k=20$	$k=25$	$k=30$
$\psi=1.25$	137.85	139.26	140.71	142.22	143.70	145.15	146.63
$\psi=1.50$	159.91	162.12	164.48	166.84	169.17	171.52	173.80
$\psi=1.75$	178.55	181.79	185.14	188.47	191.80	195.07	198.34
$\psi=2.00$	194.62	198.89	203.32	207.71	212.06	216.29	220.51

Table 7 SCC with Risk-Sensitive Preferences in US$/t$CO_2$ in Year 2020

	$k=0.25$	$k=5$	$k=10$	$k=15$	$k=20$	$k=25$	$k=30$
$\psi=1.25$	1177.77	1192.79	1174.83	1180.93	1184.92	1194.65	1203.56
$\psi=1.50$	1237.20	1214.92	1225.54	1247.83	1242.90	1249.10	1252.06
$\psi=1.75$	1300.13	1266.77	1303.95	1295.07	1304.81	1298.75	1304.14
$\psi=2.00$	1297.00	1293.97	1310.07	1336.85	1332.38	1345.93	1358.41

Table 8 SCC with Risk-Sensitive Preferences in US$/t$CO_2$ in Year 2120

	$k=5$	$k=10$	$k=15$	$k=20$	$k=25$	$k=30$
$\psi=1.25$	140.48	142.90	145.28	147.64	149.97	152.27
$\psi=1.50$	162.14	164.58	166.99	169.38	171.74	174.07
$\psi=1.75$	180.72	183.19	185.65	188.07	190.46	192.82
$\psi=2.00$	196.76	199.27	201.75	204.22	206.66	209.05

Table 9 SCC with Epstein–Zin Preferences in US$/t$CO_2$ in Year 2020

	$k=5$	$k=10$	$k=15$	$k=20$	$k=25$	$k=30$
$\psi=1.25$	1162.57	1213.43	1251.01	1246.17	1267.98	1299.35
$\psi=1.50$	1207.63	1263.98	1260.51	1311.21	1363.83	1348.33
$\psi=1.75$	1300.61	1293.22	1301.19	1335.52	1388.68	1393.79
$\psi=2.00$	1286.66	1355.80	1368.29	1361.55	1414.97	1419.41

Table 10 SCC with Epstein–Zin Preferences in US$/t$CO_2$ in Year 2120

	$k=0.5$	$k=5$	$k=10$	$k=20$	$k=30$
$\psi=1.25$	143.02	144.77	146.71	150.67	154.61
$\psi=1.50$	164.53	167.40	170.55	176.83	183.12
$\psi=1.75$	182.99	187.00	191.50	200.35	209.02
$\psi=2.00$	198.82	204.11	209.97	221.43	232.66

Table 11 SCC with Risk-Sensitive Preferences and higher tail risks in US\$/tCO_2 in Year 2020

References

Anderson E, Brock W, Sanstad AH (2016) Robust consumption and energy decisions. 2017 Allied Social Sciences Association (ASSA) Annual Meeting, January 6-8, 2017, Chicago, Illinois 250117, Agricultural and Applied Economics Association.

Anderson EW, Hansen LP, Sargent TJ (2012) Small noise methods for risk-sensitive/robust economies. *Journal of Economic Dynamics and Control* 36(4):468–500.

Bansal R, Yaron A (2004) Risks for the long run: A potential resolution of asset pricing puzzles. *The journal of Finance* 59(4):1481–1509.

Barnett M, Brock W, Hansen LP (2020) Pricing uncertainty induced by climate change. *The Review of Financial Studies* 33(3):1024–1066.

Bäuerle N, Jaśkiewicz A (2018) Stochastic optimal growth model with risk sensitive preferences. *Journal of Economic Theory* 173:181–200.

Bäuerle N, Rieder U (2014) More risk-sensitive markov decision processes. *Mathematics of Operations Research* 39(1):105–120.

Berger L, Emmerling J, Tavoni M (2017) Managing catastrophic climate risks under model uncertainty aversion. *Management Science* 63(3):749–765.

Berger L, Marinacci M (2020) Model uncertainty in climate change economics: A review and proposed framework for future research. *Environmental and Resource Economics* 77(3):475–501.

Bielecki TR, Pliska SR (2003) Economic properties of the risk-sensitive criterion for portfolio management. *Review of Accounting and Finance* 2(2):3–17.

Bommier A, Kochov A, Le Grand F (2017) On monotone recursive preferences. *Econometrica* 85(5):1433–1466.

Bommier A, Lanz B, Zuber S (2015) Models-as-usual for unusual risks? on the value of catastrophic climate change. *Journal of Environmental Economics and Management* 74:1–22.

Bommier A, Le Grand F (2019) Risk aversion and precautionary savings in dynamic settings. *Management Science* 65(3):1386–1397.

Brock W, Xepapadeas A (2020) Regional climate policy under deep uncertainty: robust control and distributional concerns. *Environment and Development Economics* 1–28.

