Economic Growth, Trade, and Environmental Quality in a Two-region World Economy

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Abstract

This paper examines the linkages between international trade, environmental degradation, and economic growth, in a dynamic North-South trade game. The North produces manufactured goods by employing capital, labor, and a natural resource that it imports from the South, using a neoclassical production function subject to an endogenously improving technology. The South extracts the resource using raw labor, in the process generating local pollution. Genetic algorithms (GA) are used to search for optimal policies in the presence of local pollution and technology spillovers from North to South. In the GA search for optimal regional policies, both noncooperative and cooperative modes of trade are considered. Noncooperative trade results in inefficiencies stemming from externalities. Though cooperative trade policies are efficient, they lack credibility. A joint maximization of the global welfare shows that transfer of technology is a viable route to improve world welfare.

Key Words:

JEL Classification:


1 Introduction

Natural resources constitute a significant source of export revenue to many nations, and in many others, are indispensable factors of production. Extraction and/or employment of resources as factors of production may also cause significant environmental damage that may long persist, or that may even be irreversible. Consequently, there exists an extensive economic literature devoted to study various aspects of natural resource economics (Levhari and Mirman, 1980; Dasgupta, 1982; Dockner and Long, 1993.).

In this study, we focus on the gaming aspects of the trade in natural resources in a complex dynamic setting. We consider the resource trade as a dynamic North/South game (See for instance Galor, 1986; Van der Ploeg and De Zeeuw, 1992 and 1994.). As customary in North/South trade models, we let North specialize in the production of manufactured goods that are consumed and invested in the North or exported to the South at a fixed world price of unity and South be the sole supplier of natural resources. Various studies have extended this basic model to address concerns related to externalities that may accompany trade (Chichilnisky, 1993,1994; Copeland and Taylor, 1994; Grossman and Helpman, 1991; Alemdar and Özyildirim, 1998 and 2002.).

...TO BE COMPLETED.

2 The Model

2.1 Non-Cooperative North/South Trade

The global economy is comprised of two regions, North and South. Each region is populated with infinitely lived identical individuals, $L_N$ in North and $L_S$ in South, which grow at the exogenous rates, $\dot{L}_N/L_N = n_N$ and $\dot{L}_S/L_S = n_S$, respectively. Since our main interest lies in the analysis of how various sources of inefficiencies interact to distort growth trajectories in an aggregative dynamic game framework, henceforth, we adopt the social planning paradigm.

The North produces a final good, $Y$, which is consumed, invested, and exported to the South at fixed price of unity. Production of manufactured goods take place according
to a Cobb-Douglas production function:

\[ Y = \phi_Y A^{\alpha_A}[u L_N]^{\alpha_L}[v K]^{\alpha_K} R^{\alpha_R} \] (2.1)

where \( \phi_Y \) is an exogenous technological shift parameter, \( 0 < \alpha_i < 1 \) (\( i = A, L, K, R \)) are the productive elasticities, \( A \) is an endogenously accumulated knowledge stock, \( u \) and \( v \) are the respective shares of labor and capital employed in the final good sector. The raw material, \( R \), is imported from the South at a monopoly price.

Employing labor and capital, the technology sector in the North produces new technologies at a rate, \( J \), while the stock of knowledge, \( A \), becomes obsolete at a constant rate, \( \delta_A \), so that the state of the technology evolves according to:

\[ \dot{A} = \phi_A A^{\beta_A}[(1 - u)L_N]^{\beta_L}[(1 - v)K]^{\beta_K} - \delta_A A = J - \delta_A A \] (2.2)

where \( \phi_A \) stands for an exogenous technological shift factor and \( 0 < \beta_i < 1 \) (\( i = A, L, K \)) are the productive elasticities. The stock of physical capital accumulates in pace with investment,

\[ \dot{K} = Y - C_N - pR - \delta_K K \] (2.3)

where \( C_N \) is the aggregate consumption, \( p \) is the relative market price of resources (Southern terms of trade) and \( 0 < \delta_K < 1 \) is the rate of depreciation of the physical capital. Equation (2.3) indicates that the rate of physical capital accumulation is affected not only by the North’s desired consumption profile, but also by the South’s. No investment takes place in the South so that the proceeds from the resource sale are totally consumed. Nonetheless, South indirectly affects the pace of physical capital accumulation in the North via the desired terms of trade.

The Northern planner takes the Souther terms of trade as given and is assumed to maximize the intertemporal utility of the representative agent, namely,

\[ \max_{C_N, u, v, R} J_N = \int_0^\infty e^{-\rho_N t} \frac{1}{\gamma_N} \left( \frac{C_N}{L_N} \right)^{\gamma_N} dt, \ 0 < \rho_N < 1, \ \gamma_N < 0 \] (2.4)

subject to the production and the accumulation constraints (2.1)-(2.3), \( K(0) = K_0 \),
\( A(0) = A_0 \), and \( C_N \geq 0 \). \( C_N/L_N \) is per capita consumption and \( \rho_N \) denotes the Northern time preference rate.

Turning to South, in resource extraction, the South uses a constant returns to scale production function which is assumed, for simplicity, to be a fixed coefficient type, \( R(t) = bL_S(t) \), \( b > 0 \). We also assume that the South is unconstrained by labor availability, \( L_S(t) \).

Resource extraction causes pollution which accumulates locally and only internalized in the South. However, technology accumulated in the North diffuses to the South to reduce this damage, albeit at a diminishing rate. Thus, the resulting patterns of trade and growth are further complicated due to the presence of local externalities.

The accumulated pollution, \( P \), evolves by:

\[
\dot{P} = \frac{1}{\theta} \frac{R^\theta}{A^e} - \delta_P P
\]

where \( \theta > 1 \), measures the exponential order of environmental damage due to extraction, \( 0 < \varepsilon < 1 \), is a technology diffusion (spillover) parameter signifying the degree of applicability of technology to pollution reduction and \( 0 < \delta_P < 1 \) denotes the constant instantaneous rate that the pollution decays naturally.

