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Titans that Clash and a State that Buffers

Serhat Doğan[1], Emin Karagözoğlu[1,2], Kerim Keskin[3], and Hüseyin Çağrı Sağlam[1]

Abstract

We present a game-theoretic approach to the analysis of the emergence or survival of buffer states. We analyze a two-stage game with three players orderly located on a linear territory, where the player in the middle is passive, and the players on the two ends are aggressive with options to declare war against the others. We conduct an equilibrium analysis and characterize the conditions under which the passive player acts as a buffer state between the aggressive players. We find various equilibrium outcomes, which can be grouped into the following categories: (*i*) peace with buffer, (*ii*) peace without buffer, and (*iii*) the last man standing. Our comparative static analyses reveal valuable insights regarding the factors affecting the existence of buffer states.

Keywords

buffer state, dynamic contest, international conflict, territorial conflict, warfare

> *“States do not choose to become buffers. It is a role thrust upon them by a hostile international environment over which they have no control. Buffer states are lesser actors sandwiched between more powerfully endowed, ambitious, and often aggressive entities. The purpose of the buffer state is established by these external competitors. They become sacrificial elements in a larger contest.” Lawrence Ziring, 1987*

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[1] Bilkent University, Ankara, Turkey
[2] CESifo Munich, Munich, Germany
[3] ADA University, Baku, Azerbaijan

Corresponding Author:
Kerim Keskin, School of Business, ADA University, 61 Ahmadbay Aghaoglu Street, Baku AZ1008, Azerbaijan.
Email: kkeskin@ada.edu.az

Introduction

A buffer state is defined as a "small independent state lying between two larger, usually rival, states" (see Mathison 1971, 107). Similarly, Menon and Snyder (2017, 966) write that buffer states lie "between the spheres of influence of two or more powerful states but are not allied with or dominated by any of them." These definitions identify three properties of a buffer state: (*i*) geography, specifying its location; (*ii*) capability, also capturing its inferiority in military power; and (*iii*) independence, indicating that it does not act under the influence of either neighboring state (see Partem 1983, 4). There are a number of states that have been considered as buffer states for certain time periods. Some examples are Morocco in the 16th century (between the Ottoman Empire, Spain, and Portugal) (see Cory 2016); Uruguay in the 19th century (between Argentina and Brazil) (see Knarr 2012); Afghanistan in the 19th century (between the British and Russian Empires) (see McLachlan 1997); Bhutan and Sikkim in the 20th century (between China and India) (see Levi 1959); Poland during World War II (between Germany and the Soviet Union) (see Suvorov 2013); and Lebanon since the 1990s (between Israel and Syria) (see Balanche 2017).[1]

This paper presents a game-theoretic approach to the analysis of the emergence or survival of buffer states. To that end, we formulate a two-stage game including three independent states located on a linear territory. The territory is divided into three regions such that each region is initially controlled by a different state. The state controlling the central region is a passive player, whereas the other states are aggressive with options to declare war against the other players. In each stage, an aggressive state chooses whether to declare war against another state, but since the passive state is located between the aggressive states, the latter cannot attack each other while the former still survives. If the passive state disappears, however, then there is a possible war between the aggressive states (modeled as a one-shot contest game), which may end with the destruction of either state. As such, although capturing the territory of the passive state has a standalone value, it may not be optimal to capture it, since it may result in the capturing state's destruction. We label the passive player as a buffer state, when neither aggressive state finds it optimal to attack (and possibly capture) the central region in equilibrium.[2] It is worth emphasizing that the assumption that the central region is controlled by a passive player, with a potential to become a buffer state, is in line with the buffer state definitions found in the literature (see Partem 1983; Ziring 1987). That is, a buffer state is expected to be a lesser actor with relatively lower ambitions and military power to extend its territory.

We analyze subgame perfect Nash equilibrium. Our analysis reveals that there are various equilibrium outcomes, which can be grouped into three categories: (*i*) peace with buffer, (*ii*) peace without buffer, and (*iii*) the last man standing. The first category, which is the only equilibrium type where the passive player surely survives as a buffer state, is realized only if the aggressive state that considers capturing the central region anticipates an attack from the third party in the future and knows that it will be in a disadvantaged position in the respective war. In the second category, we see that the passive state is attacked by at least one of the aggressive states, but there appears no further conflict between the aggressive states after the passive state is destructed. As for the third category, it is observed that the destruction of the passive state is surely followed by a warfare between the two aggressive states, such that there remains only one surviving state at the end. The last two types of equilibrium outcomes *could* indicate that the passive player was not a buffer state in the first place.

First, we identify the equilibrium conditions for war and peace in the second stage, and then we focus on the characterization of equilibrium conditions under which a peaceful equilibrium arises in the first stage. In such an equilibrium, none of the aggressive states chooses to attack the passive state, implying the emergence of a buffer state in our model. Second, we conduct comparative static analyses using the main parameters of interest. We opt to be brief here, since the detailed interpretations of these results require a deeper understanding of the modeling assumptions, which will be presented in *The Model*. To summarize our findings, we can say that the set of parameter values under which a buffer state survives gets narrower if exerting effort in warfare becomes less costly for the state that captured the central region or more costly for the state that did not capture it. As for the territorial valuations, given a fixed value of the central region, a buffer state exists for intermediate values of the other regions. Third, for the sake of completeness, we investigate the cases in which the first stage is not peaceful. After analytically showing that certain types of equilibria cannot be realized in our model, we provide a numerical analysis for a selection of parameter values to illustrate the equilibrium strategies before and after the passive state is destructed.

The organization of the paper is as follows: In *Relevant Literature*, we present a brief review of the relevant literature and emphasize our contribution to that literature. In *The Model*, we provide the details of the game model. In *The Results*, we present the analysis of subgame perfect Nash equilibrium, the equilibrium conditions for the existence of a buffer state, comparative static analyses, and a numerical analysis to further understand the possible equilibrium outcomes. We conclude in *Concluding Remarks*.

Relevant Literature

Based on its research question and methodology, our paper is potentially of interest to at least two different audiences: (i) scholars in international relations and political science working on conflict, and (ii) scholars in game theory working on the applications of dynamic contest models. In what follows, we confine ourselves to earlier works that fall into one of these broad categories.

According to Fazal (2004), the first use of the term "buffer state" appeared in the late 1800s, when describing Afghanistan's status between the British Empire (controlling a portion of South Asia) and the Russian Empire. Despite the fact that buffer states are well studied in the international relations and political science literature (e.g., Gear 1941; Levi 1959; Partem 1983; Chay and Ross 1986; McLachlan 1997; Fazal 2004; Poast 2013; Bayly 2015; Menon and Snyder 2017; Pedi 2020), to the best of our knowledge, the published works lack a mathematical description and/or a game-theoretic analysis of the concept. This paper aims to fill that gap in the literature. Hence, our contribution is mostly on the theoretical side. We are the first to formulate a dynamic game model to analytically investigate the emergence or survival of buffer states.