Brock WA, Hansen LP (2018) Wrestling with uncertainty in climate economic models. Technical Report 2019-71, University of Chicago, Becker Friedman Institute for Economics Working Paper.

Brown AL, Kim H (2014) Do individuals have preferences used in macro-finance models? an experimental investigation. *Management Science* 60(4):939–958.

Burgaard J, Steffensen M (2020) Eliciting risk preferences and elasticity of substitution. *Decision Analysis* 17(4):314–329.

Cai J, Chen X, Dai M (2018) Portfolio selection with capital gains tax, recursive utility, and regime switching. *Management Science* 64(5):2308–2324.

Cai Y (2019) Computational methods in environmental and resource economics. *Annual Review of Resource Economics* 11(1):59–82.

Cai Y (2021) The role of uncertainty in controlling climate change. *Oxford Research Encyclopedia of Economics and Finance* 1–34.

Cai Y, Lontzek TS (2019) The social cost of carbon with economic and climate risks. *Journal of Political Economy* 127(6):2684–2734.

Cai Y, Lontzek TS, Lenton TM (2016) Risk of multiple interacting tipping points should encourage rapid co2 emission reduction. *Nature Climate Change* 6(5):520–525.

Cass D (1965) Optimum growth in an aggregative model of capital accumulation. *The Review of economic studies* 32(3):233–240.

Chari VV (2018) The role of uncertainty and risk in climate change economics. Technical Report 576, Federal Reserve Bank of Minneapolis.

Chen Z, Epstein L (2002) Ambiguity, risk and asset returns in continuous time. *Econometrica* 70(4):1403–1443.

de Groot O, Richter AW, Throckmorton NA (2018) Valuation risk revisited. *Working Paper 1808, Federal Reserve Bank of Dallas* 1–22.

Di Masi GV, Stettner Ł (1999) Risk-sensitive control of discrete-time markov processes with infinite horizon. *SIAM J. Control Optim.* 38(1):61–78.

Dietz S, van der Ploeg F, Rezai A, Venmans F (2021) Are economists getting climate dynamics right and does it matter? *Journal of the Association of Environmental and Resource Economists* 8(5):895–921.

Duffie D, Epstein LG (1992) Stochastic differential utility. *Econometrica* 60(2):353–394.

Epstein L, Schneider M (2003) Recursive multiple-priors. *J. Econ. Theory* 113(1):1–31.

Epstein LG, Farhi E, Strzalecki T (2014) How much would you pay to resolve long-run risk? *American Economic Review* 104(9):2680–97.

Epstein LG, Zin SE (1989) Substitution, risk aversion, and the temporal behavior of consumption and asset returns: a theoretical framework. *Econometrica* 57(4):937–969.

Group of Thirty (2020) Mainstreaming the transtition to a net-zero economy. Technical report, Group of Thirty, Washington, D.C.

Hansen LP, Miao J (2018) Aversion to ambiguity and model misspecification in dynamic stochastic environments. *Proc. Nat. Acad. Sc.* 115(37):9163–9168.

Hansen LP, Sargent J Thomas, Turmuhambetova G, Williams N (2006) Robust control and model misspecification. *J. Econ. Theory* 128(1):45–90.

Hansen LP, Sargent TJ (1995) Discounted linear exponential quadratic gaussian control. *IEEE Transactions on Automatic Control* 40:968–971.

Hennlock M (2009) Robust control in global warming management: An analytical dynamic integrated assessment. Technical report, University of Gothenburg, Department of Economics.

Howard PH, Sylvan D (2020) Wisdom of the experts: Using survey responses to address positive and normative uncertainties in climate-economic models. *Climatic Change* 162(2):213–232.

Howard RA, Matheson JE (1972) Risk-sensitive markov decision processes. *Management science* 18(7):356–369.

IPCC (2021) Climate change 2021: The physical science basis. Masson-Delmotte V, Zhai P, Pirani A, Connors S, Péan C, Berger S, Caud N, Chen Y, Goldfarb L, Gomis M, Huang M, Leitzell K, Lonnoy E, Matthews J, Maycock T, Waterfield T, Yelekçi O, Yu R, Zhou B, eds., *Contribution of Working Group I to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change* (Cambridge University Press).

IRENA (2017) Stranded assets and renewables: how the energy transition affects the value of energy reserves, buildings and capital stock. Technical report, International Renewable Energy Agency (IRENA), Abu Dhabi, www.irena.org/remap.

Jacobson DH (1973) Optimal stochastic linear systems with exponential performance criteria and their relation to deterministic differential games. *IEEE Trans. Aut. Control* AC-18(2):124–131.

Johnsen TH, Donaldson JB (1985) The structure of intertemporal preferences under certainty and time consistent plans. *Econometrica* 53(6):1451–1458.

Judd KL (1998) *Numerical Methods in Economics*. MIT Press Books (The MIT Press).