\( P \) enters into the Southern individuals’ utility as a stock with a negative marginal utility. Facing the Northern demand for the resource, the Southern planner takes Northern policies as given and chooses terms of trade to maximize the Southern welfare, i.e.,

\[
\max_{p} J_S = \int_{0}^{\infty} e^{-\rho_S t} \left[ \frac{1}{\gamma_S} \left( \frac{C_S}{L_S} \right)^{\gamma_S} - D \frac{P}{L_S} \right] dt, \quad \gamma_S < 0, \quad D > 0, \quad \rho_S \in (0, 1)
\]

subject to (2.1), (2.3) and (2.5), \( C_S = pR \), \( P(0) = P_0 \), \( K(0) = K_0 \) and \( C_S \geq 0 \), where \( \rho_S \) is the Southern rate of time preference, \( \tau \in [0, 1] \) captures the degree to which the representative Southerner perceives pollution as a private bad and \( D \) converts pollution into utils. Following Eriksson and Zehai (2002), we shall call the ratio \( P/L_S^\tau \) the perceived

\[1\]That is, the population growth rate in the South is greater than or equal to the growth rate of the demand for the resource by the North. Also, if it is assumed that the supply of labor in the South is perfectly elastic at a fixed real wage \( w(t) \) in terms of the industrial good, the nature of the labor force coupled with the CRS production function would then determine labor income per unit of raw material as \( w(t) = b \). Competitive firms in the South will charge a price equal to the private marginal cost of resource extraction \( w(t) = b \). The assumed social planner in the South levies an export tax, not only to internalize the social cost of pollution, but also to extract monopoly profit from the North.
pollution. Note that \( \tau = 0 \) corresponds to the case where pollution exhibits pure public bad characteristics: if both pollution and population are doubled, the pollution that each individual suffers from doubles as well. \( \tau = 1 \), on the other hand, corresponds to the case where pollution exhibits pure private bad characteristics: doubling pollution and the population results in no more disutility of pollution per person.

2.2 Cooperative North/South Trade

In designing cooperative strategies, North and South must agree in advance as to how they will share the potential gains from cooperation. The distributive outcome will depend on the weights, \( \omega \), that are attached to the respective welfares. The determination of the value of \( \omega \) most likely to prevail in a cooperative agreement requires a bargaining framework which recognizes the relative power of the participants. This is outside the scope of our inquiry. Instead, to enable welfare comparisons across scenarios, we assume exogenously given weights. The Pareto efficient solution is found by choosing \( C_N, u, v, R \) and \( p \) to maximize

\[
J = \omega \int_0^\infty e^{-\rho t} \frac{1}{\gamma_N} \left( \frac{C_N}{L_N} \right)^{\gamma_N} dt + (1 - \omega) \int_0^\infty e^{-\rho t} \left[ \frac{1}{\gamma_S} \left( \frac{C_S}{L_S} \right)^{\gamma_S} - D \frac{P}{L_S} \right] dt \quad \text{(2.7)}
\]

subject to (2.1)-(2.3), (2.5), \( C_S = pR, A(0) = A_0, P(0) = P_0, K(0) = K_0 \) and \( C_N, C_S \geq 0 \).

Cooperation between North and South needs to be supported by binding agreements. Precommitment is difficult in the absence of suitable institutions which can enforce global decisions. Nonetheless, cooperative solutions, though lacking credibility, are important in so far as they indicate welfare loses that are likely to ensue given a lack of commitment.

3 Analytical Solution

The trade game, be it cooperative or noncooperative, as it is posited, would not admit a constant (Pareto or Nash) equilibrium solution in the long run. Consequently, the model needs to be appropriately scaled so that the transformed model has a steady state solution. Towards that, and irrespective of the trading regime, we envisage a steady state equilibrium in which the final output, \( Y \), the physical capital, \( K \), and consumption in the
North, $C_N$, and in the South, $C_S$, all grow at the same constant rate while the pollution stock grows at a constant rate such that, given $\tau$, the integral in South’s problem is convergent. Given these assumptions, to discover the balanced growth rates, we take logarithmic differentials of the production functions (2.1)-(2.2) and Eq. (2.5), and after few rounds of algebraic manipulations, we obtain:

\[
\hat{Y} = \hat{K} = \hat{C}_N = \hat{C}_S = \left(\alpha_A + \alpha_R(\varepsilon/\theta)\right)\beta_L + \left(\alpha_L + (\alpha_Rn_Sn_S)\right)\beta_K = g_Y n_N
\]

\[
\hat{\beta} = \beta_L + \beta_K g_Y n_N = g_{\beta} n_N
\]

\[
\hat{\beta} = (\gamma_S g_Y + (\tau - \gamma_S)(n_S/n_N)) n_N = g_{\beta} n_N
\]

\[
\hat{R} = ((\varepsilon/\theta)g_A + (1/\theta)g_P)n_N = g_R n_N
\]

\[
\hat{p} = \hat{C}_S - \hat{R} = g_p n_N
\]

where the hats denote the steady state growth rates of the respective variables. Note that even though the North is indifferent to the Southern pollution when choosing its optimal policies, nonetheless, some pollution parameters enter as determinants of the long run Northern consumption, capital, output and technology growth. Notably, in an environment where steady state growth is possible, a change in the technology spillover or the environmental damage rate in the South, affects not only the levels of Northern optimal policies, but also their permanent growth rates. More specifically, $\partial Y/\partial \varepsilon, \partial Y/\partial \theta, \partial \beta/\partial Y > 0$.

Next, we transform each variable so that it is constant in the steady state. Thus, we define the scaled variables as: $y = Y/L_{NN}^Y$, $k = K/L_{NN}^Y$, $c_n = C_N/L_{NN}^y$, $a = A/L_{NN}^A$, $r = R/L_{NN}^g$, $p^* = p/L_{NN}^{g/r}$, $P^* = P/L_{NN}^g$. For convenience, we shall refer to $y, k, c_n, a, r, p^*, c_s = p^r$ and $P^*$ as scale-adjusted quantities. Now, we can re-write the scale-adjusted output, technology, physical capital and pollution stock as:

\[
y = \phi_y a^{\alpha_A} u^{\alpha_L} (vk)^{\alpha_K} r^{\alpha_R}
\]

\[
\dot{a} = \phi_a a^{\beta_A} (1 - u)^{\beta_L} [(1 - v)k]^{\beta_K} - \delta_A^* a = j - \delta_A^* a, \delta_A^* = \delta_A + n_N g_A
\]
\[ \dot{k} = y - c_n - p^* r - \delta_K^* k, \quad \delta_K^* = \delta_K + n_N g_Y \] (3.8)

\[ \dot{P}^* = \frac{1}{\theta} \frac{r^0}{\alpha} - \delta_p^* P^*, \quad \delta_p^* = \delta_p + n_N g_p \] (3.9)

### 3.1 Open-loop Nash Equilibrium Solution

We now focus on the open-loop Nash equilibrium solution of the noncooperative trade game in terms of the scale-adjusted quantities. Taking South’s terms of trade as given, the Northern planner maximizes the Southern welfare subject to (3.6)-(3.8), \( c_n \geq 0, k(0) = k_0 \) and \( a(0) = a_0 \).