In the last few decades, the international relations literature has adopted mathematical and game-theoretic approaches to study the nature and dynamics of conflict. These include guns and butter models where each player is endowed with valuable resources to be divided between productive effort and fighting effort (see Hirshleifer 1991, 1995; Anbarci, Skaperdas, and Syropoulos 2002); bargaining and war models where warfare arises as a result of bargaining failure (see Fearon 1996; Filson and Werner 2002; Powell 2006; Leventoglu and Slantchev 2007); and spatial models of conflict where the locations, borders, and some other geographical characteristics of countries in question are taken into account (see Morrow 1986; Alesina and Spolaore 1997; Adamson and Kimbrough 2021). Among these, the current work is most closely related to the last category. Those papers study various issues such as the analysis of coercion and coalition formation for the resolution of some international issues (see Morrow 1986), how the number and borders of countries are endogenously determined (see Alesina and Spolaore 1997), whether and how geography causes state fragmentation (see Kitamura and Lagerlof 2020), the effects of country sizes and numbers in a region on violence (measured by fatalities from the conflict)(see Adamson 2021), and how the costs of conflict over spatially dispersed resources affect the geographic extent of a territory (see Adamson and Kimbrough 2021). Our paper has certain similarities with these papers and alike in that it is also concerned with an important international relations issue and investigates that issue in a model with a spatial/geographical structure.

From a game-theoretic perspective, we present a novel application of contest models in game theory.[3] We analyze a stylized dynamic contest model that focuses on the strategic dynamics between two contending parties (i.e., the aggressive states), the nature of which significantly depends on the fate of a third party (i.e., the passive state). Our model has three key modeling choices: (i) a territorial structure, (ii) endogenous attack choice, and (iii) endogenous player strength. As such, our work is related to multiple strands of literature on contest games. First, to study the emergence of buffer states, we assume that three states are orderly located on a linear territory. In terms of assuming a geographical structure, McAfee (2000) and Konrad and Kovenock (2005) study tug-of-war games between two players on a line, while Dziubinski, Goyal and Minarsch (2021) analyze contests on a network of n players. Second, we assume that the aggressive states can choose not to declare war in either stage. In that regard, Bester and Konrad (2004, 2005) and Polborn (2006) investigate models in which a player decides how long to wait before declaring a war against the other player. Third, we assume that states have different strengths governed by their cost parameters and dependent on which state captured the central region. This is

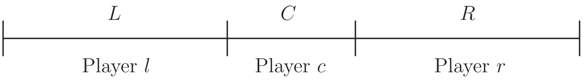

Figure 1. The linear territory structure.

similar to Polborn (2006), Bevia and Corchon (2013), Clark, Nilssen, and Sand (2020), who analyze models in which the result of a battle influences the players' strength levels in future battles.

In comparison to the abovementioned works on spatial conflict models and contest models, our difference lies in the research question analyzed. The set of our assumptions corresponds to a model that enables us to study the emergence of buffer states in a game-theoretic framework.

The Model

Located on a linear territory, there are three regions $\{L, C, R\}$ representing Left, Center, and Right, respectively (see Figure 1). There are three players, each controlling one of those regions. The players are denoted by lower-case letters corresponding to the regions they initially control: $\{l, c, r\}$. Player c is defined as a *passive* player without any strategic options, while players l and r compete in a two-stage dynamic game in which they are allowed to declare war against player c or against each other for the control of certain region(s) of the territory. However, for players l and r to be able to attack one another, region C must first be captured by either player, due to the linear territory structure. Accordingly, region C can be interpreted as a buffer zone between players l and r. Here we theoretically investigate the conditions under which player c survives as a buffer state.

In the first stage of our *tri-state buffer game*, all three players are present. Since player c is a passive player, only players l and r act in the respective stage game. Each player simultaneously chooses a strategy from the set $\{Attack, Wait\}$. The strategies indicate whether the player chooses to attack player c or not. If (*Wait*, *Wait*) is chosen, then neither player attacks player c, and the game ends peacefully and without proceeding to the second stage. If (*Attack*, *Wait*) is chosen, then players l and c engage in warfare, which is modeled as a lottery where player l becomes successful in her attack with a probability of $p_l \in (0, 1]$. If player l is indeed successful, then player c is eliminated, and region C is captured by player l. In such a case, the game proceeds to the second stage. Conversely, with a probability of $1 - p_l$, player l becomes unsuccessful in her attack,[4] and the game ends without proceeding to the second stage and without any territorial change. The latter assumption is supported by the fact that player c represents a lesser actor with no ambition and/or power to extend her territory. On the other hand, if (*Wait*, *Attack*) is chosen, then a symmetric case occurs. That is, player r attacks player c and captures the central region with a probability of $p_r \in (0, 1]$. Finally, if (*Attack*,

Attack) is chosen, then the Nature determines who gets to attack player c and whether the attacker is able to capture the central region. In particular, we assume that player l captures region C with a probability of $q_l \in (0, p_l]$, player r captures it with a probability of $q_r \in (0, p_r]$, and player c survives with the remaining probability, where $\max\{p_l, p_r\} \leq q_l + q_r \leq 1$.[5]

In the second stage, two players are present: l and r. Though, for notational simplicity, we either use "lc" or "rc" to represent one of those players. For instance, player lc would appear in this stage under the condition that player l was the one who captured region C in the first stage, and the notation indicates that player lc controls both regions L and C. The symmetric statement is true for player rc. Following this notational change, one can see that the player set in the second stage game would be either $\{lc, r\}$ or $\{l, rc\}$.

Without loss of generality, assume that it was player l who captured region C in the first stage. Then, the second stage game is played between players lc and r. Similar to above, players choose their strategies in the following stage game:

		Player *r*	
		Attack	*Wait*
Player *lc*	*Attack*	$U^2_{lc}(Attack, Attack)$, $U^2_r(Attack, Attack)$	$U^2_{lc}(Attack, Wait)$, $U^2_r(Attack, Wait)$
	Wait	$U^2_{lc}(Wait, Attack)$, $U^2_r(Wait, Attack)$	$U^2_{lc}(Wait, Wait)$, $U^2_r(Wait, Wait)$

Accordingly, each player's strategy set is still given by $\{Attack, Wait\}$. Different from the first stage game, however, *Attack* now indicates that the player chooses to attack the other player. If (*Wait*, *Wait*) is chosen, then neither player declares war against each other, and the game ends peacefully (even though there is no buffer in between). In either of the remaining outcomes, i.e., in (*Attack*, *Wait*), (*Wait*, *Attack*), or (*Attack*, *Attack*), players lc and r engage in warfare, which is now modeled as a one-shot *winner-takes-all* contest game. The players choose their contest efforts, denoted by $e_{lc,r} \in [0, \infty)$ and $e_{r,lc} \in [0, \infty)$, and the winner is determined by the following Tullock-type contest success function

$$P_{lc,r} = \frac{e_{lc,r}}{e_{lc,r} + e_{r,lc}} \text{ and } P_{r,lc} = \frac{e_{r,lc}}{e_{lc,r} + e_{r,lc}}.$$

The winner takes control of the whole territory, while the loser ends up with nothing. The game ends. In the respective contest games and for each player $i \in \{l, lc, r, rc\}$, we consider a linear cost function $C_i(e_i) = c_i e_i$ where $c_i > 0$.