Kapoor M, Ravi S (2017) Elasticity of intertemporal substitution in consumption in the presence of inertia: Empirical evidence from a natural experiment. *Management Science* 63(12):4188–4200.

Kihlstrom RE, Mirman LJ (1974) Risk aversion with many commodities. *Journal of Economic Theory* 8(3):361–388.

Klibanoff P, Marinacci M, Mukherji S (2005) A smooth model of decision making under ambiguity. *Econometrica* 73(6):1849–1892.

Koopmans TC, et al. (1963) On the concept of optimal economic growth. Technical report, Cowles Foundation for Research in Economics, Yale University.

Kreps DM, Porteus EL (1978) Temporal resolution of uncertainty and dynamic choice theory. *Econometrica* 46(1):185–200.

Kreps DM, Porteus EL (1979) Dynamic choice theory and dynamic programming. *Econometrica* 47(1):91–100.

Lemoine D, Traeger CP (2016) Ambiguous tipping points. *Journal of Economic Behavior & Organization* 132:5–18.

Li X, Narajabad B, Temzelides T (2016) Robust dynamic energy use and climate change. *Quantitative Economics* 7(3):821–857.

Lontzek TS, Cai Y, Judd KL, Lenton TM (2015) Stochastic integrated assessment of climate tipping points indicates the need for strict climate policy. *Nature Climate Change* 5(5):441–444.

Matthews HD, Gillett NP, Stott PA, Zickfeld K (2009) The proportionality of global warming to cumulative carbon emissions. *Nature* 459(7248):829–832.

Matthews HD, Solomon S, Pierrehumbert R (2012) Cumulative carbon as a policy framework for achieving climate stabilization. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* 370(1974):4365–4379.

Miftakhova A, Judd KL, Lontzek TS, Schmedders K (2020) Statistical approximation of high-dimensional climate models. *Journal of Econometrics* 214(1):67–80, annals Issue: Econometric Models of Climate Change.

Millner A, Dietz S, Heal G (2013) Scientific ambiguity and climate policy. *Environmental and Resource Economics* 55(1):21–46.

Nordhaus WD (1993) Rolling the “dice”: An optimal transition path for controlling greenhouse. *Resource and Energy Economics* 15:27–50.

Nordhaus WD (2017) Revisiting the social cost of carbon. *Proceedings of the National Academy of Sciences* 114(7):1518–1523, ISSN 0027-8424, URL `http://dx.doi.org/10.1073/pnas.1609244114`.

Pohl W, Schmedders K, Wilms O (2018) Higher order effects in asset pricing models with long-run risks. *The Journal of Finance* 73(3):1061–1111.

Ramsey FP (1928) A mathematical theory of saving. *The economic journal* 38(152):543–559.

Rezai A, van der Ploeg F (2017) Climate policies under climate model uncertainty: Max-min and min-max regret. *Energy Economics* 68:4–16.

Rudik I (2020) Optimal climate policy when damages are unknown. *American Economic Journal: Economic Policy* 12(2):340–73.

Simmons CT, Matthews HD (2016) Assessing the implications of human land-use change for the transient climate response to cumulative carbon emissions. *Environmental Research Letters* 11(035001).

Skiadas C (2003) Robust control and recursive utility. *Finance and Stochastics* 7:475–489.

Svensson LEO (1989) Portfolio choice with non-expected utility in continuous time. *Economic Letters* 30(4):313–317.

Tallarini Jr TD (2000) Risk-sensitive real business cycles. *Journal of Monetary Economics* 45(3):507–532.

United Nations DoE, Social Affairs PD (2019) World population prospects 2019, online edition. rev. 1. Technical report, United Nations.

van der Ploeg F (1984a) Economic policy rules for risk-sensitive decision making. *Zeitschrift für Nationalökonomie / Journal of Economics* 44(3):207–235.

van der Ploeg F (1984b) Risk and linear-quadratic stabilization. *Economics Letters* 15(1):73–78, ISSN 0165-1765.

van der Ploeg F (1993) A closed-form solution for a model of precautionary savings. *Review of Economic Studies* 60(2):385–395.

van der Ploeg F, de Zeeuw A (2017) Climate tipping and economic growth: Precautionary capital and the price of carbon. *Journal of the European Economic Association* 16(5):1577–1617.

Weil P (1990) Nonexpected utility in macroeconomics. *Quarterly Journal of Economics* 105(1):29–42.

Weil P (1993) Precautionary savings and the permanent income hypothesis. *Review of Economic Stuies* 60(2):367–383.

Weitzman ML (2009) On modeling and interpreting the economics of catastrophic climate change. *The Review of Economics and Statistics* 91(1):1–19.

Westermann R (2018) Measuring agency costs over the business cycle. *Management Science* 64(12):5748–5768.

Whittle P (1990) *Risk-Sensitive Optimal Control* (Chichester, England, and New York: Wiley).

World Bank (2020) State and trends of carbon pricing 2020. *World Bank, Washington, DC.* .