\[ \max_{c_n, u, v, r} J_n = \int_0^{\infty} e^{-\rho_n t} c_n^{\gamma_n} dt, \quad \rho_n = \rho_N - (g_Y - 1) \gamma_N n_N \] (3.10)

Using Pontryagin’s minimum principle, the following are the necessary conditions:

\[ c_n^{\gamma_n - 1} = \lambda_1 \] (3.11)

\[ \lambda_1 \alpha_L \frac{y}{u} = \lambda_2 \beta_L \frac{j}{1 - u} \] (3.12)

\[ \lambda_1 \alpha_K \frac{y}{v} = \lambda_2 \beta_K \frac{j}{1 - v} \] (3.13)

\[ \lambda_1 (-\alpha_R y^* \frac{r}{r} - p^*) = 0 \] (3.14)

\[ \dot{\lambda}_1 = (\rho_n + \delta_K^* - \alpha_K y^* \frac{y}{k}) \lambda_1 - \beta_K \frac{j}{k} \lambda_2 \] (3.15)

\[ \dot{\lambda}_2 = (\rho_n + \delta_A^* - \beta_A \frac{j}{a}) \lambda_2 - \lambda_1 \alpha_A \frac{y}{a} \] (3.16)

\[ \lim_{t \to \infty} e^{-\rho_n t} \lambda_1 k \to 0, \quad \lim_{t \to \infty} e^{-\rho_n t} \lambda_2 a \to 0 \] (3.17)

where \( \lambda_1, \lambda_2 \) are the shadow values of aggregate physical capital and knowledge, respectively.

Eq. (3.11) states that along the optimal paths the marginal utility from consumption should equal to that of physical capital at every point in time. Eq.s (3.12) and (3.13) determine the sectoral allocations of labor and capital so that their respective shadow values are the same across sectors. Eq. (3.15) implies that the marginal return to physical capital must equal the return on consumption, both measured in terms of the final output.
Analogously, Eq. (3.16) requires the return on technology be equated to the return on consumption, both expressed in units of knowledge, at the margin.

The demand for the resource is given by Eqs (3.11) and (3.14),

$$r = (\alpha_R \phi_Y a^{\alpha_A} u^{\alpha_L} (vk)^{\alpha_K})^{\frac{1}{1-\alpha_R}} p^{\frac{1}{1-\alpha_R}}$$  (3.18)

The respective optimal shares of labor and capital in the final good production are

$$u = u(\lambda_1, \lambda_2, a, k)$$ and $$v = v(\lambda_1, \lambda_2, a, k)$$ by Eqs (3.12) and (3.13). Consequently,

$$\text{Sgn}(\partial u/\partial k) = \text{Sgn}(\alpha_L - \beta_L); \text{Sgn}(\partial v/\partial k) = \text{Sgn}(\alpha_K - \beta_K);$$

$$\text{Sgn}(\partial u/\partial a) = \text{Sgn}(\partial v/\partial a) = \text{Sgn}(\alpha_A - \beta_A)$$ and $$\partial v/\partial u > 0.$$

Intuitively, an increase in the stock of physical capital (technology) will cause a net shift in employment and physical capital towards the sector in which physical capital (technology) is more productive.

As for the South, faced with the demand for the resource by the North, Eq. (3.18), and taking the Northern policies as given, the Southern planner chooses a path of scale-adjusted terms of trade to maximize the Southern welfare subject to (3.6), (3.8) and (3.9), $$c_s \geq 0, k(0) = k_0$$ and $$P^*(0) = P^*_0.$$ 

$$\max_{p^*} J_s = \int_0^{\infty} e^{-\rho_s t} \left( \frac{1}{\gamma_s} c_s^{\gamma_s} - D P^* \right) dt, \quad \rho_s = \rho_S - (g_Y n_N - n_S) \gamma_S$$  (3.19)

The necessary conditions for the South are:

$$c_s^{\gamma_s-1} = \frac{(1 - \alpha_R)}{\alpha_R} \mu_1 - \frac{r^{\theta-1}}{\alpha_R p^* a^\gamma} \mu_2$$  (3.20)

$$\dot{\mu}_1 = (\rho_s + \delta_K - \alpha_K \frac{y}{k}) \mu_1 - \frac{\alpha_K}{\alpha_R} \mu_2^\gamma - \frac{\alpha_K}{(1-\alpha_R) k} c_s^{\gamma_s}$$  (3.21)

$$\dot{\mu}_2 = (\rho_s + \delta_P) \mu_2 + D$$  (3.22)

$$\lim_{t \to \infty} e^{-\rho_s t} \mu_1 k \to 0, \quad \lim_{t \to \infty} e^{-\rho_s t} \mu_2 P^* \to 0$$  (3.23)

where $$\mu_1, \mu_2$$ are the respective shadow values of the stocks of aggregate physical capital and pollution.

From Eq. (3.20), optimal Southern terms of trade must be so chosen that the sum
of incremental benefits from consumption and physical capital is equal to the marginal cost of pollution. Notice from Eq. (3.21), that the value of an extra unit of physical capital stock in the South, not surprisingly, evolves differently from the North in so far as the former internalizes the interaction between capital and pollution accumulation. Eq. (3.22), on the other hand, shows how the marginal social cost of pollution will increase as pollution itself accumulates.

Next, we consider the Nash equilibrium of this game at the steady state. A joint stationary solution of the optimality conditions for both regions determine the long run equilibrium of the trade game. Hence, assuming steady state, the following set of equations constitute the Nash equilibrium where the stationary variables are denoted by “~”:

\[
\frac{\ddot{y}}{k} - \ddot{c}_n - \frac{\ddot{p}^*\ddot{r}}{k} - \delta_k^* = 0 \quad (3.24)
\]

\[
\frac{\ddot{J}}{\ddot{a}} - \delta_A^* = 0 \quad (3.25)
\]

\[
\frac{1}{\theta} \frac{\ddot{p}^\theta}{\ddot{P}^*\ddot{a}^e} - \delta_p^* = 0 \quad (3.26)
\]

\[
\alpha_R \frac{\ddot{y}}{\ddot{r}} - \ddot{p}^* = 0 \quad (3.27)
\]

\[
\dot{v} - \frac{\alpha_K \beta_L \ddot{u}}{\alpha_K \beta_L \ddot{u} + \alpha_L \beta_K (1 - \ddot{u})} = 0 \quad (3.28)
\]

\[
\rho_n + \delta_k^* - \alpha_K \frac{\ddot{y}}{k} - \frac{\beta_K \alpha_L (1 - \ddot{u}) \ddot{y}}{\beta_L \ddot{u}} = 0 \quad (3.29)
\]

\[
\rho_n + \delta_A^* - \beta_A \frac{\ddot{J}}{\ddot{a}} - \frac{\alpha_A \beta_L \ddot{u}}{(1 - \ddot{u}) \alpha_L \ddot{a}} = 0 \quad (3.30)
\]

\[
-\alpha_R (\ddot{p}^* \ddot{r})^{\gamma_S} - \frac{\alpha_K \beta_L \ddot{u}}{k} (\ddot{P}^* \ddot{a}^e) \left( \frac{\ddot{r}^D}{\ddot{a}^e (\rho_s + \delta_p^*)} \right) + \frac{D \ddot{r}^S}{\ddot{a}^e (\rho_s + \delta_p^*)} = 0 \quad (3.31)
\]