Now, to complete our model definition, we define the payoff functions for each player under each possible outcome of our tri-state buffer game. Assume that each player collects a payoff from the region(s) she controls at the end of the game. The payoff received from each region is denoted by V_L, V_C, and V_R, respectively. In case a player controls multiple regions, she gets an aggregate value of those regions. Furthermore, we assume that each player incurs a constant cost of $c_A > 0$ from declaring a war against the other player in the second stage game.[6]

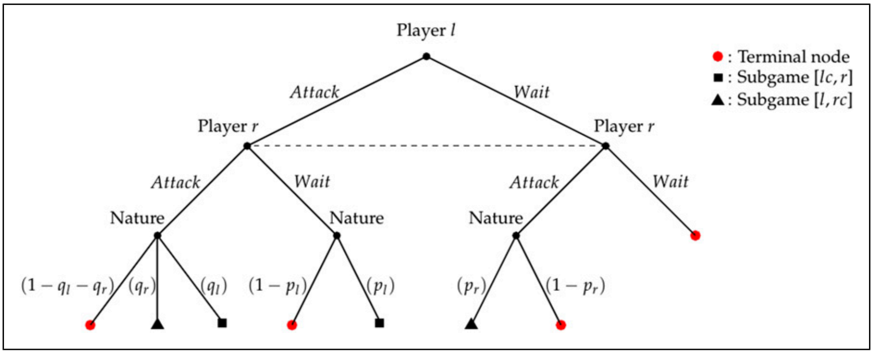

Figure 2. The first stage game.

In the second stage game, we have

$$U^2_{lc}(Wait, Wait) = V_L + V_C \text{ and } U^2_r(Wait, Wait) = V_R.$$

In the other outcomes, there is a war between the two players, where player *lc* aims to maximize

$$\mathcal{W}_{lc,r}(e_{lc,r}, e_{r,lc}) = \frac{e_{lc,r}}{e_{lc,r} + e_{r,lc}}(V_L + V_C + V_R) - c_{lc}e_{lc,r}. \tag{1}$$

and player *r* aims to maximize

$$\mathcal{W}_{r,lc}(e_{lc,r}, e_{r,lc}) = \frac{e_{r,lc}}{e_{lc,r} + e_{r,lc}}(V_L + V_C + V_R) - c_r e_{r,lc}. \tag{2}$$

By an abuse of notation, in the following payoff functions, we omit the effort choices in the respective contest game. We have

$$U^2_{lc}(Attack, Wait) = U^2_{lc}(Attack, Attack) = \mathcal{W}_{lc,r} - c_A, U^2_{lc}(Wait, Attack) = \mathcal{W}_{lc,r}$$

and

$$U^2_r(Attack, Wait) = \mathcal{W}_{r,lc}, U^2_r(Attack, Attack) = U^2_r(Wait, Attack) = \mathcal{W}_{r,lc} - c_A.$$

This completes the definition of the second stage game between players *lc* and *r*. The payoff profiles in the second stage game between players *l* and *rc* can be written in a similar manner.

As for the first stage game, Figure 2 summarizes the strategic interaction. Player *r* does not observe the action chosen by player *l*. If any player chooses to attack player *c*, then the Nature randomly determines the outcome. In case of (*Wait*, *Wait*), the game ends with no territorial change, so that players *l* and *r* collect V_L and V_R, respectively, whereas player *c* collects V_C from controlling region *C*. In the other terminal nodes, since the Nature decides in favor of player *c*, there is still no territorial change, such that all players collect the same payoffs from the respective controlled regions. In the remaining nodes, which are indicated either by a square node or a triangle node in the figure, an attack on player c is successful, and the game proceeds to the second stage. If player l captures the central region, then Subgame $[lc, r]$ starts, but if player r captures the central region, then Subgame $[l, rc]$ starts.

The expected payoff functions can be written as continuation payoffs. Given the generic forms of strategy profiles $S_j^c = (s_l, s_r)$ in the first stage game, $S_j^{lc} = (s_{lc}, s_r)$ in Subgame $[lc, r]$, and $S_j^{rc} = (s_l, s_{rc})$ in Subgame $[l, rc]$, let $S_1^k = (Wait, Wait)$, $S_2^k = (Wait, Attack)$, $S_3^k = (Attack, Wait)$, and $S_4^k = (Attack, Attack)$ for any $k \in \{c, lc, rc\}$. We have

$$\begin{aligned}
U_l^1\left(S_1^c, \ \cdot\ , \ \cdot\ \right) &= V_L, \\
U_l^1\left(S_2^c, \ \cdot\ , S_j^{rc}\right) &= p_r U_l^2\left(S_j^{rc}\right) + (1 - p_r) V_L, \\
U_l^1\left(S_3^c, S_j^{lc}, \ \cdot\ \right) &= p_l U_{lc}^2\left(S_j^{lc}\right) + (1 - p_l) V_L, \\
U_l^1\left(S_4^c, S_j^{lc}, S_j^{rc}\right) &= q_l U_{lc}^2\left(S_j^{lc}\right) + q_r U_l^2\left(S_j^{rc}\right) + (1 - q_l - q_r) V_L.
\end{aligned}$$

The payoff function for player r can be written in a similar manner.

Finally, everything is common knowledge.

The Results

In this section, we conduct a backward induction analysis to characterize all subgame perfect Nash equilibria of our tri-state buffer game. First, we focus on the war that takes place in the second stage and report the respective equilibrium behavior. Second, we investigate the conditions under which each strategy profile is realized as an equilibrium in the second stage game. Third, utilizing the equilibrium expected payoffs from the second stage as continuation payoffs, we write the expected payoffs in the first stage game and analyze the equilibrium behavior in that stage. This completes our analysis.

Equilibrium in the Second Stage

Here we analyze the equilibrium behavior in the second stage of our model. Without loss of generality, assume that the second stage game is played between players lc and r. In case there is a war between these players, player lc aims to maximize $\mathcal{W}_{lc,r}(e_{lc,r}, e_{r,lc})$, as given by equation (1) above. The expected payoff for player r can similarly be written. Given that it is a one-shot contest game with the standard Tullock contest success function and linear cost functions, there exists a unique Nash equilibrium

$$e_{lc,r}^* = \frac{c_r}{(c_{lc} + c_r)^2} W \text{ and } e_{r,lc}^* = \frac{c_{lc}}{(c_{lc} + c_r)^2} W \tag{3}$$

where $W = V_L + V_C + V_R$. The corresponding equilibrium payoffs are

$$\mathcal{W}^*_{lc,r} = \left(\frac{c_r}{c_{lc} + c_r}\right)^2 W \text{ and } \mathcal{W}^*_{r,lc} = \left(\frac{c_{lc}}{c_{lc} + c_r}\right)^2 W. \tag{4}$$

This finding reveals all the equilibrium expected payoffs to be written into the game matrix for the second stage game. The second task is to identify the equilibrium strategy profile in that stage game. Now, given that player *r* chooses to *Wait*, we can determine the best response of player *lc* by comparing

$$U^2_{lc}(Wait, Wait) \text{ and } U^2_{lc}(Attack, Wait).$$

We conclude that player *lc*'s best response is to *Attack* if

$$V_L + V_C \leq \left(\frac{c_r}{c_{lc} + c_r}\right)^2 (V_L + V_C + V_R) - c_A. \tag{5}$$

In a similar manner, we can write that if player *r* chooses to *Attack*, it would always be player *lc*'s best response to *Wait*, since

$$U^2_{lc}(Wait, Attack) = \mathcal{W}^*_{lc,r} > \mathcal{W}^*_{lc,r} - c_A = U^2_{lc}(Attack, Wait). \tag{6}$$

The latter observation is a direct consequence of our assumption, $c_A > 0$. Finally, the symmetric inequalities can be written for player *r*.

This brings us to the following equilibrium results.[7]

Lemma 1. *In the second stage of the tri-state buffer game, independent of who captured region C in the first stage, there exists a unique subgame perfect Nash equilibrium.*

Proof. Without loss of generality, assume that it was player *l* who captured region *C* in the first stage. Given the best response condition (6), it is obvious that (*Attack*, *Attack*) is not a part of any equilibrium. As such, given that the contest part of the model delivers a unique Nash equilibrium, and after neglecting the payoff-equivalent cases, the only possibility for an equilibrium multiplicity is when both (*Attack*, *Wait*) and (*Wait*, *Attack*) are parts of some subgame perfect Nash equilibria. Now, suppose that there are multiple equilibria. Then, the best response condition (5) and its symmetric version

$$V_R \leq \left(\frac{c_{lc}}{c_{lc} + c_r}\right)^2 (V_L + V_C + V_R) - c_A$$

should be satisfied at the same time. However, after adding those inequalities side-by-side, we find a contradiction, since

$$\left(\frac{c_r}{c_{lc} + c_r}\right)^2 + \left(\frac{c_{lc}}{c_{lc} + c_r}\right)^2 < 1. \qquad \square$$

Using Lemma 1, the next result follows.

Proposition 1. *Assume that it was player l who captured region C in the first stage of the tri-state buffer game. Then, in the second stage, the unique subgame perfect Nash equilibrium would include the play of (Attack, Wait) if*

$$V_L + V_C \leq \left(\frac{c_r}{c_{lc} + c_r}\right)^2 (V_L + V_C + V_R) - c_A; \tag{7}$$

(*Wait*, *Attack*) if

$$V_R \leq \left(\frac{c_{lc}}{c_{lc} + c_r}\right)^2 (V_L + V_C + V_R) - c_A; \tag{8}$$

and (*Wait*, *Wait*) if otherwise.

Finally, it is worth mentioning that if one considers the case in which it was player r who captured region C in the first stage, then the results in Proposition 1 would be preserved. However, one needs to be careful that the respective inequalities are written using V_L and $V_C + V_R$ rather than $V_L + V_C$ and V_R, as well as using c_l and c_{rc} rather than c_{lc} and c_r.

Equilibrium in the First Stage: The Emergence of a Buffer State

In this section, we analyze the equilibrium behavior in the first stage of our model, while we restrict our attention to cases in which a buffer state survives *in peace*. To put it differently, we are interested in the conditions under which there exists a subgame perfect Nash equilibrium that includes the play of (*Wait*, *Wait*) in the first stage game. This is what we label as a *peaceful equilibrium*. This is in line with Partem (1983, 9) who argues "If the buffer somehow becomes more capable in conflict with the larger neighbor, the entire system will be transformed.... The buffer, then, is no longer a buffer." In our model, when player c survives an incoming attack, this can be interpreted as becoming more capable in conflict, so that a subgame perfect Nash equilibrium where player c is attacked is not categorized as the emergence of a buffer state.

Before proceeding to our results, here we first assume that $c_{lc} < c_l$ and $c_{rc} < c_r$. The interpretation is that capturing region C leads to efficiency gains in military power, which results in a lower marginal cost of effort. Under this assumption, it can be shown that there does not exist any subgame perfect Nash equilibrium at which player c surely survives as a buffer state. To show this result, simply notice that capturing region C has two advantages: (*i*) an increase in payoff and (*ii*) a decrease in marginal cost of contest effort. Given that there is no downside of capturing the central region, so that there is no *trade-off*, it would never be optimal for either player to hold their attack against player c in the first stage game. This indeed proves that a peaceful equilibrium does not exist.

Accordingly, based on our aim to capture the survival of a buffer state in a peaceful equilibrium, from this point onward, we assume that $c_l < c_{lc}$ and $c_r < c_{rc}$.[8] The interpretation is that capturing region C leads to efficiency losses in military power, which results in a higher marginal cost of effort.

We start our analysis with the following results.[9]

Lemma 2. *If (Wait, Wait) is a part of an equilibrium in the second stage of the tri-state buffer game after player l captured region C in the first stage, then it is a best response for player l to attack player c when player r chooses not to attack player c in the first stage game.*

Proof. Under the assumption that both players lc and r choose not to attack each other in Subgame $[lc, r]$, the continuation payoffs for player l in the first stage game can be written as

$$U_l^1(Attack, Wait) = p_l(V_L + V_C) + (1 - p_l)V_L$$

and

$$U_l^1(Wait, Wait) = V_L.$$

We have $U_l^1(Attack, Wait) > U_l^1(Wait, Wait)$. This completes the proof. □

Lemma 3. *If (Attack, Wait) is a part of an equilibrium in the second stage of the tri-state buffer game after player l captured region C in the first stage, then it is a best response for player l to attack player c when player r chooses not to attack player c in the first stage game.*

Proof. Also in this case, player l knows that she will not be attacked by player r after the former captures region C. Using a similar approach as in the proof of Lemma 2 above, we can write the continuation payoffs for player l in the first stage game as

$$U_l^1(Attack, Wait) = p_l\left(\mathcal{W}_{lc,r}^* - c_A\right) + (1 - p_l)V_L$$

and

$$U_l^1(Wait, Wait) = V_L.$$

Since (*Attack*, *Wait*) is a part of an equilibrium in the second stage, by assumption, we know that $U_{lc}^2(Attack, Wait) = \mathcal{W}_{lc,r}^* - c_A > V_L + V_C = U_{lc}^2(Wait, Wait)$. We have $U_l^1(Attack, Wait) > U_l^1(Wait, Wait)$. This completes the proof. □

The symmetric results are available under the case in which it was player r who captured region C in the first stage. Accordingly, in the first stage of the tri-state buffer game, there is always a player $i \in \{l, r\}$ who chooses to attack player c, unless the capturing player would be attacked by the other player in the second stage. The next result immediately follows.

Proposition 2. *For player c to emerge as a buffer state in a peaceful equilibrium of the tri-state buffer game, it must be that (Wait, Attack) is a part of an equilibrium in the* *second stage after player l captured region C and (Attack, Wait) is a part of an equilibrium in the second stage after player r captured region C.*

Based on Propositions 1 and 2, we conclude that the following inequalities should hold for a buffer state to exist in a peaceful equilibrium

$$V_R \leq \left(\frac{c_{lc}}{c_{lc}+c_r}\right)^2 (V_L+V_C+V_R) - c_A \tag{9}$$

and

$$V_L \leq \left(\frac{c_{rc}}{c_l+c_{rc}}\right)^2 (V_L+V_C+V_R) - c_A. \tag{10}$$

Notice that these are necessary conditions, while additional conditions would appear after analyzing the best responses in the first stage. To obtain those additional conditions, one needs to write the expected payoffs as seen from the beginning of the game (and under the restrictions required in Proposition 2) and to carry out similar best response analyses for both players *l* and *r*. The aim is to obtain the conditions under which a player's best response is to *Wait* when the other player chooses to *Wait* as well.