We proceed to solve the system of equations as follows: First, we get the equilibrium growth rate of technology, \(\ddot{J}/\ddot{a} = \ddot{J}/\ddot{A}\) from Eq. (3.25). Given the growth rate of technology, we obtain the stationary sectoral allocation of labor, \(\ddot{u}\) from Eq. (3.30). Having derived \(\ddot{u}\), we use Eqs. (3.28) and (3.29) to solve for the long run sectoral allocation of capital, \(\ddot{v}\), and the output-capital ratio, \(\ddot{y}/\ddot{k}\), respectively. Given \(\ddot{y}/\ddot{k}\), the ratio of the South’s consumption to capital, \(\ddot{p}^* \ddot{r}/\ddot{k}\) can now be solved for from Eq. (3.27). Now that
we know \( \tilde{y}/\tilde{k} \) and \( \tilde{p}^*\tilde{r}/\tilde{k} \), Eq. (3.24) determines the ratio of the North’s consumption to capital, \( \tilde{c}_n/\tilde{k} \), while Eq.s (3.26) and (3.31) imply the ratio \( \tilde{P}^*/\tilde{k}^{\gamma_S} \), \( \tilde{u} \) and \( \tilde{v} \), the ratio \( \tilde{r}^{\theta}/\tilde{k}^{\gamma_S+e}\beta_k/(1-\beta_A) \) can be obtained from Eqs. (3.25) and (3.26). We use the production function for the final good and Eq. (3.25) to find the stock of capital, \( \tilde{k} \), given \( \tilde{r}^{\theta}/\tilde{k}^{\gamma_S+e}\beta_k/(1-\beta_A) \), \( \tilde{y}/\tilde{k} \), \( \tilde{u} \) and \( \tilde{v} \). Having obtained \( \tilde{k}, \tilde{a} \) and \( \tilde{r} \) are derived from Eq. (3.25) given \( \tilde{u} \) and \( \tilde{v} \) and the ratio \( \tilde{r}^{\theta}/\tilde{k}^{\gamma_S+e}\beta_k/(1-\beta_A) \). Finally, given \( \tilde{r} \) and \( \tilde{p}^*\tilde{r}/\tilde{k} \), we solve for \( \tilde{p}^* \).

3.2 Open-loop Cooperative Solution

To be able to solve the cooperative game, it is required that the scale-adjusted discount rates be the same for both regions; that is, \( \rho_n = \rho_s = \rho \). Even though this condition is not necessary under noncooperation, nevertheless, we choose parameter values to satisfy this requirement so that we can compare the numerical approximations of the cooperative and noncooperative trade games.

Written in terms of the scale-adjusted quantities, the Pareto efficient paths maximize the weighted sum of welfares subject to (3.6)-(3.9), \( c_n, c_s \geq 0, k(0) = k_0, a(0) = a_0 \) and \( P^*(0) = P_0^* \). Although, at first blush, this may seem like a straightforward optimization problem in contrast with the noncooperative mode of the game, any attempt at solution defies this early optimism. Unfortunately, the steady state of the model does not admit a closed form solution.

\[
\max_{c_n, p^*, r, u, v} J = \int_0^{\infty} e^{-\rho t} \left[ \omega \frac{1}{\gamma N} c_n^{\gamma N} + (1 - \omega) \left( \frac{1}{\gamma S} c_s^{\gamma S} - DP^* \right) \right] dt \quad (3.32)
\]

The necessary conditions are:

\[
\omega c_n^{\gamma N - 1} = v_1 \quad (3.33)
\]

\[
(1 - \omega) c_s^{\gamma S - 1} = v_1 \quad (3.34)
\]

\[
\alpha_R \frac{y}{r} v_1 + \frac{y^{\beta - 1}}{a^e} v_3 = 0 \quad (3.35)
\]

\[
v_1 \alpha_L \frac{y}{u} = v_2 \beta_L \frac{j}{(1-u)} \quad (3.36)
\]
\( v_1 \alpha_K \frac{y}{v} = v_2 \beta_K \frac{j}{1 - v} \) \hspace{1cm} (3.37)

\[ \dot{\upsilon}_1 = (\rho + \delta^*_K - \alpha_K \frac{y}{k}) \upsilon_1 - \beta_K \frac{j}{k} \upsilon_2 \] \hspace{1cm} (3.38)

\[ \dot{\upsilon}_2 = (\rho + \delta^*_A - \beta_A \frac{j}{a}) \upsilon_2 - \upsilon_1 \alpha_A \frac{y}{a} + \frac{\varepsilon}{a \theta} \frac{r^\theta}{\alpha^e} \upsilon_3 \] \hspace{1cm} (3.39)

\[ \dot{\upsilon}_3 = (\rho + \delta^*_P) \upsilon_3 + (1 - \omega) D \] \hspace{1cm} (3.40)

\[ \lim_{t \to \infty} e^{-\rho t} \upsilon_1 k \to 0, \lim_{t \to \infty} e^{-\rho t} \upsilon_2 a \to 0, \lim_{t \to \infty} e^{-\rho t} \upsilon_3 P^* \to 0 \] \hspace{1cm} (3.41)

where \( \upsilon_1, \upsilon_2 \) and \( \upsilon_3 \) are the shadow values of aggregate physical capital, technology and pollution, respectively.

We note from Eq.s (3.33) and (3.34) that along the Pareto efficient paths, the weighted marginal utilities of consumption in both regions are the same. Moreover, Eq.s (3.33), (3.34) and (3.35), imply an efficient resource price which would equate the marginal global benefits of resource use (the weighted marginal utility of consumption in both regions times the marginal product of resources) to the marginal pollution costs in the South (valued at the shadow price of pollution in the South). Also, Eq.s (3.36) and (3.37), indicate a sectoral allocation rule for labor and capital such that productivities are equalized at the margin. Finally, Eq.s (3.38), (3.39) and (3.40) indicate how the globally efficient shadow values of \( k, a \) and \( P^* \) will move over time.