Now, given that player *r* chooses to *Wait*, we can determine the best response of player *l* by comparing

$$U_l^1(Wait, Wait) \text{ and } U_l^1(Attack, Wait).$$

We conclude that player *l*'s best response is to *Wait* if

$$V_L \geq \left(\frac{c_r}{c_{lc}+c_r}\right)^2 (V_L+V_C+V_R). \tag{11}$$

Symmetrically, given that player *l* chooses to *Wait*, it is player *r*'s best response to *Wait* if

$$V_R \geq \left(\frac{c_l}{c_l+c_{rc}}\right)^2 (V_L+V_C+V_R). \tag{12}$$

These inequalities complement the necessary conditions reported earlier. Accordingly, if the inequalities (9–12) are all satisfied, there exists a peaceful equilibrium where player *c* surely survives as a buffer state.

The arguments above prove the following result.

Proposition 3. *Assume that the equilibrium conditions (9–12) are satisfied in the tri-state buffer game. There exists a unique subgame perfect Nash equilibrium, where neither player chooses to attack player c in the first stage, so that player c surely survives as a buffer state. Furthermore, if one of those inequalities is violated, player c's survival cannot be guaranteed in equilibrium.*

In the following section, we present comparative static results on these equilibrium conditions.

Comparative Static Analyses

To analyze the effects of changes in model parameters on the survival of a buffer state in a peaceful equilibrium, we now provide comparative static analyses. The following results are obtained by taking the derivatives of the terms in the equilibrium conditions, (9–12), with respect to the parameters of interest.

Keeping the other parameter values constant, the set of parameter values under which a buffer state survives gets narrower, if

- c_r increases or c_{lc} decreases, since the inequalities (9) and (11) become tighter,
- c_l increases or c_{rc} decreases, since the inequalities (10) and (12) become tighter.

These results are economically intuitive. Take c_r as an example. This parameter appears in the inequality (9), which is for the second stage after region C is captured by player l. If c_r increases and gets above a critical threshold, player r would avoid attacking player lc in the second stage. Then, knowing that she will not be attacked in the future, player l has nothing to stop her from attacking player c in the first stage. As a result, a peaceful equilibrium is not realized, so that a buffer state does not necessarily exist in equilibrium. Notice that the same cost parameter also appears in the inequality (11), which captures player l's best response behavior in the first stage game. The intuition is that even if c_r is not high enough to deter player r from attacking player lc in the second stage, player l may still choose to attack player c in the first stage. However, if c_r is sufficiently low, player l would not risk capturing the central region (anticipating that her chances of survival will be quite low if she does), so that a buffer state can survive (depending on how player r chooses to act in the first stage game). Furthermore, it can be seen that c_{lc} has a converse effect, since an increase in c_{lc} gives an advantage to player r in the contest against player lc, similar to what a decrease in c_r does. Finally, note that the intuitions for the remaining cost parameters would be symmetric.

As for the other parameters, we observe that the equilibrium conditions do not depend on the probabilities assigned for the Nature moves, so that those probability parameters do not have any influence on the emergence of a buffer state in a peaceful equilibrium. Furthermore, the attacking cost parameter, c_A, only appears in the inequalities (9)–(10), and it easily follows that as its value increases, both inequalities become tighter.

This leaves us with the comparative static analyses for the payoff parameters. Since their effects on equilibrium behavior are not as transparent, below we provide a graphical analysis. In both figures, we assume that $V_C = 100$. In Figure 3(a), we consider symmetry in cost parameters by assuming that $c_l = c_r = 1$ and $c_{lc} = c_{rc} = 2$. In Figure 3(b), we consider an asymmetric case where $c_l = 1$, $c_r = 2$, $c_{lc} = 2$, and $c_{rc} = 7$. In the symmetric case, it is visible that a buffer state exists in a peaceful equilibrium for

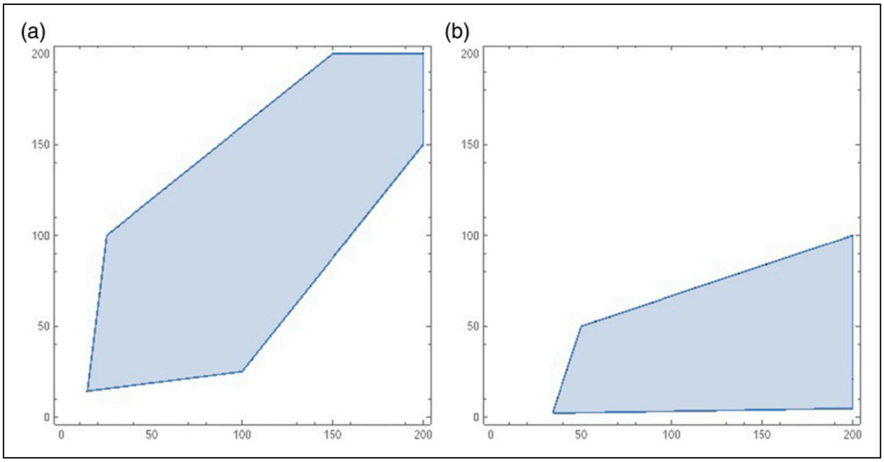

Figure 3. The region of (V_L, V_R) pairs leading to the survival of a buffer state: (a) the symmetric case, (b) the asymmetric case.

intermediate values of the (V_L, V_R) pair. If V_L is high and V_R is low, player r is more eager to attack player c in the first stage, since the future reward in the second stage is relatively high. The same is true for player l if it is vice versa. If both V_L and V_R are low, the central region is very important, and capturing it becomes worthwhile, even if it may result in the complete destruction of the capturing player. On the other hand, when both V_L and V_R get higher, the central region becomes less important, such that it is not worth capturing it and risking the loss of the high payoff from the initial territory.[10] In the asymmetric case, we see that most of these intuitions are preserved; but also, due to the cost advantage for player l, we can additionally observe that when the V_L/V_R ratio is high, player r would not attack player c easily, because now player l would have a great chance of defeating player rc in the second stage game. Yet, player r still chooses to attack player c for very small values of V_R (e.g., for $V_R \leq 2$).

Equilibrium in the First Stage: What if the Buffer State Disappears?

Until this point, we have analytically investigated the conditions under which a buffer state survives in a peaceful equilibrium. In the current section, the aim is to understand all other types of equilibria in which player c is attacked in the first stage, which implies its potential elimination from the game. Provided that the respective equilibrium conditions might not be transparent enough, here we prefer switching to a numerical analysis.

We use a more brief notation in our computational results. Each equilibrium type is summarized by a pair of terms, one term in a parenthesis and another term following that parenthesis, for example, (X)RL or (LR)XL. The latter consists of two figures that

Table 1. Computational Results for a Selection of Parameter Values.

$(c_l, c_r) = (1, 1)$				$(c_l, c_r) = (1, 10)$			
	1	20	100		1	20	100
1	(L+R) RL	(L) RL	(L) XL	1	(L) RL	(L) XL	(L) LL
20	(R) RL	(X) RL	(L) XL	20	(L) RL	(L) XL	(L) LL
100	(R) RX	(R) RX	(X) RL	100	(L) RL	(L) XL	(L) LL
$(c_l, c_r) = (10, 1)$				$(c_l, c_r) = (10, 10)$			
	1	20	100		1	20	100
1	(R) RL	(R) RL	(R) RL	1	(LR) RL	(LR) RL	(L) XL
20	(R) RX	(R) RX	(R) RX	20	(LR) RL	(LR) RL	(L) XL
100	(R) RR	(R) RR	(R) RR	100	(R) RX	(R) RX	(LR) XX

represent the unique equilibrium outcome in the second stage where player l controls region C and the same where player r controls region C, respectively. We already know that there are three possibilities for each entry: (*Wait*, *Wait*) is denoted by X, (*Wait*, *Attack*) is denoted by R, and (*Attack*, *Wait*) is denoted by L. The term in the parenthesis represents the equilibrium strategies in the first stage game. As shown in the Appendix, there are five possibilities: (*Wait*, *Wait*) is denoted by (X), (*Wait*, *Attack*) is denoted by (R), (*Attack*, *Wait*) is denoted by (L), (*Attack*, *Attack*) is denoted by (LR), and {(*Attack*, *Wait*), (*Wait*, *Attack*)} is denoted by (L+R). These correspond to a total of $5 \times 3 \times 3 = 45$ types of equilibria. In the Appendix, we focus on their equilibrium paths and show that only 10 types of equilibrium paths are possible in our model. Those include (X)RL, as the only equilibrium type in which a buffer state survives in a peaceful equilibrium. The interested reader is referred to the Appendix for the corresponding analyses.

Our numerical analysis is exhaustive in the sense that all those 10 types of equilibria are supported by some values of model parameters. Assuming an infinitesimal c_A, and setting $c_{lc} = c_l + k$ and $c_{rc} = c_r + k$ for expositional simplicity, we consider the parameter values of $c_l, c_r \in \{1, 2, 5, 10\}$, $k \in \{1, 5, 10\}$, $p_l, p_r \in \{0.25, 0.33, 0.5, 0.67, 0.75\}$, V_L, $V_R \in \{1, 10, 20, 50, 100\}$, and $V_C \in \{10, 100\}$ in the whole analysis. This corresponds to a total of 60,000 observations.[11] However, given the size of the computational results, here we report only a small portion, focusing on cases where $c_l, c_r \in \{1, 10\}$, $k = 1$, $p_l = p_r = 0.5$, V_L, $V_R \in \{1, 20, 100\}$, and $V_C = 100$. In Table 1, the rows and columns include the values of V_L and V_R, respectively.

Finally, we report the relative frequencies in Table 2. The table also presents three equilibrium categories: (*i*) peace with buffer, as studied in *Equilibrium in the First Stage: The Emergence of a Buffer State*; (*ii*) peace without buffer, in which the two states choose not to attack each other after player c is eliminated; and (*iii*) the last man standing, in which there is warfare after player c is eliminated. We see that a peaceful outcome with a buffer state occurs in more than 25% of our numerical observations. It is also interesting to observe that for a wide range of parameter values, with a relative frequency of 0.42999, the survival of a buffer state is not necessary for the aggressive states to have a peaceful coexistence.

Table 2. Relative Frequencies for Different Types of Equilibria.

Equilibrium category	Relative frequency	Equilibrium type	Relative frequency
Peace with buffer	0.25125	(X) RL	0.25125
Peace without buffer	0.42999	(L) XL	0.20208
		(R) RX	0.20208
		(LR) XX	0.02583
The last man standing	0.31873	(L) LL	0.04458
		(R) RR	0.04458
		(L) RL	0.09538
		(R) RL	0.09538
		(LR) RL	0.01736
		(L+R) RL	0.02145

Concluding Remarks

We introduce a dynamic contest game with three players located on a linear territory. In the model, each player initially controls a region of that territory. Two players are aggressive with options to declare war against the other players. The remaining player is passive, jammed between the aggressive players, so that it has a potential to become a buffer state. We conduct an extensive equilibrium analysis and characterize the conditions under which the passive player acts as a buffer state between the aggressive players. We show that there are 10 types of equilibrium outcomes that can be grouped into three categories: (*i*) peace with buffer, (*ii*) peace without buffer, and (*iii*) the last man standing. Our comparative static analyses reveal valuable insights regarding the factors affecting the existence of buffer states. For instance, the set of parameter values under which a buffer state survives gets narrower if exerting war effort becomes less (more) costly for the state that did (not) capture the central region. As for the territorial valuations, given a fixed value of the central region, a buffer state exists for intermediate values of the other regions.

Our model has a potential to provide equilibrium predictions in line with some historical examples. Switzerland acted as a buffer state between several powers in medieval and modern Europe, it has not fought an international war since 1815, and this can constitute an example for the peace with buffer category. As for the other two equilibrium categories, one needs to be more careful when mapping historical examples to the respective equilibrium predictions. In those cases, it can be claimed that the passive player (who is eventually destructed) is not necessarily a buffer state. However, a fallen buffer state of the past can still be given as an example here. To see how, suppose that our model is applied in two different time frames. A buffer state exists in peace at first, but then, due to a change in exogenous factors (e.g., military technology, power balance), a different equilibrium outcome may be realized where the buffer is destructed in the second time frame. As an example for peace without buffer, the Far Eastern Republic was a buffer state between the Bolshevik Russia and the Empire of

Japan in 1920. It was annexed by the former in 1922, yet this did not lead to any war between Russia and Japan. As an example for the last man standing, the Dulkadirids was a buffer state between the Ottoman Empire and the Mamluk Sultanate in the 15th century, but shortly after Selim I ascended the throne, the Ottomans conquered the Dulkadirids and then the Mamluk Sultanate.

Our model produces some empirically testable implications on the survival of buffer states. More precisely, our equilibrium and comparative static analyses lead to the following hypotheses: (i) The passive state in the center is more likely to survive as a buffer if the increase in war effort cost due to capturing the central region is higher; (ii) The passive state in the center is more likely to survive as a buffer if the central region's valuation is lower; and (iii) Under symmetric costs, the passive state's survival as a buffer is independent of the level of asymmetry between the valuations of the non-central regions. To test hypothesis (i), one would need to identify a proxy for the capturing state's war effort cost, which would ideally take into account the military size (e.g., the loss of armed forces during the capture) and military logistics (e.g., the difficulties in the reallocation of forces due to geography). To test hypotheses (ii) and (iii), one would need a proxy for territorial valuations, for which it would be reasonable to consider natural resources, strategic location, and other economic, strategic, and historical factors that influence the importance of the respective regions. In addition to these, one may formulate a special case of our model that considers territorial size and connects it to other model parameters, such as war effort costs and territorial valuations. Such a variant of our model would also produce testable hypotheses related to territorial size in a straightforward manner by utilizing the connections between size, cost, and valuation (e.g., a larger size implies a higher cost or valuation). Some similarities with earlier work are worth mentioning: According to Fazal (2004, 2007), a state with less power or more valuable territory is relatively more likely to die; and as mentioned in Zacher (2001) and Altman (2017, 2020), a strong opposition by third-party states operating through international organizations may increase the war effort cost and/or reduce the benefit of annexing a territory from another state, which is in line with the increase in the passive state's survival chance in our model (see hypotheses (i) and (ii) above).