The following system of equations indicate the steady state of the cooperative trade game where the efficient levels of the stationary variables are denoted by "\( \tilde{\cdot} \)";

\[ \frac{\ddot{y}}{k} - \frac{\ddot{c}_n}{k} - \frac{\ddot{\rho}^* \ddot{r}}{k} - \delta^*_K = 0 \] \hspace{1cm} (3.42)

\[ \frac{\ddot{\gamma}}{\ddot{a}} - \delta^*_A = 0 \] \hspace{1cm} (3.43)

\[ \frac{1}{\theta} \frac{\ddot{r}^\theta}{\ddot{P}^* \ddot{a}^e} - \delta^*_P = 0 \] \hspace{1cm} (3.44)

\[ \ddot{v} - \frac{\alpha_K \beta_L \ddot{u}}{\alpha_K \beta_L \ddot{u} + \alpha_L \beta_K (1 - \ddot{u})} = 0 \] \hspace{1cm} (3.45)

\[ \rho + \delta^*_K - \alpha_K \frac{\ddot{y}}{k} - \frac{\beta_K \alpha_L (1 - \ddot{u}) \ddot{y}}{\beta_L \ddot{u}} = 0 \] \hspace{1cm} (3.46)
\[
\rho + \delta_A - \beta_A \frac{j}{a} - \left( \alpha_A + \frac{\varepsilon}{\theta} \alpha_R \frac{\beta_L \tilde{u}}{\alpha_L (1 - \tilde{u})} \right) \frac{\beta_L \tilde{u}}{\alpha_L (1 - \tilde{u})} = 0
\]  
(3.47)

\[
\alpha_R \tilde{y} \omega \tilde{c}_n^{\gamma N - 1} - \frac{(1 - \omega) D_{\tilde{r} \theta}}{(\rho + \delta_p) \tilde{a} \varepsilon} = 0
\]  
(3.48)

\[
\omega \tilde{c}_n^{\gamma N - 1} - (1 - \omega) (\rho^* \tilde{r})^{\gamma s - 1} = 0
\]  
(3.49)

As previously mentioned, no closed form solutions for the steady state values exists, except for the sectoral allocations of labor and physical capital, and the output capital ratio. Using (3.43) one can get the equilibrium growth rate of knowledge, \( \frac{\dot{j}}{\dot{a}} = \frac{\dot{J}}{\dot{A}}. \)

Then, Eq. (3.47) can be used to determine the constant sectoral allocation of labor, \( \tilde{u}. \)

Subsequently, Eq.s (3.45) and (3.46) will give us the sectoral allocation of capital, \( \tilde{v}, \) and the output-capital ratio, \( \tilde{y}/\tilde{k}, \) respectively. For the rest of the variables, one needs to employ a numerical method to obtain their equilibrium values. In the next section, we describe the numerical algorithms which approximate not only the steady state but, also the transient dynamics under various parameter configurations.

### 4 Numerical Approximation of the Model

#### 4.1 Genetic Algorithms For Non-cooperative Open-loop Dynamic Games

In order to determine the open-loop Nash equilibria of the two-person differential trade game, two optimal control problems need to be solved simultaneously (Başar and Olsder, 1982). Our numerical solution strategy is first to transform the infinite horizon differential game into a finite horizon difference game using time aggregation as in Michele and Mecenier (1994), and then, to use genetic algorithms to optimize each problem using synchronous updating. That is, once best to date policies are found within each separate populations, they are passed on to the other player (GA) via the computer shared memory so that GAs can breed new and fitter populations of candidate solutions in the light of the updated policies.

\(^2\)Note from Eq.s (3.45) and (3.47) that under cooperation higher fractions of capital and labor are allocated to the technology sector due to the fact that the marginal benefit from the accelerated knowledge accumulation from the North is now internalized in both regions.
The following pseudo code shows the general outline of the algorithm for the two-region dynamic trade game:

```plaintext
procedure North GA;
begin
    Randomly initialize Pop_N(0);
    Shared memory;
    synchronize;
    evaluate Pop_N(0);
    z = 1;
    repeat
        select Pop_N(z) from Pop_N(z-1);
        copy best to shared memory;
        synchronize;
        crossover and mutate Pop_N(z);
        evaluate Pop_N(z);
        z = z + 1;
    until(termination condition);
end;

procedure South GA;
begin
    Randomly initialize Pop_S(0);
    Shared memory;
    synchronize;
    evaluate Pop_S(0);
    z = 1;
    repeat
        select Pop_S(z) from Pop_S(z-1);
        copy best to shared memory;
        synchronize;
        crossover and mutate Pop_S(z);
        evaluate Pop_S(z);
        z = z + 1;
    until(termination condition);
end;
```

In each step of this algorithm, two GAs evolve a constant size population of potential solutions. In order to reduce the time complexity, the two GAs are evolved for one generation while continuously sharing the best responses. The synchronize statement in the above algorithm is a protocol whereby each party is to wait for the other side to update their respective best structures before proceeding with a new search. This approach reduces time complexity while at the same time ensuring the convergence to the global Nash equilibrium.

### 4.2 Genetic Algorithm For Cooperative Games

In the cooperative trade game, the strategic rivalry that exists in the noncooperative trade game is eliminated by way of an “arbitration” whereby the “global fitness” as the weighted sum of each region’s respective welfare (fitness) is maximized. The cooperative trade game is thus reduced to a typical control problem which can be solved by standard GA techniques (Krishnakumar and Goldberg, 1992 and Michalewicz, 1992). The trade game involves both equality and inequality constraints. The equalities are eliminated
at the start by substitution. We penalize the remaining constraint violations by a large reduction in the fitness so as to remain within the feasible region. Generally speaking, for $s$ control variables, $T$ periods, and $l$ potential solutions, a GA performs the following steps to optimize a control problem: (1) Randomly generate an initial potential solution set, (2) Evaluate the fitness value for a solution set of $sTl$, (3) Apply selection, crossover, and mutation operations to each set of solutions to reproduce a new population, (4) Repeat steps (1), (2) and (3) until computation is terminated according to a convergence criterion, (5) Choose the solution set $sT$ based on the best fitness value from the current generations as the optimal solution set.

5 Numerical Experiments

5.1 Discretization and Numerical Parameters

We discretize problems (3.10), (3.19) and (3.32) along the lines suggested by Mercenier and Michel (1994) which ensures the steady state invariance between the continuous model and its discrete analog. The discrete-time approximation of infinite horizon noncooperative North/South trade model with steady state invariance is as follows. The cooperative model is time aggregated in a similar fashion.