To the best of our knowledge, this is the first attempt to formulate a game model to study the emergence and/or survival of buffer states, a phenomenon with an obvious strategic nature that has been overlooked in the existing literature. Our model captures the defining characteristics of a buffer state. That said, it can also be extended in several dimensions. For instance, the passive state is not given any strategic choice in our model. Future work may extend the model by giving the passive state some strategic options, such as choosing contest efforts or whether to form alliances. As another issue, the rivalry between the aggressive states is always present in our model. Future work may allow for exogenous shocks that could influence this rivalry by significantly changing the balance of power. Another possible extension is to preserve the three-part territory structure in the second stage with two states, thereby converting the second stage game into a tug-of-war model where the state who captured the central region is not destructed after losing a battle, but it only loses the central region to the rival state. Finally, territorial valuations are assumed to be constant in the current model. In a richer model, those values may depend on factors such as the efforts exerted in warfare.

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ORCID iDs

Emin Karagözoğlu https://orcid.org/0000-0003-2442-6949
Kerim Keskin https://orcid.org/0000-0001-8112-5574

Notes

1. The interested reader is referred to Chay and Ross (1986) for further readings on the examples of buffer states.
2. From a different perspective, it can be argued that the passive player is already a buffer state from the very beginning, and thus the respective outcome indicates the survival of the buffer state. Although such an alternative interpretation is acceptable, we must note that it may be problematic, since the two aggressive states do not necessarily attack each other in equilibrium after the passive player loses its territory.
3. See Corchon (2007), Garfinkel and Skaperdas (2007), Konrad (2009), Kimbrough, Laughren, and Sheremeta (2020) for extensive reviews of the contest theory literature.
4. If the war between players l and c was modeled as a contest game, where each player chooses a war effort, $1 - p_l$ would be the equilibrium probability that player c wins that contest. We do not explicitly model those dynamics here, to simply keep our focus on the *Attack-Wait* decisions. For example, see Fearon (1995) and Smith and Stam (2004) for a similar approach in the relevant literature. As a matter of fact, the war interpretation can be extended further. For instance, consider that the attacking state has multiple chances to defeat player c, so that even if it loses a battle, it can attack again and again, in which case $1 - p_l$ would be the equilibrium probability that player c wins all of the respective sequence of battles.

5. The latter inequality gives player c a survival chance in (*Attack*, *Attack*); while the former inequality implies that being under attack by two states is worse than by only one state, i.e., (*Attack*, *Attack*) is the worst outcome for player c.
6. Note that since the contest game we consider is a rent-seeking model, it will never be the case that both players prefer warfare over peace. However, if there is no such attack cost, i.e., if $c_A = 0$, then (*Attack*, *Attack*) is trivially a part of an equilibrium in the second stage. To address this concern, we assume a positive attack cost, which guarantees that warfare is not a trivial outcome of the model. It can even be assumed that c_A is infinitesimal. As it will be revealed in our equilibrium analysis, as long as there is such an attack cost, a higher c_A does not change the qualitative results, but it only raises the payoff threshold for choosing to attack. Similarly, Bester and Konrad (2005) state that an equilibrium where two parties attack each other always exists, but it is of limited interest due to the fact that it constitutes a coordination failure. Differently from our approach, instead of assuming an attack cost that makes such an equilibrium non-trivial, they focus on equilibria where a war takes place with delay.
7. For measure-zero selections of parameter values, there may be cases in which a player is indifferent between attacking and waiting. For the sake of expositional simplicity, we do not consider such payoff-equivalent cases.
8. We assume symmetry in the direction of the changes in marginal costs caused by the capture of region C. Even if asymmetry was assumed here, it would mean that one player gets a cost advantage from capturing the central region, removing the same trade-off as discussed above, so that the same equilibrium result follows trivially under such an assumption.
9. In the following equilibrium analysis, by an abuse of notation, we omit explicitly specifying the equilibrium strategies in the second stage game when we write the payoff functions for the first stage game, $U_l^1(\cdot)$ and $U_r^1(\cdot)$.
10. Note that if the payoff parameters are too high (e.g., V_L, $V_R > 400$), a buffer state does not necessarily exist in equilibrium. This happens because now that the central region's relative value is too low, the second stage always turns out to be peaceful, implying that capturing the central region does not have any downside.
11. The results of the whole analysis can be found in the corresponding author's website.
12. If both players choose not to attack player c in the first stage game, which is (*Wait*, *Wait*), what happens in either of the second stage games would not affect the equilibrium path; or if only player l chooses to attack player c in the first stage game, which is (*Attack*, *Wait*), what happens after player r captures region C would not affect the equilibrium path.

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Appendix A

In this Appendix, we show that there exists no type of equilibrium other than those covered by our numerical analysis.

By Proposition 1, it is already known that there exist three types of equilibria in any of the two possible second stages of our model. The following proposition deals with the first stage game.

Proposition A.1. *In the first stage of the tri-state buffer game, without loss of generality, assume that player l prefers to wait when she anticipates that player r would wait. Then, it is also true that player l prefers to wait when she anticipates that player r would attack.*

Proof. Assume that waiting is a best response for player l if player r chooses not to attack player c in the first stage game. This implies that player l prefers collecting V_L to receiving her equilibrium continuation payoff from capturing region C, denoted by $\mathrm{U}^{2*}_{lc,r}$, which also implies that a war would start between players lc and r in Subgame $[lc, r]$. Now, we fix player r's action to *Attack* in the first stage game. If player l chooses to attack, then she collects

$$q_l \mathrm{U}^{2*}_{lc,r} + q_r \mathrm{U}^{2*}_{l,rc} + (1 - q_l - q_r)V_L.$$

where $\mathrm{U}^{2*}_{l,rc}$ denotes player l's equilibrium continution payoff after player r captures region C in the first stage. On the other hand, if player l chooses to wait, then she collects

$$p_r \mathrm{U}^{2*}_{l,rc} + (1 - p_r)V_L.$$

Given that $q_l + q_r \geq p_r \geq q_r$ by modeling assumptions, $V_L > \mathrm{U}^{2*}_{lc,r}$ as discussed above, and $\mathrm{U}^{2*}_{l,rc} > \mathrm{U}^{2*}_{lc,r}$ due to the cost inefficiency of capturing the central region, we conclude that waiting should be a best response for player l, also when she anticipates that player r would attack. □

Now we state the uniqueness of certain types of equilibria in the first stage.

Proposition A.2. *In our tri-state buffer game, consider the reduced version of the first stage game after all equilibrium continuation payoffs from the second stage game are taken into account. In that reduced game,*

- *(Wait, Wait) can only be a dominant strategy equilibrium,*
- *if (Attack, Attack) is a Nash equilibrium, then it is unique.*

Proof. First, assume that (*Wait*, *Wait*) is an equilibrium strategy profile in the reduced version of the first stage game. Then, by Proposition A.1, we know that waiting is a best response for a player when she anticipates that the other player would attack. This makes *Wait* a dominant strategy for each player, which in turn implies that (*Wait*, *Wait*) is a dominant strategy equilibrium.