North:

$$\max J_n = \sum_{h=0}^{H-1} \mu^{N}_{h} \left( \frac{1}{\gamma N} c^N_n \right) + \mu^{N}_{H-1} G^N(k(t_H), a(t_H))$$

subject to

$$k(t_{h+1}) - k(t_h) = \Delta_h(y(t_h) - c_N(t_h) - p^*(t_h)r(t_h) - \delta_k k(t_h))$$

$$a(t_{h+1}) - a(t_h) = \Delta_h(\phi_A a^A(1 - u)^{\beta A} [(1 - v)k]^{\beta K} - \delta^* A a(t_h))$$

$$k(0) \text{ and } a(0) \text{ given.}$$

South:

$$\max J_s = \sum_{h=0}^{H-1} \mu^{S}_{1,h} \left( \frac{1}{\gamma S} c^S_s - DP^* \right) + \mu^{S}_{H-1} G^S(k(t_H), P^*(t_H))$$

subject to

$$\text{See Michalewicz (1992) for various GA approaches to handle linear constraints.}$$

$$\text{See Alendar and Özyıldırım (1998) for derivation.}$$
\[ k(t_{h+1}) - k(t_h) = \Delta_h (g(t_h) - c_N(t_h) - p^*(t_h)r(t_h) - \delta_K k(t_h)) \]

\[ P^*(t_{h+1}) - P^*(t_h) = \Delta_h \left( \frac{1}{\theta} \frac{r^\theta}{a^\varepsilon} - \delta_p^* P^* \right) \]

\[ k(0) \text{ and } P^*(0) \text{ given.} \]

where \( H \) is the assumed terminal time when the stationary state is reached, \( \Delta_h \) is a scalar factor that converts the continuous flow into stock increments, \( \Delta_h = t_{h+1} - t_h \) and \( \mu^j \) is the sequence of discount factors of the region \( i = N, S \) for which the stationary solution of the discrete-time problem is equivalent to the corresponding continuous-time problem. These sequences are given by the following recursions:

\[ \mu^N_h = \frac{\mu^N_{h-1}}{1 + \Delta_h \rho_n}, \quad \mu^S_h = \frac{\mu^S_{h-1}}{1 + \Delta_h \rho_s} \]

where \( \mu^N_0 > 0 \) and \( \mu^S_0 > 0 \). The functions \( G^j(\cdot), j = N, S \), denote the terminal values.

Displayed in Table 1 are the set of benchmark parameter values used in the numerical simulations. For the North, the baseline parameter values are adopted from earlier studies using calibration exercises.\(^5\)

<table>
<thead>
<tr>
<th>Table 1: Benchmark parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production: ( \phi_Y = 1.0, \alpha_K = 0.35, \alpha_L = 0.65, \alpha_A = 0.15, \alpha_R = 0.10 )</td>
</tr>
<tr>
<td>Technology: ( \phi_A = 1.0, \beta_A = 0.60, \beta_L = 0.50, \beta_K = 0.20 )</td>
</tr>
<tr>
<td>Pollution: ( \delta_P = 0.08, \varepsilon = 0.10, \theta = 2.0 )</td>
</tr>
<tr>
<td>Preferences: ( \rho_N = 0.04, \rho_S = 0.04, \gamma_N = -0.5, \gamma_S = -0.5, D = 0.01, \tau = 0 )</td>
</tr>
<tr>
<td>Depreciation &amp; population: ( \delta_K = 0.05, \delta_A = 0.01, n_N = 0.015, n_S = 0.015 )</td>
</tr>
</tbody>
</table>

Both the final good and the new technologies exhibit increasing returns to scale.\(^6\) In both regions, we assume a rate of time preference of 4 percent and an intertemporal elasticity of substitution of 67 percent.\(^7\) Physical capital is assumed to depreciate at 5 percent, which is a common benchmark in the real business cycle literature; see e.g. Cooley (1995).

\(^5\)See, for example, Lucas (1988), Ortigueira and Santos (1997), and Jones (1995).

\(^6\)There does not exist much empirical literature on research functions, especially if separate elasticities for labor, capital and technology are required. For example, Adams (1990) and Caballero and Jaffee (1993) provide thorough empirical investigations that are ultimately unsuccessful in reporting separate elasticities for labor and technology.

\(^7\)The empirical evidence on the intertemporal elasticity of substitution for the United States is quite varied, ranging between 0.1 (Hall, 1988) and 1 (Beaudry and van Wincoop, 1995).
The knowledge, on the other hand, depreciates at a slower rate of 1 percent. Populations are assumed to grow at 1.5 percent. Information on pollution parameter values is sparse, and therefore, we conduct some sensitivity analysis with alternative parameter values. Equal weights, \( \omega = 0.5 \), are assigned to both regions in the cooperative game. With these benchmark parameter values, pollution exhibits pure public bad characteristics, and we have \( g_Y = 1.454, g_A = 1.977, g_P = -0.227 \) and \( g_R = -0.015 \). The benchmark equilibrium values are reported in Table 2.

<table>
<thead>
<tr>
<th>Table 2: Benchmark Equilibrium Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Non-cooperation</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>( \hat{p}^* )</td>
</tr>
<tr>
<td>0.307</td>
</tr>
</tbody>
</table>

| Cooperation                          |
|                                      |
| \( \hat{p}^* \) | \( \hat{r} \) | \( \hat{u} \) | \( \hat{v} \) | \( \hat{k} \) | \( \hat{P}^* \) | \( \hat{\tilde{u}} \) | \( \hat{\tilde{c}}_{\tilde{u}} \) |
| 1.165 | 1.861 | 0.926 | 0.944 | 18.161 | 13.955 | 124.037 | 2.169 |

As for the genetic operators in the numerical experiments, we use the public domain GENESIS as a platform (Grefenstette, 1990) and modify the GENESIS codes as we need them. All experiments are run on a IBM RS/6000 running AIX 5.2. A typical run uses population size, \( j = 50 \), runs 2 million generations, crossover rate is 0.60 and mutation rate is 0.001. None of the results depends on the values of genetic operators other than run time by the choice of number of generations.

For each parameter configuration, we have to implement two separate experiments. Hence, we are limited by the increased computational costs in our scope for a complete sensitivity analysis. The selection strategy is elitist so that the best performing strategy in the population of survivors is retained. Were it not for the elitist selection, the best structures may disappear making for a nonconvergence.