Second, assume that (*Attack*, *Attack*) is an equilibrium strategy profile in the reduced version of the first stage game. Ignoring payoff equivalences, this already suggests that attacking is a unique best response for a player when she anticipates that the other player would attack. Then, (*Wait*, *Attack*) or (*Attack*, *Wait*) cannot be an equilibrium. Moreover, suppose that (*Wait*, *Wait*) is also an equilibrium. By the former result, it is a dominant strategy equilibrium, which implies that (*Attack*, *Attack*) is not an equilibrium. This is a contradiction. □

Proposition A.2 implies that there can be five possible types of Nash equilibria in the reduced version of the first stage game: (*Wait*, *Wait*), (*Wait*, *Attack*), (*Attack*, *Wait*), (*Attack*, *Attack*), and $\{(Attack, Wait), (Wait, Attack)\}$. Also considering all possible subgame perfect Nash equilibria in the second stage games, it appears that there is a total of 5 × 3 × 3 = 45 possible equilibrium types in our model. As a matter of fact, concentrating only on the equilibrium paths, this number reduces to 25 types.[12]

In what follows, we present five additional propositions to prove that some of those 25 possible equilibrium types cannot be observed as a subgame perfect Nash equilibrium in our model.

Proposition A.3. *In our tri-state buffer game, if (Attack, Wait) is a part of an equilibrium in Subgame [lc, r], then (Attack, Wait) is a part of an equilibrium also in Subgame [l, rc].*

Proof. This result follows because of the cost inefficiency of capturing the central region. Simply observe that if one prefers to attack after experiencing an increase in one's marginal cost while her opponent's marginal cost remains the same, one would prefer to attack also after the converse occurs. □

Proposition A.4. *In our tri-state buffer game, if (Attack, Wait) is a part of an equilibrium in Subgame [lc, r], then (Attack, Wait) is the unique Nash equilibrium in the reduced version of the first stage game.*

Proof. Assume that (*Attack*, *Wait*) is a part of an equilibrium in Subgame [*lc*, *r*]. By Proposition A.3, we know that (*Attack*, *Wait*) is a part of an equilibrium also in Subgame [*l*, *rc*]. We have

$$\mathcal{W}^*_{lc,r} - c_A > V_L + V_C \text{ and } \mathcal{W}^*_{l,rc} - c_A > V_L.$$

Now, consider the first stage game. The relevant expected payoff comparisons are as follows

$$U^1_l(Attack, Wait) = p_l\left(\mathcal{W}^*_{lc,r} - c_A\right) + (1 - p_l)V_L > V_L = U^1_l(Wait, Wait)$$

and

$$U^1_r(Attack, Wait) = p_l\mathcal{W}^*_{r,lc} + (1 - p_l)V_R > \\ q_l\mathcal{W}^*_{r,lc} + q_r\mathcal{W}^*_{rc,l} + (1 - q_l - q_r)V_R = U^1_r(Attack, Attack).$$

The former inequality trivially follows, whereas the latter inequality holds because $q_l + q_r \geq p_l \geq q_l$ and $V_R > \mathcal{W}^*_{r,lc} > \mathcal{W}^*_{rc,l}$, where the latter is due to the fact that there is rent dissipation in the contest game considered here and due to the cost inefficiency of capturing the central region, respectively. This proves that (*Attack*, *Wait*) is a Nash equilibrium in the reduced version of the first stage game. However, we still need to show that (*Wait*, *Attack*) is not a Nash equilibrium in the same game. For that, we simply observe that

$$U_r^1(Wait, Wait) = V_R > p_r\left(\mathcal{W}^*_{rc,l} - c_A\right) + (1 - p_r)V_R = U_l^1(Wait, Attack).$$

This completes the proof. □

Proposition A.5. *In our tri-state buffer game, if (Wait, Wait) is a part of an equilibrium in Subgame [lc, r] and (Attack, Wait) is a part of an equilibrium in Subgame [l, rc], then (Attack, Attack) is not a Nash equilibrium in the reduced version of the first stage game for a sufficiently small* c_A.

Proof. Assume that (*Wait*, *Wait*) is a part of an equilibrium in Subgame [*lc*, *r*] and (*Attack*, *Wait*) is a part of an equilibrium in Subgame [*l*, *rc*]. We then have

$$\mathcal{W}^*_{lc,r} - c_A < V_L + V_C, \ \mathcal{W}^*_{r,lc} - c_A < V_R, \ \text{ and } \mathcal{W}^*_{l,rc} - c_A > V_L.$$

Now, consider the first stage game. The relevant expected payoff comparison is as follows

$$U_r^1(Attack, Wait) = V_R > q_r\mathcal{W}^*_{rc,l} + (1 - q_r)V_R = U_r^1(Attack, Attack).$$

This inequality holds because $V_R > \mathcal{W}^*_{r,lc} > \mathcal{W}^*_{rc,l}$, which is due to the fact that c_A is sufficiently small and due to the cost inefficiency of capturing the central region, respectively. Accordingly, (*Attack*, *Attack*) is not a Nash equilibrium in the reduced version of the first stage game, since player *r* deviates from that strategy profile. This completes the proof. □

Proposition A.6. *In the second stage of the tri-state buffer game, if there is no warfare along the equilibrium path independent of who captured region C, then (Attack, Attack) is the unique Nash equilibrium in the reduced version of the first stage game.*

Proof. The proof is quite straightforward. Under our assumptions that $c_l < c_{lc}$ and $c_r < c_{rc}$, there is a trade-off of capturing the central region: an increase in payoff versus an increase in marginal cost of contest effort. However, notice that if (*Wait*, *Wait*) is a part of an equilibrium in Subgame [*lc*, *r*], then the disadvantage of capturing the central region disappears for player *l*. The same is true for player *r*, since (*Wait*, *Wait*) is a part of an equilibrium in Subgame [*lc*, *r*]. Both players always choose to attack in the reduced version of the first stage game, implying that (*Attack*, *Attack*) is the unique Nash equilibrium. □

Proposition A.7. *In our tri-state buffer game, if there exist multiple Nash equilibria in the reduced version of the first stage game, then (Wait, Attack) is a part of an equilibrium in Subgame [lc, r] and (Attack, Wait) is a part of an equilibrium in Subgame [l, rc].*

Proof. Assume that (*Attack*, *Wait*) and (*Wait*, *Attack*) are both Nash equilibria in the reduced version of the first stage game. Without loss of generality, consider the second stage game after player *l* captured region *C*, i.e., Subgame [*lc*, *r*]. Suppose, for a contradiction, that (*Wait*, *Attack*) is not a part of an equilibrium in the second stage. Then, two possibilities remain for the unique equilibrium, either (*Wait*, *Wait*) or (*Attack*, *Wait*).

If it is the former, then knowing that capturing region *C* is peaceful, player *l* would attack player *c* in the first stage. This follows from Lemma 2. Hence, (*Wait*, *Attack*) cannot be an equilibrium in the reduced version of the first stage game. This is a contradiction.

On the other hand, if it is the latter, i.e., if (*Attack*, *Wait*) is a part of an equilibrium in Subgame [*lc*, *r*], then it follows from Proposition A.3 that (*Attack*, *Wait*) is a part of an equilibrium also in Subgame [*l*, *rc*]. Now, by Proposition A.4, we know that (*Attack*, *Wait*) is the unique Nash equilibrium in the reduced version of the first stage game. This is also a contradiction.

This completes the proof. □

Propositions A.3–A.7 jointly reduce the number of equilibrium types that might be observed in our model to 10. Note once again that all such equilibria are already reported in *Equilibrium in the First Stage: What if the Buffer State Disappears?*