Since GAs work with constant-size populations of candidate solutions, GA searches are initialized from a number of points. Initialization routines may vary. We however start from randomly generated populations so as not to prejudice the convergence of the populations on the initial ones. Therefore, a randomly initialized GA is less prone to numerical instability that may be caused by initialization. For the GA parameters which might cause instability, we used the parameters chosen and studied on various optimiza-
tion experiments by Grefenstette (1986). From the result of the experiments in the paper, the convergence is self evident. The termination conditions are specified beforehand as a certain number of iterations. We gradually increase the number of iterations until no further improvements are observed. In the time-aggregated model, we assume 31 periods ($H = 31$) with a dense equally spaced gridding of the time horizon $T(t(H) = 360)$, which is sufficient to capture the convergence over time.

5.2 Numerical Findings

Figures 1-16 of Appendix and Tables 3-6 summarize our numerical findings based on the assumed parameter values. First, with benchmark parameter values, global welfare under cooperation is 36.96% higher compared to noncooperation. Furthermore, South has more to gain from such a regime switch (Southern welfare increases by 51.18% while Northern welfare decreases by 21.18%) indicating the severity of the Northern noncooperation. Moreover, as depicted in Table 3, the welfare improvements in response to the variations in the benchmark parameter values are usually larger under a cooperative trade regime attesting to the severity of loss of efficiency due to policy spillover.

Table 3: Percentage welfare changes from benchmark under alternate parameter values

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon = 0.50$</td>
<td>6.14</td>
<td>6.33</td>
<td>6.04</td>
<td>6.17</td>
<td>6.07</td>
<td>6.25</td>
</tr>
<tr>
<td>$\theta = 4.0$</td>
<td>-1.58</td>
<td>-2.02</td>
<td>-0.33</td>
<td>1.13</td>
<td>-0.67</td>
<td>-0.37</td>
</tr>
<tr>
<td>$\phi_Y = 2.0$</td>
<td>35.30</td>
<td>35.26</td>
<td>34.62</td>
<td>34.14</td>
<td>34.80</td>
<td>34.67</td>
</tr>
<tr>
<td>$\phi_A = 2.0$</td>
<td>4.85</td>
<td>4.75</td>
<td>4.43</td>
<td>4.71</td>
<td>4.54</td>
<td>4.73</td>
</tr>
</tbody>
</table>

Studying Figures 1-16, a number of results stand out. To wit, in the long-run cooperation leads to sizable increases in physical capital and knowledge stocks, resource use and Southern terms of trade (hence in the Southern consumption) at the expense of more polluted Southern environment. The shares of labor and capital in the final good sector and Northern consumption, on the other hand, are lower with cooperation regardless of the parameter values.

Thus, noncooperative Northern investment plans are globally inefficient as they understate the true world marginal benefit by the amount of the marginal improvement in the Southern welfare due to the incremental reduction in the pollution level. On the other
hand, Southern noncooperation adds to the dynamic inefficiency to the extent her market power limits growth in the North. This deleterious effect of resource monopoly, however, is mitigated to the degree South internalizes the knowledge spillovers.

The dynamic inefficiency of the noncooperative trade regime and the failure of cooperation can be explained as follows. In the noncooperative mode, the shadow value (the marginal benefit) of the physical capital stock differs for the two regions as the regions have different preferences (fitnesses) leading to conflicting policies and harmful ‘policy externalities’. Moreover, when policies are chosen with a view to maximize own fitnesses taking the rival’s as given, the ‘incentive’ effects of the policies are ignored. The South chooses resource prices for any ‘given’ investment policy for physical capital of the North, thus, ignoring the fact that a lower price today (lower Southern consumption) may ‘induce’ the North to invest in physical capital more today which then leads to higher knowledge stock and higher prices (higher Southern consumption) as the higher capital stock shifts the demand for resources tomorrow. The North, on the other hand, ignores the fact that an initially higher investment in physical capital (hence in knowledge stock) at the expense of lower Northern consumption may induce South to ask for lower resource prices today in return for higher prices tomorrow (as the demand for resources will shift) and also lead to higher Northern consumption in the future as the amount to be invested will be lower in the future (higher Northern consumption).

Parties ignore the incentive effects for the fact that promises are not credible. If South were to offer cooperative prices, it would not be optimal for the North to invest in physical capital as much promised as along the cooperative path: North will consume more and invest less. Likewise, if North were to commit itself to the investment plan along the cooperative path, then it would not be optimal for the South to ask for the cooperative prices: South will raise prices and consume more. Failing to cooperate, the parties will revert to their respective Nash strategies.

5.3 Changes in the Knowledge Diffusion

We first investigate effect of an increased knowledge diffusion rate from $\varepsilon = 0.10$ to $\varepsilon = 0.50$. With the new rate of knowledge spillovers, the growth rates of final output, $g_Y$, technology, $g_A$, and pollution, $g_P$, increase to 1.521, 2.011 and 0.372, respectively,
while the growth rate of resource, $g_R$, decreases to -0.261. The equilibrium values with the new knowledge spillover rate are reported in Table 4.8

<table>
<thead>
<tr>
<th>Table 4: Equilibrium Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Non-cooperation</td>
</tr>
<tr>
<td>$\hat{p}^*$</td>
</tr>
<tr>
<td>0.143</td>
</tr>
<tr>
<td>Cooperation</td>
</tr>
<tr>
<td>$\hat{p}^*$</td>
</tr>
<tr>
<td>0.530</td>
</tr>
</tbody>
</table>

First, note from the first order conditions that an increase in $g_Y$ causes a shift in employment and physical capital from the final goods sector to the technology sector under both cooperative and noncooperative modes of trade. Additionally, from a global efficiency point of view, technology is less pollutant, this shift in employment and capital to the technology sector is stronger under a cooperative trading regime.

Observe from Table 3 the increase in regional welfares attendant with stronger technology diffusion. We see about %6.07 improvement in global welfare with uncoordinated trading policies attesting to the importance of access to knowledge. The policy implication is that even if the parties fail, say due to enforcement problems, to realize the first best solution, they may still achieve significant improvements in global welfare by strengthening the knowledge flows from North to South. It may be costly to set up global institutions to monitor and enforce North/South cooperations. To the extent that knowledge diffusion can be enhanced relatively cheaply, regions may opt to cooperate on sharing knowledge related to pollution control.

Figures 1-4 show that when knowledge spillovers get stronger, the marginal cost of pollution falls, thus inducing South to ask for lower prices for the resource. Consequently, lower resource prices enable North to accumulate not only more physical but, also more technology shifting labor and capital to the technology sector thereby leading to a lower level of pollution. These effects are more significant in the cooperative game since the positive effect of knowledge spillovers on pollution accumulation is internalized in both

---

8Because of the chosen parameter values, when noncooperation is assumed, changes in equilibrium $u$ and $v$ values under alternate knowledge spillover rates are small and not reflected in Table 4. Here equilibrium values of $u$ and $v$ decrease by 0.00005 and 0.00004 respectively. This can be observed from Figure 2.
regions.

5.4 Changes in resource damage rate

We now increase the order of environmental damage due to resource extraction from \( \theta = 2.0 \) to \( \theta = 4.0 \). With this change, the growth rates of final output, \( g_Y \), technology, \( g_A \), and resource, \( g_R \) increase to 1.455, 1.978 and -0.007, respectively, while the growth rate of pollution, \( g_P \), decreases to -0.228. The equilibrium values with the new rate of environmental damage are displayed in Table 5.\(^9\)

<table>
<thead>
<tr>
<th>Table 5: Equilibrium Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-cooperation</td>
</tr>
<tr>
<td>( \hat{p}^* )</td>
</tr>
<tr>
<td>0.376</td>
</tr>
<tr>
<td>Cooperation</td>
</tr>
<tr>
<td>( \hat{p}^* )</td>
</tr>
<tr>
<td>1.492</td>
</tr>
</tbody>
</table>

Observe from Figures 5-8 that along with increase in the order of environmental damage, the marginal cost of pollution rises in the South thus increasing the prices for the resource. With higher resource prices, North slows down accumulation of not only physical capital but also knowledge by shifting labor and capital away from the technology sector. However, the effect of the decrease in resource use dominates the effect of the decrease in knowledge stock, leading to lower pollution in the South. Again, these effects are stronger under cooperation as since pollution is globally internalized.

Furthermore, observe from Table 3 the decrease in regional welfares due to stronger environmental damage. Under noncooperation, the Northern and the Southern welfares decrease by 1.58% and 0.33%, respectively. With cooperation, while the Northern welfare further deteriorates, the Southern welfare further improves. Whereas the global welfare worsens by about a 0.67% with noncooperation, it declines by 0.37% with cooperation.

\(^9\)Due to the chosen parameter values, changes in equilibrium \( u \) and \( v \) values under noncooperation with alternate environmental damage rates are very small and not reflected in Table 5. Here equilibrium values of \( u \) and \( v \) decrease by 0.000001 and 0.0000008 respectively.
5.5 Productivity shocks in final good sector

We now investigate the long run effects of an exogenous change in the productivity of the final output. With a productivity shock, the growth rates remains the same.

<table>
<thead>
<tr>
<th>Table 6: Equilibrium Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Non-cooperation</td>
</tr>
<tr>
<td>$\tilde{\rho}^*$</td>
</tr>
<tr>
<td>1.273</td>
</tr>
<tr>
<td>Cooperation</td>
</tr>
<tr>
<td>$\check{\rho}^*$</td>
</tr>
<tr>
<td>4.838</td>
</tr>
</tbody>
</table>

First, observe the substantial improvements in Northern, Southern and global welfares under both cooperative and noncooperative trading policies from Table 3. Though percent welfare improvements with noncooperative and cooperative policies are almost the same, cooperative trading policy leads to higher levels of welfare for both regions.

Note from Figures 9-12 that increased productivity in the final good sector results higher levels of both technology and capital. Increased knowledge stock decreases pollution. Also, demand for the resource by the North decreases, leading to higher Southern terms of trade.

5.6 Productivity shocks in technology sector

Finally, we investigate the long run effects of an exogenous change in the productivity of the technology. The growth rates again remains the same.

First, observe the substantial improvements in Northern, Southern and global welfares under both cooperative and noncooperative trading policies from Table 3. Also note that cooperation again leads to higher welfare improvements.

Notice from Figures 13-16 that the initial shift in employment and physical capital to technology sector becomes more significant due to increased productivity. Also, since the flow of knowledge stock increases, final output (hence physical capital) increases and pollution falls. Moreover, due to higher flow of technology, the demand for the resource by the North decreases, leading to higher Southern terms of trade.
Table 7: Equilibrium Values

<table>
<thead>
<tr>
<th>Non-cooperation</th>
<th>Cooperation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{p}^*$</td>
<td>$\hat{p}^*$</td>
</tr>
<tr>
<td>0.489</td>
<td>1.858</td>
</tr>
<tr>
<td>$\tilde{r}$</td>
<td>$\tilde{r}$</td>
</tr>
<tr>
<td>1.617</td>
<td>1.827</td>
</tr>
<tr>
<td>$\tilde{u}$</td>
<td>$\tilde{u}$</td>
</tr>
<tr>
<td>0.928</td>
<td>0.926</td>
</tr>
<tr>
<td>$\tilde{v}$</td>
<td>$\tilde{v}$</td>
</tr>
<tr>
<td>0.946</td>
<td>0.944</td>
</tr>
<tr>
<td>$\tilde{k}$</td>
<td>$\tilde{k}$</td>
</tr>
<tr>
<td>22.381</td>
<td>28.384</td>
</tr>
<tr>
<td>$\tilde{P}^*$</td>
<td>$\tilde{P}^*$</td>
</tr>
<tr>
<td>8.814</td>
<td>11.062</td>
</tr>
<tr>
<td>$\tilde{c}$</td>
<td>$\tilde{c}$</td>
</tr>
<tr>
<td>743.929</td>
<td>875.091</td>
</tr>
<tr>
<td>$\tilde{n}$</td>
<td>$\tilde{n}$</td>
</tr>
<tr>
<td>5.471</td>
<td>3.393</td>
</tr>
</tbody>
</table>

Note that even though the directions of changes with a technology shock are same as the directions of changes with a productivity shock, a technology shock affects the final good production through its productive elasticity whereas a productivity shock has a direct and more substantial effect.

6 Conclusion

TO BE COMPLETED.
References


Appendix: Figures

The benchmark equilibrium values are represented with a dotted line in the pictures below. In Figures 1 and 5, however, the benchmark equilibrium values are not displayed as otherwise the dynamics become invisible due to the differences in magnitudes.

Figure 1. Scale-adjusted price and resource

Figure 2. Sectoral allocations of labor and capital
Figure 3. Scale-adjusted capital and technology

Figure 4. Scale-adjusted pollution
Figure 5. Scale-adjusted price and resource

Figure 6. Sectoral allocations of labor and capital
Figure 7. Scale-adjusted capital and technology

Figure 8. Scale-adjusted pollution
Figure 9. Scale-adjusted price and resource

Figure 10. Sectoral allocations of labor and capital
Figure 11. Scale-adjusted capital and technology

Figure 12. Scale-adjusted pollution
Figure 13. Scale-adjusted price and resource

Figure 14. Sectoral allocations of labor and capital
Figure 15. Scale-adjusted capital and technology

Figure 16. Scale-adjusted pollution